Inclusive Semi-Leptonic B-Decays to Order $1/m_b^4$

Sascha Turczyk

in collaboration with

Benjamin M. Dassinger Thomas Mannel

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Motivation

- Precision measurement of Standardmodel parameters: Reduction of theoretical errors
- Test of the Standardmodell: Precise determination of V_{cb}
- Nowadays high experimental precision reached:
  Buchmüller and Flächer, Phys. Rev. D **73** (2006) 073008

  New: B. Aubert *et al.* [BABAR Collaboration], arXiv:0707.2670 [hep-ex]
- In fit Heavy Quark Expansion (HQE) is used up to $\mathcal{O}(1/m_b^3)$ and radiative corrections up to α_s on parton level

Fit	$\|V_{cb}\|$	m_b / GeV	m_c / GeV
RESULT	41.96	4.590	1.142
Δ exp	0.23	0.025	0.037
Δ HQE	0.35	0.030	0.045

$\Rightarrow$ High theoretical precision demanded

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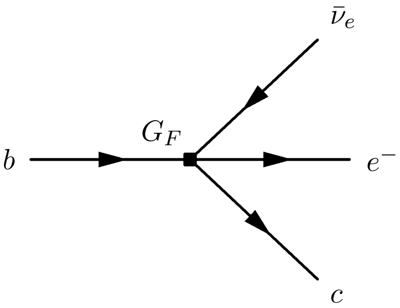
The Effective Hamilton-Operator



- Propagator expanded into local terms

$$\mathcal{H}_W = \frac{4\,G_F}{\sqrt{2}} V_{cb} \bar{c} \gamma^\mu P_L b \bar{e} \gamma_\mu P_L \nu_e$$

- With left-handed projector

$$P_L = 1/2 \left(1 - \gamma^5\right)$$

Differential Rate

$$\mathrm{d}\Gamma = \frac{G_F^2 |V_{cb}|^2}{4 M_B} L_{\mu\nu} W^{\mu\nu} \mathrm{d}\phi_{\mathrm{PS}}$$

- Leptonic tensor $L_{\mu\nu}$
- Hadronic tensor $W^{\mu\nu}$

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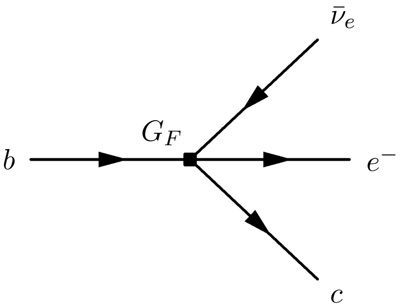

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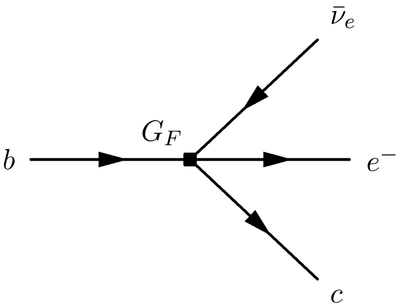

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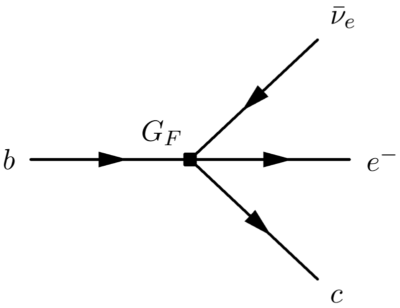

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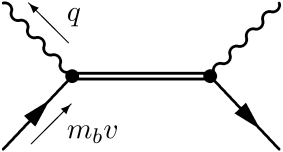
Optical Theorem

- Starting point: correlator of two hadronic currents

$$iT_{\mu\nu} = \frac{1}{2M_B}\int d^4x\, e^{-ix(m_b v - q)}\langle B|\bar{b}_v(x)\Gamma_\mu c(x)\bar{c}(0)\Gamma_\nu^\dagger b_v(0)|B\rangle$$



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$$-\frac{1}{\pi}\text{Im}\, T_{\mu\nu} = W_{\mu\nu}$$

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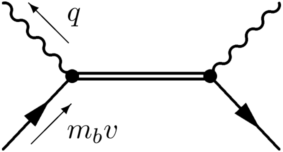

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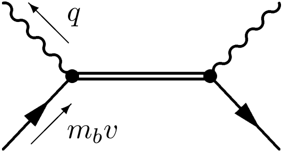

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Operator Product Expansion I

Operator Product Expansion

$$\int d^4x\, e^{iQ\cdot x}\, T\left[\mathcal{O}_\alpha(x)\mathcal{O}_\beta(0)\right] \approx \sum_{i=0}^{7} C_i(Q)\mathcal{O}_i(0)$$

- OPE allows transition to "free" quark fields

- Remove large momentum

$$p_b = m_b v + k$$
$$b_v(x) = e^{im_b v\cdot x} b(x)$$

- $k \leftrightarrow iD$

- Background field propagator:

$$S_{\mathrm{BGF}} = \frac{1}{m_b \not{v} - \not{q} + i\not{D} - m_c}$$

- Expand S_{BGF} in small quantitiy $i\not{D}$

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Operator Product Expansion II

Expansion of the Propagator (Geometric Series)

$$S_{\mathrm{BGF}} = \frac{1}{\not{Q} + i\not{D} - m_c}$$
$$= \frac{1}{\not{Q} - m_c} - \frac{1}{\not{Q} - m_c}(i\not{D})\frac{1}{\not{Q} - m_c}$$
$$+ \frac{1}{\not{Q} - m_c}(i\not{D})\frac{1}{\not{Q} - m_c}(i\not{D})\frac{1}{\not{Q} - m_c} + \cdots$$

- $Q = m_b v - q$
- Keeps track on the ordering of the covariant derivatives: $i\not{D}$ does not commute (color generators and Dirac matrices)

Time-Ordered Product

$$2M_B\, iT_{\mu\nu} = \langle B(p)|\bar{b}_v \gamma_\mu P_L i S_{\mathrm{BGF}} \gamma_\nu P_L b_v|B(p)\rangle$$

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Operator Product Expansion III

Separate Contributions

- Separate Dirac-matrices from matrix-element with derivatives

$\Rightarrow$ $$T_{\mu\nu} \propto \Gamma^{\rho_1 \ldots \rho_n}_{\alpha\beta} \langle B(p)|\bar{b}_{v,\alpha}(iD_{\rho_1})\ldots(iD_{\rho_n})b_{v,\beta}|B(p)\rangle$$

Remaining Task

- Evaluate forward matrix-element with general parametrisation (" trace-formula")

$$\langle B(p)|\bar{b}_{v,\alpha}(iD_{\mu_1})\ldots(iD_{\mu_n})b_{v,\beta}|B(p)\rangle$$

- Start from highest dimension, evaluation recursively

Calculate Time-Ordered Product

- Contract Dirac-matrix coefficients with trace-formula
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▸ Parametrisation

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Identification of Matrix-Elements

Spin-Independent Basic Parameters of Dimension 7

$$2M_B\ s_1 = \langle B(p)|\bar{b}_v iD_\rho (iv\cdot D)^2 iD^\rho b_v|B(p)\rangle$$
$$2M_B\ s_2 = \langle B(p)|\bar{b}_v iD_\rho (iD)^2 iD^\rho b_v|B(p)\rangle$$
$$2M_B\ s_3 = \langle B(p)|\bar{b}_v ((iD)^2)^2 b_v|B(p)\rangle$$

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$$2M_B\ s_4 = \langle B(p)|\bar{b}_v iD_\mu (iD)^2 iD_\nu (-i\sigma^{\mu\nu}) b_v|B(p)\rangle$$
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Physical Interpretation of the s_i

Spin-Independent

$$2M_B s_1 = -g^2 \langle \boldsymbol{E}^2 \rangle$$

$$2M_B s_2 = g^2 (\langle \boldsymbol{E}^2 \rangle - \langle \boldsymbol{B}^2 \rangle) + \langle ((\boldsymbol{p})^2)^2 \rangle$$

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$$2M_B s_4 = -3g \langle (\boldsymbol{S} \cdot \boldsymbol{B})(\boldsymbol{p})^2 \rangle + 2g \langle (\boldsymbol{p} \cdot \boldsymbol{B})(\boldsymbol{S} \cdot \boldsymbol{p}) \rangle$$

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Already Known Parameters

Up to Order $1/m_b^2$

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Up to Order $1/m_b^3$

$$2M_B\hat{\rho}_D^3 = \langle B(p)|\bar{b}_v(iD_\mu)(iv\cdot D)(iD^\mu)b_v|B(p)\rangle$$
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Relation to Normally Defined Parameters

- Parameters $\mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3$ defined with spatial component

⇒ Relation between “our” and normal definition

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▸ Relation

Estimation of the Size of the New Parameters

Physical Interpretation of the Old Parameters

$$2M_B\mu_\pi^2 = \langle \boldsymbol{p}^2 \rangle \qquad 2M_B\mu_G^2 = g\langle \boldsymbol{S}\cdot\boldsymbol{B}\rangle$$
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"Guestimates" of the New Parameters

$$2M_B s_1 = -g^2\langle \boldsymbol{E}^2\rangle \sim -\frac{(g\langle \boldsymbol{p}\cdot\boldsymbol{E}\rangle)^2}{\langle \boldsymbol{p}^2\rangle} \sim -\frac{\rho_D^6}{\mu_\pi^2}, \quad \text{etc.}$$
$$s_2 \sim \frac{\rho_D^6}{\mu_\pi^2} - \mu_G^4 + \mu_\pi^4, \quad s_3 \sim \mu_\pi^4, \quad s_4 \sim s_5 \sim -\mu_G^2\mu_\pi^2$$

Our Guess for Basic Parameters s_i (μ_π^2, μ_G^2, ρ_D^3, ρ_{LS}^3 from Buchmüller and Flächer)

s_1 / GeV^4	s_2 / GeV^4	s_3 / GeV^4	s_4 / GeV^4	s_5 / GeV^4
-0.08 ± 0.03	0.15 ± 0.06	0.16 ± 0.06	-0.12 ± 0.04	-0.12 ± 0.04

Good agreement with I. I. Bigi, N. Uraltsev and R. Zwicky, Eur. Phys. J. C 50 (2007) 539

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Moments of Lepton Energy Spectrum

Definition of the Moment

$$\delta^{(4)}\langle E_\ell^n\rangle = \frac{1}{\Gamma_0}\int dM_X \int_{E_{\mathrm{cut}}} dE_\ell\, E_\ell^n \frac{d^2\Gamma^{(4)}}{dM_x\, dE_\ell}$$

$$\Gamma_0 = \frac{G_F^2 m_b^5 |V_{cb}|^2}{192\pi^3}(1 - 8\rho - 12\log(\rho)\rho^2 + 8\rho^3 - \rho^4)$$

$$\rho = \frac{m_c^2}{m_b^2}$$

Define Dimensionless Function

$$\delta^{(4)}\langle E_\ell^n\rangle = \sum_{i=1}^{5} m_b^n\, g_i^{(n)}(E_{\mathrm{cut}})\, \frac{s_i}{m_b^4}$$

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Values for the Coefficients $g_i^{(n)}$, quoted for $E_{\rm cut} = 0.8$ GeV

i \ n	1	2	3	4
1	- 11.570	- 6.314	- 3.418	- 1.845
2	2.073	1.076	0.556	0.288
3	- 5.969	- 2.801	- 1.320	- 0.620
4	- 0.102	- 0.126	- 0.105	- 0.074
5	- 3.377	- 1.042	- 0.174	0.089

Moments of the Hadronic Invariant Mass Spectrum

Definition of the Moment

$$\delta^{(4)}\langle M_X^n\rangle = \frac{1}{\Gamma_0}\int dM_X\, M_X^n \int_{E_{\rm cut}} dE_\ell\, \frac{d^2\Gamma^{(4)}}{dM_x\, dE_\ell}$$

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$$\rho = \frac{m_c^2}{m_b^2}$$

Define Dimensionless Function

$$\delta^{(4)}\langle M_X^n\rangle = \sum_{i=1}^{5} m_b^n\, f_i^{(n)}(E_{\rm cut})\frac{s_i}{m_b^4}$$

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Moments of the Hadronic Invariant Mass Spectrum

Values for the Coefficients $f_i^{(n)}$, Quoted for $E_{cut} = 0.8$ GeV

i\n	1	2	3	4
1	see below	6.214	- 6.633	- 1.322
2	see below	- 1.343	1.026	0.203
3	see below	2.472	1.358	- 0.033
4	see below	- 0.059	0.315	0.133
5	see below	- 0.377	- 0.129	0.019

Coefficient $f_i^{(1)}$

- Only coefficient with significant dependence on E_{cut}
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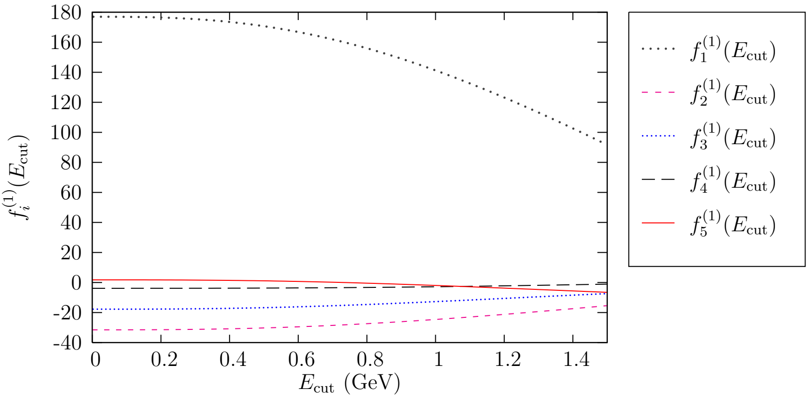
Moments of the Hadronic Invariant Mass Spectrum



Overall Impact on the Theoretical Error

Contribution from $1/m_b^4$ Terms to the Moments

- Used central value for s_i and quoted for $E_{cut} = 0.8$ GeV

n	1 /GeV	2 / GeV^2	3 / GeV^3	4 / GeV^4
$\delta^{(4)}\langle M_X^n \rangle$	-0.1835	-0.0104	0.1850	0.1064
$\delta^{(4)}\langle E_e^n \rangle$	0.0066	0.0154	0.0351	0.0803

Contribution to the Total Rate

- We use the guestimate for the s_i parameters
- Other values taken from Buchmüller and Flächer

$$\frac{\delta^{(4)}\Gamma}{\Gamma} \approx 0.25\%$$

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- In principle calculation for arbitrary $\mathcal{O}(1/m_b^n)$ possible
- Complete result for $\mathcal{O}(1/m_b^4)$ contributions at tree-level known
- Size of $1/m_b^4$ terms is "normal" in tree-level calculation
- Small experimentel errors: Analysis is already sensitive to the new $\mathcal{O}(1/m_b^4)$ contributions
- Theoretical uncertainty (HQE) of V_{cb} determination reduced to $< 1\%$

Outlook

- Investigation of phasespace logarithms
- Radiative corrections to $1/m_b^2$ contributions

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Connection between the two definitions

Parameters in $1/m_b^2$

$$\hat{\mu}_\pi^2 = \mu_\pi^2 + \frac{1}{8m_b^2}[s_2 + s_3 + 4s_5]$$

$$\hat{\mu}_G^2 = \mu_G^2 - \frac{1}{m_b}[\rho_D^3 + \rho_{\mathrm{LS}}^3] - \frac{1}{4m_b^2}[s_2 + s_3 + 4s_5]$$

Parameters in $1/m_b^3$

$$\hat{\rho}_D^3 = \rho_D^3$$

$$\hat{\rho}_{\mathrm{LS}}^3 = \rho_{\mathrm{LS}}^3 - \frac{s_1}{m_b}$$

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Parametrisation of Matrix-Elements

Static Case

$$\langle B(p)|b_{v,\alpha}(iD_{\mu_1})....(iD_{\mu_n})b_{v,\beta}|B(p)\rangle$$
$$= \langle \mathrm{B}_v|h_{v,\alpha}(iD_{\mu_1})....(iD_{\mu_n})h_{v,\beta}|\mathrm{B}_v\rangle + \mathcal{O}(1/m_b)$$
$$= 1_{\beta\alpha}A_{\mu_1\mu_2\cdots\mu_n} + s_\lambda B^{\lambda}_{\mu_1\mu_2\cdots\mu_n}$$

- With the generalization of the Pauli matrices $s_\lambda = P_+\gamma_\lambda\gamma_5 P_+$

Nonstatic Case

$$\langle B(p)|b_{v,\alpha}(iD_{\mu_1})....(iD_{\mu_{n-1}})b_{v,\beta}|B(p)\rangle$$
$$= \sum_i \hat{\Gamma}^{(i)}_{\beta\alpha} A^{(i)}_{\mu_1\mu_2\cdots\mu_{n-1}}$$

- Where $\hat{\Gamma}^{(i)}$ are the complete set of the 16 Dirac matrices

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Example for Trace-Formula

$$
\begin{aligned}
\langle B(p)|\bar{b}_v(iD^\rho)b_v|B(p)\rangle = & -\frac{M_B}{2m_b}P_+\left\{v^\rho(\hat{\mu}_G^2-\hat{\mu}_\pi^2)\right\} \\
& +\frac{M_B}{6m_b}\left\{(\gamma^\rho - v^\rho \not{v})(\hat{\mu}_G^2-\hat{\mu}_\pi^2)\right\} \\
& +\frac{M_B}{12m_b^2}\left\{(\gamma^\rho - 4\ v^\rho \not{v})(\hat{\rho}_{\mathrm{LS}}^3+\hat{\rho}_D^3)\right\} \\
& +\mathcal{O}(1/m_b^4)
\end{aligned}
$$

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Total Rate

$$\Gamma = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 \Big\{ -\rho^4 + 8\rho^3 - 12\log(\rho)\rho^2 - 8\rho + 1$$

$$- \frac{1}{2m_b^2}\hat{\mu}_\pi^2 \quad \Big(-\rho^4 + 8\rho^3 - 12\log(\rho)\rho^2 - 8\rho + 1 \Big)$$

$$+ \frac{1}{2m_b^2}\hat{\mu}_G^2 \quad \Big(-5\rho^4 + 24\rho^3 - 12(\log(\rho) + 2)\rho^2 + 8\rho - 3 \Big)$$

$$+ \frac{2}{3m_b^3}\hat{\rho}_D^3 \quad \Big(-5\rho^4 + 16\rho^3 - 12\rho^2 - 16\rho + 12\log(\rho) + 17 \Big)$$

Total Rate

$$
\begin{aligned}
&+\frac{8}{9m_b^4}s_1\left(9\rho^4-20\rho^3+9\rho^2+6\log(\rho)+2\right)\\
&+\frac{1}{9m_b^4}s_2\left(-27\rho^4+76\rho^3-72\rho^2+36\rho-12\log(\rho)-13\right)\\
&+\frac{4}{9m_b^4}s_3\left(3\rho^4-7\rho^3+9\rho^2-21\rho+4(3\log(\rho)+4)\right)\\
&+\frac{1}{3m_b^4}s_5\left(-5\rho^4+16\rho^3-12\rho^2-16\rho+12\log(\rho)+17\right)\Bigg\}
\end{aligned}
$$

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