A Snowball's Chance in Hell: The Hierarchy Problem in Particle Physics

Andrei Angelescu
LPT Orsay, Univ. Paris-Sud XI

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What is the Hierarchy Problem?

Figure: Magnet levitating above a superconductor.

How is it possible that an electromagnetic force coming from a cm-sized superconductor manages to overcome the gravitational pull from the (6×10^3 km radius) Earth?

Contents

Overview

A Classical Version: The Electron Self-Energy

- Electron mass = "bare mass" + Coulomb contribution (energy cost for assembling a thin spherical shell of radius r_e and electrical charge e):

$$(m_e c^2)_{\rm phys} = (m_e c^2)_{\rm bare} + \Delta E_{\rm EM}, \quad \Delta E_{\rm EM} \propto \frac{e^2}{r_e}. \tag{1}$$

- Experiments indicate that $r_e < 10^{-17}$ cm, for which $\Delta E_{\rm EM} \simeq 10^4$ MeV. But $(m_e c^2)_{\rm phys} \simeq 0.511$ MeV! $\Rightarrow$ The bare mass needs to be fine-tuned:

$$0.511\,{\rm MeV} = -9999.489\,{\rm MeV} + 10000\,{\rm MeV}. \tag{2}$$

- Solution: add the positron and take into account quantum effects $\rightarrow$ virtual electron-positron pairs spontaneously created in the electron's EM field "smear out" the electron's charge.

$$\Delta E = \Delta E_{\rm EM} + \Delta E_{\rm pair} \propto \alpha m_e c^2 \log \frac{\hbar c / r_e}{m_e c^2} \Rightarrow \tag{3}$$

$$(m_e c^2)_{\rm phys} = (m_e c^2)_{\rm bare} \left[1 + {\rm const.} \times \alpha \log \frac{\hbar c / r_e}{m_e c^2} \right] \tag{4}$$

- Morality:
  1) for $r_e \lesssim 10^{-13}$ cm or $\hbar c / r_e \equiv \Lambda \gtrsim m_e c^2$, classical EM no longer provides an accurate description of nature;
  2) $(m_e c^2)_{\rm phys} \propto (m_e c^2)_{\rm bare}$ because of chiral symmetry.

The Quantum Field Theory Version

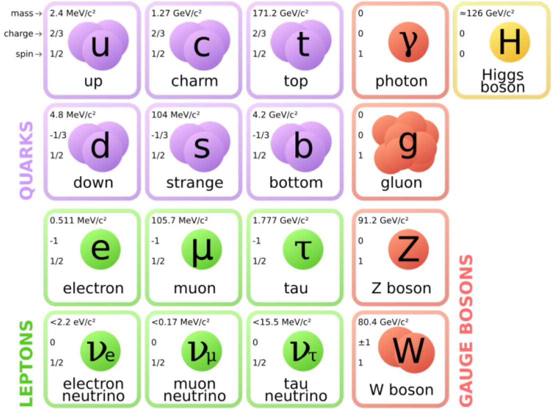


Figure: Standard Model Particles.

The Quantum Field Theory Version

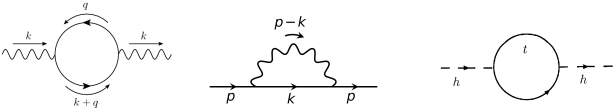


Figure: Quantum corrections to various particle masses.

- Chiral symmetry and gauge symmetry protect fermion and gauge boson masses respectively (in natural units: $\hbar = c = 1$):

$$m_{\rm phys} - m_{\rm bare} \propto m_{\rm bare} \log \frac{\Lambda}{m} \tag{5}$$

- No symmetry protecting the Higgs mass → correction proportional to Λ (the cutoff):

$$m^2_{h,\rm phys} - m^2_{h,\rm bare} \propto \Lambda^2 \tag{6}$$

- Higgs mass measured to be ~ 125 GeV.
- Assume the Standard Model is valid up to $\Lambda \simeq M_{\rm Pl} \simeq 10^{18}$ GeV ⇒ huge fine tuning needed:

$$(125\,{\rm GeV})^2 \simeq m^2_{h,\rm bare} + 10^{36}\,{\rm GeV}^2. \tag{7}$$

Overview

Supersymmetry

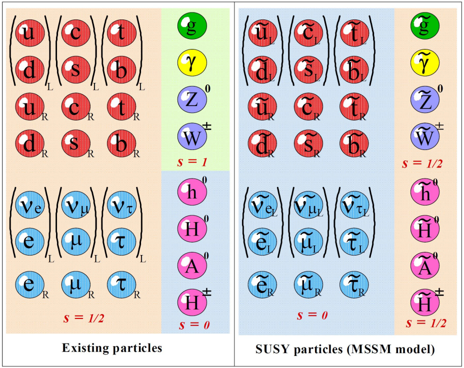


Figure: Minimal Supersymmetric Standard Model (MSSM) particle content.

Supersymmetry

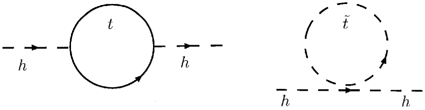


Figure: Top and stop (scalar top) contributions to the Higgs mass.

- The top and stop contributions to the Higgs mass give

$$m^2_{h,\mathrm{phys}} - m^2_{h,\mathrm{bare}} \propto (m^2_t - m^2_{\tilde{t}}) \log \frac{\Lambda}{m_{\tilde{t}}}. \quad (8)$$

- Mass corrections are no longer $\propto \Lambda^2 \Rightarrow$ fine-tuning not needed anymore (unless $m^2_t - m^2_{\tilde{t}} \gg m^2_h$)!

Composite Higgs

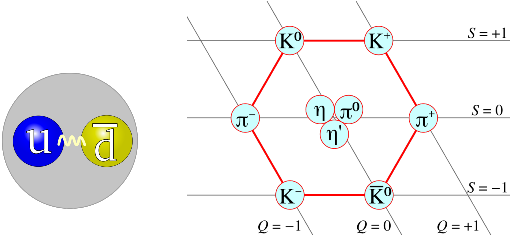


Figure: Left: A π^+ meson's substructure. Right: various mesons.

- Composite Higgs theories postulate that the Higgs scalar is a bound state of a new strong interaction.
- The Higgs can be thought of as a meson $\rightarrow$ analogy with the strong nuclear interaction.
- At higher energies, one does not "see" the Higgs anymore, but its constituents $\rightarrow$ the concept of Higgs mass is no longer defined at very high energies $\Rightarrow$ no fine tuning!
- Prediction $\rightarrow$ other bound states (conceptually similar to resonances in nuclear physics).

Extra Dimensions

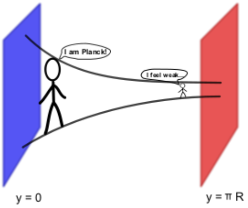


Figure: Warped extra dimension.

- World looks 3D ⇒ extra dimensions should be microscopical.
- Two classes of extra dimensions → flat or warped.
- Main idea → one still has $m^2_{h,\mathrm{phys}} - m^2_{h,\mathrm{bare}} \propto \Lambda^2$, but the cutoff Λ is expected to be of order 10^3 GeV (the extra dimension(s) become "visible" at an energy scale close to Λ) ⇒ no fine tuning.
- Gravity appears weaker because:
  1) it gets "diluted" ("leaks") into the flat extra dimension(s);
  2) the high curvature (warping) of the extra dimension creates an exponential hierarchy between the weak and gravitational interactions.

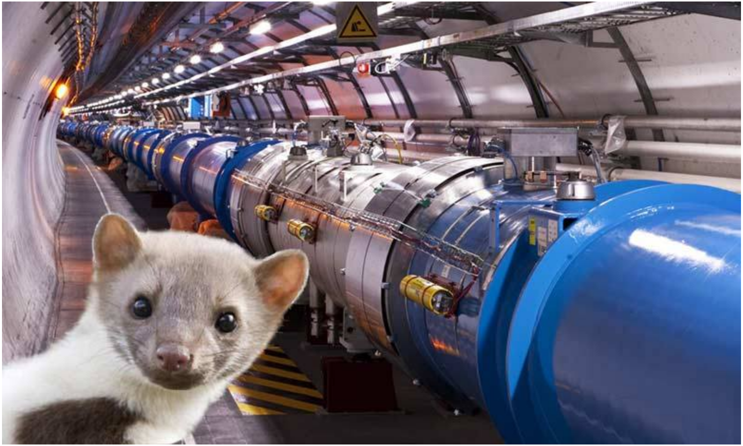
Overview

Summary

Figure: The Large Hadron Collider (LHC).

- The Hierarchy/Fine-Tuning Problem has been a fruitful playground in particle physics: several interesting ideas were put forward as solutions to it (supersymmetry, extra dimensions etc.).
- Resolution of this problem implies the appearance of new particles in the TeV range → the LHC is actively probing their existence.
- Nothing new found at the LHC till now, but there is still hope!

For further reading, a nice non-technical discussion: http://arxiv.org/abs/0801.2562v2.



Thank you for your attention !