D Decays on the lattice

Benjamin Haas

in collaboration with

Damir Becirevic (LPT - Orsay, France), Federico Mescia (LNF, Roma, Italy)

LPT, Universite ParisXI/CNRS

November 15, 2007

Why *D*-decays?

Large CP VIOLATION was expected in K- and B- physics
$\rightarrow$ community focused on K- and B- experiments
$\rightarrow$ But since no serious discrepancy was observed wrt the S.M.

$\downarrow$

BACK TO D-DECAYS

What about CKM matrix elements in the charm sector?

1. So far the CKM unitarity assumed: $|V_{cd}| = |V_{us}|$ and $|V_{cs}| = |V_{ud}|$
2. CKM entries $|V_{cs}|$ and $|V_{cd}|$ must be determined by confronting theory vs. experiment

REVIVAL OF THE CHARM PHYSICS

Charm physics and recent experiment

1. **Charm spectroscopy**: New states observed, including the suprinsingly narrow “strange states”
   - 0^+ state : $D_0^*(2400)$, $\Gamma = 261 \pm 50\mathrm{MeV}$; $\mathrm{D}_{s0}^*(2317)$, $\Gamma < 4.6$ MeV
   - 1^+ state : $D_1(2420)$, $\Gamma = 20.4 \pm 1.7\mathrm{MeV}$; $D_{s1}(2460)$, $\Gamma < 5.5$ MeV
2. **$\mathbf{D} - \bar{\mathbf{D}}$** mixing
3. **BaBar and BELLE** provide high statistic of charm events $\rightarrow$ good for studying D-decays (less good systematics)
4. **Charm factories**: Cleo-c working at the $\psi(3770)$ resonnance; BESS-III starts operating in 2008 (good systematics)

$\downarrow$

Better theoretical estimates needed

Lattice QCD and charm physics

Analytic methods hard to control:

1. m_c not large enough for the Heavy Quark Expansion
2. But still $m_c \gg m_{uds}$ (no LE effective theory)

$$\downarrow$$

Lattice QCD (LQCD)

2 scale problem: π/a vs. $m_c \approx 1$ GeV

1. Relativistic charm quark: beware of discretization errors
2. Use of an effective action: renormalization (??)

Leptonic decays

1. Most direct way to determine the CKM matrix element :

$$\Gamma(D_q^+ \to \ell^+ \nu_\ell) = |V_{cq}|^2 \frac{G_F^2}{8\pi} f_{D_q^+}{}^2 m_{D_q} m_\ell^2 \left(1 - \frac{m_\ell^2}{m_{D_q^+}^2}\right)$$

$$\langle 0|\bar{c}\gamma_\mu\gamma_5 q|D_q^+(p)\rangle = ip_\mu f_{D_q^+}$$

2. Channels Available :

	D decays	D_s decays
CLEO[1]	$D \to e^+\nu_e$	$D_s \to e^+\nu_e$
	$D \to \mu^+\nu_\mu$	$D_s \to \mu^+\nu_\mu$
		$D_s \to \tau^+\nu_\tau$
BaBar[2]	n.a.	$D_s \to \mu^+\nu_\mu$
BELLE[3]	n.a.	$D_s \to \mu^+\nu_\mu$

[1] Stone(2006), Pedlar(2007)

[2] Aubert(2006)

Decay constant

1. Extraction of the decay constants

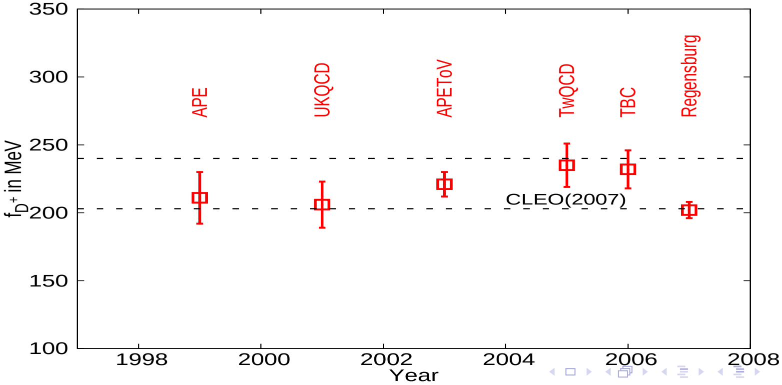
IMPOSING $|V_{us}| = |V_{cd}|$

	f_D(MeV)	f_{D_s}(MeV)
CLEO	$222.6(16.7)^{+2.8}_{-3.4}$	280(12)(6)
BaBar	n.a.	283(14)(4)(14)
BELLE	n.a.	275(16)(12)

2. Computation from Lattice QCD leading to the extraction of $|V_{cq}|$
   - $\rightarrow$ Checking unitarity
   - $\rightarrow$ Extraction of the CP-violating phase

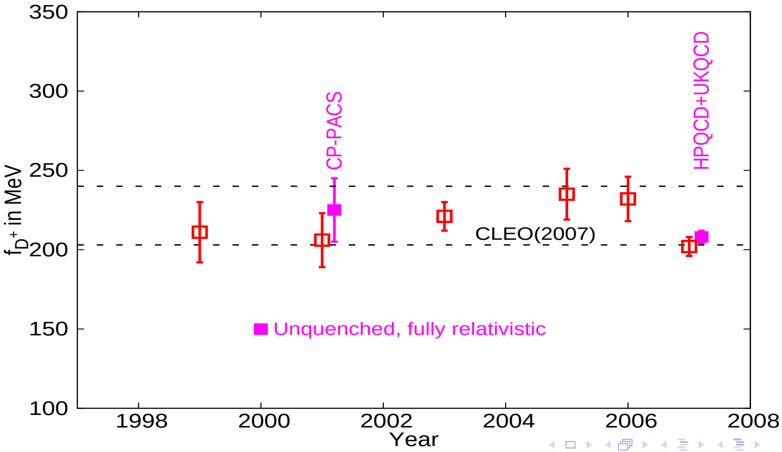
Lattice results : quenched and fully relativistic

Collaboration	Year	f_D in MeV	Action	$a \to 0$
APE	1999	211(18)	Wilson	No
UKQCD	2001	206(17)	Wilson	No
APEToV	2003	221(9)	Wilson	Yes
TwQCD	2005	235(16)	Overlap	No
RBC	2006	232(14)	Domain-Wall	No
Regensburg	2007	202(6)	Wilson	No



Lattice results : unquenched and fully relativistic

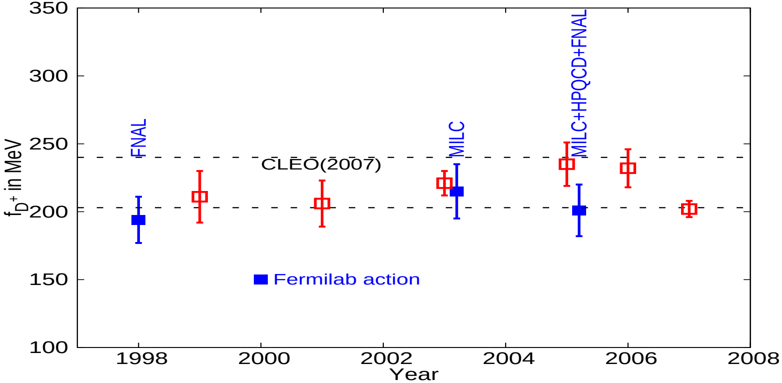
Collaboration	Year	f_D in MeV	Action	$a \longrightarrow 0$	N_F
CP-PACS	2001	225(20)	Wilson	No	2
HPQC+UKQCD	2007	208(4)	Imp. staggered	No	2+1



Lattice results : heavy quark lattice action*

* a la FERMILAB

Collaboration	Year	f_D in MeV	Light-quark action	$a \to 0$	N_F
FNAL	1998	194(17)	Wilson	No	0
MILC	2003	215(17)	Wilson	No	2
MILC+HPQCD+FNAL	2005	201(19)	Staggered	Yes	2+1



Our calculation

Gauge configurations produced by the QCDSF collaboration

1. Wilson Action, $\mathcal{O}(a)$ improved with $N_F = 2$
2. Both light and heavy quarks are FULLY RELATIVISTIC
3. ALPHA showed good scaling of $f_{D_s}^{N_F=0}$ (in 2003)
4. $a^{-1} \simeq 2.5\ \mathrm{GeV}$
5. 3 values of m_π from 770 MeV down to 390 MeV

$\rightarrow$ The light *u* and *d* quark are unquenched (no partial quenching)
$\rightarrow$ No study of D_s - strange quark would be quenched

Computation of f_D and f_π

1. $\langle 0|\bar{c}\gamma_\mu\gamma_5 q|D(p)\rangle = ip_\mu f_D$ is extracted from the fit of two point functions for various $m_q \gg m_{u,d}^{\text{phys.}}$
2. Chiral extrapolation

Linear Fit: Data show a linear behaviour

HMχPT:

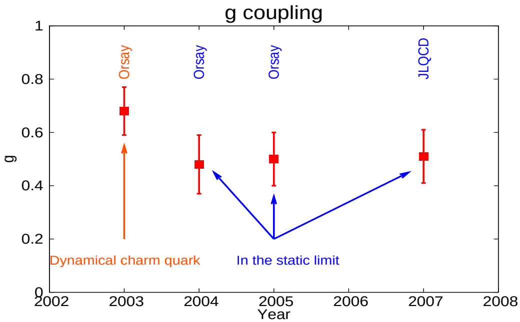
$$\Phi_D \equiv f_D\sqrt{m_D} = \Phi_0\left[1 - \frac{1+3g^2}{(4\pi f_0)^2} m_\pi^2 \log m_\pi^2 + c_\Phi m_\pi^2\right]$$

$$f_\pi = f_0\left[1 - \frac{2}{(4\pi f_0)^2} m_\pi^2 \log m_\pi^2 + c_f m_\pi^2\right]$$

g is the strong coupling $\propto g_{DD^*\pi}$ in the chiral limit

On importance of g

$$\Phi_D = \Phi_0 \left[1 - \frac{1+3g^2}{(4\pi f_0)^2} m_\pi^2 \log m_\pi^2 + c_\Phi m_\pi^2 \right]$$



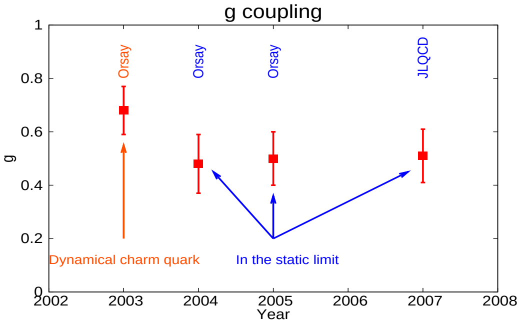
$g \simeq 0.5(m_c \text{ static})$
$g \simeq 0.67(m_c \text{ prop.})$

$$\downarrow$$

$1 + 3g^2 \simeq 1.7$
$1 + 3g^2 \simeq 2.3$

On importance of g

$$\frac{\Phi_D}{f_\pi} = \frac{\Phi_0}{f_0}\left[1 + \frac{1-3g^2}{(4\pi f_0)^2} m_\pi^2 \log m_\pi^2 + c_1 m_\pi^2\right]$$



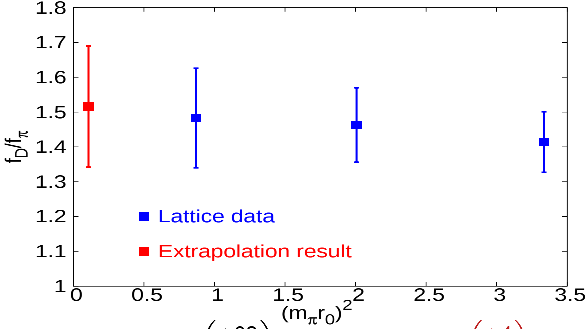
$g \simeq 0.5(m_c \text{ static})$
$g \simeq 0.67(m_c \text{ prop.})$

$\downarrow$

$1 - 3g^2 \simeq 0.2$
$1 - 3g^2 \simeq -0.3$

Our results

Fit used	f_D/f_π
Linear	1.52(17)
HMχPT g=0.5	1.55(17)
HMχPT g=0.68	1.45(17)



$$\Rightarrow f_D/f_\pi = 1.52(17)\begin{pmatrix}+03\\-07\end{pmatrix} \quad f_D = 201(22)\begin{pmatrix}+4\\-9\end{pmatrix}\ \mathrm{MeV}$$

Semileptonic decay $D \to \pi \ell \nu_\ell$

1. Extraction of the CKM matrix element:

$$\frac{d\Gamma}{dq^2}(D \to \pi \ell \nu_\ell) = |V_{cd}|^2 \frac{G_F^2}{192\pi^2 m_D^3} \lambda^{3/2}(q^2) |F_+(q^2)|^2$$

where

$$\langle \pi(\vec{k}) | V_\mu^{qc} | D(\vec{p}) \rangle = (p + k - q\frac{m_D^2 + m_P^2}{q^2})_\mu F_+(q^2) + q_\mu \frac{m_D^2 + m_P^2}{q^2} F_0(q^2)$$

and

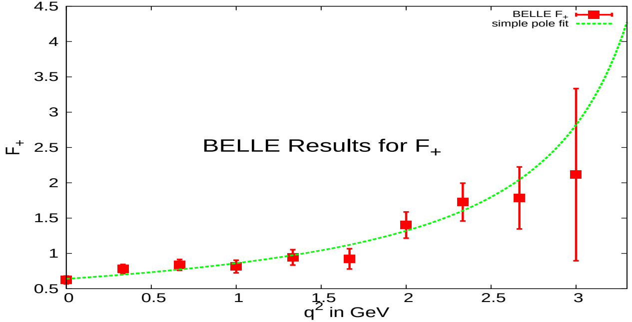
$$F_+ \left(q^2 = 0\right) = F_0 \left(q^2 = 0\right)$$

2. Results reported by **BELLE**, and those from **BaBar** and **CLEO-c** expected soon.

BELLE experimental results

1. Extraction of $F_+(q^2)$

IMPOSING $|V_{us}| = |V_{cd}|$



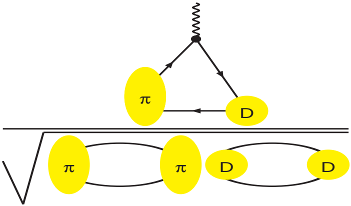
2. Computation from Lattice QCD leading to the extraction of $|V_{cd}|$.

Lattice calculations

Collaboration	Year	Action	N_F	$F_+(0)$
UKQCD	1994	Wilson	0	0.61(12)(11)
APE	2001	Wilson	0	0.57(6)
FNAL	2001	Wilson+Fermilab	0	$0.94\left(^{+10+12}_{-05-10}\right)$
FNAL+MILC+HPQCD	2004	Staggered+Fermilab	2+1	0.64(3)(6)
Regensburg	2007	Wilson	0	n.a.(yet)

USUAL STRATEGY:

$R =$



$$\longrightarrow \langle \pi(\vec{k}) | V_\mu^{dc} | D(\vec{p}) \rangle \times \frac{1}{\sqrt{\mathcal{Z}_D}\sqrt{\mathcal{Z}_\pi}}$$

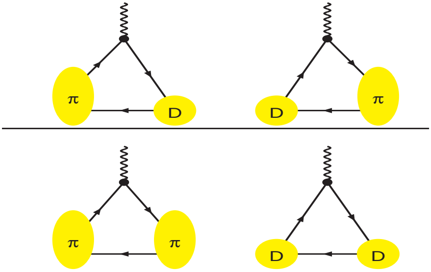
ACCURACY LIMITED BY THE DETERMINATION OF EXTRA QUANTITIES:

1. Renomalization of the vector current
2. Pseudoscalar density factors: $\mathcal{Z}_D$ and $\mathcal{Z}_\pi$ in c

Improved extraction for *D*-decays

Based on double ratios of triangle diagrams

$R_0 =$



$$\longrightarrow \frac{m_D m_\pi}{2\sqrt{m_D m_\pi}} \left(F_0\left(q^2_{\mathrm{max}}\right)\right)^2$$

1

1. AUTOMATIC CANCELLATION OF RENORMALIZATION FACTOR
2. AUTOMATIC CANCELLATION OF PSEUDOSCALAR DENSITY NORMALIZATIONS

↓

Combining various double ratios allows to access F_+ and F_0 see(0710.1741)

Form factors - shape

$\rightarrow$ We want to access q^2's in $(0, (m_D - m_\pi))^2]$ to study the shapes of $F_+(q^2)$ and $F_0(q^2)$

1. Periodic boundary conditions: $\psi(x_i) = \psi(x_i + L_i)$

$$k_i = n_i \frac{2\pi}{L} \geq k_{\min} = \frac{2\pi}{L} \simeq 650 \text{ MeV}$$

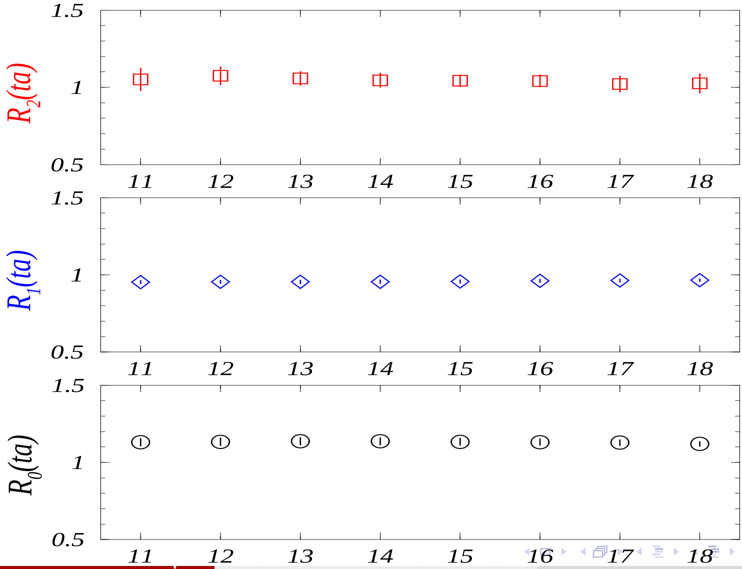
   On usual lattices, $k > k_{\min} \Rightarrow q^2 \lesssim 0$

2. Twisted boundary conditions (TwBC): $\psi(x_i) = \psi(x_i + L_i)e^{i\frac{\theta_i}{L_i}}$
   - Uplift the overall quark momenta by $\frac{\theta_i}{L_i}$:

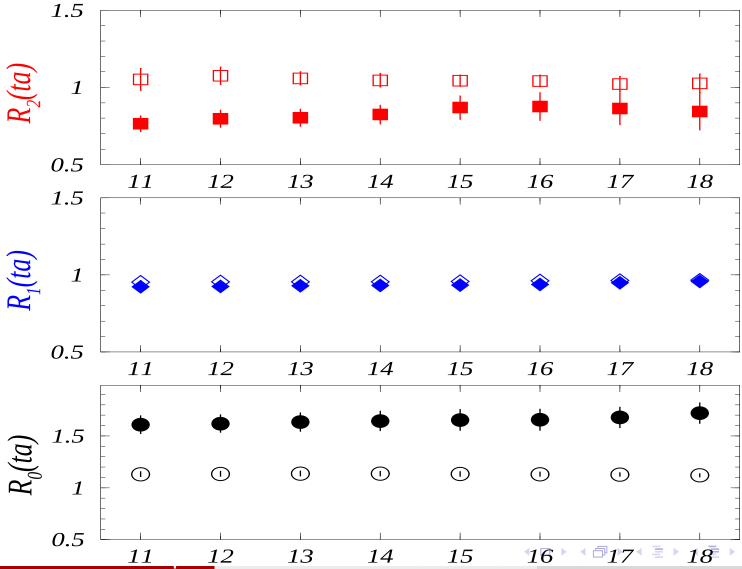
$$k_i = \frac{\theta_i}{L_i}$$

   - No need to redo the Monte Carlo for each θ_i
   - Instead, quark propagators must be re-computed for each θ_i

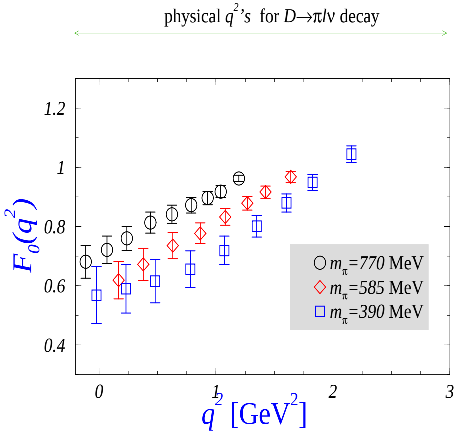
Double ratios at $m_\pi = 770\ \mathrm{MeV}$

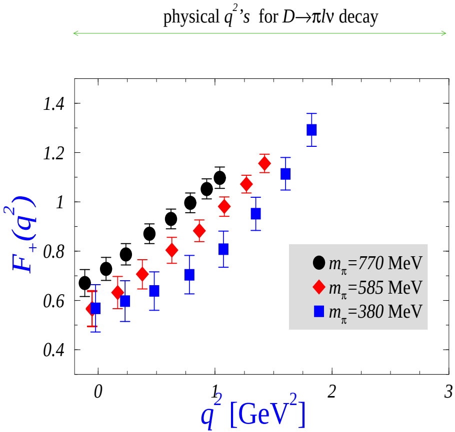


Double ratios at $m_\pi = 770\ \mathrm{MeV}$ and $m_\pi = 390\ \mathrm{MeV}$



Results


physical q^2's for $D \to \pi l \nu$ decay
$F_0(q^2)$
1.2
1
0.8
0.6
0.4
0
1
2
3
q^2 [GeV^2]
m_π=770 MeV
m_π=585 MeV
m_π=390 MeV


physical q^2's for $D \to \pi l \nu$ decay
$F_+(q^2)$
1.4
1.2
1
0.8
0.6
0.4
0
1
2
3
q^2 [GeV^2]
m_π=770 MeV
m_π=585 MeV
m_π=380 MeV

Chiral Extrapolation with $N_F = 2$ *

*Valid in the static limit

$$\frac{F_+(E_\pi)}{\sqrt{m_D}} = \frac{(f_D\sqrt{m_D})_\chi\, g}{2f_0(E_P+\Delta)}\left[1 + \frac{4g^2}{(4\pi f_0)^2}J_1(m_\pi, E_\pi) - \frac{1+3g^2}{(4\pi f_0)^2}\frac{3}{4}m_\pi^2 \log m_\pi^2 + c_+(E_\pi)m_\pi^2\right]$$

$$F_0(E_\pi)\sqrt{m_D} = \frac{(f_D\sqrt{m_D})_\chi}{f_0}\left[1 + \frac{1}{(4\pi f_0)^2}\left(\frac{15-27g^2}{12}m_\pi^2 \log m_\pi^2 + 2I_2(m_\pi, E_\pi)\right) + c_0 m_\pi^2\right]$$

g: is the strong coupling $\propto g_{DD^*\pi}$ in the chiral limit f_D: D decay constant

f_0: π decay constant in the chiral limit

Result of chiral extrapolations

Fit used	$F_+(1\mathrm{GeV}^2)$
Linear	0.75(8)
HMχPT g=0.5	0.88(8)
HMχPT g=0.68	0.96(8)

We absolutly need the value g $\rightarrow$ work in progress

Can g be measured experimentally?

GOLD RATIO to confront LQCD vs. experiment

$$\frac{\frac{d\Gamma}{dq^2}(D \rightarrow \pi\ell\nu_\ell)}{\Gamma(D \rightarrow \ell\nu_\ell)} = \frac{F_+(q^2)}{f_D} \frac{\lambda^{3/2}(q^2)}{24\pi m_D^2 m_\ell^2 \left(1 - \frac{m_\ell^2}{m_D^2}\right)}$$

$$\frac{F_+(1\mathrm{GeV}^2)}{f_D} = 3.76(54) \qquad \mathrm{LinearFit}$$

$$4.32(56) \qquad \mathrm{HM}\chi\mathrm{PT}$$

in c

Conclusion

- There is room for making substantial progress in D-physics
- We show how to improve extraction of the form factors
- From $\mathcal{O}(a)$ improved Wilson action (at $a \approx 0.08\ \mathrm{fm}$) with $N_F = 2$, we obtain $f_D = 200(22)\ \mathrm{MeV}$ and

$$\frac{F_+(1\mathrm{GeV}^2)}{f_D} = \begin{array}{ll} 3.76(54) & \mathrm{LinearFit} \\ 4.32(56) & \mathrm{HM}\chi\mathrm{PT} \end{array}$$

- We plan to study saturation of the form factors by their first poles $(f_{D_0^*}, f_{D^*},\ g_{DD^*\pi}, g_{DD_0^*\pi})$
- Employ effective treatment of c-quark à la Fermilab
  need to address the issue of renormalization
  $\rightarrow$Workshop in Paris in April 2008