
LC Theory Calculations: Status and Prospects

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Outline

- Introduction
- Precision observables
- Cross section calculations, Higgs
- Parametric Uncertainties
- Conclusions

Apologies for mostly covering SM calculations and problems.

Apologies for being even very selective for SM calculations.

Apologies for not providing all citations and appraising many developments

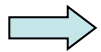
. . . due to lack of time

See also 1504.01726: “Physics at the e^+e^- Linear Collider”

Introduction

ILC / CLIC / FCC-ee / CEPC: (note: LC = “Lepton Collider”)

- Non-QCD initial state (less QCD issues in general)
- Collision energy tunable
- Polarization of e^+ / e^- (very powerful when combined with polarization/spin measurements)
- Less background (although not background-free)
- (Lower energy reach)
- In general higher precision than at LHC – IF a measurement is possible.

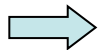


Distinctly different from LHC

More powerful in higher precision of measurements (for indirect BSM search)

Smaller reach in direct search reach

- Requirements for theoretical particle physics more focused on precision to get more (or most?) out of experimental data → urge to [reduce uncertainties in predictions of known quantities](#).
- [Rewarding for theorist because there is a well-defined problem to tackle mathematically.](#)



LC not necessarily an easier environment to work, since there are also brick walls that are not necessarily easier to deal with than at the LHC. They just look different.

Sometimes problems known from LHC and not expected at lepton collider reappear.

... hard work in any case ...

Introduction

General requirements (everyone knows):

- More higher order perturbative loop calculations
- Higher precision for theoretical input parameters (couplings, masses)

But also:

- Fully differential cross sections
- Automatic N^kLO calculations
- Model independent parametrization of BSM physics
- Beyond on-shell or narrow width approximation for unstable particles
- Better understanding of non-perturbative effects
- Improvements in MC event generators
- Improved jet algorithms / jet finding

And even (for some measurements):

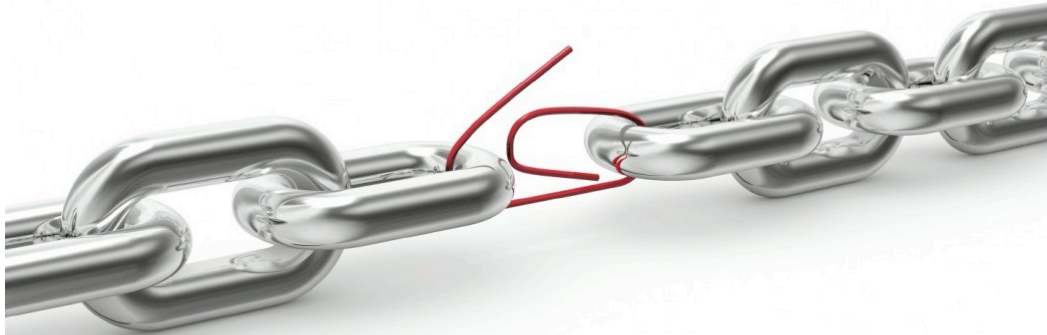
- Beam effects and interference with ISR
- Development of completely new theoretical methods



A lot of profit for LC physics from work being invested already at LHC

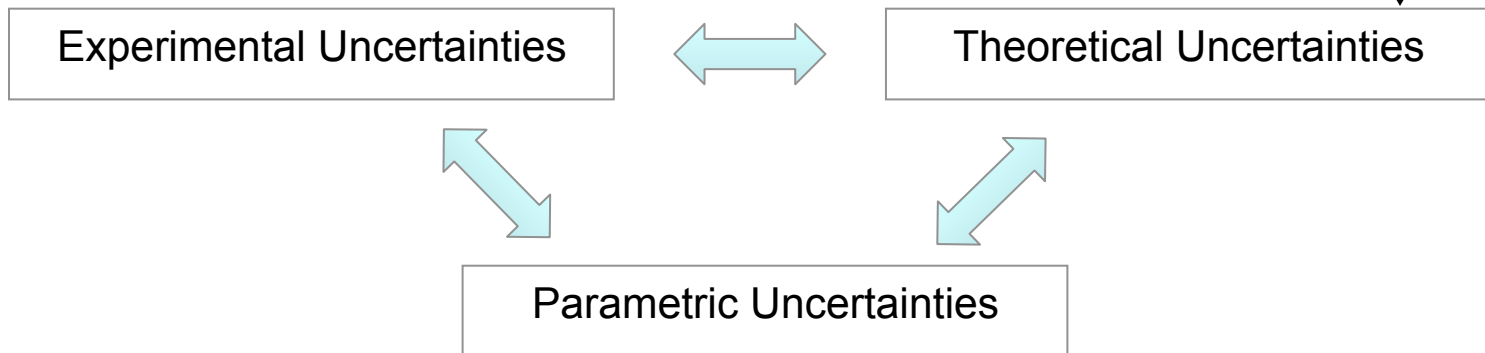
Many issues, however, unique to lepton colliders.

The more precision required, the more involved the theoretical description.



The chain is only as strong as weakest link.

Be aware: not every theorist
has the same standards



Precision Observables

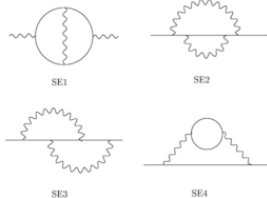
Precision Observables:

- Can be measured very precisely
- Can be calculated very precisely at quantum level

$M_W, \sin^2 \theta_{\text{eff}}, a_\mu, M_h, \dots$

Rare B decays, ...

Through loop calculations
sensitive to many other
(eventually all) parameters
of the theory



Experiment

Theory model
(SM, MSSM,...)

- Test of consistency of theory model with experimental data
- Can constrain parameter space of theory model
- Can falsify a theory model

Experimental precision and theoretical precision (often = number of loops calculated) have to match.



Precision Observables

Examples:

W boson mass: Theoretical prediction for M_W in terms

of $M_Z, \alpha, G_\mu, \Delta r$:

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi \alpha}{\sqrt{2} G_\mu} (1 + \Delta r)$$

Tree level

Loop corrections

One-loop result for M_W in the SM:

[A. Sirlin '80] , [W. Marciano, A. Sirlin '80]

$$\begin{aligned} \Delta r_{1\text{-loop}} = & \Delta\alpha & - & \frac{c_W^2}{s_W^2} \Delta\rho & + & \Delta r_{\text{rem}}(M_H) \\ & \sim \log \frac{M_Z}{m_f} & & \sim m_t^2 & & \log(M_H/M_W) \\ & \sim 6\% & & \sim 3.3\% & & \sim 1\% \end{aligned}$$

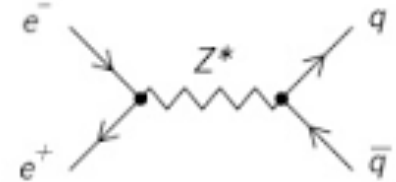
Sensitivity to: gauge couplings, fermion masses, gauge boson masses, Higgs mass

Precision Observables

Examples:

Effective mixing angles: Parametrizes relative strength of V and AV couplings and their loop corrections in

$e^+e^- \rightarrow Z^* \rightarrow f\bar{f}$ at the Z pole



$$\sin^2 \theta_{\text{eff}} = \frac{1}{4|Q_f|} \left(1 - \frac{\text{Re } g_V^f}{\text{Re } g_A^f} \right)$$

Higher order contributions:

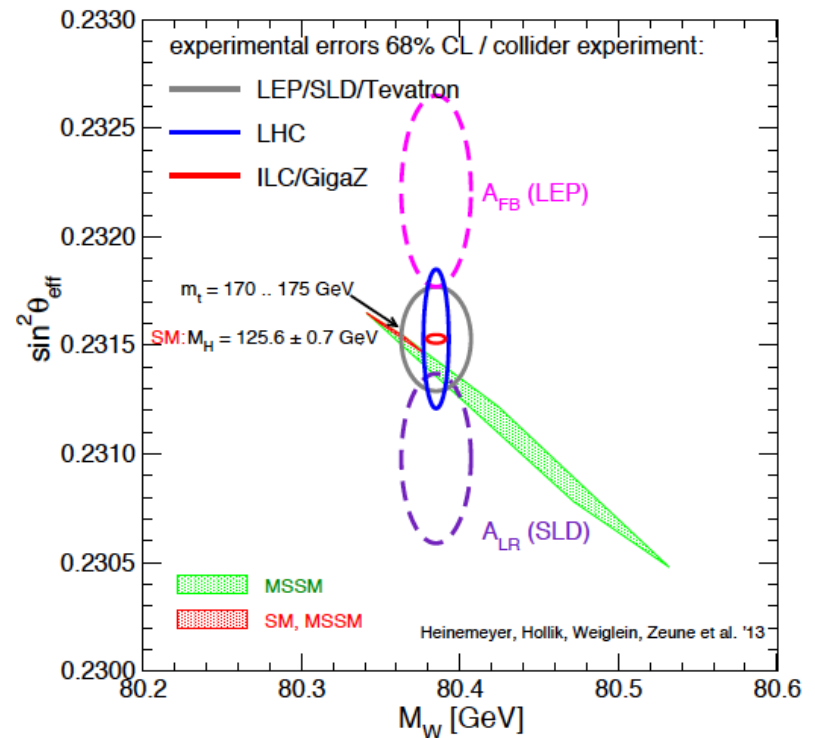
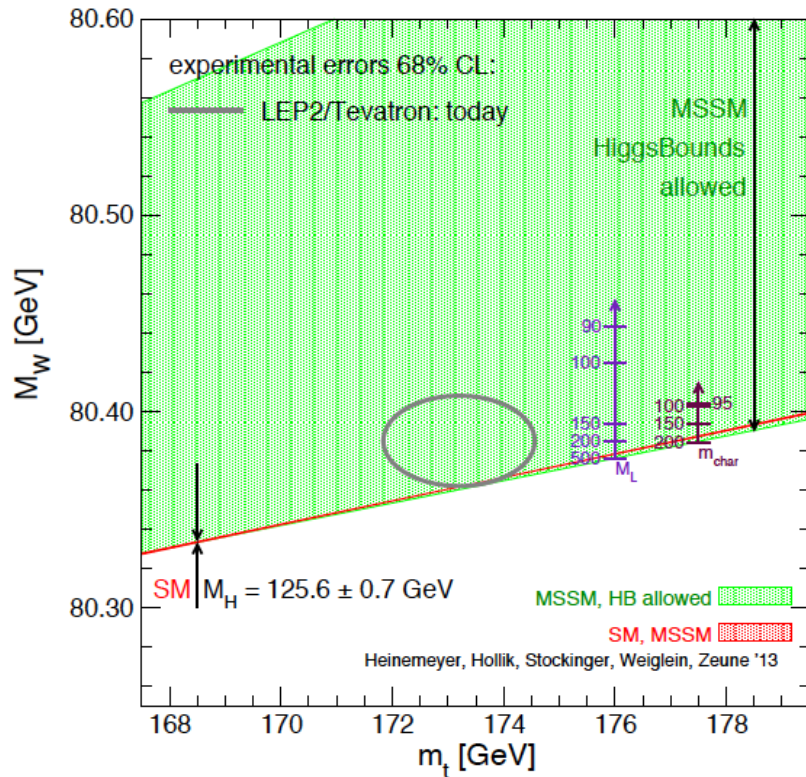
$$g_V^f \rightarrow g_V^f + \Delta g_V^f, \quad g_A^f \rightarrow g_A^f + \Delta g_A^f$$

Polarization of beams crucial to get high experimental precision!

Extensively studied in the SM and the MSSM

Precision Observables

Extensively studied in the SM and the MSSM (Gfitter, Zfitter, ...)



Precision Observables

The W boson mass

Experimental accuracy:

Today: LEP2, Tevatron: $M_W^{\text{exp}} = 80.385 \pm 0.015 \text{ GeV}$

ILC/FCC-ee: – polarized threshold scan

– kinematic reconstruction of W^+W^-

[G. Wilson '13]

– hadronic mass (single W)

$$\delta M_W^{\text{exp,ILC(FCC-ee)}} \lesssim 3(1) \text{ MeV (from thr. scan)} \quad \Leftarrow \text{ TU neglected}$$

Theoretical accuracies:

intrinsic today: $\delta M_W^{\text{SM,theo}} = 4 \text{ MeV}$, $\delta M_W^{\text{MSSM,today}} = 5 - 10 \text{ MeV}$

intrinsic future: $\delta M_W^{\text{SM,theo,fut}} = 1 \text{ MeV}$, $\delta M_W^{\text{MSSM,fut}} = 2 - 4 \text{ MeV}$

parametric today: $\delta m_t = 0.9 \text{ GeV}$, $\delta(\Delta\alpha_{\text{had}}) = 10^{-4}$, $\delta M_Z = 2.1 \text{ MeV}$

$$\delta M_W^{\text{para},m_t} = 5.5 \text{ MeV}, \quad \delta M_W^{\text{para},\Delta\alpha_{\text{had}}} = 2 \text{ MeV}, \quad \delta M_W^{\text{para},M_Z} = 2.5 \text{ MeV}$$

parametric future: $\delta m_t^{\text{fut}} = 0.05 \text{ GeV}$, $\delta(\Delta\alpha_{\text{had}})^{\text{fut}} = 5 \times 10^{-5}$, $\delta M_Z^{\text{ILC/FCC-ee}} = 1/0.1 \text{ MeV}$

$$\Delta M_W^{\text{para,fut},m_t} = 0.5 \text{ MeV}, \quad \Delta M_W^{\text{para,fut},\Delta\alpha_{\text{had}}} = 1 \text{ MeV}, \quad \Delta M_W^{\text{para,fut},M_Z} = 0.2/0.02 \text{ MeV}$$

Precision Observables

The effective weak leptonic mixing angle: $\sin^2 \theta_{\text{eff}}$

Experimental accuracy:

Today: LEP, SLD: $\sin^2 \theta_{\text{eff}}^{\text{exp}} = 0.23153 \pm 0.00016$

GigaZ/TeraZ: both beams polarized, Blondel scheme

$$\delta \sin^2 \theta_{\text{eff}}^{\text{exp,ILC(FCC-ee)}} = 13 (3) \times 10^{-6} \quad \Leftarrow \text{TU neglected}$$

Theoretical accuracies: $[10^{-6}]$

intrinsic today: $\delta \sin^2 \theta_{\text{eff}}^{\text{SM,theo}} = 47 \quad \delta \sin^2 \theta_{\text{eff}}^{\text{MSSM,today}} = 50 - 70$

intrinsic future: $\delta \sin^2 \theta_{\text{eff}}^{\text{SM,theo,fut}} = 15 \quad \delta \sin^2 \theta_{\text{eff}}^{\text{MSSM,fut}} = 25 - 35$

parametric today: $\delta m_t = 0.9 \text{ GeV}$, $\delta(\Delta\alpha_{\text{had}}) = 10^{-4}$, $\delta M_Z = 2.1 \text{ MeV}$

$$\delta \sin^2 \theta_{\text{eff}}^{\text{para},m_t} = 30, \quad \delta \sin^2 \theta_{\text{eff}}^{\text{para},\Delta\alpha_{\text{had}}} = 36, \quad \delta \sin^2 \theta_{\text{eff}}^{\text{para},M_Z} = 14$$

parametric future: $\delta m_t^{\text{fut}} = 0.05 \text{ GeV}$, $\delta(\Delta\alpha_{\text{had}})^{\text{fut}} = 5 \times 10^{-5}$, $\delta M_Z^{\text{ILC/FCC-ee}} = 1/0.1 \text{ MeV}$

$$\Delta \sin^2 \theta_{\text{eff}}^{\text{para,fut},m_t} = 2, \quad \Delta \sin^2 \theta_{\text{eff}}^{\text{para,fut},\Delta\alpha_{\text{had}}} = 18, \quad \Delta \sin^2 \theta_{\text{eff}}^{\text{para,fut},M_Z} = 6.5/0.7$$

Precision Observables

Current uncertainties for EWPOs

6/19

[from A. Freitas '16]

	Experiment	Theory error	Main source
M_W	80.385 ± 0.015 MeV	4 MeV	$\alpha^3, \alpha^2\alpha_s$
Γ_Z	2495.2 ± 2.3 MeV	0.5 MeV	$\alpha_{\text{bos}}^2, \alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2$
σ_{had}^0	41540 ± 37 pb	6 pb	$\alpha_{\text{bos}}^2, \alpha^3, \alpha^2\alpha_s$
$R_b \equiv \Gamma_Z^b / \Gamma_Z^{\text{had}}$	0.21629 ± 0.00066	0.00015	$\alpha_{\text{bos}}^2, \alpha^3, \alpha^2\alpha_s$
$\sin^2 \theta_{\text{eff}}^\ell$	0.23153 ± 0.00016	4.5×10^{-5}	$\alpha^3, \alpha^2\alpha_s$

Extrapolation to future:

Parametric inputs:

* **ILC**: $\delta m_t = 100$ MeV, $\delta\alpha_s = 0.001$, $\delta M_Z = 2.1$ MeV

****FCC-ee**: $\delta m_t = 50$ MeV, $\delta\alpha_s = 0.0001$, $\delta M_Z = 0.1$ MeV

also: $\delta(\Delta\alpha) \sim 5 \times 10^{-5}$

	ILC	FCC-ee	perturb. error with 3-loop [†]	Param. error ILC*	Param. error FCC-ee**
M_W [MeV]	3–5	~ 1	1	2.6	1
Γ_Z [MeV]	~ 1	~ 0.1	$\lesssim 0.2$	0.5	0.06
R_b [10^{-5}]	15	$\lesssim 5$	5–10	< 1	< 1
$\sin^2 \theta_{\text{eff}}^\ell$ [10^{-5}]	1.3	0.6	1.5	2	2

[†] **Theory scenario**: $\mathcal{O}(\alpha\alpha_s^2)$, $\mathcal{O}(N_f\alpha^2\alpha_s)$, $\mathcal{O}(N_f^2\alpha^2\alpha_s)$

(N_f^n = at least n closed fermion loops)

VERY
CHALLENGING

Assumed: all next order
missing corrections
calculated. (2-loop ew)

Cross Section Calculations

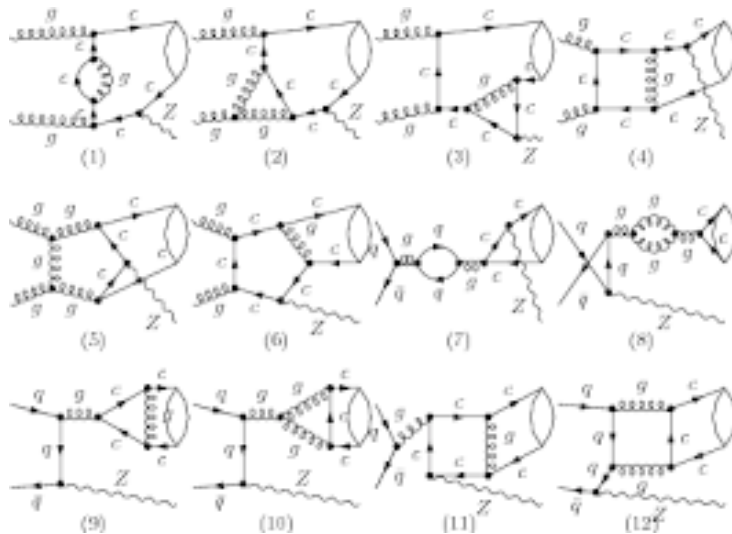
General Situation / Demands:

- Problem of fully differential NLO (QCD + ew) calculations in principle solved¹: virtual + real radiation
- Limitations due to technical complexity and manpower (e.g. number of external legs, BSM models)
- Resulting typical precision away from extrem kinematical regions of phase space:
 $O(\text{few } \%)$ for electroweak ($\alpha_{\text{em}} \sim 0.01$)
 $O(10 \%)$ for QCD ($\alpha_S \sim 0.1$)
- NNLO QCD effects desirable in general to reach close to $O(\text{few } \%)$,
but hard and can only be expected for very important processes (likely with in approximations
and fully differential only in exceptional cases)
- NNLO electroweak corrections desirable for few high-precision observables (e.g. W pair threshold)
but even harder and only achievable with very high motivation and for inclusive/global observables.
- QED effects a different issue: (ISR, beam effects \rightarrow “The PDF problem of Lepton Colliders”
“QED theory” still at level of LEP and needs to be improved to higher precision for many obs.
- A lot of profit from work that already goes into LHC (in principle immediately available for LC), but
there are a number of observables that are very special for Lepton Colliders and require own
dedicated methods.

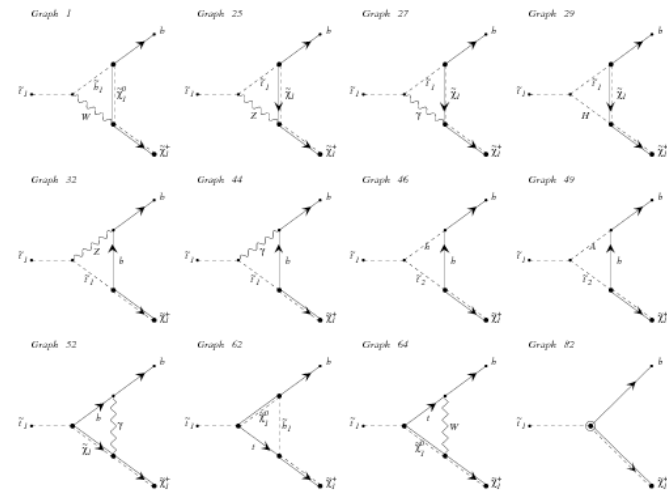
¹ Many processes and NLO corrections already available in automatized MC event
generators: Madgraph, Whizard, ...
or as dedicated special computer codes.

Cross Section Calculations

QCD/QED Loops



Elektroweak Loops

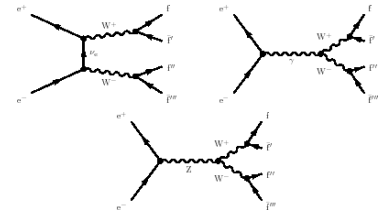


- QCD loops have more lines with zero mass particles (“more mass scales”).
- Can lead to more singularities, but simplifies computations enormously.

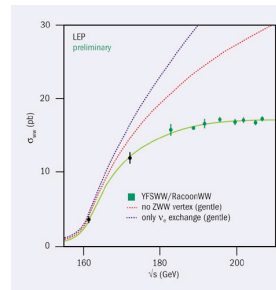
Pure Electroweak Process: WW Threshold

W mass measurement:

- Full final state process $e^+e^- \rightarrow 4 f$ (incl. non-W) needed
- Full NLO corrections $2 \rightarrow 4$ (RACOONxx) + QED-ISR
 \Rightarrow Theoretical error on M_W : $\delta^{\text{th}} M_W \sim M_W \alpha^2 \sim 5 \text{ MeV}$
- Expected experimental LC errors: 5 MeV (or even 1 MeV ?)
 (from statistical+systematical+beam errors)
- Full NNLO calculation would resolve all foreseen theory uncertainties
- Theory approximations to NNLO and higher orders maybe reach 1 MeV:
 - Finite W lifetime corrections: unstable particle effective methods
 - Nonrelativistic Coulomb effects (QED)
 - Sudakov log summation (all for total cross section only)



Denner, Dittmaier, Roth, Wackerroth, 99-02



Actis, Beneke, Falgari, Schwinn, 08

Overall, experimental and theoretical precision match well, but higher precision possible with more efforts:

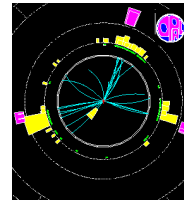
Overall desired precision aim: $\delta^{\text{full}} M_W \sim 1 \text{ MeV}$

QCD Processes: Massless Quarks

R-ratio / Z-width (inclusive):

- Four-loop corrections known (theory error gone)
- Precision $\delta\alpha_s(M_Z) \sim 0.003$ for error $\delta R/R \sim 0.001$
- More useful for hadronic τ decays.

$$\Gamma_Z = \frac{G_F M_Z^3}{24\pi\sqrt{2}} R^{nc} \quad \text{Baikov et al. '12}$$
$$R^{nc} = 20.1945 + 20.1945 a_s \quad a_s = \alpha_s(M_Z)/\pi$$
$$+ (28.4587 - 13.0575 + 0) a_s^2$$
$$+ (-257.825 - 52.8736 - 2.12068) a_s^3$$
$$+ (-1615.17 + 262.656 - 25.5814) a_s^4$$



3-jet production ($e^+e^- \rightarrow q\bar{q}g+X$):

- Much higher sensitivity to α_s already as leading order.
- NNLO $\mathcal{O}(\alpha_s^3)$ fully differential cross section (up to 5 partons in final state) Gehrmann et al. '07
Weinzierl '08
- Applications for event shape distributions (thrust, C=parameter) + NNLL resummation of logs + power correction using dispersion approach Gehrmann et al. '09
+ LEP data: $\delta\alpha_s(M_Z) \sim 0.003$ (dominated by theory error)
- SCET-EFT factorization + NNNLL resummation of logs Abbate et al. '10
Hoang et al. '15
+ shape funct + analysis of thrust and C-parameter LEP data : $\delta\alpha_s(M_Z) \sim 0.0015$ (theory)

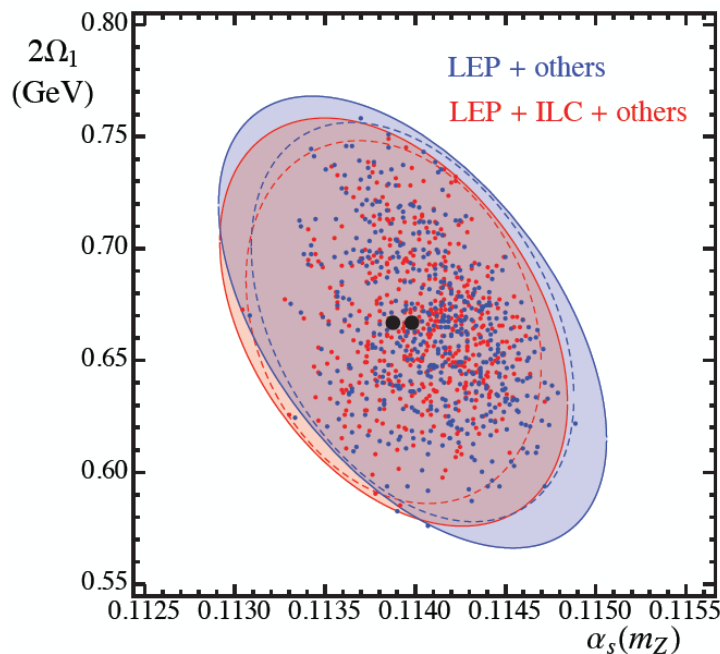
Prospects to improve theory:

- NNNLO perturbative corrections: technology breakthrough needed, many years !
- Improved subleading SCET factorization: studied and complicated. Useful ?

QCD Processes: Massless Quarks

What would a precise measurement of event shapes at $Q=500$ GeV with current QCD calculations contribute?

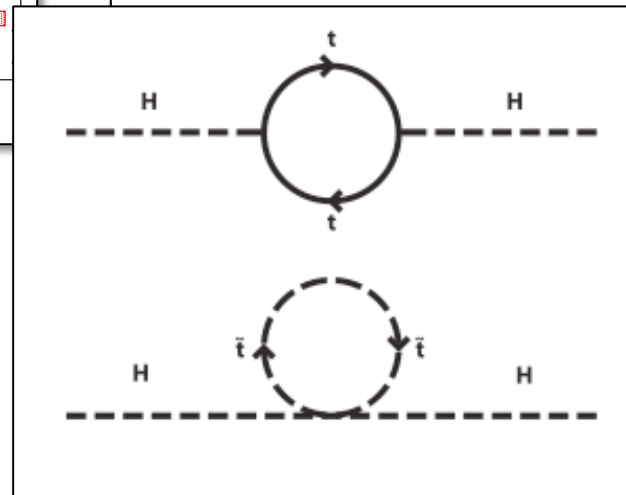
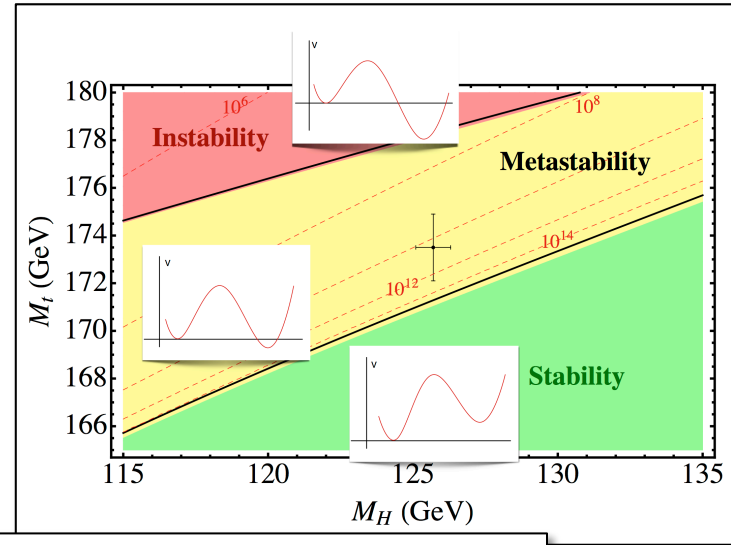
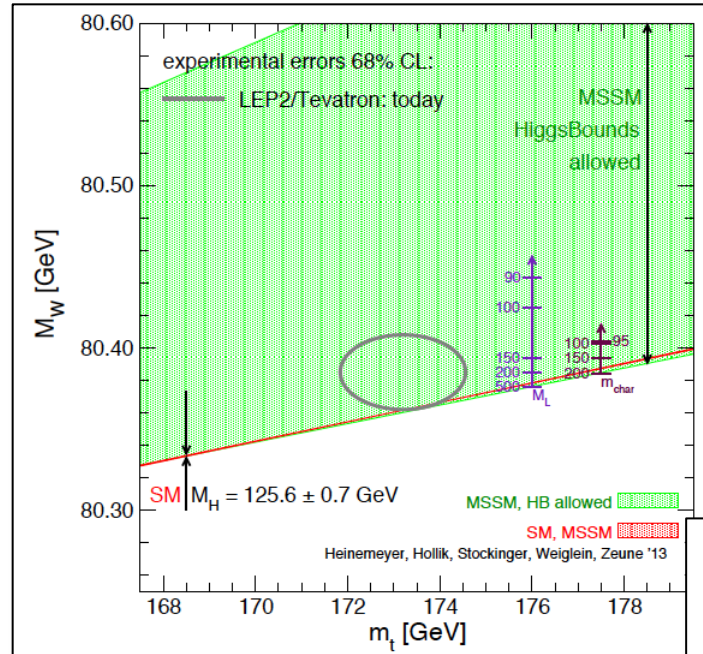
Exercise: Make up fictitious ILC data at 500 GeV, with assumed 1% statistical and 1% systematical uncertainties. Repeat fits.



- Limited impact on precision of α_s because even small high-energy uncertainties blown up in the evolution to Z mass
- Nevertheless, would be very important consistency test of our understanding of QCD: α_s determinations.

Dedicated jet program at LC still very important.

QCD Processes: Top Quark



QCD Processes: Top Quark

Continuum ($Q > 2 m_t$):

- Same situation as for massless quarks, but just one order less!
- Everything done for LHC in principle available for LC (great profit):
 - $\mathcal{O}(\alpha_s^2)$ fully differential stable top cross section
Czakon, Fiedler et al '13-16
Alioli, Fernandes, et al '13
 - $\mathcal{O}(\alpha_s)$ fully differential unstable top NW approx
Bernreuther, Si '10
Melnikov, Schulze '09
Campbell, Ellis '15
Denner et al '12
 - $\mathcal{O}(\alpha_s)$ fully differential unstable top (Wb)
Bavilequa et al '11
Heinrich et al '14
 - $\mathcal{O}(\alpha_s)$ top pair spin density matrix
Bernreuther, Si '13 - 15
 -

Top mass determination:

- Top mass reconstruction + template fits
- Endpoint / lepton moment methods
- . . .

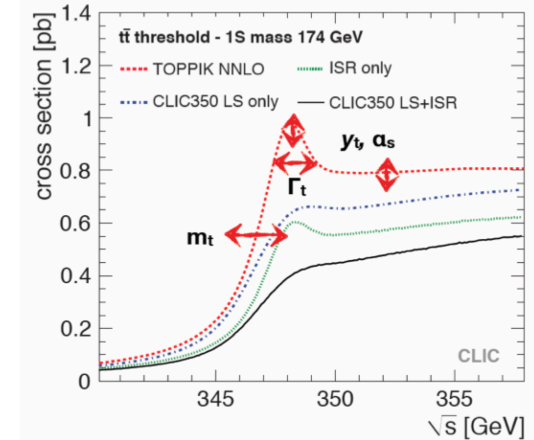
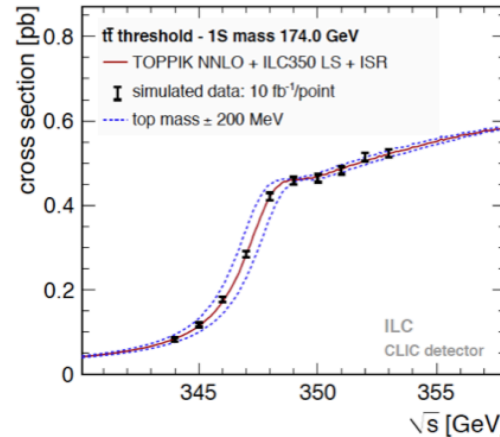
Wanted: Theory how
masses are related to field
theory masses ($\overline{\text{MS}}$)

Wanted: $\mathcal{O}(\alpha_s^2)$ precision for essentially all calculations to have $\mathcal{O}(\%)$ theory precision for measurements (anomalous couplings, spin correlations,...)

QCD Processes: Top Quark

Threshold Scan ($Q \sim 2 m_t$):

- Would-be toponium threshold
- Total cross section: no hadronization corrections.
- Very sensitive to the top mass
- Also important for $t\bar{t}h$ at 500 GeV



$$(\delta m_t^{1S})^{\text{exp, beam}} \sim 30 \text{ MeV}$$

$$(\delta m_t^{1S})^{\text{exp, rest}} \sim 20 \text{ MeV}$$

$$(\delta m_t^{1S})^{\text{th}} \sim 45 \text{ MeV}$$

(based on regular MC's)

- Total cross section results (σ_{tot}):
 - NNNLO QCD “fixed order” (10 years)
 - NNLL $\log(v)$ renormalization group improved (8 years)
 - NNLO electroweak and finite lifetime effects ($e^+e^- \rightarrow WWbb$)

Beneke et al '15

Hoang, Stahlhofen '13

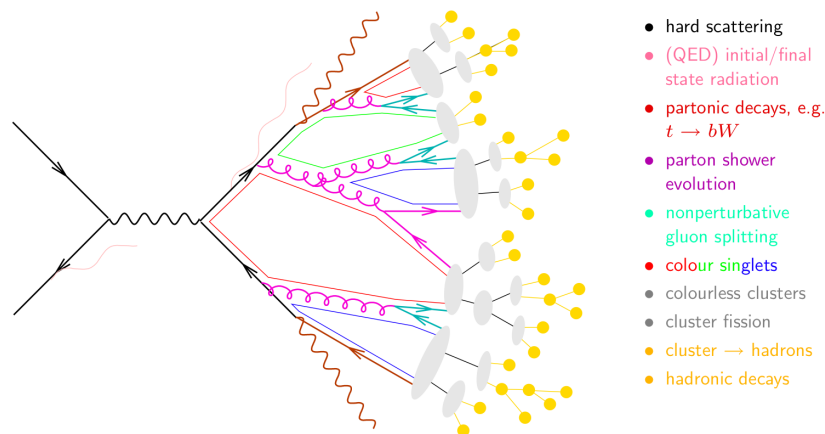
Hoang, Reisser, Ruiz '08

Beneke, Jantzen, Ruiz '10

Desired:

- QCD Next-order total cross section \rightarrow very hard
- Elektroweak, finite lifetime next order \rightarrow doable
- Threshold Monte-Carlo (crucial) (NLO/NLL) \rightarrow talk by Bijan Choukufe
- Modern treatment of ISR / beam (crucial) \rightarrow doable

Just to mention it: Monte Carlo Generators



- Full simulation of all processes (all experimental aspects accessible)
- QCD-inspired: partly first principles QCD \Leftrightarrow partly model (observable-dependent)
- Description power of data better than intrinsic theory accuracy.
- Top quark: treated like a real particle ($m_t^{\text{MC}} \approx m_t^{\text{pole}} + ?$).

But pole mass ambiguous by $O(1 \text{ GeV})$ due to confinement.

Better mass definition needed.

Uncertainty (a): But how precise is modelling? \rightarrow Part of exp. Analyses

Uncertainty (b): What is the meaning of MC QCD parameters? \rightarrow

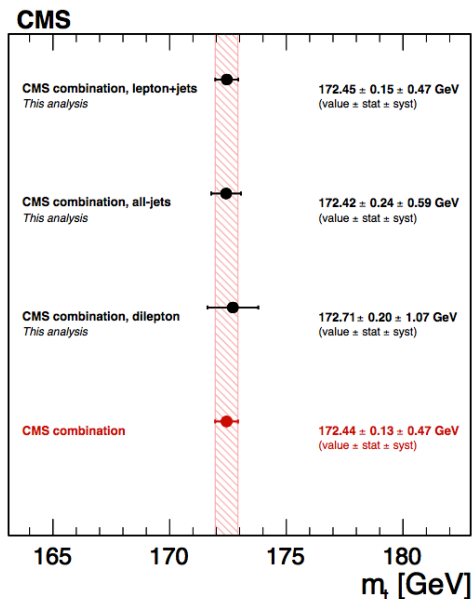
Depends strictly speaking on the observable, because of model character of MCs !

Must be addressed for each type of observable (until we have better MCs).

Just to mention it: MC Improvements

CMS, 1509.04044

Top mass from reconstruction 7+8 TeV combination



- MC modeling uncertainties usually considered as experimental issues.
- But **they are actually theoretical uncertainties** that need dedicated conceptual work to be resolved (e.g. LO+NLO parton shower)

Combined m_t result	δm_t (GeV)
Experimental uncertainties	
Method calibration	0.03
Jet energy corrections	
- JEC: Intercalibration	0.01
- JEC: In situ calibration	0.12
- JEC: Uncorrelated non-pileup	0.10
Lepton energy scale	0.01
E_T^{miss} scale	0.03
Jet energy resolution	0.03
b tagging	0.05
Pileup	0.06
Backgrounds	0.04
Trigger	<0.01
Modeling of hadronization	
JEC: Flavor	0.33
b jet modeling	0.14
Modeling of perturbative QCD	
PDF	0.04
Ren. and fact. scales	0.10
ME-PS matching threshold	0.08
ME generator	0.11
Top quark p_T	0.02
Modeling of soft QCD	
Underlying event	0.11
Color reconnection modeling	0.10
Total systematic	0.47
Statistical	0.13
Total Uncertainty	0.48

LC: Seidel et al, 1303.3758

Experimental uncertainties
below 100 MeV possible

Biggest experimental error

Monte-Carlo uncertainties

⊕

How is the relation of the
MC top quark mass to field
theory masses ?

MC top mass calibration for LC:
→ Talk by Vicent Mateu

Calibration of the MC Top Mass

Method:

- ✓ 1) Strongly mass-sensitive hadron level observable (as closely as possible related to reconstructed invariant mass distribution !)
- ✓ 2) Accurate analytic hadron level QCD predictions at \geq NLL/NLO with full control over the quark mass scheme dependence.
- ✓ 3) QCD masses as function of m_t^{MC} from fits of observable.
- 4) Cross check observable independence

$$m_t^{\text{MC}} = m_t^{\text{MSR}}(R = 1 \text{ GeV}) + \Delta_{t,\text{MC}}(R = 1 \text{ GeV})$$

$$\Delta_{t,\text{MC}}(1 \text{ GeV}) = \bar{\Delta} + \delta\Delta_{\text{MC}} + \delta\Delta_{\text{pQCD}} + \delta\Delta_{\text{param}}$$

Monte Carlo errors:

- different tunings
- parton showers
- color reconnection
- Intrinsic error, ...

QCD errors:

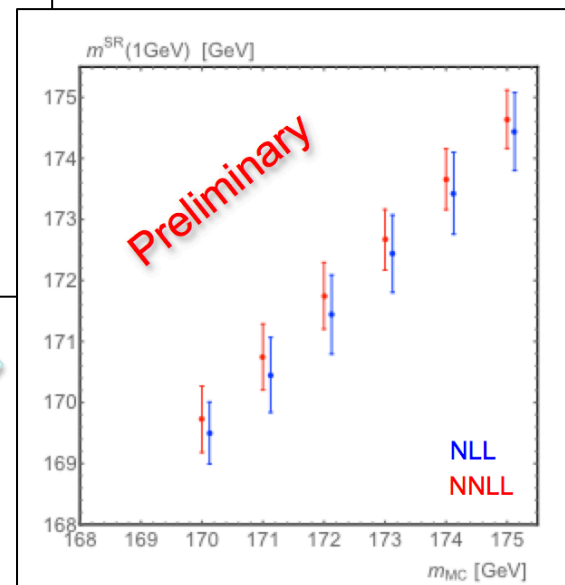
- perturbative error
- scale uncertainties
- electroweak effects

Parametric errors:

- strong coupling α_s
- Non-perturbative parameters

NLO+NNLL hadron level QCD calculation of the 2-jetiness distribution for $e^+e^- \rightarrow t\bar{t}+X$

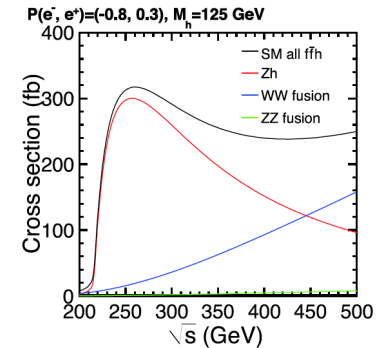
Butenschön, Dehnadi, AHH, Mateu, Stewart w.i.p.



Higgs Precision Physics

Issues to clarify:

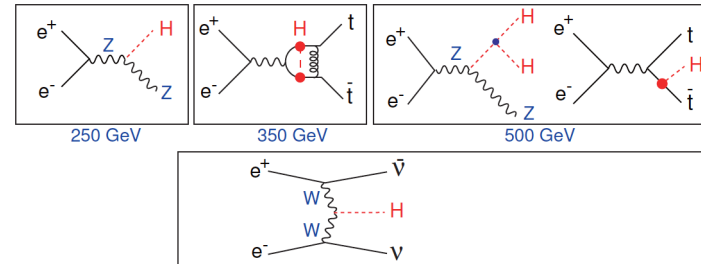
- Couplings to all SM particles: SM Higgs or not?
- Higgs properties: Consistent with SM or inconsistencies?
- Higgs self coupling consistent with SM Higgs potential?
- Are there more than one?



Production mechanisms:

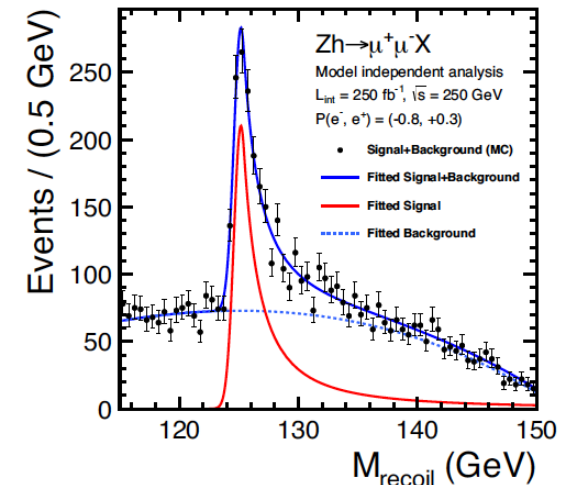
Higgs-strahlung : $e^+e^- \rightarrow Z + H$

W-boson fusion : $e^+e^- \rightarrow \bar{\nu}_e \nu_e + H$

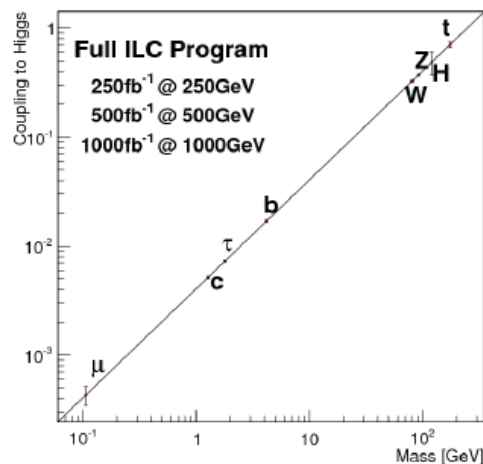


LC special capabilities:

- **Model-independent measurements** of hZZ coupling, total width, branching ratios, couplings, total cross section, spin + CP quantum numbers (250 GeV)
- **Self coupling and top Yukawa coupling** measurements above 500 GeV through $e^+e^- \rightarrow Zh$
- Cross section measurements (recoil method)

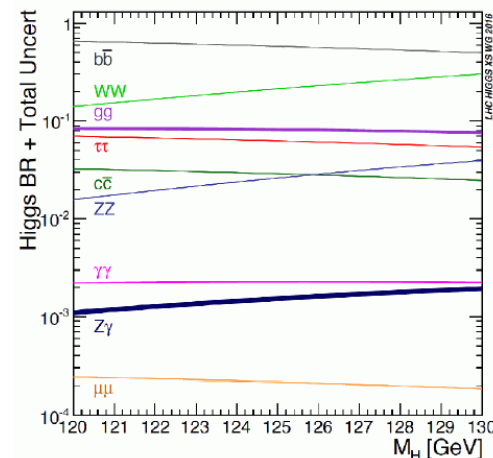


Higgs Precision Physics



Coupling	LHC	HL-LHC	LC	HL-LHC + LC
HWW	4-6%	2-5%	0.3%	0.1%
HZZ	4-6%	2-4%	0.5%	0.3%
Htt	14-15%	7-10%	1.3%	1.3%
Hbb	10-13%	4-7%	0.6%	0.6%
$H\tau\tau$	6-8%	2-5%	1.3%	1.2%
$H\gamma\gamma$	5-7%	2-5%	3.8%	3.0%
Hgg	6-8%	3-5%	1.2%	1.1%
H_{invis}	—	—	0.9%	0.9%

- Higgs physical properties become **new precision observables** that have to be matched by theoretical calculations:
cross sections, partial widths → lot of work to do
- SM perturbative calculations at least **2 loop** will become mandatory (particular when QCD corrections important):
→ Already an industry at LHC with tremendous successes (e.g. Higgs production at NNNLO).
I'm rather optimistic here
- Automatization for BSM models (NLO for sure): Hdecay, Prphecy4f
NNLO (electroweak) very hard, but at least approximation might be possible



Higgs Precision Physics

Higgs branching fractions / couplings / partial widths:

Current theoretical uncertainties:

[LHCHXSWG BR group '15]

Partial Width	QCD	Electroweak	Total
$H \rightarrow b\bar{b}/c\bar{c}$	$\sim 0.2\%$	$\sim 0.5\%$ for $M_H \lesssim 500$ GeV	$\sim 0.5\%$
$H \rightarrow \tau^+\tau^-/\mu^+\mu^-$		$\sim 0.5\%$ for $M_H \lesssim 500$ GeV	$\sim 0.5\%$
$H \rightarrow t\bar{t}$	$\lesssim 5\%$	$\sim 0.5\%$ for $M_H < 500$ GeV	$\sim 5\%$
$H \rightarrow gg$	$\sim 3\%$	$\sim 1\%$	$\sim 3\%$
$H \rightarrow \gamma\gamma$	$< 1\%$	$< 1\%$	$\sim 1\%$
$H \rightarrow Z\gamma$	$< 1\%$	$\sim 5\%$	$\sim 5\%$
$H \rightarrow WW/ZZ \rightarrow 4f$	$< 0.5\%$	$\sim 0.5\%$ for $M_H < 500$ GeV	$\sim 0.5\%$

- QCD corrections: scale change by factor 2 and 1/2
 - EW corrections: missing HO estimation based on the known structure and size of the NLO corrections
 - Different uncertainties on a given channel added linearly
- ⇒ Strong improvement in ~ 20 years possible, but ...
- ... they have to be consistently implemented into codes!
- ⇒ intrinsic uncertainty can/will be sufficiently under control?!

[one of Sven's slides]

Higgs Precision Physics

Higgs branching fractions / couplings / partial widths:

[LHCHSWG YR3]

$M_H = 126 \text{ GeV}$			
Decay	TU	PU	Total
	[%]	[%]	[%]
$H \rightarrow \gamma\gamma$	± 2.7	± 2.2	± 4.9
$H \rightarrow b\bar{b}$	± 1.5	± 1.9	± 3.3
$H \rightarrow \tau\tau$	± 3.5	± 2.1	± 5.6
$H \rightarrow WW$	± 2.0	± 2.2	± 4.1
$H \rightarrow ZZ$	± 2.0	± 2.2	± 4.2



Theory uncertainty



Parametric uncertainty

Higgs Precision Physics

Parametric uncertainties: What about those?

Current uncertainties on decay widths:

[LHCHXSWG YR4]

Channel	Γ [MeV]	$\Delta\alpha_s$	Δm_b	Δm_c	Δm_t	THU
$H \rightarrow b\bar{b}$	2.38	-1.4% +1.4%	+1.7% -1.7%	+0.0% -0.0%	+0.0% -0.0%	+0.5% -0.5%
$H \rightarrow \tau^+\tau^-$	$2.56 \cdot 10^{-1}$	+0.0% +0.0%	+0.0% -0.0%	+0.0% -0.0%	+0.1% -0.1%	+0.5% -0.5%
$H \rightarrow \mu^+\mu^-$	$8.90 \cdot 10^{-4}$	+0.0% +0.0%	+0.0% -0.0%	-0.1% -0.0%	+0.0% -0.1%	+0.5% -0.5%
$H \rightarrow c\bar{c}$	$1.18 \cdot 10^{-1}$	-1.9% +1.9%	-0.0% -0.0%	+5.3% -5.2%	+0.0% -0.0%	+0.5% -0.5%
$H \rightarrow gg$	$3.35 \cdot 10^{-1}$	+3.0% -3.0%	-0.1% +0.1%	+0.0% -0.0%	-0.1% +0.1%	+3.2% -3.2%
$H \rightarrow \gamma\gamma$	$9.28 \cdot 10^{-3}$	+0.0% -0.0%	+0.0% -0.0%	+0.0% -0.0%	+0.0% -0.0%	+1.0% -1.0%
$H \rightarrow Z\gamma$	$6.27 \cdot 10^{-3}$	+0.0% -0.0%	+0.0% -0.0%	+0.0% -0.1%	+0.0% -0.1%	+5.0% -5.0%
$H \rightarrow WW^*$	$8.74 \cdot 10^{-1}$	+0.0% -0.0%	+0.0% -0.0%	+0.0% -0.0%	+0.0% -0.0%	+0.5% -0.5%
$H \rightarrow ZZ^*$	$1.07 \cdot 10^{-1}$	+0.0% -0.0%	+0.0% -0.0%	+0.0% -0.0%	+0.0% -0.0%	+0.5% -0.5%

Data available for $M_H = 124$ GeV, 125 GeV, 126 GeV

⇒ substantially larger than κ precision at ILC/FCC-ee

[one of Sven's slides]

Higgs Precision Physics

Parametric uncertainties: Seem to be a brickwall

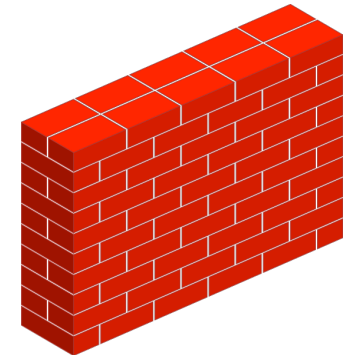
Parameter	Central value	$\overline{\text{MS}}$ masses	Uncertainty
$\alpha_s(M_Z)$	0.118		± 0.0015
m_c	1.403 GeV	$m_c(3 \text{ GeV}) = 0.986 \text{ GeV}$	$\pm 0.026 \text{ GeV}$
m_b	4.505 GeV	$m_b(m_b) = 4.18 \text{ GeV}$	$\pm 0.03 \text{ GeV}$
m_t	172.5 GeV	$m_t(m_t) = 162.7 \text{ GeV}$	$\pm 0.8 \text{ GeV}$

Basically follow the PDG averages

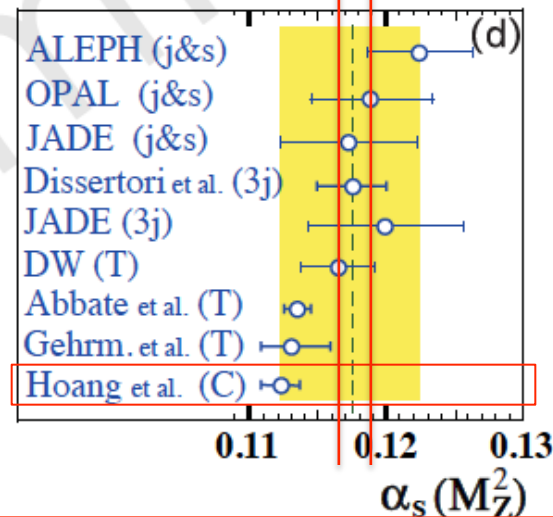
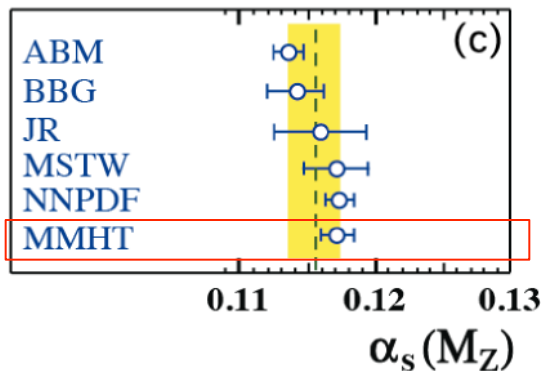
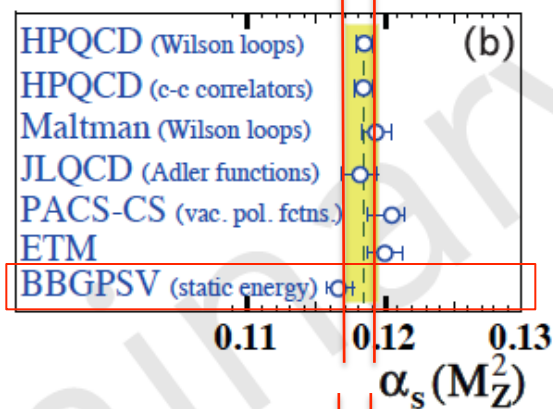
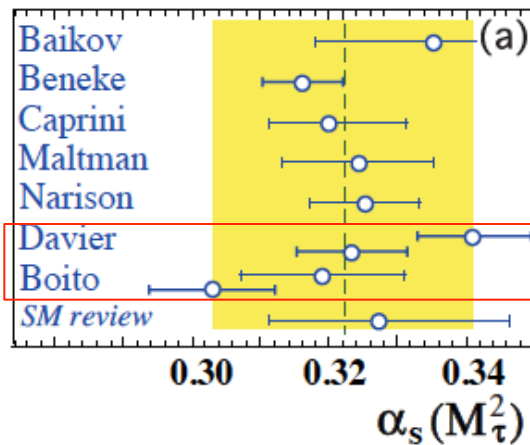
Are the error too conservative or even too optimistic?

Some breakthrough needed: situation deadlocked.

At this time there is NO argument that justifies changing the parameter uncertainties.



Strong Coupling Story



(e) hadron collider ($t\bar{t}$): $\alpha_s(M_Z) = 0.1151^{+0.0033}_{-0.0032}$

(f) e.w. precision fit (GFitter): $\alpha_s(M_Z) = 0.1196 \pm 0.0030$

 = new

2015 World Average:

- New available results were contradicting and lead for the first time to an increase in the world average
- Quoted error in strong contradiction to some high precision analyses.
- Who is wrong?
- Or have we been a bit too optimistic in the past?
- We have to first understand $\delta\alpha_s \sim 0.001$ before going for $\delta\alpha_s \sim 0.0001$

$$\alpha_s(M_Z) = 0.1185 \pm 0.0006 \quad (2013)$$

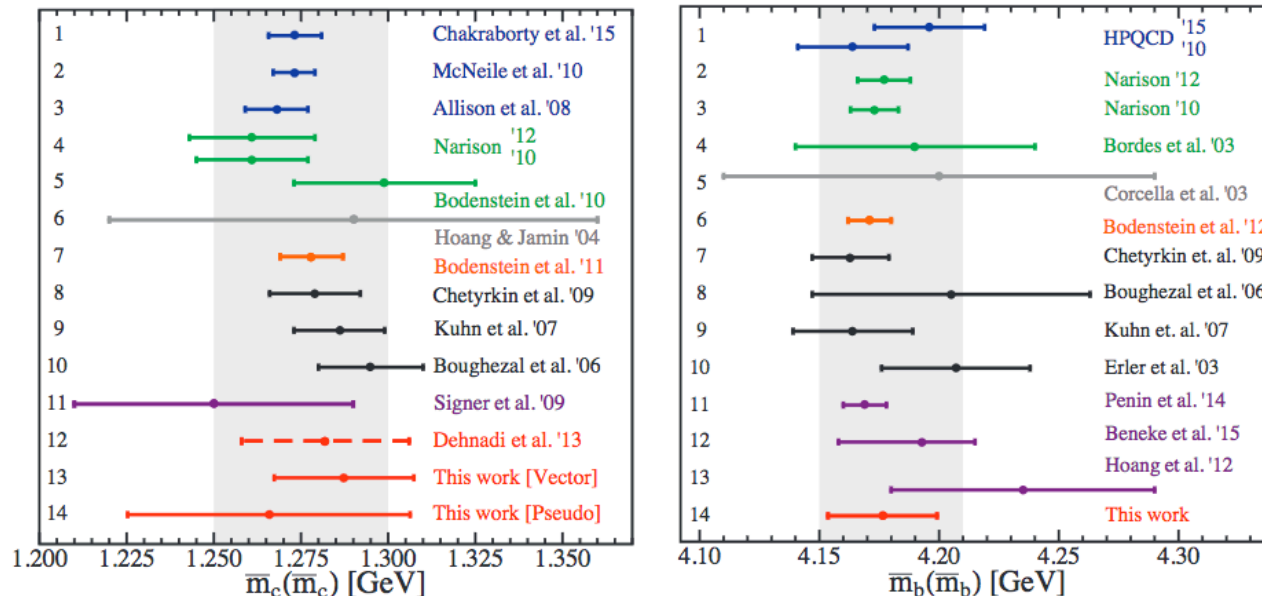


$$\alpha_s(M_Z) = 0.1177 \pm 0.0013 \quad (2015)$$

Charm and Bottom Quark Mass Story

Most precise methods based on QCD sum rules:

Moments on current-current correlator (3-loops) vs. Data on hadronic R-ratio



Dehnadi et al, 1504.07638

- Most recent analysis with same theory input leading to larger theory uncertainties.
- Maybe go on as before: ask for better data, ask for one more loop?

$$\bar{m}_c(\bar{m}_c) = 1.288 \pm (0.006)_{\text{stat}} \pm (0.009)_{\text{syst}} \pm (0.014)_{\text{pert}} \\ \pm (0.010)_{\alpha_s} \pm (0.002)_{\langle GG \rangle} \text{ GeV},$$

$$\bar{m}_b(\bar{m}_b) = 4.176 \pm (0.004)_{\text{stat}} \pm (0.019)_{\text{syst}} \pm (0.010)_{\text{pert}} \\ \pm (0.007)_{\alpha_s} \pm (0.0001)_{\langle GG \rangle} \text{ GeV}.$$

Dehnadi et al, 1504.07638

Summary

- Lots of progress in theory calculations motivated by LHC physics that can be recycled for LC
 - NLO calculations for in principle any (not too complicated) process possible in automatized manner (numerical, computer intensive)
 - NNLO QCD calculations are desired for essentially all processes, but are only available for specific processes
 - Multileg methods very impressive (limited use for LC)
 - Conceptual developments: factorization, resummation of corrections, parton shower matching to NLO results, ...
- NNLO electroweak calculations desired for many processes: systematic tackle of this problem somewhat dormant, but momentum should pick up once LC becomes reality. (Well known what to do in principle)
- LC-specific processes (e.g. top pair threshold) which are very theory demanding and still require many more developments.
- Almost all next big steps (e.g. next order) involve breakthrough developments that appear not easy.
- Parametric uncertainties (α_s , m_c , m_b , $\alpha(Q)$,...) reflect overall consensus of many different independent methods: No obvious path what to do next
(Maybe the real ultimate precision of QCD?)