Testing the left-handedness of the b⟶c transition

Robert Feger
Theoretische Physik 1, Universität Siegen

in collaboration with
Thomas Mannel and Benjamin Dassinger

EuroFlavour '07

Orsay, 14.11.2007

Outline

1. Introduction
   - Testing the standard model
   - Enhancement via higher dimensional operators
   - New Ansatz
2. The lepton-energy spectrum at Tree-level
3. Radiative corrections
   - Real corrections
   - Virtual corrections

Problem

History

1. Experiments → neutrinos have helicity -1 (lefthanded) and a small or no rest mass
2. standard model assumption : neutrinos are massless
   ⇒ helicity is a Lorentz invariant
3. Neutrinos are only generated from the weak interaction
4. Absence of righthanded neutrinos
   ⇒ weak interaction couples only with lefthanded leptonic currents

Quark-lepton universality

The coupling to only lefthanded currents is assumed to be a fundamental, universal property of the weak interaction and transferred to weak quark decays.

Problem

History

1. Experiments → neutrinos have helicity -1 (lefthanded) and a small or no rest mass
2. standard model assumption : neutrinos are massless
   ⇒ helicity is a Lorentz invariant
3. Neutrinos are only generated from the weak interaction
4. Absence of righthanded neutrinos
   ⇒ weak interaction couples only with lefthanded leptonic currents

Quark-lepton universality

The coupling to only lefthanded currents is assumed to be a fundamental, universal property of the weak interaction and transferred to weak quark decays.

Testing the standard model

B factories BaBar and Belle

- High precision at the B factories BaBar and Belle allows also a test of charged currents
- This test is known from the lepton sector as the „Michel parameter analysis“.

Approach

- Extension of the standard model
- Calculation of the moments of the lepton energy spectrum and of the spectrum of the hadronic invariant mass of the inclusive decay $\bar{B} \to X_c e^- \bar{\nu}_e$
- Comparision with data from the B factories

Testing the standard model

B factories BaBar and Belle

- High precision at the B factories BaBar and Belle allows also a test of charged currents
- This test is known from the lepton sector as the „Michel parameter analysis“.

Approach

- Extension of the standard model
- Calculation of the moments of the lepton energy spectrum and of the spectrum of the hadronic invariant mass of the inclusive decay $\bar{B} \to X_c \, e^- \, \bar{\nu}_e$
- Comparision with data from the B factories

Extension of the standard model

Assumption

- There exists an **unknown**, parent theory at the scale Λ.
- The standard model is an effective theory of this superior theory.

Required properties of the enhanced theory

Reproduction of the standard model at low scales:

- $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetry
- V-A interaction

Extension of the standard model

Assumption

- There exists an **unknown**, parent theory at the scale Λ.
- The standard model is an effective theory of this superior theory.

Required properties of the enhanced theory

Reproduction of the standard model at low scales:

- $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetry
- V-A interaction

Extension of the standard model

Expansion of the parent theory

$$\mathcal{L} = \mathcal{L}_{4\text{D}} + \frac{1}{\Lambda}\mathcal{L}_{5\text{D}} + \frac{1}{\Lambda^2}\mathcal{L}_{6\text{D}} + \dots \quad (1)$$

- $\mathcal{L}_{4\text{D}} = \mathcal{L}_{SM}$: lagrangian of the standard model
- $\mathcal{L}_{5D}$, $\mathcal{L}_{6D}$: lagrangian with dimension 5 and 6
- Λ: scale parameter of new physics

Contruction

- Expansion of the parent theory in standard model fields
  $\Rightarrow$ minimal expansion (no SUSY etc.)
- Construction of higher dimensional operators by hand.

Extension of the standard model

Expansion of the parent theory

$$\mathcal{L} = \mathcal{L}_{4\mathrm{D}} + \frac{1}{\Lambda}\mathcal{L}_{5\mathrm{D}} + \frac{1}{\Lambda^2}\mathcal{L}_{6\mathrm{D}} + \dots \quad (1)$$

- $\mathcal{L}_{4\mathrm{D}} = \mathcal{L}_{SM}$: lagrangian of the standard model
- $\mathcal{L}_{5D}$, $\mathcal{L}_{6D}$: lagrangian with dimension 5 and 6
- Λ: scale parameter of new physics

Contruction

- Expansion of the parent theory in standard model fields
  $\Rightarrow$ minimal expansion (no SUSY etc.)
- Construction of higher dimensional operators by hand.

Constituents and transition behavior

Quark fields

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \quad \begin{pmatrix} t_L \\ b_L \end{pmatrix} \tag{2}$$

$$q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}, \quad \begin{pmatrix} c_R \\ s_R \end{pmatrix}, \quad \begin{pmatrix} t_R \\ b_R \end{pmatrix} \tag{3}$$

Higgs field

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_0 - i\chi_0 & \sqrt{2}\phi_+ \\ -\sqrt{2}\phi_- & \phi_0 + i\chi_0 \end{pmatrix} \tag{4}$$

$SU(2)_L \otimes SU(2)_R$ symmetry

Construction of the new operators

Limitations

- $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetry
  $\Rightarrow$ no dimension 5 operators possible
- Neglect of all operators $\mathcal{O}(1/\Lambda^3)$

Structure of the lagrangian

$$L = L_{\text{SM}} + \frac{1}{\Lambda^2}\sum_i (O_{\text{LL}}^{(i)} + O_{\text{LR}}^{(i)} + O_{\text{RR}}^{(i)}) \tag{5}$$

- Interaction of left and right handed particles
- New operators are of dimension 6

List of independent operators

LL operators

$$O_{LL}^{(1)} = \bar{Q}_A \not{L} G_{AB}^{(1)} Q_B$$
$$O_{LL}^{(2)} = \bar{Q}_A \not{L}_3 G_{AB}^{(2)} Q_B$$

$$L^\mu = H \left(iD^\mu H\right)^\dagger + \left(iD^\mu H\right) H^\dagger$$
$$L_3^\mu = H \tau_3 \left(iD^\mu H\right)^\dagger + \left(iD^\mu H\right) \tau_3 H^\dagger$$

RR operators

$$O_{RR}^{(1)} = \bar{q}_A \not{R} F_{AB}^{(1)} q_B$$
$$O_{RR}^{(2)} = \bar{q}_A \left\{\tau_3, \not{R}\right\} F_{AB}^{(2)} q_B$$
$$O_{RR}^{(3)} = i\bar{q}_A \left[\tau_3, \not{R}\right] F_{AB}^{(3)} q_B$$
$$O_{RR}^{(4)} = \bar{q}_A \tau_3 \not{R} \tau_3 F_{AB}^{(4)} q_B$$

$$R^\mu = H^\dagger \left(iD^\mu H\right) + \left(iD^\mu H\right)^\dagger H$$

LR operators

$$O_{LR}^{(1)} = \bar{Q}_A \, H H^\dagger H \widehat{K}_{AB}^{(1)} q_B + \text{h.c.}$$
$$O_{LR}^{(2)} = \bar{Q}_A \left(\sigma_{\mu\nu} B^{\mu\nu}\right) H \widehat{K}_{AB}^{(2)} q_B + \text{h.c.}$$
$$O_{LR}^{(3)} = \bar{Q}_A \left(\sigma_{\mu\nu} W^{\mu\nu}\right) H \widehat{K}_{AB}^{(3)} q_B + \text{h.c.}$$
$$O_{LR}^{(4)} = \bar{Q}_A \left(iD_\mu H\right) iD^\mu \widehat{K}_{AB}^{(4)} q_B + \text{h.c.}$$

$$\widehat{K}_{AB}^{(i)} = K_{AB}^{(i)} + \tau_3 K_{AB}^{(i)\prime}$$

The terms with τ_3 take care about the explicit breaking of the $SU(2)_L \otimes SU(2)_R$ symmetry down to the observed $SU(2)_L \otimes U(1)_Y$ symmetry.

Further proceeding

Further proceeding

- Perform the spontaneous symmetry breaking
- Application to the decay $\bar{B} \longrightarrow X_c\, e^-\, \bar{\nu}_e$
  $\Rightarrow$ many of the operators drop out
- Integrating out the W boson $\Rightarrow$ Fermi coupling
- Using the unaltered lefthanded leptonic current assuming massless leptons

Effective hamiltonian

$$\mathcal{H}_{\text{eff}} = \frac{4G_{\text{F}}V_{cb}}{\sqrt{2}} J_{q,\mu} J_l^{\mu} \qquad J_l^{\mu} = \bar{e}\, \gamma^{\mu} P_- \, \nu_e \tag{6}$$

Ansatz for the quark current structure

Ansatz

The most general form of the current contains all possible dirac structures:

$$\begin{aligned} J_{h,\mu} &= c_L \bar{c}\,\gamma_\mu P_L b + c_R\,\bar{c}\,\gamma_\mu P_R b \\ &+ g_L \bar{c}\,\frac{iD_\mu}{m_b} P_L b + g_R\,\bar{c}\,\frac{iD_\mu}{m_b} P_R b \\ &+ d_L \frac{i\partial^\nu}{m_b}(\bar{c}\,i\sigma_{\mu\nu} P_L b) + d_R \frac{i\partial^\nu}{m_b}(\bar{c}\,i\sigma_{\mu\nu} P_R b), \end{aligned} \tag{7}$$

$$\text{with} \quad P_L = \left(\frac{1-\gamma^5}{2}\right) \quad \text{and} \quad P_R = \left(\frac{1+\gamma^5}{2}\right).$$

standard model

In the standard model:

$$J_{q,\mu} = \bar{c}\,\gamma_\mu \left(\frac{1-\gamma^5}{2}\right) b, = \bar{c}\,\gamma_\mu P_L\, b$$

so $c_L = 1$ and $c_R = g_L = g_R = d_L = d_R = 0$.

Ansatz for the quark current structure

Ansatz

The most general form of the current contains all possible dirac structures:

$$\begin{aligned} J_{h,\mu} = {} & c_L \bar{c}\,\gamma_\mu P_L b + c_R\,\bar{c}\,\gamma_\mu P_R b \\ & + g_L \bar{c}\,\frac{iD_\mu}{m_b} P_L b + g_R\,\bar{c}\,\frac{iD_\mu}{m_b} P_R b \\ & + d_L \frac{i\partial^\nu}{m_b}(\bar{c}\, i\sigma_{\mu\nu} P_L b) + d_R \frac{i\partial^\nu}{m_b}(\bar{c}\, i\sigma_{\mu\nu} P_R b), \end{aligned} \tag{7}$$

$$\text{with} \quad P_L = \left(\frac{1-\gamma^5}{2}\right) \quad \text{and} \quad P_R = \left(\frac{1+\gamma^5}{2}\right).$$

order of magnitude of the parameters

$$\begin{aligned} c_L &\propto 1; & c_R &\propto \frac{v^2}{\Lambda^2}; \\ d_{R/L} &\propto \frac{v\, m_b}{\Lambda^2}; & g_{R/L} &\propto \frac{v\, m_b}{\Lambda^2}; \end{aligned} \tag{8}$$

Outline

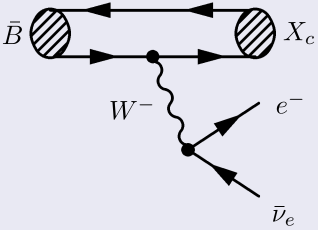
1 Introduction
- Testing the standard model
- Enhancement via higher dimensional operators
- New Ansatz

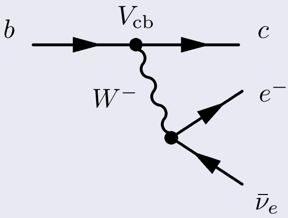
2 The lepton-energy spectrum at Tree-level

3 Radiative corrections
- Real corrections
- Virtual corrections

Calculation of the lepton-energy spectrum

Meson decay and parton-level





Coefficient decomposition of the lepton-energy spectrum

The electron's energy spectrum as a sum of single contributions:

$$\frac{d\Gamma}{dy} = \sum_i i \, \frac{d\Gamma^{(i)}}{dy} \qquad \text{mit} \qquad y = \frac{2E_l}{m_b} \qquad \text{und} \tag{9}$$

$$i = c_L c_L,\, c_L c_R,\, c_L d_L,\, c_L d_R,\, c_L g_L,\, c_L g_R$$

Results at tree-level

$c_L c_L$ contribution

This contribution contains the standard model lepton-energy spectrum:

$$\frac{\mathrm{d}\Gamma^{c_L c_L}}{\mathrm{d}y} = \frac{G_\mathrm{F}^2 |V_{cb}|^2 m_b^5}{192\pi^3}\left[2y^2(3-2y) - 6y^2\rho - \frac{6y^2\rho^2}{(1-y)^2} + \frac{2y^2(3-y)\rho^3}{(1-y)^3}\right]$$

New contributions

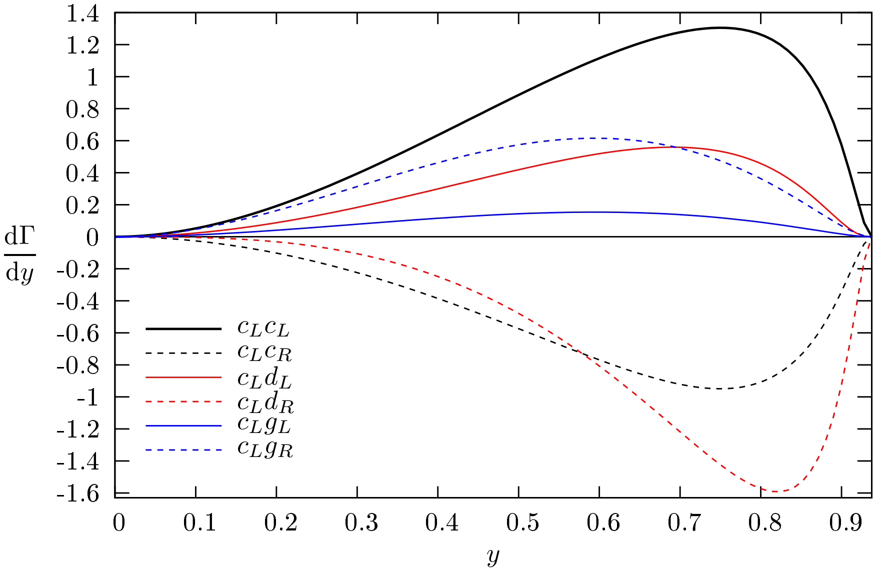
$$\frac{\mathrm{d}\Gamma^{c_L c_R}}{\mathrm{d}y} = -\frac{G_\mathrm{F}^2 |V_{cb}|^2 m_b^5}{192\,\pi^3}\sqrt{\rho}\left[12y^2 - \frac{24y^2\rho}{1-y} + \frac{12y^2\rho^2}{(1-y)^2}\right].$$

$$\frac{\mathrm{d}\Gamma^{c_L d_R}}{\mathrm{d}y} = -\frac{G_\mathrm{F}^2 |V_{cb}|^2 m_b^5}{192\pi^3}\left[4y^3 - \frac{12y^3\rho^2}{(1-y)^2} + \frac{8y^3\rho^3}{(1-y)^3}\right]$$

$$\frac{\mathrm{d}\Gamma^{c_L d_L}}{\mathrm{d}y} = -\frac{G_\mathrm{F}^2 |V_{cb}|^2 m_b^5}{192\pi^3}\sqrt{\rho}\left[\frac{4y^2(3-y)(y-1+\rho)^3}{(1-y)^3}\right]$$

$$\frac{\mathrm{d}\Gamma^{c_L g_L}}{\mathrm{d}y} = \frac{G_\mathrm{F}^2 |V_{cb}|^2 m_b^5}{192\pi^3}\sqrt{\rho}\left[\frac{6y^2(y-1+\rho)^2}{1-y}\right] = \sqrt{\rho}\,\frac{\mathrm{d}\Gamma^{c_L g_R}}{\mathrm{d}y}$$

LEPTON-ENERGY SPECTRUM

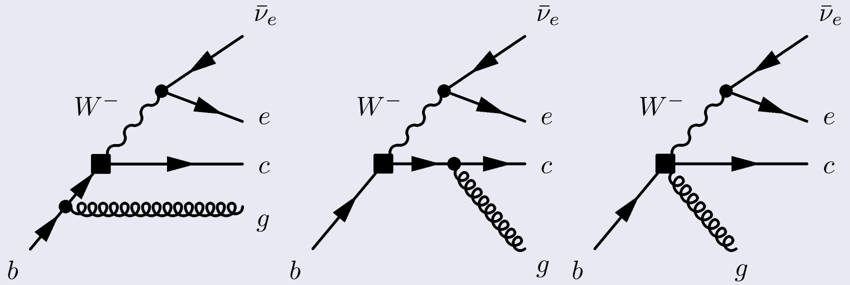


Outline

1. Introduction
   - Testing the standard model
   - Enhancement via higher dimensional operators
   - New Ansatz
2. The lepton-energy spectrum at Tree-level
3. Radiative corrections
   - Real corrections
   - Virtual corrections

Real corrections

Feynman diagrams



Quark-quark-gluon-boson vertex in the scalar contribution

The covariant derivative $D_\mu = \partial_\mu + i g_3 A^a_\mu \lambda_a / 2$ in the scalar contribution to the current

$$g_L \bar{c} \frac{i D_\mu}{m_b} P_- b + g_R \bar{c} \frac{i D_\mu}{m_b} P_+ b,$$

generates a Quark-quark-gluon-boson vertex (diagram on the right-hand side)

Calculation of the moments

Leptonic moments

$$L_n = \frac{1}{\Gamma_0}\int_{E_{\rm cut}} dE_\ell\, E_\ell^n\, \frac{\mathrm{d}\Gamma}{\mathrm{d}E_\ell} \quad \text{with} \quad \Gamma_0 = \frac{G_{\rm F}^2 |V_{cb}|^2 m_b^5}{192\pi^3}\left[1-8\rho-12\rho^2\ln\rho+8\rho^3-\rho^4\right]$$

Hadronic moments

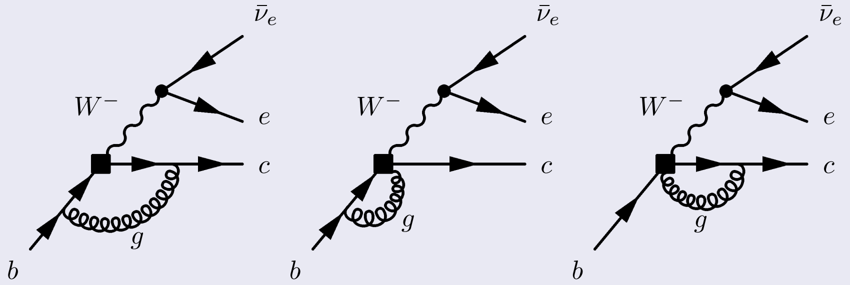
$$H_{ij} = \frac{1}{\Gamma_0}\int_{E_{\rm cut}} \mathrm{d}E_\ell \int \mathrm{d}E_{\rm had}\,\mathrm{d}M_{\rm had}^2 (M_{\rm had}^2 - m_c^2)^i\, E_{\rm had}^j\, \frac{\mathrm{d}^3\Gamma}{\mathrm{d}E_\ell\,\mathrm{d}E_{\rm had}\mathrm{d}M_{\rm had}^2}$$

α_s/π coefficients of the hadronic moments

i	j	c_l^2	$c_l c_r$	$c_l d_l$	$c_l d_r$	$c_l g_l$	$c_l g_r$
1	0	0.09009	−0.03629	0.01697	−0.05009	0.01286	0.06051
1	1	0.04700	−0.01782	0.00789	−0.02426	0.00679	0.03264
1	2	0.02509	−0.00903	0.00377	−0.01205	0.00364	0.01794
2	0	0.00911	−0.00330	0.00117	−0.00418	0.00121	0.00660
2	1	0.00534	−0.00188	0.00062	−0.00229	0.00071	0.00396
3	0	0.00181	−0.00063	0.00018	−0.00070	0.00023	0.00138

Virtual corrections

Feynman diagrams



Special properties

- Quark-quark-gluon-boson vertex in the scalar part of the current generates two new diagrams (right).
- The results with virtual corrections are only calculated for $c_L c_L$ and $c_L c_R$, because the others have an anomalous dimension

Virtual corrections

α_s/π coefficients of the leptonic moments (left) and hadronic moments for $i = 0$ (right)

n	c_l^2	$c_l c_r$
0	−1.778	2.198
1	−0.551	0.666
2	−0.188	0.222
3	−0.068	0.079

i	j	c_l^2	$c_l c_r$
0	0	−1.778	2.198
0	1	−0.719	0.867
0	2	−0.292	0.349
0	3	−0.118	0.143

Special properties

- Quark-quark-gluon-boson vertex in the scalar part of the current generates two new diagrams (right).
- The results with virtual corrections are only calculated for $c_L c_L$ and $c_L c_R$, because the others have an anomalous dimension

Summary and outlook

Conclusion

- The effects of the new couplings on the moments can be sizable.
- Example for a parent theory: Multi-Higgs model with charged Higgs
- The calculation of the non-perturbative corrections is straight forward, but the perturbative QCD-effects give the main contributions.

Outlook

- Calculation of the anomalous dimensions is almost finished and to be published soon
- Combined fit of all parameters, including quark masses and HQE-parameters