Lattice chiral extrapolations in processes of positive and negative parity heavy mesons

Jernej F. Kamenik

INFN, Laboratori Nazionali di Frascati, Italy

Jozef Stefan Institute, Ljubljana, Slovenia

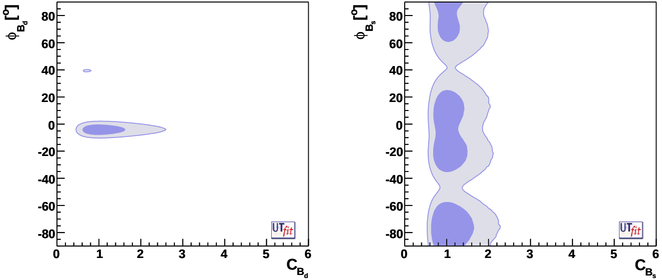
In collaboration with Damir Bećirević, Jan O. Eeg and Svjetlana Fajfer.

EuroFlavour '07
November 15, 2007

1. Motivation
   - Theoretical uncertainties in $B_{s,d} - \overline{B}_{s,d}$ mixing
2. Framework
3. Chiral logarithmic corrections
   - Impact of the $1/2^+$-mesons
4. Conclusions

Example: $B_{s,d} - \overline{B}_{s,d}$ mixing

- Experimental measurement of *large* Δm_{B_d} correctly indicated a very heavy top quark
- Nowadays Δm_{B_d} and Δm_{B_s} are used to constrain the shape of the CKM unitarity triangle and thereby determine the amount of the CP-violation in the SM as well as constrain new physics contributions.



$$C_{B_q} e^{2i\phi_{B_q}} = \frac{\langle B_q|\mathcal{H}_{\rm eff}^{\rm full}|\overline{B}_q\rangle}{\langle B_q|\mathcal{H}_{\rm eff}^{\rm SM}|\overline{B}_q\rangle}$$

UTfit Collaboration (2007) arXiv:0707.0636 [hep-ph]

- The program requires accurate control of theoretical uncertainties, especially related to the hadronic quantities involved

Theoretical framework: OPE

- One evolves the relevant $\Delta B = 2$ operators and Wilson coefficients from the weak scale down to $\nu \sim m_b$.
- "SUSY basis" of operators

$$
\begin{aligned}
O_1 &= \bar{b}^i \gamma_\mu (1-\gamma_5) q^i \, \bar{b}^j \gamma^\mu (1-\gamma_5) q^j \,, \\
O_2 &= \bar{b}^i (1-\gamma_5) q^i \, \bar{b}^j (1-\gamma_5) q^j \,, \\
O_3 &= \bar{b}^i (1-\gamma_5) q^j \, \bar{b}^j (1-\gamma_5) q^i \,, \\
O_4 &= \bar{b}^i (1-\gamma_5) q^i \, \bar{b}^j (1+\gamma_5) q^j \,, \\
O_5 &= \bar{b}^i (1-\gamma_5) q^j \, \bar{b}^j (1+\gamma_5) q^i \,.
\end{aligned}
$$

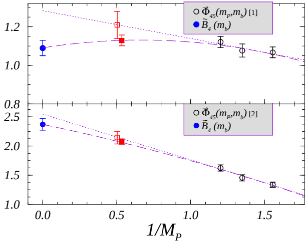
- In SM, only O_1 (left-left) operator is relevant in describing Δm_{B_q}.
- What is needed are matrix elements of O_i between B-meson states.
- Introducing bag-parameters, B_{q1-5} as measures of the difference with respect to the vacuum saturation approximation (VSA)

$$
\frac{\langle \bar{B}^0_q | O_{1-5}(\nu) | B^0_q \rangle}{\langle \bar{B}^0_q | O_{1-5}(\nu) | B^0_q \rangle_{\mathrm{VSA}}} = B_{q1-5}(\nu)\,.
$$

Theoretical uncertainties

$B_{B_{s,d}}$ (and $f_{B_{s,d}}$) can, in principle, be computed on the lattice:

- For m_b one can combine LQCD caluclations for $m_c < m_Q \leq 2m_c$, with those obtained in the static HQET limit

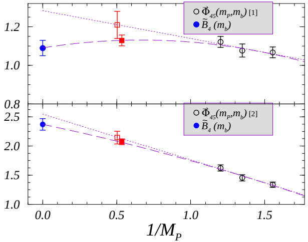


D. Bećirević et al. JHEP 0204:025 (2002) [hep-lat/0110091]

Theoretical uncertainties

$B_{B_{s,d}}$ (and $f_{B_{s,d}}$) can, in principle, be computed on the lattice:

- For m_b one can combine LQCD caluclations for $m_c < m_Q \leq 2m_c$, with those obtained in the static HQET limit
- For the light quarks one can afford $m_s > m_q > m_s/4$, but the physical limit is $m_d \simeq m_s/25$
- Extrapolation of results with larger light quark masses is needed – Induces systematic uncertainties.

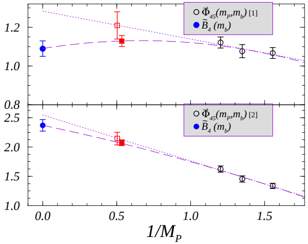


D. Bećirević et al. JHEP 0204:025 (2002) [hep-lat/0110091]

Theoretical uncertainties

$B_{B_{s,d}}$ (and $f_{B_{s,d}}$) can, in principle, be computed on the lattice:

- For m_b one can combine LQCD caluclations for $m_c < m_Q \leq 2m_c$, with those obtained in the static HQET limit
- For the light quarks one can afford $m_s > m_q > m_s/4$, but the physical limit is $m_d \simeq m_s/25$
- Extrapolation of results with larger light quark masses is needed – Induces systematic uncertainties.



D. Bećirević et al. JHEP 0204:025 (2002) [hep-lat/0110091]

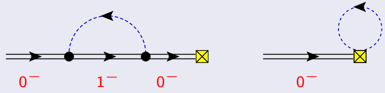
HMχPT allows us to gain some control over these uncertainties:

- Combines HQET and spontaneous breaking of chiral symmetry, $SU(3)_L \otimes SU(3)_R \to SU(3)_V$.
- One computes chiral logarithmic corrections which are expected to be relevant at $m_q \ll \Lambda_\chi$.
  - Predicts the chiral behavior of the hadronic quantities.
  - Can be implemented to guide the extrapolation of the lattice results.

Chiral logarithmic corrections to decay constants

Revisit the known result in the $SU(3)$ theory

Wavefunction and weak vertex corrections



$$\hat{f}_d = \alpha \left[1 - \frac{1}{(4\pi f)^2} \left(\frac{3}{4} m_\pi^2 \log \frac{m_\pi^2}{\mu^2} + \frac{1}{2} m_K^2 \log \frac{m_K^2}{\mu^2} + \frac{1}{12} m_\eta^2 \log \frac{m_\eta^2}{\mu^2} \right) + \varkappa_1(\mu) m_d + \varkappa_2(\mu)(m_u + m_d + m_s) + \frac{1}{2}\delta Z_d \right] ,$$

$$\hat{f}_s = \alpha \left[1 - \frac{1}{(4\pi f)^2} \left(m_K^2 \log \frac{m_K^2}{\mu^2} + \frac{1}{3} m_\eta^2 \log \frac{m_\eta^2}{\mu^2} \right) + \varkappa_1(\mu) m_s + \varkappa_2(\mu)(m_u + m_d + m_s) + \frac{1}{2}\delta Z_s \right] .$$

Such expressions are inverted and fitted to the LQCD data.

Bosonization of $\Delta B = 2$ operators in HMχPT

In HQET, heavy quark spin symmetry ($h^\dagger \gamma_0 = h^\dagger$) imposes relation

$$\langle \bar{B}_q^0 | \widetilde{O}_3 + \widetilde{O}_2 + \frac{1}{2}\widetilde{O}_1 | B_q^0 \rangle = 0.$$

In HMχPT $\left\{ H_q^Q(v) = \frac{1+\not v}{2}\left[P_\mu^{Q*}(v)\gamma^\mu - P^Q(v)\gamma_5\right]_q, H_q^{\bar{Q}}(v) = \left[P_\mu^{\bar{Q}*}(v)\gamma^\mu - P^{\bar{Q}}(v)\gamma_5\right]_q \frac{1-\not v}{2} \right\}$, the bosonized operators are color blind

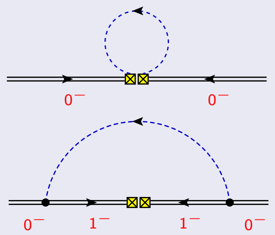
$$\begin{aligned}
\widetilde{O}_1 &= \sum_X \beta_{1X} \mathrm{Tr}\left[(\xi \overline{H}^Q)_q \gamma_\mu (1-\gamma_5) X\right] \mathrm{Tr}\left[(\xi H^{\bar{Q}})_q \gamma^\mu (1-\gamma_5) X\right] + \mathrm{c.t.}\,, \\
\widetilde{O}_2 &= \sum_X \beta_{2X} \mathrm{Tr}\left[(\xi \overline{H}^Q)_q (1-\gamma_5) X\right] \mathrm{Tr}\left[(\xi H^{\bar{Q}})_q (1-\gamma_5) X\right] + \mathrm{c.t.}\,, \\
\widetilde{O}_4 &= \sum_X \beta_{4X} \mathrm{Tr}\left[(\xi \overline{H}^Q)_q (1-\gamma_5) X\right] \mathrm{Tr}\left[(\xi^\dagger H^{\bar{Q}})_q (1+\gamma_5) X\right] \\
&\quad + \bar{\beta}_{4X} \mathrm{Tr}\left[(\xi H^{\bar{Q}})_q (1-\gamma_5) X\right] \mathrm{Tr}\left[(\xi^\dagger \overline{H}^Q)_q (1+\gamma_5) X\right] + \mathrm{c.t.}\,,
\end{aligned}$$

where $X \in \{1, \gamma_5, \gamma_\nu, \gamma_\nu\gamma_5, \sigma_{\nu\rho}\}$.

Chiral logarithmic corrections to SUSY basis bag parameters

All factorisable chiral loop corrections can be absorbed into HMχPT bag-parameter and decay constant definitions ($\beta_x \propto \tilde{B}_x/\hat{f}^2$).

Two nonfactorisable contributions



$$\widetilde{B}_{1d} = \widetilde{B}_1^{\rm Tree}\left[1 - \frac{1-3g^2}{(4\pi f)^2}\left(\frac{1}{2}m_\pi^2\log\frac{m_\pi^2}{\mu^2} + \frac{1}{6}m_\eta^2\log\frac{m_\eta^2}{\mu^2}\right) + {\rm c.t.}\right],$$

$$\widetilde{B}_{1s} = \widetilde{B}_1^{\rm Tree}\left[1 - \frac{1-3g^2}{(4\pi f)^2}\frac{2}{3}m_\eta^2\log\frac{m_\eta^2}{\mu^2} + {\rm c.t.}\right],$$

$$\widetilde{B}_{2,4d} = \widetilde{B}_{2,4}^{\rm Tree}\left[1 + \frac{3g^2 Y \mp 1}{(4\pi f)^2}\left(\frac{1}{2}m_\pi^2\log\frac{m_\pi^2}{\mu^2} + \frac{1}{6}m_\eta^2\log\frac{m_\eta^2}{\mu^2}\right) + {\rm c.t.}\right]$$

$$\widetilde{B}_{2,4s} = \widetilde{B}_{2,4}^{\rm Tree}\left[1 + \frac{2}{3}\frac{3g^2 Y \mp 1}{(4\pi f)^2}m_\eta^2\log\frac{m_\eta^2}{\mu^2} + {\rm c.t.}\right],$$

Also computed by W. Detmold and C.J.D. Lin Phys.Rev.D76 (2007) 014501 [hep-lat/0612028].

Inspection of results in the $SU(3)$ limit

- Chiral logarithmic corrections are expected to be relevant at $m_q \ll \Lambda_\chi$.
  - Condition is satisfied for u- and d-quarks.
  - Ambiguous size of the chiral symmetry breaking scale Λ_χ:
    - $4\pi f_\pi \simeq 1$ GeV
    - $m_\rho = 0.77$ GeV

Inspection of results in the $SU(3)$ limit

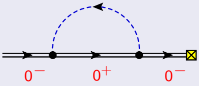
- Chiral logarithmic corrections are expected to be relevant at $m_q \ll \Lambda_\chi$.
  - Condition is satisfied for u- and d-quarks.
  - Ambiguous size of the chiral symmetry breaking scale Λ_χ:
    - $4\pi f_\pi \simeq 1$ GeV
    - $m_\rho = 0.77$ GeV

- Heavy-light quark systems are more complicated – first orbital excitations ($j_\ell^P = 1/2^+$) are not far from the ground ($j_\ell^P = 1/2^-$) states.
- Experimental evidence for scalar D_{0s}^* and axial D_{1s} mesons indicates $\Delta_{S_s} \equiv m_{D_{0s}^*} - m_{D_s} = 350$ MeV, $\Delta_{S_q} = 430$ MeV
- Both Δ_{S_s} and Δ_{S_q} are smaller than Λ_χ, m_η, and even m_K.
- Requires revisiting predictions based on HMχPT.

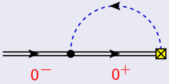
Impact of the $1/2^+$-mesons

- New $J_\ell^P = 1/2^+$ field operators $S_q(v) = \frac{1+\not v}{2}\left[P^*_{1\ \mu}(v)\gamma_\mu\gamma_5 - P_0(v)\right]_q$.
- New scale parameter $\Delta_S \approx 400$ MeV.

Corrections to the decay constants



Wavefunction renormalization



Weak vertex correction

- In the limit $m_\pi/\Delta_S \to 0$ all leading order corrections due to $1/2^+$-mesons are analytic in m_π.
- Kaon and eta logarithms are competitive in size with the terms proportional to $\Delta_S^2 \log(4\Delta_S^2/\mu^2)$
- Relevant chiral logarithmic corrections are those coming from the $SU(2)_L \otimes SU(2)_R \to SU(2)_V$ theory (below the Δ_S scale)

$$\boxed{\hat{f}_q = \alpha\left[1 - \frac{1+3g^2}{2(4\pi f)^2}\frac{3}{2}m_\pi^2 \log\frac{m_\pi^2}{\mu^2} + c_f(\mu)m_\pi^2\right]},$$

$$\boxed{\hat{f}_q^+ = \alpha^+\left[1 - \frac{1+3\tilde{g}^2}{2(4\pi f)^2}\frac{3}{2}m_\pi^2 \log\frac{m_\pi^2}{\mu^2} + c_f^+(\mu)m_\pi^2\right]}.$$

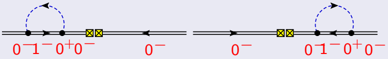
Bag-parameters

Operator bosonization receives new contributions

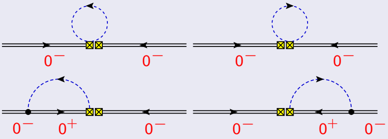
$$\begin{aligned}\widetilde{O}_1 \quad = \quad & \sum_X \beta_{1X} \mathrm{Tr}\left[\left(\xi \overline{H}^Q\right)_q \gamma_\mu (1-\gamma_5) X\right] \mathrm{Tr}\left[\left(\xi H^{\bar{Q}}\right)_q \gamma^\mu (1-\gamma_5) X\right] \\ & +\beta'_{1X} \left\{ \mathrm{Tr}\left[\left(\xi \overline{H}^Q\right)_q \gamma_\mu (1-\gamma_5) X\right] \mathrm{Tr}\left[\left(\xi S^{\bar{Q}}\right)_q \gamma^\mu (1-\gamma_5) X\right] + \mathrm{h.c.} \right\} \\ & +\beta''_{1X} \mathrm{Tr}\left[\left(\xi \overline{S}^Q\right)_q \gamma_\mu (1-\gamma_5) X\right] \mathrm{Tr}\left[\left(\xi S^{\bar{Q}}\right)_q \gamma^\mu (1-\gamma_5) X\right] .\end{aligned}$$

Similarly for $\widetilde{O}_2$ and $\widetilde{O}_4$.

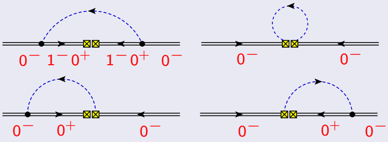
Chiral loop corrections



Wavefunction renormalization



Factorisable contributions cancel against weak vertex corrections



Nonfactorisable contributions

Bag-parameters

Typical loop integrals involving the new Δ_S scale probe large pion momenta in the chiral limit. The two scales (m_π and Δ_S) do not decouple as in the case of $\hat{f}$. We attempt an expansion:

$$\begin{aligned} & -2(4\pi)^2 v_\mu v_\nu \times i\mu^\epsilon \int \frac{d^{4-\epsilon}p}{(2\pi)^{4-\epsilon}} \frac{p^\mu p^\nu}{(p^2 - m_\pi^2)(\Delta_S - v\cdot p)^2} \\ = & -\frac{2(4\pi^2)}{\Delta_S^2} v_\mu v_\nu \left[i\mu^\epsilon \int \frac{d^{4-\epsilon}p}{(2\pi)^{4-\epsilon}} \frac{p^\mu p^\nu}{p^2 - m_\pi^2} + \mathcal{O}(1/\Delta_S^2) \right] \\ \to & -\frac{m_\pi^4}{2\Delta_S^2} \log \frac{m_\pi^2}{\mu^2} + \dots , \end{aligned}$$

- Expansion around the decoupling limit of the positive parity states.
- Series of local operators with Δ_S dependent prefactors.
- Effective counter terms of a theory with no positive parity mesons.
- Like for the decay constants, the relevant chiral expansion of the bag-parameters is the one derived in the $SU(2)_L \otimes SU(2)_R \to SU(2)_V$ theory.

$$\widetilde{B}_{1q} = \widetilde{B}_1^{\rm Tree} \left[1 - \frac{1-3g^2}{2(4\pi f)^2} m_\pi^2 \log \frac{m_\pi^2}{\mu^2} + c_{B_1}(\mu) m_\pi^2 \right]$$

$$\widetilde{B}_{2,4q} = \widetilde{B}_{2,4}^{\rm Tree} \left[1 + \frac{3g^2 Y \mp 1}{2(4\pi f)^2} m_\pi^2 \log \frac{m_\pi^2}{\mu^2} + c_{B_{2,4}}(\mu) m_\pi^2 \right]$$

Conclusions

- Precision determination of the $B_{s,d} - \overline{B}_{s,d}$ mixing parameters requires LQCD simulations data with light quarks corresponding to $m_\pi < \Delta_S$.
- $\Delta_S \approx 400 MeV$, therefore the lattice results obtained with light quarks lighter than $m_s/3$ can be fit to the χPT formulas and extrapolated down to $m_s/25$.
- Going beyond $m_s/3$ requires inclusion of heavy scalars, which means a whole plethora of new couplings and counterterms to be fixed plus complicated matching at resonance thresholds.
- Similar conclusions regarding the impact of the $1/2^+$-mesons on leading chiral logarithms have been reached in other processes.
  - Effective HMχPT couplings between heavy and light mesons (g,h,$\tilde{g}$)
  - Isgur-Wise functions in semileptonic B to $D^{(*)}$ meson decays (ξ,$\tau_{1/2}$,$\tilde{\xi}$)

References

- S. Fajfer and J.K., Phys.Rev.D74 (2006) 074023 [hep-ph/0606278]
- D. Bećirević, S. Fajfer and J.K. JHEP 0706:003 (2007) [hep-ph/0612224]
- J.O.Eeg, S. Fajfer and J.K., JHEP 0707:078 (2007) [arXiv:0705.4567]
- J.K. (2007) [arXiv:0709.3494]

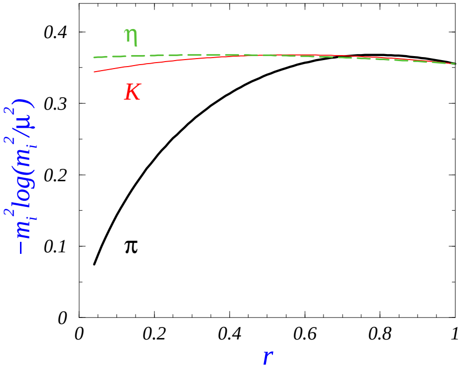
Backup Slides

Chiral extrapolation

$$m_\pi^2 = \frac{8\lambda_0 m_s}{f^2} r,$$

$$m_K^2 = \frac{8\lambda_0 m_s}{f^2} \frac{r+1}{2},$$

$$m_\eta^2 = \frac{8\lambda_0 m_s}{f^2} \frac{r+2}{3},$$



Chiral extrapolation

Extracting the effective coupling dependence on the pseudo-Goldstone masses.

$$\frac{1}{m_j^2}\frac{\mathrm{d}g^{\mathrm{eff.}}_{P_a^* P_b \pi^i}}{\mathrm{d}\log m_j^2} = \frac{\mathbf{g}}{(4\pi f)^2}$$
$$\times\left\{\frac{\lambda^j_{ac}\lambda^j_{ca}+\lambda^j_{bc}\lambda^j_{cb}}{2}\left[-3\mathbf{g}^2-\mathbf{h}^2\left(1-\frac{6\Delta^2_{SH}}{m_j^2}\right)\right]\right.$$
$$\left.+\frac{\lambda^j_{ac}\lambda^i_{cd}\lambda^j_{db}}{\lambda^i_{ab}}\left[\mathbf{g}^2-\mathbf{h}^2\frac{\tilde{\mathbf{g}}}{\mathbf{g}}\left(1-\frac{6\Delta^2_{SH}}{m_j^2}\right)\right]\right\}$$

Chiral extrapolation

Extracting the effective coupling dependence on the pseudo-Goldstone masses.

$$\frac{1}{m_j^2}\frac{\mathrm{d}g^{\mathrm{eff.}}_{P_a^* P_b \pi^i}}{\mathrm{d}\log m_j^2} = \frac{\mathbf{g}}{(4\pi f)^2}$$
$$\times\left\{\frac{\lambda^j_{ac}\lambda^j_{ca}+\lambda^j_{bc}\lambda^j_{cb}}{2}\left[-3\mathbf{g}^2-\mathbf{h}^2\left(1-\frac{6\Delta_{SH}^2}{m_j^2}\right)\right]\right.$$
$$\left.+\frac{\lambda^j_{ac}\lambda^i_{cd}\lambda^j_{db}}{\lambda^i_{ab}}\left[\mathbf{g}^2-\mathbf{h}^2\frac{\tilde{\mathbf{g}}}{\mathbf{g}}\left(1-\frac{6\Delta_{SH}^2}{m_j^2}\right)\right]\right\}$$

Large Δ_{SH} dependence.

Chiral extrapolation

$$\mu^{(4-D)} \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{q^\mu q^\nu}{(q^2 - m^2)(v \cdot q - \Delta)}$$

Chiral extrapolation

$$\mu^{(4-D)} \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{q^\mu q^\nu}{(q^2 - m^2)(v \cdot q - \Delta)}$$

Loop integral expansion in $1/\Delta_{SH}$

$$\Rightarrow \quad \mu^{(4-D)} \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{q^\mu q^\nu}{(q^2 - m^2)} \frac{-1}{\Delta} (1 + \frac{q \cdot v}{\Delta} + \ldots)$$

(All even orders vanish.)

$$\left(1 - \frac{6\Delta_{SH}^2}{m_j^2}\right) \Rightarrow \frac{m_j^2}{4\Delta_{SH}^2}$$

- Expansion around the decoupling limit of the positive parity states.
- Series of local operators with Δ_{SH} dependent prefactors.
- Effective counter terms of a theory with no positive parity mesons.

Restrictions on bosonized operators matching to HQET

Contraction of Lorentz indices and HQET parity conservation requires the same X to appear in both traces of a summation term. Any insertions of $\not{v}$ can be absorbed via $\not{v}H = H$, while any nonfactorisable contribution with a single trace over Dirac matrices can be reduced to this form by using the 4×4 matrix identity

$$\begin{aligned} 4\mathrm{Tr}(AB) &= \mathrm{Tr}(A)\mathrm{Tr}(B) + \mathrm{Tr}(\gamma_5 A)\mathrm{Tr}(\gamma_5 B) + \mathrm{Tr}(A\gamma_\mu)\mathrm{Tr}(\gamma^\mu B) \\ &+ \mathrm{Tr}(A\gamma_\mu\gamma_5)\mathrm{Tr}(\gamma_5\gamma^\mu B) + 1/2\mathrm{Tr}(A\sigma_{\mu\nu})\mathrm{Tr}(\sigma^{\mu\nu} B). \end{aligned}$$