Data Processing and Imaging for Cas A

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Observed Visibility

The observed visibility is

$$
\begin{aligned}
V_{ij} &= \langle F_i F_j^* \rangle \\
&= g_i g_j^* \int d^2\hat{\boldsymbol{n}} d^2\hat{\boldsymbol{n}}' \, A_i(\hat{\boldsymbol{n}}) A_j^*(\hat{\boldsymbol{n}}') \langle \varepsilon(\hat{\boldsymbol{n}}) \varepsilon(\hat{\boldsymbol{n}}') \rangle e^{2\pi i (\hat{\boldsymbol{n}} \cdot \boldsymbol{u}_i - \hat{\boldsymbol{n}}' \cdot \boldsymbol{u}_j)} + n_{ij} \\
&= g_i g_j^* \int d^2\hat{\boldsymbol{n}} d^2\hat{\boldsymbol{n}}' \, A_i(\hat{\boldsymbol{n}}) A_j^*(\hat{\boldsymbol{n}}') I(\hat{\boldsymbol{n}}) \delta(\hat{\boldsymbol{n}} - \hat{\boldsymbol{n}}') e^{2\pi i (\hat{\boldsymbol{n}} \cdot \boldsymbol{u}_i - \hat{\boldsymbol{n}}' \cdot \boldsymbol{u}_j)} + n_{ij} \\
&= g_i g_j^* \int d^2\hat{\boldsymbol{n}} \, A_i(\hat{\boldsymbol{n}}) A_j^*(\hat{\boldsymbol{n}}) I(\hat{\boldsymbol{n}}) e^{2\pi i \hat{\boldsymbol{n}} \cdot \boldsymbol{u}_{ij}} + n_{ij}, \qquad (1)
\end{aligned}
$$

where $\boldsymbol{u}_{ij} = \boldsymbol{u}_i - \boldsymbol{u}_j = (\boldsymbol{r}_i - \boldsymbol{r}_j)/\lambda$.

Calibrator

For the calibrator $I(\hat{\boldsymbol{n}}) = S_c \delta(\hat{\boldsymbol{n}} - \hat{\boldsymbol{n}}_0)$,

$$V_{ij}^{\text{calibrator}} = g_i g_j^* A_i(\hat{\boldsymbol{n}}_0) A_j^*(\hat{\boldsymbol{n}}_0) S_c e^{2\pi i \hat{\boldsymbol{n}}_0 \cdot \boldsymbol{u}_{ij}} + n_{ij}, \tag{2}$$

so

$$V'_{ij} = \frac{V_{ij}}{S_c A_i(\hat{\boldsymbol{n}}_0) A_j^*(\hat{\boldsymbol{n}}_0) e^{2\pi i \hat{\boldsymbol{n}}_0 \cdot \boldsymbol{u}_{ij}}} = g_i g_j^* + n'_{ij}. \tag{3}$$

Calibration

The eigen-decomposition of the matrix V'_{ij} is

$$V'_{ij} x_j = \lambda x_i, \tag{4}$$

that is

$$V'_{ij} = \lambda x_i x_j^*, \tag{5}$$

Compare 3 and 5, we get

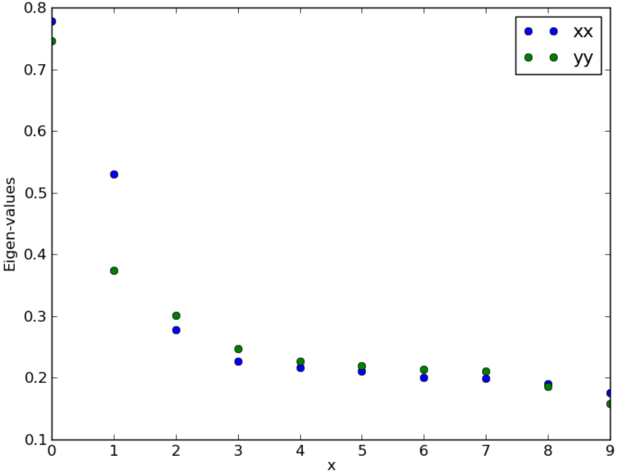
$$g_i = \sqrt{\lambda} x_i. \tag{6}$$

Do the calibration by using the gain in 6,

$$V_{ij}^{\text{cal}} = \frac{V_{ij}}{g_i g_j^*} = \int d^2\hat{\boldsymbol{n}}\, A_i(\hat{\boldsymbol{n}}) A_j^*(\hat{\boldsymbol{n}}) I(\hat{\boldsymbol{n}}) e^{2\pi i \hat{\boldsymbol{n}} \cdot \boldsymbol{u}_{ij}}, \tag{7}$$

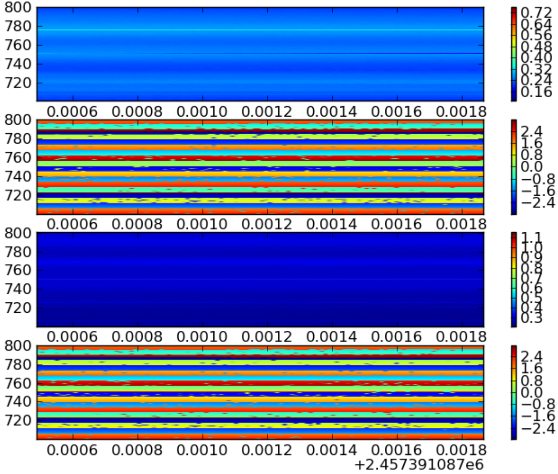
here I have ignored the noise term.

Eigen-Values



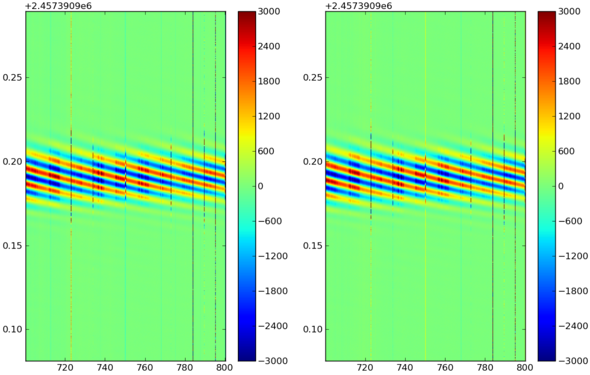
Gain

Gain for Antenna 3 — Amplitude and Phase



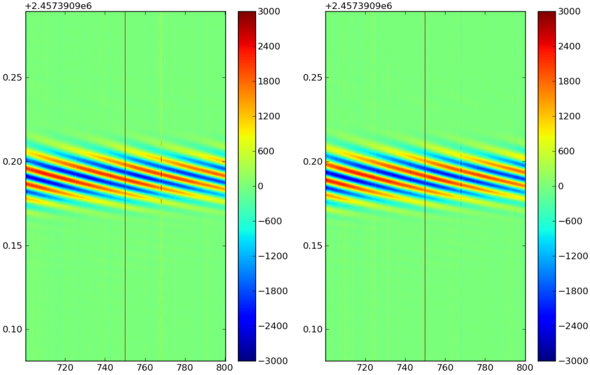
Visibility After Calibration

XX correlation



Visibility After Calibration

YY correlation



Phase to Cas A

Now choose a phase reference point (Cas A) $\hat{\boldsymbol{s}}_0$ such that $\hat{\boldsymbol{n}} = \hat{\boldsymbol{s}}_0 + \boldsymbol{\sigma}$, then the calibrated visibility can be expressed as

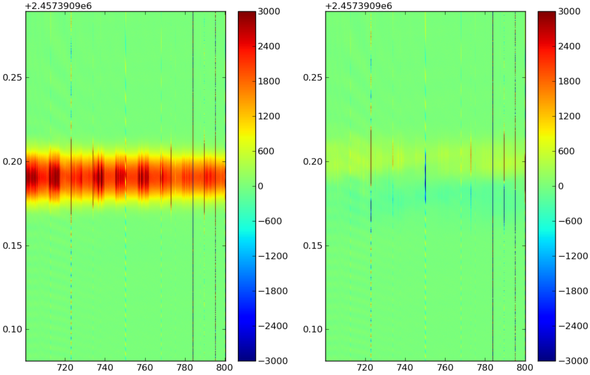
$$V_{ij}^{\mathsf{cal}} = e^{2\pi i \hat{\boldsymbol{s}}_0 \cdot \boldsymbol{u}_{ij}} \int d^2\boldsymbol{\sigma} A_i(\boldsymbol{\sigma}) A_j^*(\boldsymbol{\sigma}) I(\boldsymbol{\sigma}) e^{2\pi i \boldsymbol{\sigma} \cdot \boldsymbol{u_{ij}}}. \tag{8}$$

Phase the calibrated visibility to the phase reference center by divide V_{ij}^{cal} by $e^{2\pi i \hat{\boldsymbol{s}}_0 \cdot \boldsymbol{u}_{ij}}$,

$$V_{ij} = \int d^2\boldsymbol{\sigma} A_i(\boldsymbol{\sigma}) A_j^*(\boldsymbol{\sigma}) I(\boldsymbol{\sigma}) e^{2\pi i \boldsymbol{\sigma} \cdot \boldsymbol{u_{ij}}}. \tag{9}$$

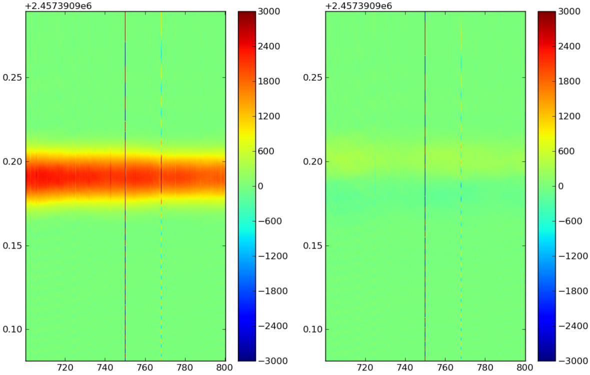
Visibility After Phased to Cas A

XX correlation



Visibility After Phased to Cas A

YY correlation



Imaging

In the flat-sky approximation, and ignore the w-term,

$$V_{ij} = \iint A_i(l,m)A_j^*(l,m)I(l,m)e^{-2\pi i(ul+vm)}dldm, \qquad (10)$$