Integrating out strange quarks in ChPT

Jürg Gasser[a], Christoph Haefeli[b], Mikhail A. Ivanov[c], Martin Schmid[a]

[a]University of Bern, [b]University of Valencia, IFIC, [c]JINR Dubna

EuroFlavour 07, 14.– 16. November 2007

Contents

Introduction

PDG06 [$\overline{\mathrm{MS}}$-scheme at $\mu = 2\mathrm{GeV}$]:

$$m_u = 1.5 - 3.0\,\mathrm{MeV}\,, \quad m_d = 3 - 7\,\mathrm{MeV}\,, \quad m_s = 95 \pm 25\,\mathrm{MeV}$$

- ChPT exploits systematically quark mass dependence at low–energies
- Two options for strange quark
  - Treat $m_s \bar{s}s$ as perturbation — 3 flavour ChPT
  - Treat m_s on same footing as heavy quarks — 2 flavour ChPT

Gasser, Leutwyler (84),(85)

Introduction

PDG06 [$\overline{\mathrm{MS}}$-scheme at $\mu = 2\mathrm{GeV}$]:

$$m_u = 1.5 - 3.0\,\mathrm{MeV}\,, \quad m_d = 3 - 7\,\mathrm{MeV}\,, \quad m_s = 95 \pm 25\,\mathrm{MeV}$$

- ChPT exploits systematically quark mass dependence at low–energies
- Two options for strange quark
  - Treat $m_s \bar{s}s$ as perturbation — 3 flavour ChPT
  - Treat m_s on same footing as heavy quarks — 2 flavour ChPT

Gasser, Leutwyler (84),(85)

- The degrees of K and η freeze for

$$|p^2| \ll M_K^2\,, \qquad m_u, m_d \ll m_s$$

- In this limit: relations among the 2 flavour vs. the 3 flavour low–energy constants of the effective Lagrangians.

- These relations give additional information on the values of the low–energy constants. E.g.

$$\underbrace{\ell_6^r(\mu)}_{(\star)} = -2\,\underbrace{L_9^r(\mu)}_{(\star\star)} + \frac{1}{192\pi^2}(\ln B_0 m_s/\mu^2 + 1)$$

($\star$) 2 flavour low–energy constant

($\star\star$) 3 flavour low–energy constant

Two one-loop derivations as illustrations:

i) Pion decay constant

ii) Vector formfactor

I) The pion decay constant

At one-loop:

2 *flavours* : $$F_\pi = F\left(1 + \frac{1}{F^2}\left[-2\mu_\pi + 2B\hat{m}\ell_4^r\right] + \mathcal{O}(\hat{m}^2)\right)$$

3 *flavours* : $$F_\pi = F_0\left(1 + \frac{1}{F_0^2}\left[-2\mu_\pi - \mu_K + 8\hat{m}B_0L_5^r + 8(2\hat{m} + m_s)B_0L_4^r\right]\right)$$

$$\hat{m} = \tfrac{1}{2}(m_u + m_d)\ , \qquad \mu_P = \tfrac{1}{32\pi^2}M_P^2\ln(M_P^2/\mu^2)$$

I) The pion decay constant

At one-loop:

2 *flavours* : $F_\pi = F\left(1 + \frac{1}{F^2}\left[-2\mu_\pi + 2B\hat{m}\ell_4^r\right] + \mathcal{O}(\hat{m}^2)\right)$

3 *flavours* : $F_\pi = F_0\left(1 + \frac{1}{F_0^2}\left[-2\mu_\pi - \mu_K + 8\hat{m}B_0L_5^r + 8(2\hat{m} + m_s)B_0L_4^r\right]\right)$

For $\hat{m} = 0$,

$$F = F_0\left(1 + \frac{B_0 m_s}{16\pi^2 F_0^2}\left[128\pi^2 L_4^r(\mu) - \frac{1}{2}\ln B_0 m_s/\mu^2\right]\right) + \mathcal{O}(m_s^2)$$

Gasser, Leutwyler (85)

$\hat{m} = \frac{1}{2}(m_u + m_d)$, $\mu_P = \frac{1}{32\pi^2}M_P^2\ln(M_P^2/\mu^2)$

I) The pion decay constant

$$F = F_0 \left(1 + \frac{B_0 m_s}{16\pi^2 F_0^2} \left[128\pi^2 L_4^r(\mu) - \tfrac{1}{2} \ln B_0 m_s/\mu^2 \right] \right) + \mathcal{O}(m_s^2)$$

i) For physical m_s, $\frac{B_0 m_s}{16\pi^2 F_0^2} \approx 0.20$

I) The pion decay constant

$$F = F_0 \left(1 + \frac{B_0 m_s}{16\pi^2 F_0^2}\left[128\pi^2 L_4^r(\mu) - \tfrac{1}{2}\ln B_0 m_s/\mu^2\right]\right) + \mathcal{O}(m_s^2)$$

i) For physical m_s, $\frac{B_0 m_s}{16\pi^2 F_0^2} \approx 0.20$

ii) Different patterns of chiral symmetry breaking

- Pattern I: $L_4^r \approx 0$

Amoros, Bijnens, Talavera (01,main fit)

- Pattern II: ($L_4^r \neq 0$) assumes large $\bar{s}s$ corrections that need to be summed up

Descotes, Girlanda, Stern (00), Descotes (07), talk Kolesar

Here: Do not investigate, which pattern is favoured by nature
The focus is set on the derivation of the matching relations

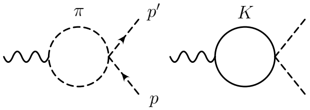
II) The vector formfactor $F_V(t)$

$$\langle \pi^+(p') \,|\, \tfrac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) | \pi^+(p)\rangle = (p+p')_\mu F_V(t) \;;\; t = (p'-p)^2 \,,$$

For $m_u = m_d = 0$:

2 flavours : $$F_{V,2}(t) = 1 + \frac{t}{F^2}\Phi(t,0;d) - \frac{\ell_6 t}{F^2}$$

3 flavours : $$F_{V,3}(t) = 1 + \frac{t}{F_0^2}\left[\Phi(t,0;d) + \tfrac{1}{2}\Phi(t,M_K;d)\right] + \frac{2L_9 t}{F_0^2}$$



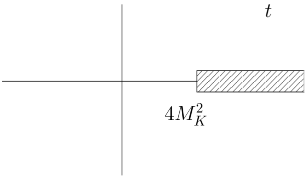
II) The vector formfactor $F_V(t)$

$$\langle \pi^+(p') \,|\, \tfrac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)|\pi^+(p)\rangle = (p+p')_\mu F_V(t) \;;\; t = (p'-p)^2 \,,$$

For $m_u = m_d = 0$:

2 flavours : $$F_{V,2}(t) = 1 + \frac{t}{F^2}\Phi(t,0;d) - \frac{\ell_6 t}{F^2}$$

3 flavours : $$F_{V,3}(t) = 1 + \frac{t}{F_0^2}\left[\Phi(t,0;d) + \tfrac{1}{2}\Phi(t,M_K;d)\right] + \frac{2L_9 t}{F_0^2}$$



$$\Phi(t,M_K;d) = \sum_{n=0}^{\infty} \Phi_n(M_K,d)\left(\frac{t}{M_K^2}\right)^n$$

II) The vector formfactor $F_V(t)$

$$\langle \pi^+(p') \,|\, \tfrac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)|\pi^+(p)\rangle = (p+p')_\mu F_V(t) \;;\; t = (p'-p)^2 \,,$$

For $m_u = m_d = 0$:

2 flavours : $$F_{V,2}(t) = 1 + \frac{t}{F^2}\Phi(t,0;d) - \frac{\ell_6 t}{F^2}$$

3 flavours : $$F_{V,3}(t) = 1 + \frac{t}{F_0^2}\left[\Phi(t,0;d) + \tfrac{1}{2}\Phi(t,M_K;d)\right] + \frac{2L_9 t}{F_0^2}$$

$F_{V,3}(t)$ reduces to $F_{V,2}(t)$, provided

$$-\ell_6 = 2L_9 + \frac{1}{2}\Phi_0(M_K,d)$$

At $d=4$,

$$\ell_6^r(\mu) = -2L_9^r(\mu) + \frac{1}{192\pi^2}(\ln B_0 m_s/\mu^2 + 1)$$

Gasser, Leutwyler (85)

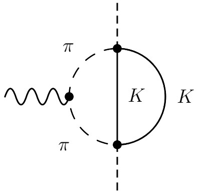
Matching at two loops

What about a matching at two–loop order? Some remarks:

- For ℓ_6 one can extract its strange quark mass dependence at two–loops from the literature

2 flavours:	$F_{V,2}(t)$,	Gasser, Leutwyler (84)
3 flavours:	$F_{V,3}(t)$	Bijnens, Talavera (02)

- Despite literature, still an exhaustive work, because two–loop diagrams need to be known analytically in an expansion in $t/B_0 m_s$
  [up to logarithms $\ln(-t/B_0 m_s)$]



Matching at two loops

Relations at two–loops:

chiral order	LECs	
p^2	F^2B	Moussallam (00)
p^2	B	Kaiser, Schweizer (06)
p^2, p^4	$F, B, \ell_1, \ldots, \ell_{10}$	Gasser, C.H., Ivanov, Schmid (07)
p^6	$c_1, \ldots, c_{56}$	Gasser *et al.*, work in progress

- One also likes to know $c_i(m_s)$; $i = 1, \ldots, 56$
- Analogous procedure would be possible, however we favour a more general approach. . .
- . . . which shall be introduced now

  [at one–loop level only]

. . . in the footsteps of Nyffeler and Schenk (95)

Generating functional

- Euclidean generating functional of all Green's functions with v, a, s, p sources

3 flavours
$$e^{-Z[v,a,s,p]} = \langle 0_{\rm out} | 0_{\rm in} \rangle_{v,a,s,p} = \mathcal{N} \int [\mathrm{d}u]\, e^{-S_{\rm eff}^{(3)}}$$

- Low–energy expansion

$$Z = \bar{S}_{\rm eff}^{(3)} + \frac{1}{2} \ln \frac{\det D}{\det D^0} + \mathcal{O}(p^6)$$

- differential operator D is associated to propagator G

$$D(x) G(x,y) = \delta(x-y)$$

Generating functional

- Euclidean generating functional of all Green's functions with v, a, s, p sources

3 flavours
$$e^{-Z[v,a,s,p]} = \langle 0_{\rm out} | 0_{\rm in} \rangle_{v,a,s,p} = \mathcal{N} \int [\mathrm{d}u] \, e^{-S^{(3)}_{\rm eff}}$$

- Low–energy expansion

$$Z = \bar{S}^{(3)}_{\rm eff} + \tfrac{1}{2} \ln \frac{\det D}{\det D^0} + \mathcal{O}(p^6)$$

- For two flavours

$$z = \bar{s}^{(2)}_{\rm eff} + \tfrac{1}{2} \ln \frac{\det d}{\det d^0} + \mathcal{O}(p^6)$$

- Consider framework where Z reduces to z
  - same external two–flavour sources [no sources with strangeness]
  - external momenta $|p^2| \ll B_0 m_s$

Generating functional

- In this limit

$$\bar{S}_{\text{eff}}^{(3)} + \frac{1}{2}\ln\frac{\det D}{\det D^0} = \bar{s}_{\text{eff}}^{(2)} + \frac{1}{2}\ln\frac{\det d}{\det d^0}$$

Determinant:

- Separation of heavy and light fields

$$\ln\det D = \ln\det d + \ln\det D_\eta + \underbrace{\ln\det D_K}_{(1)} + \underbrace{\ln\det(1 - D_\pi^{-1} D_{\pi\eta} D_\eta^{-1} D_{\eta\pi})}_{(2)}$$

Generating functional

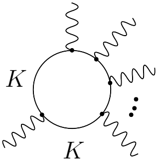
- In this limit

$$\bar{S}_{\text{eff}}^{(3)} + \frac{1}{2} \ln \frac{\det D}{\det D^0} = \bar{s}_{\text{eff}}^{(2)} + \frac{1}{2} \ln \frac{\det d}{\det d^0}$$

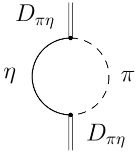
Determinant:

- Separation of heavy and light fields

$$\ln \det D = \ln \det d + \ln \det D_\eta + \underbrace{\ln \det D_K}_{(1)} + \underbrace{\ln \det(1 - D_\pi^{-1} D_{\pi\eta} D_\eta^{-1} D_{\eta\pi})}_{(2)}$$



(1)



(2)

- (1) Short distance expansion with heat–kernel
  → manifestly covariant throughout all steps
- (2) $\pi - \eta$ mixing

[gives no headaches at this order]

Generating functional

$$\bar{S}_{\rm eff}^{(3)} + \frac{1}{2}\ln\frac{\det D}{\det D^0} = \bar{s}_{\rm eff}^{(2)} + \frac{1}{2}\ln\frac{\det d}{\det d^0}$$

- Eg. for ℓ_6:

$$\Big(-2L_9 \underbrace{-\frac{1}{12}\int\frac{\mathrm{d}q}{(2\pi)^d}\frac{1}{[M_K^2+q^2]^2}}_{\text{from } detD_K}\Big)\int \mathrm{d}x\langle f_{+\mu\nu}[u_\mu,u_\nu]\rangle = \ell_6\int \mathrm{d}x\,\underbrace{\langle f_{+\mu\nu}[u_\mu,u_\nu]\rangle}_{\text{chiral operator}}$$

- From which one verifies again

$$-2L_9^r(\mu) + \frac{1}{192\pi^2}(\ln B_0 m_s/\mu^2 + 1) = \ell_6^r(\mu)$$

Universality of approach with generating functional

- m_s dependence of all two–flavour LECs ℓ_i in one go

Gasser, C.H., Ivanov, Schmid (07)

- adaptive to $\mathcal{O}(p^6)$ LECs $c_i(m_s)$

Gasser, C.H., Ivanov, Schmid, work in progress

- adaptive to ChPT including virtual photons

C.H., Ivanov, Schmid (07)

Summary/Outlook

- The degrees of K and η freeze for

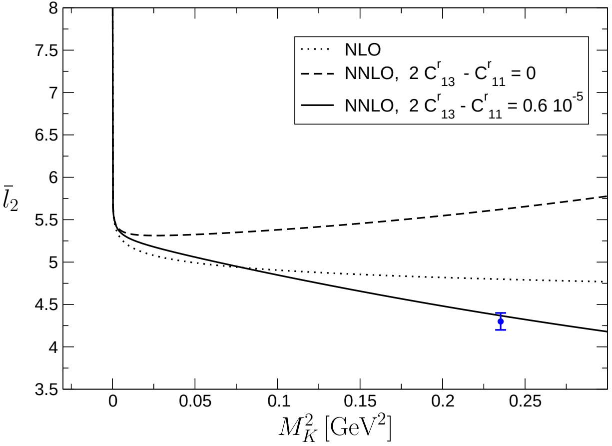
$$|p^2| \ll M_K^2\,, \qquad m_u, m_d \ll m_s$$

- In this limit, one can establish relations among the 2 flavour vs. the 3 flavour low energy constants

Results of matching at two–loops

chiral order	LECs	
p^2	$F^2 B$	Moussallam (00)
p^2	B	Kaiser, Schweizer (06)
p^2, p^4	$F, B, \ell_1, \ldots, \ell_{10}$	Gasser, C.H., Ivanov, Schmid (07)
p^6	$c_1, \ldots, c_{56}$	Gasser *et al.*, work in progress

Backup


NLO
NNLO, 2 C^r_13 - C^r_11 = 0
NNLO, 2 C^r_13 - C^r_11 = 0.6 10^-5
$\bar{l}_2$
M_K^2 [GeV^2]
8
7.5
7
6.5
6
5.5
5
4.5
4
3.5
0
0.05
0.1
0.15
0.2
0.25