

Very Rare, Exclusive Higgs Decays in QCD Factorization

Matthias König THEP, Johannes Gutenberg-University (Mainz)

Higgs Hunting Paris, 31 August, 2016







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The premise for new physics searches nowadays: **Leave no stone unturned!** 

Exclusive hadronic decays can serve as probes for new physics, revealing more information when combined with "more conventional" searches!

Based on: JG|U

#### Exclusive Radiative Decays of W and Z Bosons in QCD Factorization

Yuval Grossman, MK, Matthias Neubert

JHEP 1504 (2015) 101, arXiv:1501.06569

### Exclusive Radiative Z-Boson Decays to Mesons with

Flavor-Singlet Components

Stefan Alte, MK, Matthias Neubert

JHEP 1602 (2016) 162, arXiv:1512.09135

## Exclusive Radiative Higgs Decays as Probes of Light-Quark Yukawa Couplings

MK, Matthias Neubert

JHEP 1508 (2015) 012, arXiv:1505.03870

# Exclusive Weak Radiative Higgs Decays and Flavor-Changing Higgs-Top Couplings Stefan Alte, MK, Matthias Neubert

arXiv:160x.soon

**Outline** JG|U

- QCD-factorization
  - The factorization formula

- 2 Hadronic Higgs decays
  - Radiative hadronic Higgs decays
  - Weak radiative hadronic Higgs decays
- 3 Conclusions

QCD-factorization
The factorization formula

The framework of QCD factorization was originally developed by Brodsky, Efremov, Lepage and Radyushkin in the beginning of the 1980's.

[Brodsky, Lepage (1979), Phys. Lett. B 87, 359]
 [Brodsky, Lepage (1980), Phys. Rev. D 22, 2157]
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The factorization formula was **derived using light-cone perturbation theory**.

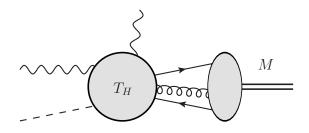
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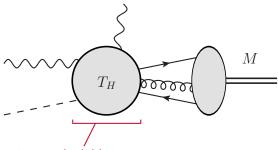
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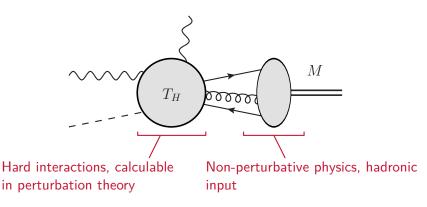
The derivation can also be phrased in the language of soft-collinear effective theory.

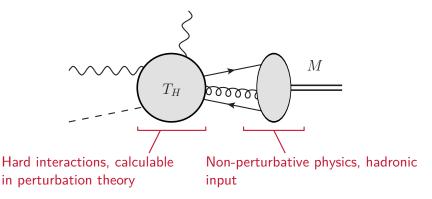
```
[Bauer et al. (2001), Phys. Rev. D 63, 114020]
[Bauer Pirjol, Stewart (2002), Phys. Rev. D 65, 054022]
[Beneke, Chapovsky, Diehl, Feldmann (2002), Nucl. Phys. B 643, 431]
```





Hard interactions, calculable in perturbation theory





The scale separation in the case at hand calls for an effective theory description!

$$i\mathcal{A} = \int \mathcal{C}(t,\dots)\langle M(k)| J_q(t,\dots) |0\rangle dt$$
  
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The LCDAs are expanded in Gegenbauer polynomials:

$$\phi_M^q(x,\mu) = 6x \,\bar{x} \left[ 1 + \sum_{n=1}^{\infty} a_n^M(\mu) C_n^{(3/2)}(2x - 1) \right]$$

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Large logarithms  $\alpha_s \log \mu_H / \Lambda_{\rm QCD}$  are resummed through renormalization group evolution.

Hadronic Higgs decays Radiative hadronic Higgs decays **Idea:** Use hadronic Higgs decays to probe non-standard Higgs couplings.

```
[Isidori, Manohar, Trott (2013), Phys. Lett. B 728, 131]
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Work with the effective Lagrangian:

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**blue terms**:  $\rightarrow 1$  in SM, red terms:  $\rightarrow 0$  in SM!

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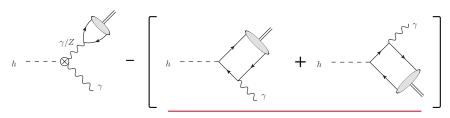
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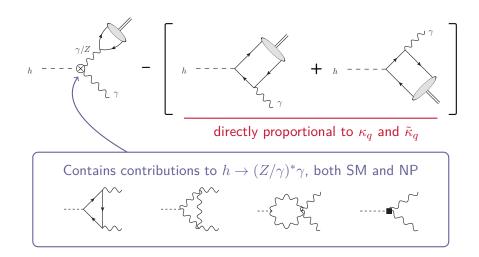
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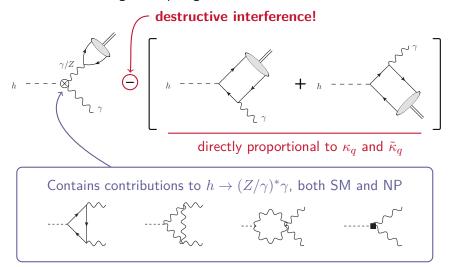
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 $\rightarrow$  Provides a model independent analysis of NP effects in  $h \rightarrow V \gamma$  decays!



directly proportional to  $\kappa_q$  and  $\tilde{\kappa}_q$ 









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We normalize the branching ratio to the  $h \to \gamma \gamma$  branching ratio, which also makes our prediction insensitive to the total Higgs width:

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this contains the direct amplitude!

We want to probe the **Higgs couplings to light fermions**. The indirect contributions however are **sensitive to many other couplings**, like  $\kappa_{\gamma\gamma}$ ,  $\kappa_{Z\gamma}$ ,  $\kappa_W$ ,  $\kappa_f$ ...



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corrections from the indirect contributions due to off-shellness

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ightarrow only very weak sensitivity to the indirect contributions!

Assuming SM couplings of all particles, we find:

$$BR(h \to \rho^{0}\gamma) = (1.68 \pm 0.02_{f} \pm 0.08_{h\to\gamma\gamma}) \cdot 10^{-5}$$

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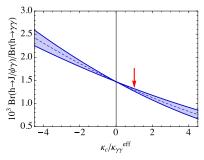
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**But:** What is wrong with the  $\Upsilon$ -channels?

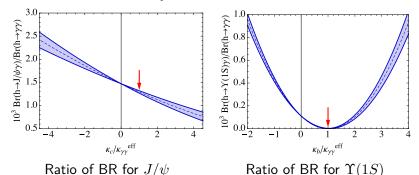
Allowing deviations of the  $\kappa_q$  and no CP-odd couplings:



Ratio of BR for  $J/\psi$ 

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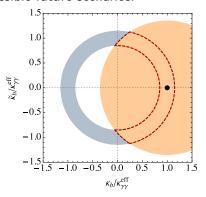
Allowing deviations of the  $\kappa_q$  and no CP-odd couplings:



Usually, the indirect contributions are the dominant ones, however for the  $\Upsilon$ , the direct contribution is comparable, leading to a cancellation between the two.

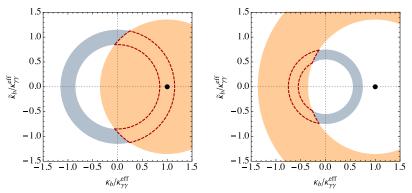
⇒ This leads to a **strong sensitivity to NP effects**!

Possible future scenarios:



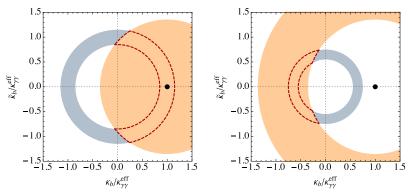
Blue circles: direct measurements of  $h \to q \bar q$  constrain  $\kappa_q^2 + \tilde \kappa_q^2$  Red circles: measurements of  $h \to \Upsilon \gamma$  constrain  $(1-\kappa_q)^2 + \tilde \kappa_q^2$ 

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 $\Rightarrow$  From the **overlap** one can find information on the CP-odd coupling, **even the sign** of the CP-even coupling!

Hadronic Higgs decays Weak radiative hadronic Higgs decays

[Isidori, Manohar, Trott (2014), Phys.Lett. B728 131-135]

[Gao (2014), Phys.Lett. B737 366-368]

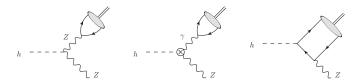
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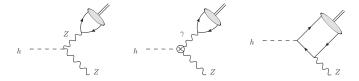
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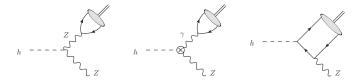
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While the diagrams  $h\to Z(\gamma^*\to V)$  are **loop-suppressed**, the photon is off-shell only by  $m_V^2$ , **lifting** the **suppression**.

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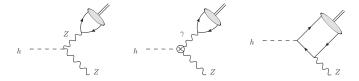


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The indirect diagrams interfere destructively, enhancing the sensitivity to the effective coupling  $\kappa_{\gamma Z}$ .  $(\mathcal{O} \sim h F_{\mu\nu} Z^{\mu\nu})$ 

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The direct contributions are only important for **heavy quarkonia**.

The bound on  $\kappa_{\gamma Z}$  from CMS is:

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From this we get (for SM and for saturated bounds):

Mode	SM Branching ratio $[10^{-6}]$					NP range
$h  o \pi^0 Z$	(2.30)	+	$0.01_{f}$	+	$0.09_{\Gamma})$	
$h  o \eta Z$	(0.83)	+	$0.08_{f}$	+	$0.03_{\Gamma})$	
$h  o \eta' Z$	(1.24)	+	$0.12_{f}$	+	$0.05_{\Gamma})$	
$h  o  ho^0 Z$	(7.19)	+	$0.09_{f}$	+	$0.28_{\Gamma})$	1.83 – 53.3
$h  o \omega Z$	(0.56)	+	$0.01_{f}$	+	$0.02_{\Gamma})$	0.06 - 4.56
$h  o \phi Z$	(2.42)	+	$0.05_{f}$	+	$0.09_{\Gamma})$	1.77 – 9.12
$h  o J/\psi Z$	(2.30)	+	$0.06_{f}$	+	$0.09_{\Gamma})$	1.59 – 13.10
$h \to \Upsilon(1S)Z$	(15.38)	+	$0.21_{f}$	+	$0.60_{\Gamma})$	13.7 – 20.8
$h \to \Upsilon(2S)Z$	(7.50)	+	$0.14_{f}$	+	$0.29_{\Gamma})$	
$h \to \Upsilon(3S)Z$	(5.63)	+	$0.10_{f}$	+	$0.22_{\Gamma})$	



## **Conclusions**

Exclusive hadronic decays of heavy electroweak bosons are an interesting application of the QCD factorization approach in a theoretically clean environment due to the high factorization scale (power corrections tiny, RGE suppresses hadronic parameters).

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- $\blacksquare$  Radiative decays  $h\to M\gamma$  can probe Yukawa couplings along with CP phases and not just the absolute value.
- Weak radiative decays  $h \to MZ$  are sensitive to the coupling of the effective operator  $hF_{\mu\nu}Z^{\mu\nu}$ .
- The downside are the small branching ratios which make these modes challenging. But HL-LHC should be able to see some of them and we don't know what kind of machine the future brings...

Conclusions

 Exclusive hadronic decays of heavy electroweak bosons are an interesting application of the QCD factorization approach in a theoretically clean environment due to the high factorization scale (power corrections tiny, RGE suppresses hadronic parameters).

■ Hadronic decays of the Higgs exhibit interesting dependences on

# Thank you for your attention!

- wheak radiative decays  $n \to m Z$  are sensitive to the coupling of the effective operator  $hF_{\mu\nu}Z^{\mu\nu}$ .
- The downside are the small branching ratios which make these modes challenging. But HL-LHC should be able to see some of them and we don't know what kind of machine the future brings...

# **Backup slides**

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The non-perturbative hadronization is encoded in the matrix element of the current operators between the QCD vacuum and the hadronic final state  $\langle M \mid J \mid 0 \rangle$ .

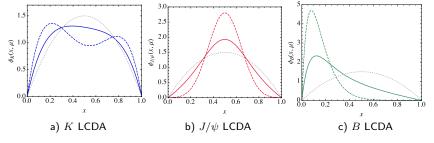
When scale-evolved to high scales, all Gegenbauer moments decrease:

$$\mu \to \infty \quad \Rightarrow \quad a_n, b_n \to 0 \quad \Leftrightarrow \quad \phi_q \to 6x(1-x)$$

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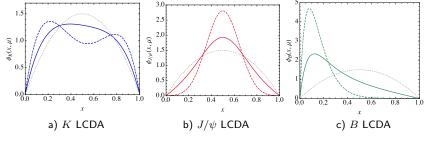


LCDAs for mesons at different scales, dashed lines:  $\phi_M(x,\mu=\mu_0)$ , solid lines:  $\phi_M(x,\mu=m_Z)$ , grey dotted lines:  $\phi_M(x,\mu\to\infty)$ 

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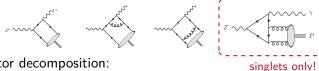
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At high scales compared to  $\Lambda_{\rm QCD}$  (e.g.  $\mu\sim m_Z)$  the sensitivity to poorly-known  $a_n^M$  ,  $b_n^M$  is greatly reduced!

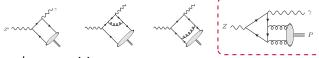
The decay amplitude is governed by diagrams:



Form factor decomposition:

$$i\mathcal{A} = \pm \frac{egf_M}{2\cos\theta_W} \left[ i\epsilon_{\mu\nu\alpha\beta} \frac{k^\mu q^\nu \varepsilon_Z^\alpha \varepsilon_\gamma^{*\beta}}{k \cdot q} F_1^M - \left( \varepsilon_Z \cdot \varepsilon_\gamma^* - \frac{q \cdot \varepsilon_Z k \cdot \varepsilon_\gamma^*}{k \cdot q} \right) F_2^M \right]$$

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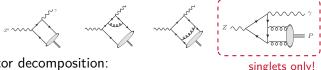
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singlets only!

The form factors contain the contain the convolution integrals:

$$F^{M} \sim \int_{0}^{1} dx \, H(x,\mu) \phi_{M}(x,\mu) = \sum_{n} C_{2n}(\mu) a_{2n}^{M}(\mu)$$
$$C_{n}(\mu) = 1 + \frac{C_{F} \alpha_{s}(\mu)}{4\pi} \left\{ 3 \log \frac{m_{Z}^{2}}{\mu^{2}} + \dots \right\}$$

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Evaluating the hard function at  $\mu=m_Z$  and evolving it down to  $\mu_{\rm hadr}$ resums large logarithms  $\left[\alpha_s \log(m_Z^2/\mu^2)\right]^n$ .

$Z \to \dots$	Branching ratio		asym.	LO
$\pi^0\gamma$	$(9.80 ^{+0.09}_{-0.14 \mu} \pm 0.03_f \pm 0.61_{a_2} \pm 0.82_{a_4})$	$\cdot 10^{-12}$	7.71	14.67
$\eta\gamma$	$(2.36 \begin{array}{c} +0.02 \\ -0.04 \ \mu \end{array} \pm 1.19_f \qquad \pm 0.04_{\phi})$	$\cdot 10^{-10}$		
$\eta'\gamma$		$\cdot 10^{-9}$		
$ ho^0\gamma$	$(4.19  {}^{+ 0.04}_{- 0.06  \mu}  \pm 0.16_f  \pm 0.24_{a_2} \pm 0.37_{a_4})$	$\cdot 10^{-9}$	3.63	5.68
$\phi\gamma$	$(8.63 ^{+0.08}_{-0.13} _{\mu} \pm 0.41_{f} \pm 0.55_{a_{2}} \pm 0.74_{a_{4}})$	$\cdot 10^{-9}$	7.12	12.31
$\omega\gamma$	$(2.89  {}^{+ 0.03}_{- 0.05  \mu}  \pm 0.15_f  \pm 0.29_{a_2} \pm 0.25_{a_4})$	$\cdot 10^{-8}$	2.54	3.84
$J/\psi \gamma$	$(8.02  {}^{+ 0.14}_{- 0.15  \mu}  \pm 0.20_f  {}^{+ 0.39}_{- 0.36  \sigma})$	$\cdot 10^{-8}$	10.48	6.55
$\Upsilon(1S) \gamma$	$(5.39  {}^{+0.10}_{-0.10}  {}^{\mu}  \pm 0.08_f  {}^{+0.11}_{-0.08}  {}^{\sigma})$	$\cdot 10^{-8}$	7.55	4.11
$\Upsilon(4S) \gamma$		$\cdot 10^{-8}$	1.71	0.93
$\Upsilon(nS) \gamma$	$(9.96  {}^{+ 0.18}_{- 0.19}  {}^{\mu}  \pm 0.09_f  {}^{+ 0.20}_{- 0.15}  {}^{\sigma})$	$\cdot 10^{-8}$	13.96	7.59

$Z \to \dots$	Branching ratio	asym.	LO
$\pi^0\gamma$	$(9.80 + 0.09 \atop -0.14 \mu) \pm 0.03_f \pm 0.61_{a_2} \pm 0.82_{a_4}) \cdot 10^{-12}$	7.71	14.67
$\eta\gamma$	$(2.36 \begin{vmatrix} +0.02 \\ -0.04 \mu \end{vmatrix} \pm 1.19_f \qquad \pm 0.04_{\phi}) \qquad \cdot 10^{-10}$		
$\eta'\gamma$	$(6.68 \begin{array}{c} +0.08 \\ -0.11 \end{array}) \pm 0.49_f \qquad \pm 0.12_{\phi}) \qquad \cdot 10^{-9}$		
$ ho^0\gamma$	$(4.19 \begin{vmatrix} +0.04 \\ -0.06 \mu \end{vmatrix} \pm 0.16_f \pm 0.24_{a_2} \pm 0.37_{a_4}) \cdot 10^{-9}$	3.63	5.68
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$\Upsilon(4S) \gamma$	$(1.22 \begin{array}{c} +0.02 \\ -0.02 \\ \mu \end{array}) \pm 0.13_f \qquad \begin{array}{c} +0.02 \\ -0.02 \\ \sigma \end{array}) \qquad \cdot 10^{-8}$	1.71	0.93
$\Upsilon(nS) \gamma$		13.96	7.59



$Z \to \dots$	Branching ratio	asym.	LO
$\pi^0\gamma$	$(9.80 + 0.09 \atop -0.14 \mu) \pm 0.03 f \pm 0.61 \atop -0.14 \mu) \pm 0.03 f \pm 0.61 \atop -0.14 \mu) \pm 0.03 f$	7.71	14.67
$\eta\gamma$	$(2.36 \begin{vmatrix} +0.02 & \mu \\ -0.04 & \mu \end{vmatrix} \pm 1.19_f$ $\pm 0.04_{\phi}$ $\cdot 10^{-10}$		
$\eta'\gamma$	$(6.68 \begin{vmatrix} +0.08 \\ -0.11 \mu \end{vmatrix} \pm 0.49_f \qquad \pm 0.12_{\phi}) \qquad \cdot 10^{-9}$		
$ ho^0\gamma$	$(4.19 \begin{array}{c c} +0.04 & \pm 0.16 \\ -0.06 & \mu \end{array} \pm 0.16 \begin{array}{c} \pm 0.24 \\ -0.06 & \mu \end{array} \cdot 10^{-9}$	3.63	5.68
$\phi\gamma$	$(8.63 \begin{array}{c} +0.08 \\ -0.13 \end{array}) \pm 0.41 $ $\pm 0.55 $ $_{a_2} \pm 0.74 $ $_{a_4}) \cdot 10^{-9}$	7.12	12.31
$\omega\gamma$	$(2.89 \begin{array}{ c c c c c c c c c c c c c c c c c c c$	2.54	3.84
$J/\psi \gamma$	$ \begin{pmatrix} 8.02 & +0.14 & \pm 0.20 & +0.39 & -0.36 & \sigma \\ -0.15 & \mu & \pm 0.20 & \pm 0.10 & +0.11 \\ -0.26 & \pm 0.10 & \pm 0.08 & \pm 0.11 \end{pmatrix} $	10.48	6.55
$\Upsilon(1S) \gamma$	$(0.39 - 0.10 \mu) \pm 0.00 f$ $-0.08 \sigma) 10$	7.55	4.11
$\Upsilon(4S) \gamma$	$(1.22 \begin{vmatrix} +0.02 \\ -0.02 \mu \end{vmatrix} \pm 0.13_f \begin{vmatrix} +0.02 \\ -0.02 \sigma \end{vmatrix} $ $\cdot 10^{-8}$	1.71	0.93
$\Upsilon(nS) \gamma$	$(9.96 \begin{array}{c} +0.18 \\ -0.19 \end{array}) \begin{array}{c} +0.09 \\ -0.15 \end{array} \begin{array}{c} +0.20 \\ -0.15 \end{array} \sigma) $ $\cdot 10^{-8}$	13.96	7.59
	<u> </u>		

scale dependence decay constant

$Z \to \dots$	Branching ratio	asym.	LO
$\pi^0\gamma$	$(9.80 + 0.09 \atop -0.14 \atop +0.02) \pm 0.03 \atop +0.02 \atop +0.02) \pm 0.01 \atop +0.02$	7.71	14.67
$\eta\gamma$	$(2.30  _{-0.04 \mu}  _{\pm 1.19 f}  _{\pm 0.04 \phi})$ ·10		
$\eta'\gamma$	$(6.68 \begin{array}{c c} +0.08 \\ -0.11 \\ \mu \end{array} \pm 0.49_f \qquad \pm 0.12_{\phi}) \qquad \cdot 10^{-9}$		
$ ho^0 \gamma$	$(4.19 \begin{vmatrix} +0.04 \\ -0.06 \ \mu \end{vmatrix} \pm 0.16_f \pm 0.24_{a_2} \pm 0.37_{a_4}) \cdot 10^{-9}$	3.63	5.68
$\phi\gamma$	$(8.63 \begin{array}{c} +0.08 \\ -0.13 \end{array}) \pm 0.41 \begin{array}{c} \pm 0.55 \\ -0.13 \end{array} \pm 0.74 \\ -0.13 \end{array}) \cdot 10^{-9}$	7.12	12.31
$\omega\gamma$	$(2.89 \begin{vmatrix} +0.03 \\ -0.05 \mu \end{vmatrix} \pm 0.15_f \pm 0.29_{a_2} \pm 0.25_{a_4}) \cdot 10^{-8}$	2.54	3.84
$J/\psi \gamma$	$(8.02 \begin{vmatrix} +0.14 \\ -0.15 \mu \end{vmatrix} \pm 0.20_f \begin{vmatrix} +0.39 \\ -0.36 \sigma \end{vmatrix}) \cdot 10^{-8}$	10.48	6.55
$\Upsilon(1S) \gamma$	$(5.39 \begin{array}{c c} +0.10 \\ -0.10 \\ \mu \end{array} \pm 0.08 f \begin{array}{c} +0.11 \\ -0.08 \\ \sigma \end{array}) $ $\cdot 10^{-8}$	7.55	4.11
$\Upsilon(4S) \gamma$	(1.99 + 0.02 + 0.02) + 0.02)	1.71	0.93
$\Upsilon(nS) \gamma$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	13.96	7.59
	↑ ↑ <b>↑</b>		

scale dependence LCDA shape decay constant

$Z \to \dots$	Branching ratio	asym.	LO
$\pi^0\gamma$	$(9.80^{+0.09}_{-0.14} \pm 0.03_f \pm 0.61_{a_2} \pm 0.82_{a_4}) \cdot 10^{-12}$	7.71	14.67
$\eta\gamma$	$(2.36^{+0.02}_{-0.04} _{\mu} \pm 1.19_f \pm 0.04_{\phi}) \cdot 10^{-10}$		
$\eta'\gamma$	$(6.68  {}^{+ 0.08}_{- 0.11  \mu}  \pm 0.49_f  \pm 0.12_{\phi})  \cdot 10^{-9}$		
$ ho^0\gamma$	$(4.19  {}^{+ 0.04}_{- 0.06  \mu}  \pm 0.16_f  \pm 0.24_{a_2} \pm 0.37_{a_4}) \cdot 10^{-9}$	3.63	5.68
$\phi\gamma$	$(8.63  ^{+ 0.08}_{- 0.13  \mu}  \pm 0.41_{f}  \pm 0.55_{a_{2}} \pm 0.74_{a_{4}}) \cdot 10^{-9}$	7.12	12.31
$\omega\gamma$	$(2.89  {}^{+ 0.03}_{- 0.05  \mu}  \pm 0.15_{f}  \pm 0.29_{a_{2}} \pm 0.25_{a_{4}}) \cdot 10^{-8}$	2.54	3.84
$J/\psi \gamma$	$(8.02  {}^{+0.14}_{-0.15  \mu}  \pm 0.20_f  {}^{+0.39}_{-0.36  \sigma})  \cdot 10^{-8}$	10.48	6.55
$\Upsilon(1S) \gamma$	$(5.39  {}^{+0.10}_{-0.10}  {}_{\mu}  \pm 0.08_f  {}^{+0.11}_{-0.08}  {}_{\sigma})  \cdot 10^{-8}$	7.55	4.11
$\Upsilon(4S) \gamma$		1.71	0.93
$\Upsilon(nS) \gamma$	$(9.96  ^{+0.18}_{-0.19}  _{\mu}  \pm 0.09_{f}  ^{+0.20}_{-0.15}  _{\sigma})  \cdot 10^{-8}$	13.96	7.59
$\Upsilon(nS)\gamma$	$(0.06 \pm 0.18) + 0.00 + 0.20$	13.96	7.59

#### obtained when using only asymptotic form of LCDA

$$\phi_{\mathbf{M}}(\mathbf{x}) = 6\mathbf{x}(\mathbf{1} - \mathbf{x})$$

$Z \to \dots$	Branching ratio		asym.	LO
$\pi^0\gamma$		$\cdot 10^{-12}$	7.71	14.67
$\eta\gamma$		$\cdot 10^{-10}$		
$\eta'\gamma$		$\cdot 10^{-9}$		
$ ho^0\gamma$	$(4.19  {}^{+ 0.04}_{- 0.06  \mu}  \pm 0.16_f  \pm 0.24_{a_2} \pm 0.37_{a_4})$	$\cdot 10^{-9}$	3.63	5.68
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$\Upsilon(1S) \gamma$		$\cdot 10^{-8}$	7.55	4.11
$\Upsilon(4S) \gamma$	$(1.22  {}^{+ 0.02}_{- 0.02  \mu}  \pm 0.13_f  {}^{+ 0.02}_{- 0.02  \sigma})$	$\cdot 10^{-8}$	1.71	0.93
$\Upsilon(nS) \gamma$	$(9.96  {}^{+ 0.18}_{- 0.19}  {}^{\mu}  \pm 0.09_f  {}^{+ 0.20}_{- 0.15}  {}^{\sigma})$	$\cdot 10^{-8}$	13.96	7.59
			×	

# obtained when using only LO hard functions

$Z \to \dots$	Branching ratio	asym.	LO
$\pi^0\gamma$	$\left[ (9.80  {}^{+ 0.09}_{- 0.14  \mu}  \pm 0.03_f  \pm 0.61_{a_2} \pm 0.82_{a_4})  \cdot 10^{-12} \right]$	7.71	14.67
$\eta\gamma$	$\left[ (2.36  {}^{+ 0.02}_{- 0.04  \mu}  \pm 1.19_f  \pm 0.04_\phi)  \cdot 10^{-10} \right]$		
$\eta'\gamma$	$(6.68^{+0.08}_{-0.11}{}_{\mu} \pm 0.49_{f} \pm 0.12_{\phi})$ $\cdot 10^{-9}$		
$ ho^0\gamma$	$\left[ (4.19  {}^{+ 0.04}_{- 0.06}  \mu  \pm 0.16_{f}  \pm 0.24_{a_{2}} \pm 0.37_{a_{4}} \right] \cdot 10^{-9}  \right]$	3.63	5.68
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$J/\psi \gamma$	$\left(8.02  {}^{+0.14}_{-0.15  \mu}  \pm 0.20_f  {}^{+0.39}_{-0.36  \sigma}\right)  \cdot 10^{-8}  \right]$	10.48	6.55
$\Upsilon(1S) \gamma$	$\left[ (5.39  {}^{+ 0.10}_{- 0.10}  \mu  \pm 0.08_f  {}^{+ 0.11}_{- 0.08}  \sigma)  {}^{+ 0.10}_{0.08}  \sigma \right]$	7.55	4.11
$\Upsilon(4S) \gamma$	$(1.22  {}^{+0.02}_{-0.02  \mu}  \pm 0.13_f  {}^{+0.02}_{-0.02  \sigma})  {}^{\cdot 10^{-8}}$	1.71	0.93
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#### The form factors become:

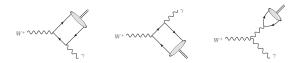
$$\operatorname{Re} F_{1}^{M} = \mathcal{Q}_{M} \left[ 0.94 + 1.05 \, a_{2}^{M}(m_{Z}) + 1.15 \, a_{4}^{M}(m_{Z}) + 1.22 \, a_{6}^{M}(m_{Z}) + \dots \right]$$
$$= \mathcal{Q}_{M} \left[ 0.94 + 0.41 \, a_{2}^{M}(\mu_{h}) + 0.29 \, a_{4}^{M}(\mu_{h}) + 0.23 \, a_{6}^{M}(\mu_{h}) + \dots \right]$$

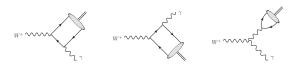
$Z \to \dots$	Branching ratio	asym.	LO
$\pi^0\gamma$	$\left[ (9.80  {}^{+ 0.09}_{- 0.14  \mu}  \pm 0.03_f  \pm 0.61_{a_2} \pm 0.82_{a_4})  \cdot 10^{-12} \right]$	7.71	14.67
$\eta\gamma$	$\left[ (2.36  {}^{+ 0.02}_{- 0.04  \mu}  \pm 1.19_f  \pm 0.04_\phi)  \cdot 10^{-10} \right]$		
$\eta'\gamma$	$(6.68^{+0.08}_{-0.11}{}_{\mu} \pm 0.49_{f} \pm 0.12_{\phi})$ $\cdot 10^{-9}$		
$ ho^0\gamma$	$\left[ (4.19  {}^{+ 0.04}_{- 0.06}  \mu  \pm 0.16_{f}  \pm 0.24_{a_{2}} \pm 0.37_{a_{4}} \right] \cdot 10^{-9}  \right]$	3.63	5.68
$\phi\gamma$	$\left(8.63  {}^{+ 0.08}_{- 0.13  \mu}  \pm 0.41_{f}  \pm 0.55_{a_{2}} \pm 0.74_{a_{4}}\right) \cdot 10^{-9}$	7.12	12.31
$\omega\gamma$	$\left[ (2.89  {}^{+ 0.03}_{- 0.05}  {}^{\mu}  \pm 0.15_{f}  \pm 0.29_{a_{2}} \pm 0.25_{a_{4}} \right] \cdot 10^{-8}  \right]$	2.54	3.84
$J/\psi \gamma$	$\left(8.02  {}^{+0.14}_{-0.15  \mu}  \pm 0.20_f  {}^{+0.39}_{-0.36  \sigma}\right)  \cdot 10^{-8}  \right]$	10.48	6.55
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The form factors become:

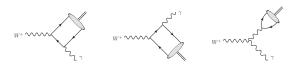
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$$= \mathcal{Q}_{M} \left[ 0.94 + 0.41 \, a_{2}^{M}(\mu_{h}) + 0.29 \, a_{4}^{M}(\mu_{h}) + 0.23 \, a_{6}^{M}(\mu_{h}) + \ldots \right]$$

 $\rightarrow$  RGE from high to low scale reduces sensitivity to  $a_n^M!$ 



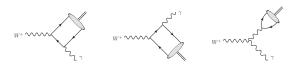


mode	Branching ratio	asym.	LO
$\pi^{\pm}\gamma$	$(4.00^{+0.06}_{-0.11} _{\mu} \pm 0.01_f \pm 0.49_{a_2} \pm 0.66_{a_4}) \cdot 10^{-9}$	2.45	8.09
$\rho^{\pm}\gamma$	$(8.74^{+0.17}_{-0.26 \mu} \pm 0.33_f \pm 1.02_{a_2} \pm 1.57_{a_4}) \cdot 10^{-9}$	6.48	15.12
$K^{\pm}\gamma$	$\left(3.25^{+0.05}_{-0.09}_{\mu} \pm 0.03_{f} \pm 0.24_{a_{1}} \pm 0.38_{a_{2}} \pm 0.51_{a_{4}}\right) \cdot 10^{-10}$	1.88	6.38
$K^{*\pm}\gamma$	$\left[ (4.78^{+0.09}_{-0.14} _{\mu} \pm 0.28_{f} \pm 0.39_{a_{1}} \pm 0.66_{a_{2}} \pm 0.80_{a_{4}}) \cdot 10^{-10} \right]$	3.18	8.47
$D_s \gamma$	$(3.66^{+0.02}_{-0.07} _{\mu} \pm 0.12_{\text{CKM}} \pm 0.13_{f} _{-0.82}^{+1.47} _{\sigma}) \cdot 10^{-8}$	0.98	8.59
$D^{\pm}\gamma$	$(1.38^{+0.01}_{-0.02} _{\mu} \pm 0.10_{\rm CKM} \pm 0.07_{f} ^{+0.50}_{-0.30} _{\sigma}) \cdot 10^{-9}$	0.32	3.42
$B^{\pm}\gamma$	$(1.55^{+0.00}_{-0.03}  \mu \pm 0.37_{\text{CKM}} \pm 0.15_{f -0.45  \sigma}^{+0.68}) \cdot 10^{-12}$	0.09	6.44



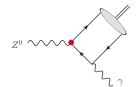
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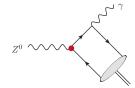
# flavour off-diagonal mesons allowed

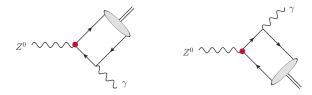


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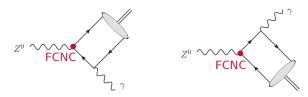
#### introduces uncertainties from CKM elements





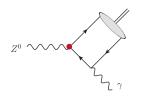


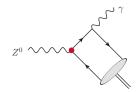
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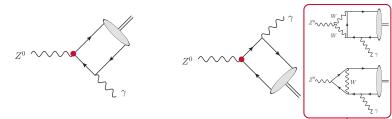
Introducing FCNC couplings allows the production of flavor off-diagonal mesons





### Model independent predictions for flavor off-diagonal mesons:

Decay mode	Branching ratio	SM background
$Z^0  o K^0 \gamma$	$\left[ (7.70 \pm 0.83)  v_{sd} ^2 + (0.01 \pm 0.01)  a_{sd} ^2 \right] \cdot 10^{-8}$	$\frac{\lambda}{\sin^2 \theta_W} \frac{\alpha}{\pi} \sim 2 \cdot 10^{-3}$
$Z^0  o D^0 \gamma$	$\left[ (5.30^{+0.67}_{-0.43})   v_{cu} ^2 + (0.62^{+0.36}_{-0.23})   a_{cu} ^2 \right] \cdot 10^{-7}$	$\frac{\lambda}{\sin^2 \theta_W} \frac{\alpha}{\pi} \sim 2 \cdot 10^{-3}$
$Z^0  o B^0 \gamma$	$\left[ (2.08^{+0.59}_{-0.41})  v_{bd} ^2 + (0.77^{+0.38}_{-0.26})  a_{bd} ^2 \right] \cdot 10^{-7}$	$\frac{\lambda^3}{\sin^2\theta_W} \frac{\alpha}{\pi} \sim 8 \cdot 10^{-5}$
$Z^0  o B_s \gamma$	$\left[ (2.64 + 0.82)  v_{bs} ^2 + (0.87 + 0.51)  a_{bs} ^2 \right] \cdot 10^{-7}$	$\frac{\lambda^2}{\sin^2\theta_W} \frac{\alpha}{\pi} \sim 4 \cdot 10^{-4}$



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FCNCs would induce tree-level neutral-meson mixing, strongly constrained:

$$\begin{array}{|c|c|c|c|c|} |\operatorname{Re}\left[(v_{sd} \pm a_{sd})^2\right]| & < 2.9 \cdot 10^{-8} \\ |\operatorname{Im}\left[(v_{sd} \pm a_{sd})^2\right]| & < 1.0 \cdot 10^{-10} \\ |\operatorname{Im}\left[(v_{sd} \pm a_{sd})^2\right]| & < 1.0 \cdot 10^{-10} \\ |(v_{cu} \pm a_{cu})^2| & < 2.2 \cdot 10^{-8} \\ |(v_{bd} \pm a_{bd})^2| & < 4.3 \cdot 10^{-8} \\ |(v_{bd} \pm a_{bs})^2| & < 5.5 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\ |(v_{bs})^2 - (a_{bs})^2| & < 1.4 \cdot 10^{-7} \\$$

[Bona et al. (2007), JHEP 0803, 049] [Bertone et al. (2012), JHEP 1303, 089] [Carrasco et al. (2013), JHEP 1403, 016]

These bounds push our branching ratios down to  $10^{-14}$ , rendering them unobservable.

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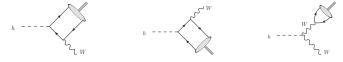
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■ However: Future lepton machines like ILC or TLEP might produce  $10^{12}Z$ 's and  $10^7W$ 's at the corresponding thresholds  $\rightarrow$  This enables an experimental program to test QCDF in a theoretically clean environment!

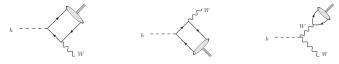
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For this to work, we need two criteria to be fulfilled:

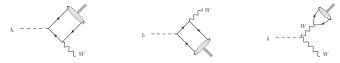
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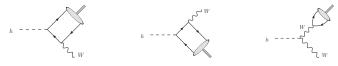
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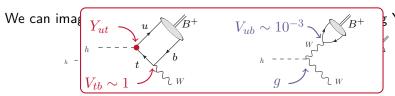
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JGU

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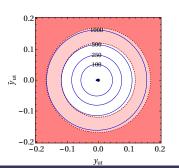
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