$B \longrightarrow D^{(*)}$ Form Factors from QCD Sum Rules with B-Meson Distribution Amplitudes

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Motivation

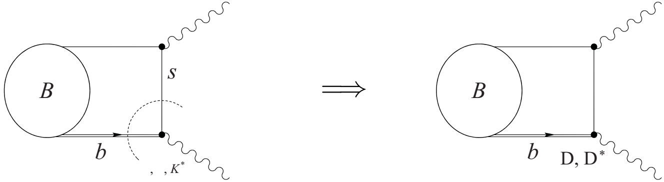
- $|V_{cb}|$ from exclusive $B \to D^{(*)}$ decay

$$\frac{d\Gamma(\bar{B} \to D e \bar{\nu}_e)}{dw} = \frac{G_F^2 |V_{cb}|^2 m_B^5}{48\pi^3} \sqrt{(w^2-1)^3} r^3 (1+r)^2 \mathcal{F}_D(w)^2 ,$$

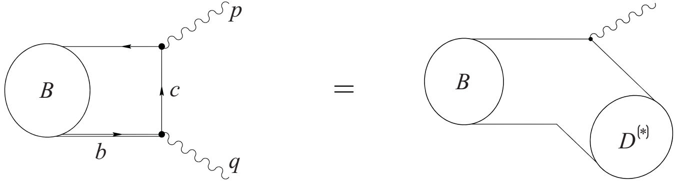
$$\frac{d\Gamma(\bar{B} \to D^* e \bar{\nu}_e)}{dw} = \frac{G_F^2 |V_{cb}|^2 m_B^5}{48\pi^3} \sqrt{(w-1)}(w+1) r^{*3} (1-r^*)^2 \cdot \left[1 + \frac{4w}{w+1} \frac{1-2wr^*+r^{*2}}{(1-r^*)^2}\right] \mathcal{F}_{D^*}(w)^2 .$$

- $r = m_D/m_B$, $r^* = m_{D^*}/m_B$
- $\mathcal{F}_D$ and $\mathcal{F}_{D^*}$ combinations of form factors $h_+(w)$, $h_-(w)$ and $h_V(w)$
- $v^\mu = (p+q)^\mu/m_B$ and $v'^\mu = p^\mu/m_{D^{(*)}}$ four-velocities
- $w = (v \cdot v') = [m_B^2 + m_{D^{(*)}}^2 - q^2]/[2 m_B m_{D^{(*)}}]$
- $1/m_c$, $1/m_b$-corrections to form factors, need full QCD

- new method: LCSR with B-meson distribution amplitudes (DA′s)
- form factors $B \to \pi, K$ and $B \to \rho, K^*$ [A. Khodjamirian, Th. Mannel and N. Offen (2007)]



- apply this technique for $B \to D^{(*)}$ form factors



Form Factors

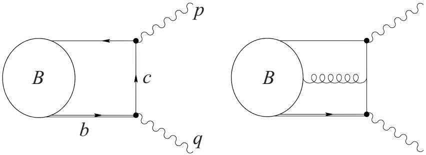
- standard definition

$$\langle D(p)|\bar{c}\gamma_\mu b|\bar{B}(p+q)\rangle = 2p_\mu f^{+}_{BD}(q^2) + q_\mu f^{\pm}_{BD}$$
$$\langle D^*(p)|V^\mu|\bar{B}(p+q)\rangle = \frac{2V^{BD^*}}{m_B + m_{D^*}} \epsilon^{\mu\nu\alpha\beta} \epsilon^*_\nu q_\alpha p_\beta$$

- form factors adjusted to Isgur-Wise limit
  [Isgur and Wise (1989, 1990), Neubert (1994)]

$$\frac{\langle D(p)|V^\mu|\bar{B}(p+q)\rangle}{\sqrt{m_B m_D}} = h_+(w)(v+v')^\mu + h_-(w)(v-v')^\mu$$
$$\frac{\langle D^*(v',\epsilon)|V^\mu|\bar{B}(v)\rangle}{\sqrt{m_B m_D^*}} = h_V(w)\epsilon^{\mu\nu\alpha\beta} \epsilon^*_\nu v'_\alpha v_\beta$$

- $B \to D^{(*)}$ correlation functions, contributions of two- and three-particle B-meson DA′s



$$\langle 0|\bar{q}_\alpha(x)[x,0]h_{v\beta}(0)|\bar{B}_v\rangle = -\frac{if_B m_B}{4}\int_0^\infty d\omega e^{-i\omega v\cdot x}\left[(1+\not{v}) \cdot \left\{\phi_+^B(\omega) - \frac{\phi_+^B(\omega)-\phi_-^B(\omega)}{2v\cdot x}\not{x}\right\}\gamma_5\right]_{\beta\alpha}$$

- finite c-quark mass, c-quark virtual
- B-meson DA′s defined in heavy-quark effective theory (HQET)
- two particle DA′s $\phi_+^B(\omega)$ and $\phi_-^B(\omega)$ [Grozin and Neubert (1997), Braun et al. (2004)]

	current	form factor		model
$B \to D$	$i\gamma_5$	f_{BD}^+, $f_{BD}^\pm$	h_+, h_-	$\phi_+ = \frac{\omega}{\omega_0^2}e^{-(\omega/\omega_0)} = \frac{\omega}{\omega_0}\phi_-^B$
$B \to D^*$	γ_μ	V_{BD^*}	h_V	$\omega_0 = \lambda_B = 460 \pm 110$ MeV

Isgur-Wise Limit

- $f_+(q^2)$, $f_-(q^2)$ and $V(q^2) \to h_+(w)$, $h_-(w)$ and $h_V(w)$, heavy quark limit
  - $m_B = m_b + \bar{\Lambda}$, $m_D = m_c + \bar{\Lambda}$
  - Borel-parameter $M^2 = 2m_c\tau$
  - threshold $s_0^{D^{(*)}} = m_c^2 + m_c\beta_0$
  - $f_{B,D^{(*)}} \to \tilde{f}_{B,D^{(*)}}/\sqrt{m_{B,D^{(*)}}}$
- Isgur Wise limit
  - $m_{b,c} \to \infty$, but $\frac{m_c}{m_b} = const. = \kappa^2$
  - $h_+(w) = h_V(w) = \xi(w)$ and $h_-(w) = 0$
  - zero recoil point $\xi(1) = 1$
- kinematic range for w

$$1 \leq w \leq 1 + \frac{[m_B - m_{D^{(*)}}]^2}{2m_B m_{D^{(*)}}}$$

Results for $B \longrightarrow D^*$

- sum rule for the form factor V^{BD^*} [Khodjamirian et al. (2005)]
- Isgur-Wise function: $h_V(w) = \frac{2\sqrt{m_B m_{D^*}}}{m_B + m_{D^*}} V^{BD^*}(w)$

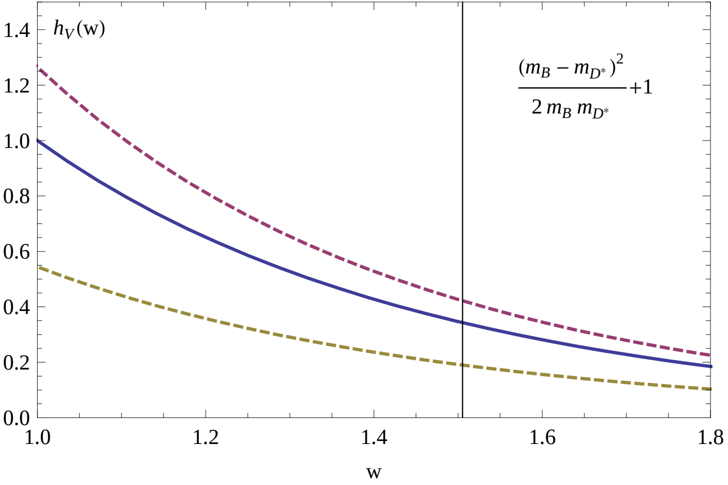
Isgur-Wise limit

$$h_V(w) = \frac{\tilde{f}_B}{\tilde{f}_{D^*}} \int_0^{\beta_0/w} d\omega \exp\left\{ \frac{\omega}{\tau}\left(\frac{\kappa^2}{2} - w\right) + \frac{\bar{\Lambda}}{\tau} \right\} \cdot \left[\frac{1}{2w}\phi_-^B(\omega) + \left(1 - \frac{1}{2w}\right)\phi_+^B(\omega) \right]$$

where $\kappa = \sqrt{\frac{m_c}{m_b}}$ and $\omega_0 = \frac{\beta_0}{w}$.

- three-particle contribution suppressed

Isgur-Wise limit: $h_V(w)$



$3 < M^2 \leq 6$ GeV^2

Results for $B \to D$

- Isgur-Wise $h_+(w)$

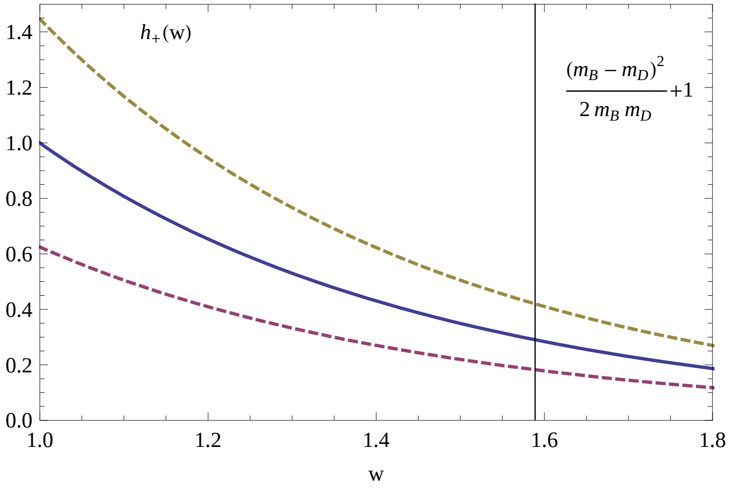
$$h_+^{BD}(w) = \frac{f_B m_B m_c}{2\kappa m_D^2 f_D} \int_0^{\sigma(s_0,q^2)} \exp\left\{ \frac{-s(\sigma, q^2(w) + m_D^2}{M^2} \right\} \cdot \left[\frac{m_c m_B [m_c + m_B(\kappa^2 - \sigma)]}{\bar\sigma^2 m_B^2 + m_c^2 - q^2} \phi_-^B(\omega) \right.$$
$$+ \left(\frac{m_c + m_B(\kappa^2 - \sigma)}{\bar\sigma} - \frac{m_B m_c [m_c + m_B(\kappa^2 - \sigma)]}{\bar\sigma^2 m_B^2 + m_c^2 - q^2} \right) \phi_+^B(\omega)$$
$$\left. - \left(\frac{1}{\bar\sigma} - \frac{m_c m_B}{\bar\sigma^2 m_B^2 + m_c^2 - q^2} + \frac{2 m_B^2 \bar\sigma [m_c + m_B(\kappa^2 - \sigma)]}{(\bar\sigma^2 m_B^2 + m_c^2 - q^2)^2} \right) \Phi_\pm^B(\omega) \right] + \Delta h_+^{BD}$$

Isgur-Wise limit

$$h_+(w) = \frac{\tilde f_B}{\tilde f_D} \int_0^{\beta_0/w} d\omega \exp\left\{ \frac{\omega}{\tau} \left(\frac{\kappa^2}{2} - w \right) + \frac{\bar\Lambda}{\tau} \right\} \cdot \left[\frac{1}{2w} \phi_-^B(\omega) + \left(1 - \frac{1}{2w} \right) \phi_+^B(\omega) \right]$$

- Isgur-Wise limit: three particle contribution $\Delta h_+^{BD} \to 0$

Isgur-Wise limit: $h_+(w)$



Results for $B \to D$

- Isgur-Wise function $h_-(w)$

$$h_-^{BD}(w) = \frac{f_B m_B m_c}{2\kappa m_D^2 f_D} \int_0^{\sigma(s_0,q^2)} \exp\left\{ \frac{-s(\sigma, q^2(w) + m_D^2}{M^2} \right\} \cdot \left[\frac{m_c m_B [m_c + m_B(\kappa^2 - \sigma)]}{\bar\sigma^2 m_B^2 + m_c^2 - q^2} \phi_-^B(\omega) \right.$$
$$\cdot \left[\frac{m_c m_B [m_c - m_B(\kappa^2 + \sigma)]}{\bar\sigma^2 m_B^2 + m_c^2 - q^2} \phi_-^B(\omega) \right.$$
$$+ \left(\frac{m_c - m_B(\kappa^2 + \sigma)}{\bar\sigma} + \frac{m_B m_c [m_B(\kappa^2 + \sigma) - m_c]}{\bar\sigma^2 m_B^2 + m_c^2 - q^2} \right) \phi_+^B(\omega)$$
$$\left. - \left(\frac{1}{\bar\sigma} - \frac{m_c m_B}{\bar\sigma^2 m_B^2 + m_c^2 - q^2} - \frac{2\bar\sigma m_B^2 m_c [m_B(\kappa^2 + \sigma) - m_c}{(\bar\sigma^2 m_B^2 + m_c^2 - q^2)^2} \right) \bar\Phi_\pm^B(\omega) \right] + \Delta h_-^{BD}$$

Isgur-Wise limit

$$h_-(w) \equiv 0 \quad \text{and} \quad \Delta h_-^{BD} \to 0$$

Summary and Outlook

- Summary
  - new light-cone sum rules for $B \to D^{(*)}$ in terms of B-meson DA′s
  - obey Isgur-Wise limit for the form factors h_+, h_- and h_V
- Outlook
  - $1/m_c$ and partial $1/m_b$-expansion of the Isgur-Wise functions
  - A_1^{BD}, A_2^{BD} and A_3^{BD} form factors for the differential decay rates $d\Gamma(\bar{B} \to D^{(*)} e\bar{\nu}_e) \to |V_{cb}|$
  - dynamics at the zero-recoil point ($w = 1$)
  - perturbative α_s-corrections