Atomic Mass Evaluation

Wenjia HUANG

Centre de Sciences Nucléaires et de Sciences de la Matière
huang@csnsm.in2p3.fr

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Outline

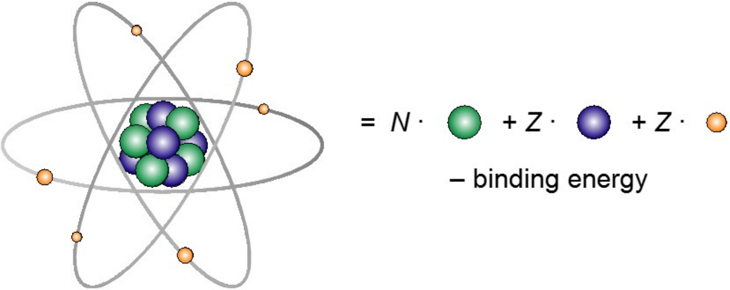
1. Introduction
   - Atomic Mass
   - Mass Measurements

2. Evaluation technique
   - History
   - Input data
   - Least-squares method
   - Correlations

Outline

Atomic Mass

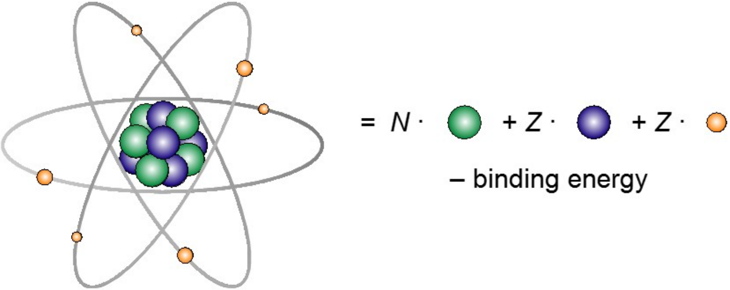
Mass $\longrightarrow$ Binding energy $\longrightarrow$ Interaction



- Nuclear physics : shells, shapes, pairng, nuclear models . . .
- Nuclear astrophysics : r,rp,νp-process
- Atomic physics : QED
- Metrology : Fundamental constants

Atomic Mass

Mass ⟶ Binding energy ⟶ Interaction



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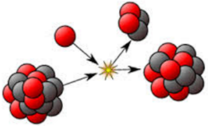
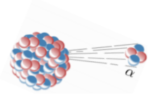
Why “Atomic” mass?

- Almost all measurements give the mass of the atom or of the single charge
- easy to calculate Q-value. . .

Mass Measurements

- **Indirect methods: Energy**
  - Reaction Energies $A(a,b)B$: $Q_r = M_A + M_a - M_b - M_B$
    - ★ (n,γ) and (p,γ) are the backbones
    - ★ close to stability
  - Decay Energies: α, β, p decays
    - ★ far from stability





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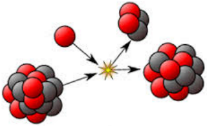
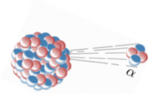


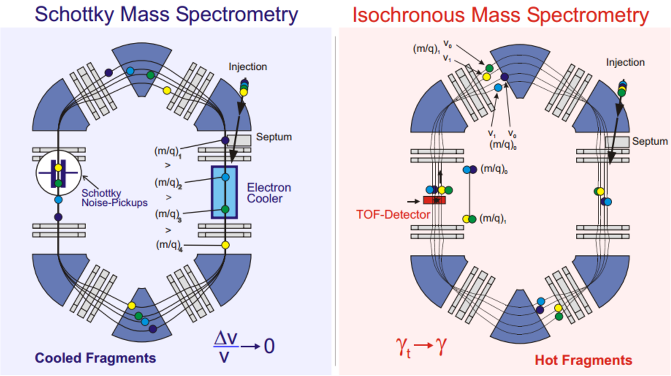


- **Direct Methods: Flight time or frequency**
  - TOF (MSU, GANIL et al)
  - Penning Traps (ISOLDE, TRIUMF et al)
  - Storage Rings (GSI, IMP)

Storage Ring

Theorem

$$\frac{\Delta T}{T} = \frac{1}{\gamma_t^2}\frac{\Delta \frac{m}{q}}{\frac{m}{q}} + \left(1 - \frac{\gamma^2}{\gamma_t^2}\right)\frac{\Delta V}{V}$$



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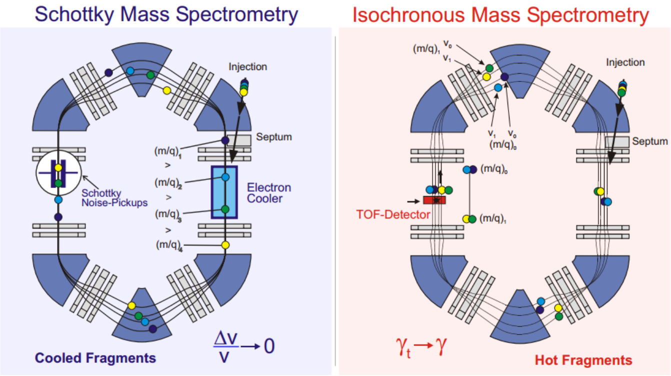
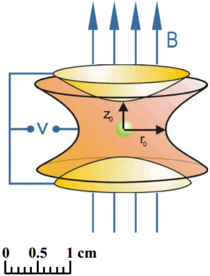


Penning Trap



Cyclotron frequency

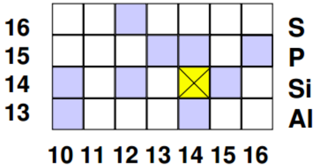
$$\omega_c = q/mB$$

The free cyclotron frequency is inversely proportional to the mass of the ions

$A = 100u, q = +1, B = 6\text{T} \Rightarrow v_c = \omega_c/2\pi \approx 1$ MHz

60 years of mass evaluation

- In the early 1950's, it was found that many relations (direct and indirect) overdetermined the mass value of many nuclides.



^{27}Al (p,γ) ^{28}Si
^{28}Si (^{3}He,^{8}Li) ^{23}Al
^{28}Si (^{4}He,^{8}He) ^{24}Si
^{28}Si (p,t) ^{26}Si
^{28}Si (p,n) ^{28}P
^{28}Si (d,p) ^{29}Si
^{28}Si (p,γ) ^{29}P
^{28}Si (π^{+},π^{-}) ^{28}S
^{31}P (p,α) ^{28}Si
and ^{28}Si / ^{12}C

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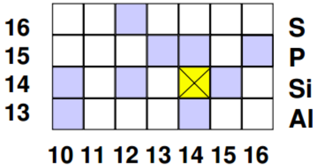

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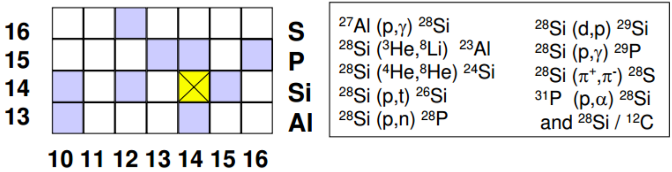

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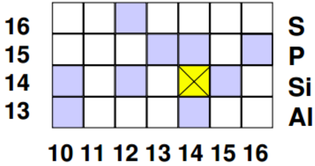

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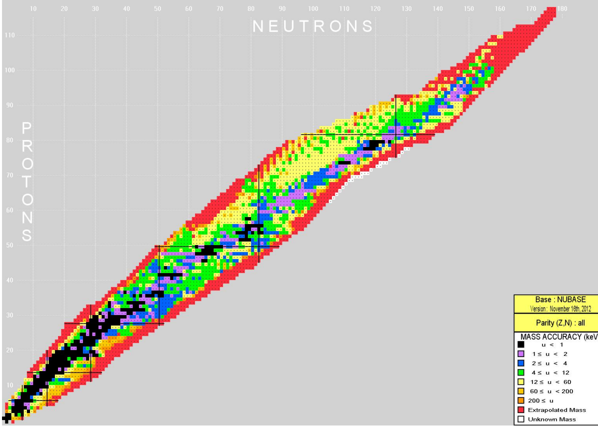
- AME1955, AME1961, AME1964, AME1971, AME1977, AME1983, AME1993, AME2003, **AME2012**

AME2012



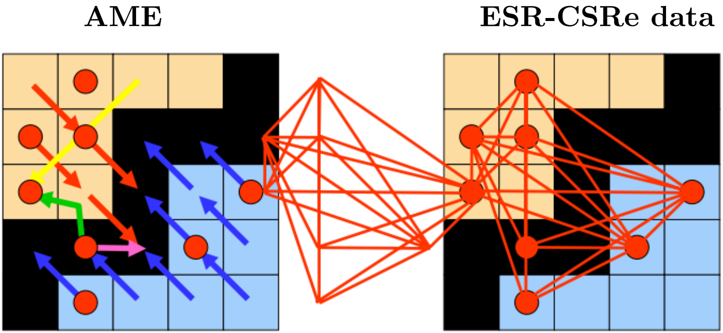
$$3827 \text{ masses} = \text{AME2003} + 323$$

$$3353 \text{ ground states} = \text{AME2003} + 174$$

$$474 \text{ isomers}(> 100\text{ns}) = \text{AME2003} + 149$$

AME2016 is coming...

Input data for AME



- **Reaction Energies (eV)**
  - (**p,n**), (**n,γ**)
- **Desintegration Energies (eV)**
  - β-decay(β^-, β^+)
  - α-decay(α)
- **Mass Spectrometry (u)**
  - Frequency correlations between all known and unknown

Data Connections–Schematic

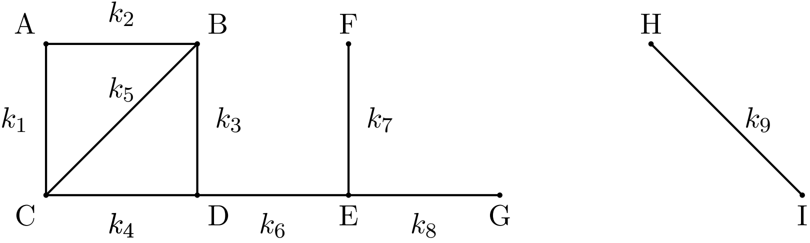


Figure: Connections plot with primary, secondary and unconnected items.

- primary : A,B,C,D $\Rightarrow$ used in LSM
- secondary : E,F,G $\Rightarrow$ deduced from primaries
- unconnected : H, I $\Rightarrow$ systematic #

Data Connections–Real

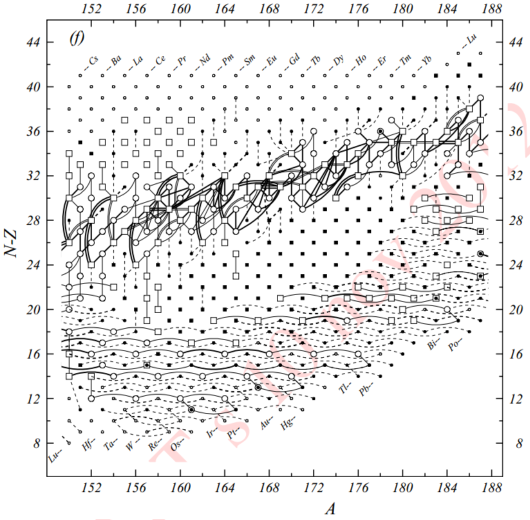


Figure: Diagram of connections for input data.

Treatment of Data–LSM-1

Q equations to N parameters ($Q > N$)

$$\sum_{j=1}^{N} k_{ij} m_j = q_i \pm dq_i \quad i = 1, \ldots, Q \quad \Rightarrow \quad K\left|m\right\rangle = \left|q\right\rangle$$

Simple construction

$$\begin{aligned} {}^{t}KWK\left|m\right\rangle &= {}^{t}KW\left|q\right\rangle \\ A\left|m\right\rangle &= {}^{t}KW\left|q\right\rangle \end{aligned}$$

A: normal matrix, W: error matrix $\omega_i = 1/(dq_i dq_i)$

Parameters–Masses

$$\left|\bar{m}\right\rangle = A^{-1\,t}KW\left|q\right\rangle \quad \Rightarrow \quad \left|\bar{m}\right\rangle = R\left|q\right\rangle$$

Treatment of Data–LSM-2

Flow-of-information matrix

$$F = {}^{t}R \otimes K$$

G.Audi(1986)

- The(i, j) element of F represents the influence of datum i on mass m_j
- A column of F represents all the contribution brought by all data to a given mass m_j
- A row of F represents the influences given by a single piece of data to each nuclide, their sum is the significance of the data

Adjusted input data

$$|\bar{q}\rangle = KR\,|q\rangle$$

Correlations

Variance and Covariance (in nano-amu**2)

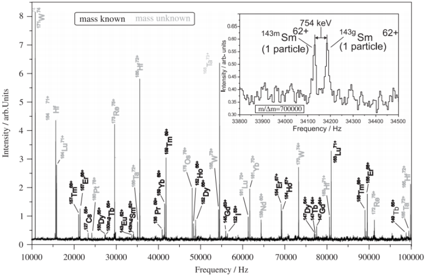
n	H	D	4He	28Si	40Ar	107Ag	109Ag	133Cs
0.241391								
-0.00617	0.00879							
0.01217	0.00262	0.01480						
-0.00000	-0.00000	-0.00000	0.00401					
-0.00508	0.00950	0.00441	0.00000	0.27456				
0.03548	0.02669	0.06219	-0.00000	0.04041	5.76521			
0.14414	0.02641	0.02317	0.00001	0.13582	0.53570	654005		
-0.16098	0.01971	-0.00122	0.00000	0.08311	0.23177	26.30821	2022748.7	
-0.02979	0.04845	0.01877	0.00000	0.18089	0.37448	4.66230	6.32697	73.75107

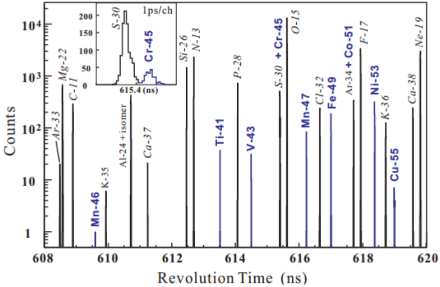
Covariance matrix provided to Codata group as request

now available on AMDC website http://amdc.in2p3.fr/

Correlations in Storage rings

Covariance matrix used in ESR and CSRe data ?





$$\frac{m}{q} = a + bT + cT^2$$

Thank you!