Quark mass dependence of the heavy-strange meson decay constant in quenched QCD

ALPHA Collaboration

FlaviA net

Jochen Heitger

institut für theoretische physik

Universität Münster, Wilhelm-Klemm-Str. 9, D-48149 Münster

EuroFlavour 07

Annual Meeting of the European Flavour Physics Network *FLAVIAnet*

Université Paris-Sud, Faculté des Sciences, Orsay, France

14 – 16 November 2007

Motivation

- A precise & non-perturbative (NP) determination of B–physics matrix elements within the unitarity triangle analysis is a task for lattice QCD
- Why lattice *HQET*?
  - The B–meson system is characterized by two disparate scales: the heavy quark mass ($m_b \approx 5\,\mathrm{GeV}$) plus a QCD scale linked to m_{light}
  - However, a propagating b-quark on the lattice needs *very* small lattice spacings ($a < 1/m_b$), still beyond today's computing resources

Motivation

- A precise & non-perturbative (NP) determination of B–physics matrix elements within the unitarity triangle analysis is a task for lattice QCD
- Why lattice *HQET* ?
  - The B–meson system is characterized by two disparate scales: the heavy quark mass ($m_b \approx 5\,\mathrm{GeV}$) plus a QCD scale linked to m_{light}
  - However, a propagating b-quark on the lattice needs *very* small lattice spacings ($a < 1/m_b$), still beyond today's computing resources

⇒ Recourse to an *effective* theory for the b-quark:

Heavy Quark Effective Theory

[Eichten, 1988; Eichten & Hill, 1990]

$$\overline{\psi}_b\{\gamma_\mu D_\mu + m_b\}\psi_b$$

$$\downarrow$$

$$\mathcal{L}_{\mathrm{HQET}}(x) = \overline{\psi}_h(x)\Big[\underbrace{D_0 + m_b}_{\text{static limit}} - \frac{\omega_{\mathrm{kin}}}{2m_b}\mathbf{D}^2 - \frac{\omega_{\mathrm{spin}}}{2m_b}\boldsymbol{\sigma}\cdot\mathbf{B}\Big]\psi_h(x) + \ldots$$

- HQET $=$ (continuum) asymptotic expansion of QCD
  $\longrightarrow$ Explicit pure theory tests that HQET is an *effective* theory of QCD?
- Phenomen. applications by constraining the large-mass behaviour of QCD quantities (at $m_h \approx m_{charm} < m_b$) with computations in HQET
  $\longrightarrow$ Needs renormalizations entering the *matching* with high precision

- HQET $=$ (continuum) asymptotic expansion of QCD
  $\rightarrow$ Explicit pure theory tests that HQET is an *effective* theory of QCD?
- Phenomen. applications by constraining the large-mass behaviour of QCD quantities (at $m_h \approx m_{charm} < m_b$) with computations in HQET
  $\rightarrow$ Needs renormalizations entering the *matching* with high precision

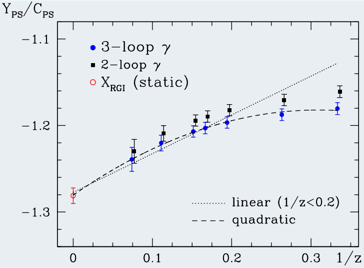
NB: A demonstration, how in this way *extra*polations turn into *inter*polations, has been given within the (Tor Vergata) step scaling method for B-physics in

[Guazzini, Sommer & Tantalo, arXiv:0710.2229]

- HQET $=$ (continuum) asymptotic expansion of QCD
  $\rightarrow$ Explicit pure theory tests that HQET is an *effective* theory of QCD?
- Phenomen. applications by constraining the large-mass behaviour of QCD quantities (at $m_h \approx m_{charm} < m_b$) with computations in HQET
  $\rightarrow$ Needs renormalizations entering the *matching* with high precision

Non-perturbative tests in *finite* volume

[ALPHA Collaboration, JHEP0411(2004)048]



Example:

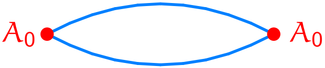
Finite-volume decay constant Y_{PS}

$Y_{PS}(L,M) \overset{M\to\infty}{\sim} C_{PS}(M/\Lambda) X_{RGI}^{stat}(L) + O(1/z)$

- HQET applicable as long as $z = ML \gg 1$
- Composite fields are renormalized NP'ly & CL taken at large quark masses
- Finite-mass (QCD) observables smoothly turn into the HQET predictions
- Coefficient of z^{-n}–corrections small: $O(1)$
- "Conversion function" C_{PS} reduces mass dependence of $Y_{PS}(L,M)$ by a factor > 2 ($z^{-1} = 0.1 - 0.2$: power $>$ perturb. corrections)

Composite fields involving b-quarks in HQET

$$A_0(x) = \overline{\psi}_l(x)\gamma_0\gamma_5\psi_b(x) \quad \longrightarrow \quad A_0^{\rm stat} = \overline{\psi}_l(x)\gamma_0\gamma_5\psi_h(x)$$



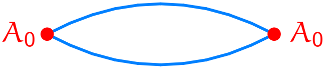
: Correlation function of the axial current

$$\int d^3x \left\langle A_0(x) A_0^\dagger(0) \right\rangle_{\rm QCD} \overset{x_0 \gg 1/M_b}{\sim} \left[C_{\rm PS}\left(\tfrac{M_b}{\Lambda}\right) Z_{\rm RGI} \right]^2 \int d^3x \left\langle A_0^{\rm stat}(x) \left(A_0^{\rm stat}\right)^\dagger(0) \right\rangle_{\rm stat}$$

$$+ \, \mathrm{O}\left(1/M_b\right) \qquad \Lambda \equiv \Lambda_{\rm QCD}$$

Composite fields involving b-quarks in HQET

$$A_0(x) = \overline{\psi}_l(x)\gamma_0\gamma_5\psi_b(x) \quad \longrightarrow \quad A_0^{\text{stat}} = \overline{\psi}_l(x)\gamma_0\gamma_5\psi_h(x)$$



: Correlation function of the axial current

$$\int d^3x \left\langle A_0(x) A_0^\dagger(0) \right\rangle_{\text{QCD}} \overset{x_0 \gg 1/M_b}{\sim} \left[C_{\text{PS}}\left(\tfrac{M_b}{\Lambda}\right) Z_{\text{RGI}} \right]^2 \int d^3x \left\langle A_0^{\text{stat}}(x) \left(A_0^{\text{stat}}\right)^\dagger(0) \right\rangle_{\text{stat}}$$

$$+\, \mathrm{O}\left(1/M_b\right) \qquad \Lambda \equiv \Lambda_{\text{QCD}}$$

Generic structure of the HQET-expansion of QCD matrix elements

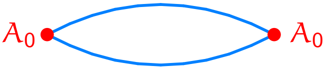
$$\Phi = \langle\, B \,|\, A_0 \,|\, 0 \,\rangle : \quad \Phi^{\text{QCD}} \equiv F_B\sqrt{m_B} = \underbrace{C_{\text{PS}}\left(M_b/\Lambda\right)} \times \underbrace{\Phi_{\text{RGI}}} + \mathrm{O}\left(1/M_b\right)$$

conversion function $\Leftarrow$ renormalization — RGI matrix element in effective theory

- In HQET: Absence of chiral symmetry as it is met in (massless) QCD implies a scale dependence $\Phi^{\text{stat}}(\mu) \equiv Z_A^{\text{stat}}(\mu)\,\langle\, B \,|\, A_0^{\text{stat}} \,|\, 0 \,\rangle$
- M_b = scale & scheme independent (RG-invariant) quark mass

Composite fields involving b-quarks in HQET

$$A_0(x) = \overline{\psi}_l(x)\gamma_0\gamma_5\psi_b(x) \quad \longrightarrow \quad A_0^{\rm stat} = \overline{\psi}_l(x)\gamma_0\gamma_5\psi_h(x)$$



: Correlation function of the axial current

$$\int d^3x \left\langle A_0(x) A_0^\dagger(0) \right\rangle_{\rm QCD} \overset{x_0 \gg 1/M_b}{\sim} \left[C_{\rm PS}\left(\frac{M_b}{\Lambda}\right) Z_{\rm RGI} \right]^2 \int d^3x \left\langle A_0^{\rm stat}(x) \left(A_0^{\rm stat}\right)^\dagger(0) \right\rangle_{\rm stat}$$

$$+ \, O\left(1/M_b\right) \qquad \Lambda \equiv \Lambda_{\rm QCD}$$

b–quark mass dependence is a consequence of the *matching step*:
(Finite) renormalization s.th. matrix elements of the current in the effective theory are equal to the QCD ones up to $O(1/m_b)$

$$\Phi^{\rm QCD} = F_B\sqrt{m_B} = C_{\rm match}(m_b,\mu) \times \Phi_{\overline{\rm MS}}(\mu) + O(1/m_b)$$

$$C_{\rm match}(m_b,\mu) = 1 + c_1(m_b/\mu)\,\bar{g}^2_{\overline{\rm MS}}(\mu) + \ldots$$

μ–dependence removed by passing $\Phi_{\overline{\rm MS}}(\mu) = Z^{\rm stat}_{\rm A,\overline{MS}}(\mu)\,\langle\, B \,|\, A_0^{\rm stat} \,|\, 0 \,\rangle \longmapsto$ RGI

$$\Phi_{\rm RGI} = \lim_{\mu\to\infty} \left[2b_0\bar{g}^2(\mu)\right]^{-\gamma_0/(2b_0)} \Phi_{\overline{\rm MS}}(\mu)\ , \quad \gamma(\bar{g}) \equiv \frac{\mu}{Z_{\rm A}^{\rm stat}} \frac{dZ_{\rm A}^{\rm stat}}{d\mu} = -\gamma_0\bar{g}^2 + O(\bar{g}^4)$$

Evaluation of the conversion function for the axial current:

$$\Phi^{\mathrm{QCD}} = C_{\mathrm{PS}}(M_b/\Lambda) \times \Phi_{\mathrm{RGI}} + \mathrm{O}(1/M_b) \qquad (\star)$$

$$\stackrel{!}{=} C_{\mathrm{match}}(m_b/\mu) \times \Phi_{\overline{\mathrm{MS}}}(\mu) + \mathrm{O}(1/m_b)$$

$$\Rightarrow \quad C_{\mathrm{PS}}(M_b/\Lambda) = C_{\mathrm{match}}(1) \times \frac{\Phi_{\overline{\mathrm{MS}}}(m_b)}{\Phi_{\mathrm{RGI}}} \qquad \bar{g} = \bar{g}_{\overline{\mathrm{MS}}}\,,\ \Lambda = \Lambda_{\overline{\mathrm{MS}}}$$

$$= \left[2b_0\bar{g}^2(m_b)\right]^{\gamma_0/(2b_0)} \exp\left\{\int_0^{\bar{g}(m_b)} dg \left[\frac{\gamma^{\mathrm{match}}(g)}{\beta(g)} - \frac{\gamma_0}{b_0 g}\right]\right\}$$

Evaluation of the conversion function for the axial current:

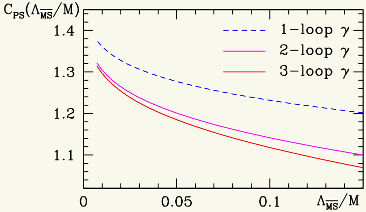
$$\Phi^{QCD} = C_{PS}(M_b/\Lambda) \times \Phi_{RGI} + O(1/M_b) \qquad (\star)$$

$$\stackrel{!}{=} C_{match}(m_b/\mu) \times \Phi_{\overline{MS}}(\mu) + O(1/m_b)$$

$$\Rightarrow \quad C_{PS}(M_b/\Lambda) = C_{match}(1) \times \frac{\Phi_{\overline{MS}}(m_b)}{\Phi_{RGI}} \qquad \bar{g} = \bar{g}_{\overline{MS}}\,,\ \Lambda = \Lambda_{\overline{MS}}$$

$$= \left[2b_0\bar{g}^2(m_b)\right]^{\gamma_0/(2b_0)} \exp\left\{\int_0^{\bar{g}(m_b)} dg\left[\frac{\gamma^{match}(g)}{\beta(g)} - \frac{\gamma_0}{b_0 g}\right]\right\}$$

C_{PS} perturbatively under control [3-loop AD by Chetyrkin & Grozin, 2003]



$N_f = 0$

uncertainty for $M \geqslant M_{charm}$:

$\lesssim 2\% = 0.5 \cdot$ (3-loop − 2-loop)

- Anom. dimension in the matching scheme:

$$\gamma^{match}(\bar{g}) = -\gamma_0\bar{g}^2 - [\gamma_1^{\overline{MS}} + 2b_0c_1(1)]\,\bar{g}^4 + \ldots$$

(with contributions from $\gamma^{\overline{MS}}$ and C_{match})

- RGI-*ratio* M/Λ: can be fixed in numerical simulations without perturbative errors
- Full (logarithmic) mass dependence $\in C_{PS}$
- C_{PS} resp. γ^{match} defined beyond PT via $(\star)$ upon replacing $\bar{g}_{\overline{MS}}$ by a non-perturb. $\bar{g}$

Heavy-strange meson decay constants in the continuum limit of quenched QCD

M. Della Morte, S. Dürr, D. Guazzini, J. H., A. Jüttner, R. Sommer

arXiv:0710.2201 [hep-lat]

Strategy of the non-perturbative study

1. Calculate the RGI matrix element of $A_0^{\rm stat}$ in the lowest order of lattice HQET (= static approximation) in large volume

$$\Phi_{\rm RGI} \equiv \Phi_{\rm RGI}^{\rm stat} = Z_{\rm RGI} \langle B \,|\, A_0^{\rm stat} \,|\, 0 \rangle$$

$Z_{\rm RGI}$: NP'ly known ($\approx 1\%$ accuracy) [$N_f = 0$: H., Kurth & Sommer, 2003]

$$\Phi_{\rm RGI} = F_B \sqrt{m_B} / C_{\rm PS}(M_b/\Lambda) =$$

$$(2L^3)^{1/2} \langle B \,|\, Z_{\rm RGI} A_0^{\rm stat} \,|\, 0 \rangle = Z_{\rm RGI} \lim_{x_0 \to \infty} \left[2 \exp\left\{ x_0 \, E_{\rm stat}^{\rm eff}(x_0) \right\} C_{\rm AA}^{\rm stat}(x_0) \right]^{1/2}$$

Strategy of the non-perturbative study

1. Calculate the RGI matrix element of $A_0^{\rm stat}$ in the lowest order of lattice HQET (= static approximation) in large volume

$$\Phi_{\rm RGI} \equiv \Phi_{\rm RGI}^{\rm stat} = Z_{\rm RGI} \langle B | A_0^{\rm stat} | 0 \rangle$$

$Z_{\rm RGI}$: NP'ly known ($\approx 1\%$ accuracy) [$N_f = 0$: H., Kurth & Sommer, 2003]

$$\Phi_{\rm RGI} = F_B \sqrt{m_B} / C_{PS}(M_b/\Lambda) =$$

$$(2L^3)^{1/2} \langle B | Z_{\rm RGI} A_0^{\rm stat} | 0 \rangle = Z_{\rm RGI} \lim_{x_0 \to \infty} \left[2 \exp\left\{ x_0 E_{\rm stat}^{\rm eff}(x_0) \right\} C_{AA}^{\rm stat}(x_0) \right]^{1/2}$$

2. Combine this with large-volume QCD results for $F_{PS}(m_{PS})$, obtained for heavy quark mass values covering a range within the charm region

$$\left[\Phi^{\rm QCD}\right]^2 \equiv$$

$$F_B^2 m_B = 2L^3 \left[\langle PS | Z_A A_0 | 0 \rangle \right]^2 = \lim_{x_0 \to \infty} \left[2 \exp\left\{ x_0 m_{PS}^{\rm eff}(x_0) \right\} C_{AA}(x_0) \right]$$

$$C_{AA}(x_0) = Z_A^2 a^3 \sum_{\mathbf{x}} \langle A_0(x) (A_0)^\dagger(0) \rangle$$

Strategy of the non-perturbative study

1. Calculate the RGI matrix element of $A_0^{\rm stat}$ in the lowest order of lattice HQET (= static approximation) in large volume

$$\Phi_{\rm RGI} \equiv \Phi_{\rm RGI}^{\rm stat} = Z_{\rm RGI} \langle B | A_0^{\rm stat} | 0 \rangle$$

$Z_{\rm RGI}$: NP'ly known ($\approx$ 1% accuracy) [$N_f = 0$: H., Kurth & Sommer, 2003]

$$\Phi_{\rm RGI} = F_B \sqrt{m_B} / C_{PS}(M_b/\Lambda) =$$

$$(2L^3)^{1/2} \langle B | Z_{\rm RGI} A_0^{\rm stat} | 0 \rangle = Z_{\rm RGI} \lim_{x_0 \to \infty} \left[2 \exp\left\{ x_0 E_{\rm stat}^{\rm eff}(x_0) \right\} C_{AA}^{\rm stat}(x_0) \right]^{1/2}$$

2. Combine this with large-volume QCD results for $F_{PS}(m_{PS})$, obtained for heavy quark mass values covering a range within the charm region

$$\left[\Phi^{\rm QCD}\right]^2 \equiv$$

$$F_B^2 m_B = 2L^3 \left[\langle PS | Z_A A_0 | 0 \rangle \right]^2 = \lim_{x_0 \to \infty} \left[2 \exp\left\{ x_0 m_{PS}^{\rm eff}(x_0) \right\} C_{AA}(x_0) \right]$$

$$C_{AA}(x_0) = Z_A^2 a^3 \sum_{\mathbf{x}} \langle A_0(x) (A_0)^\dagger(0) \rangle$$

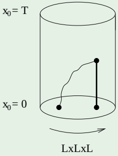
$\Rightarrow$ The physical b-region can be reached through an *interpolation* in the inverse of the heavy-light meson mass, $1/m_{PS}$

Results in the static-light sector

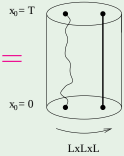
Computational setup

- $V = L^3 \times T$ with $L \approx 2\,\text{fm}$, $T = 3L/2, 5L/4$, $a \approx (0.09 - 0.05)\,\text{fm}$
- Schrödinger functional boundary conditions:
  Dirichlet in time, periodic in space
  $\longrightarrow$ Correlation functions

$f_A^{\text{stat}}(x_0) =$



$f_1^{\text{stat}} =$



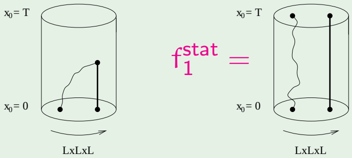
- Construction of interpolating B–meson fields with wave functions $\omega_i(\mathbf{x})$ to suppress contributions from excited states to the CFs

Results in the static-light sector

Computational setup

- $V = L^3 \times T$ with $L \approx 2\,\mathrm{fm}$, $T = 3L/2, 5L/4$, $a \approx (0.09 - 0.05)\,\mathrm{fm}$
- Schrödinger functional boundary conditions:
  Dirichlet in time, periodic in space
  $\rightarrow$ Correlation functions

$f_A^{\mathrm{stat}}(x_0) =$ … $f_1^{\mathrm{stat}} =$ …

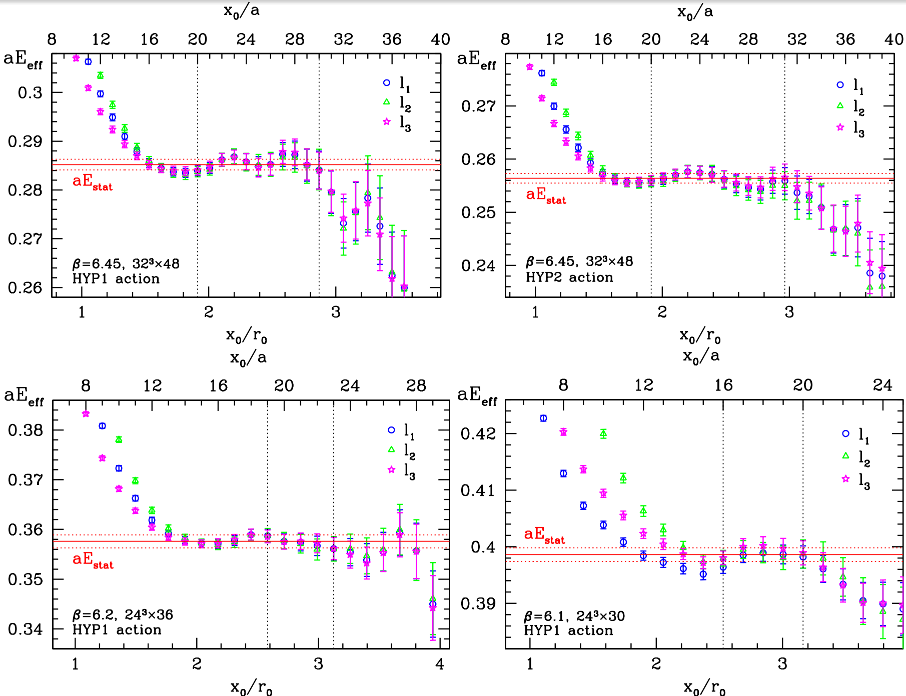


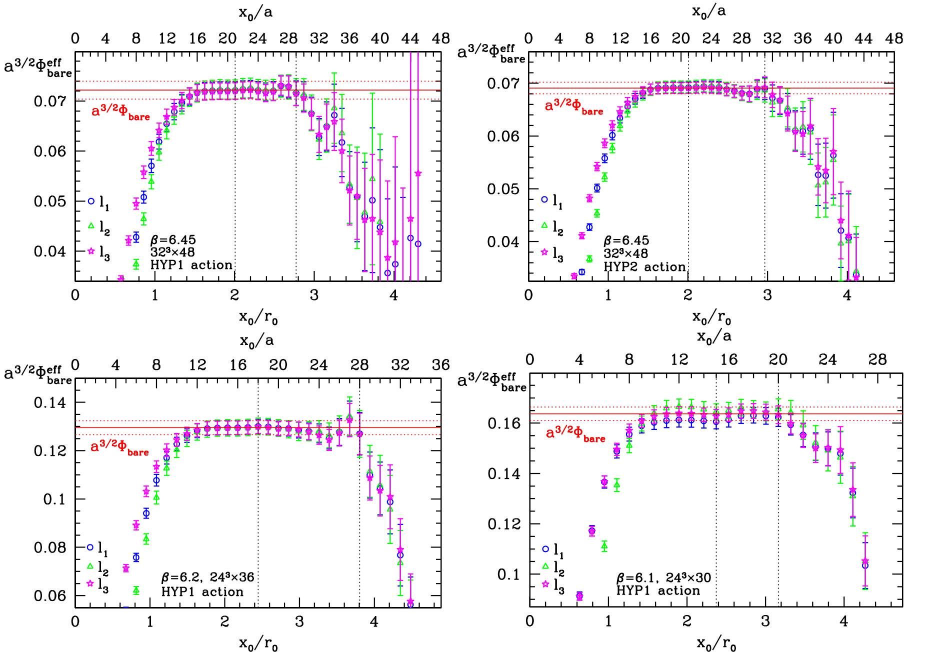
- Construction of interpolating B–meson fields with wave functions $\omega_i(\mathbf{x})$ to suppress contributions from excited states to the CFs

Observables:

$$\Phi^{\mathrm{eff}}_{\mathrm{bare}}(x_0,\omega_i) = -2L^{3/2}\,\frac{f_A^{\mathrm{stat}}(x_0,\omega_i)}{\sqrt{f_1^{\mathrm{stat}}(T,\omega_i,\omega_i)}}\,e^{(x_0-T/2)E_{\mathrm{eff}}(x_0,\omega_i)}$$

$$\Phi_{\mathrm{RGI}} = Z_{\mathrm{RGI}}\left(1 + b_A^{\mathrm{stat}} a m_q\right)\Phi_{\mathrm{bare}} \qquad E_{\mathrm{eff}}(x_0,\omega_i) = \frac{1}{2a}\ln\left[\frac{f_A^{\mathrm{stat}}(x_0-a,\omega_i)}{f_A^{\mathrm{stat}}(x_0+a,\omega_i)}\right]$$


x_0/a
aE_{eff}
l_1
l_2
l_3
aE_{stat}
β=6.45, $32^3\times48$
HYP1 action
x_0/r_0
β=6.45, $32^3\times48$
HYP2 action
β=6.2, $24^3\times36$
HYP1 action
β=6.1, $24^3\times30$
HYP1 action



x_0/a
$a^{3/2}\Phi^{eff}_{bare}$
$a^{3/2}\Phi_{bare}$
l_1
l_2
l_3
β=6.45
$32^3\times48$
HYP1 action
x_0/r_0
x_0/a
$a^{3/2}\Phi^{eff}_{bare}$
$a^{3/2}\Phi_{bare}$
l_1
l_2
l_3
β=6.45
$32^3\times48$
HYP2 action
x_0/r_0
x_0/a
$a^{3/2}\Phi^{eff}_{bare}$
$a^{3/2}\Phi_{bare}$
l_1
l_2
l_3
β=6.2, $24^3\times36$
HYP1 action
x_0/r_0
x_0/a
$a^{3/2}\Phi^{eff}_{bare}$
$a^{3/2}\Phi_{bare}$
l_1
l_2
l_3
β=6.1, $24^3\times30$
HYP1 action
x_0/r_0

Analysis details:

- 1– and 2–state fits guided by the expected large-time asymptotics

$$
\begin{aligned}
-2f_A^{\rm stat}(x_0,\omega_i) &\overset{x_0\to\infty}{\sim} \beta_i^{(0)}\,e^{-x_0 E_{\rm stat}} + \beta_i^{(1)}\,e^{-x_0(E_{\rm stat}+\Delta^{\rm stat})} \\
2f_1^{\rm stat}(T',\omega_i,\omega_j) &\overset{T'\to\infty}{\sim} \alpha_i^{(0)}\alpha_j^{(0)}\,e^{-T' E_{\rm stat}}
\end{aligned}
$$

Analysis details:

- 1– and 2–state fits guided by the expected large-time asymptotics

$$E_{\mathrm{eff}}(x_0,\omega_i) \overset{x_0\to\infty}{\sim} E_{\mathrm{stat}} + \frac{\beta_i^{(1)}}{\beta_i^{(0)}}\,\frac{\sinh\left(a\Delta^{\mathrm{stat}}\right)}{a}\,e^{-x_0\Delta^{\mathrm{stat}}}$$

$$\Phi_{\mathrm{RGI}}^{\mathrm{eff}}(x_0,\omega_i) \overset{x_0\to\infty}{\sim} \Phi_{\mathrm{RGI}}\left\{1+\frac{\beta_i^{(1)}}{\beta_i^{(0)}}\,e^{-x_0\Delta^{\mathrm{stat}}}\left[1+\left(x_0-\tfrac{T'}{2}\right)\frac{\sinh(a\Delta^{\mathrm{stat}})}{a}\right]\right\}$$

Analysis details:

- 1– and 2–state fits guided by the expected large-time asymptotics

$$E_{\rm eff}(x_0,\omega_i) \overset{x_0\to\infty}{\sim} E_{\rm stat} + \frac{\beta_i^{(1)}}{\beta_i^{(0)}}\,\frac{\sinh\left(a\Delta^{\rm stat}\right)}{a}\,e^{-x_0\Delta^{\rm stat}}$$

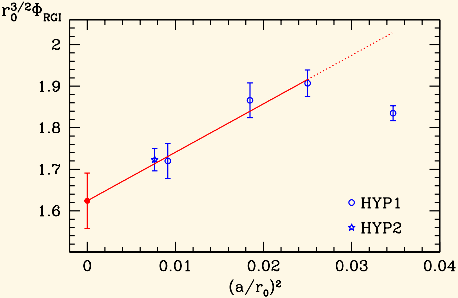
$$\Phi_{\rm RGI}^{\rm eff}(x_0,\omega_i) \overset{x_0\to\infty}{\sim} \Phi_{\rm RGI}\left\{1+\frac{\beta_i^{(1)}}{\beta_i^{(0)}}\,e^{-x_0\Delta^{\rm stat}}\left[1+\left(x_0-\frac{T'}{2}\right)\frac{\sinh(a\Delta^{\rm stat})}{a}\right]\right\}$$

- $f_A^{\rm stat}$, $f_1^{\rm stat}$ optimized via linearly combining the ω_i s.th. quality & extent of the plateau in $E_{\rm eff}$ are enhanced by eliminating the 1st excited state contribution
- To tame the dominance of the statistical uncertainty of $f_1^{\rm stat}(T)$ in $\Phi_{\rm bare}^{\rm eff}(x_0)$, the former is extracted from runs on lattices with smaller time extents $T' < T$

Analysis details:

- 1– and 2–state fits guided by the expected large-time asymptotics
- f_A^{stat}, f_1^{stat} optimized via linearly combining the ω_i s.th. quality & extent of the plateau in E_{eff} are enhanced by eliminating the 1st excited state contribution
- To tame the dominance of the statistical uncertainty of $f_1^{stat}(T)$ in $\Phi_{bare}^{eff}(x_0)$, the former is extracted from runs on lattices with smaller time extents $T' < T$

Continuum limit extrapolation



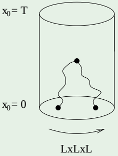
- Non-perturb. $O(a)$ improvement
- Light quark mass is fixed to the strange quark mass
- Static actions with reduced errors (HYP1 & HYP2)

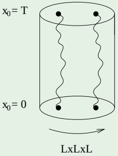
$$\Rightarrow \quad r_0^{3/2}\Phi_{RGI} = 1.624(67)$$

Decay constant at finite heavy quark mass

Computational setup

- $V = L^3 \times T$ with $L \approx 2\,\mathrm{fm}$, $T = 2L$, $a \approx (0.09 - 0.03)\,\mathrm{fm}$ (!)
- Non-degenerate *relativistic* quarks
  $\rightarrow$ SF Correlation functions
  involving
  $(A_I)_0(x) = A_0(x) + a\, c_A \tilde{\partial}_0 P(x)$

$f_A(x_0) =$ 

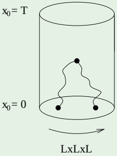
$f_1 =$ 

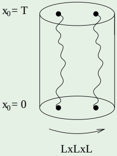
- $F_{PS}(x_0) = -Z_A \left(1 + \frac{b_A}{2}(am_{q,i} + am_{q,s})\right) \frac{2}{\sqrt{m_{PS} L^3}} \frac{f_A(x_0)}{\sqrt{f_1}}\, e^{(x_0 - T/2) m_{PS}}$

Decay constant at finite heavy quark mass

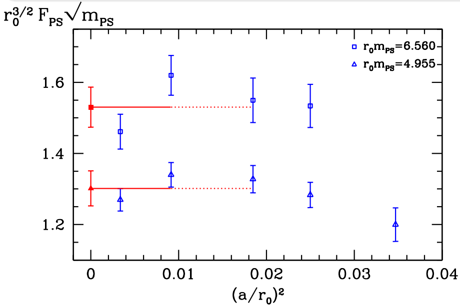
Computational setup

- $V = L^3 \times T$ with $L \approx 2\,\text{fm}$, $T = 2L$, $a \approx (0.09 - 0.03)\,\text{fm}$ (!)
- Non-degenerate *relativistic* quarks
  $\rightarrow$ SF Correlation functions involving
  $(A_I)_0(x) = A_0(x) + a\,c_A\,\tilde{\partial}_0 P(x)$

$f_A(x_0) =$ 

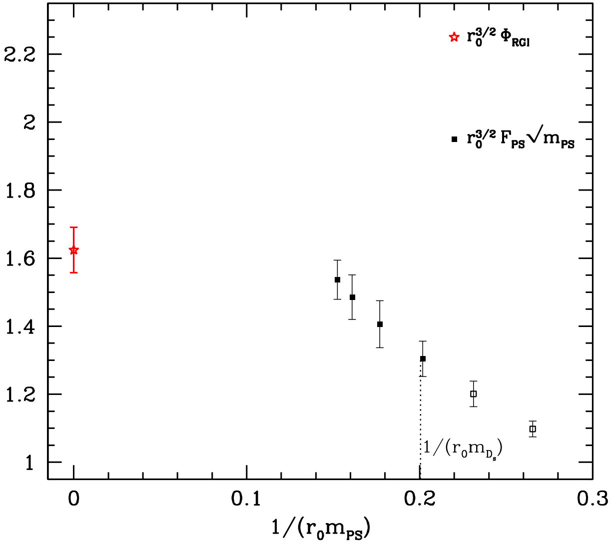
$f_1 =$ 

- $F_{PS}(x_0) = -Z_A\left(1 + \frac{b_A}{2}(am_{q,i} + am_{q,s})\right)\frac{2}{\sqrt{m_{PS}L^3}}\,\frac{f_A(x_0)}{\sqrt{f_1}}\,e^{(x_0 - T/2)m_{PS}}$

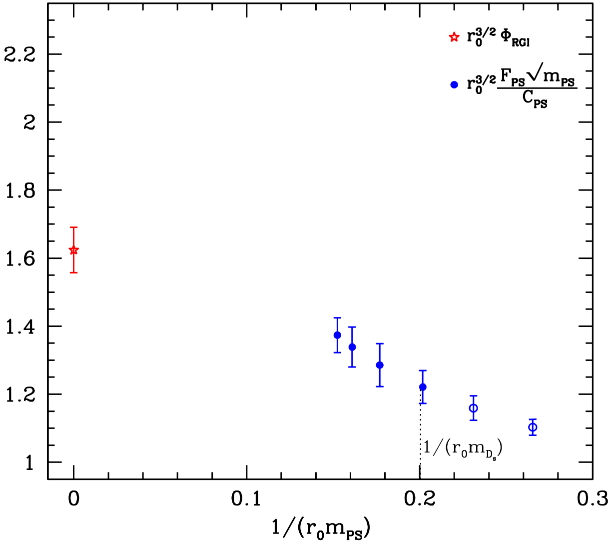


- Non-perturb. $O(a)$ improvement
- $m_{light} = m_{strange}$
- $F_{PS}(m_{PS})$ for PS-meson masses fixed around the physical charm, $m_{PS} = (1.5\text{-}2.6)\text{GeV} \approx (0.8\text{-}1.3)m_{D_s}$
- CL for $am_{heavy} \lesssim \frac{1}{2}$ to avoid area of breakdown of $O(a^2)$ scaling

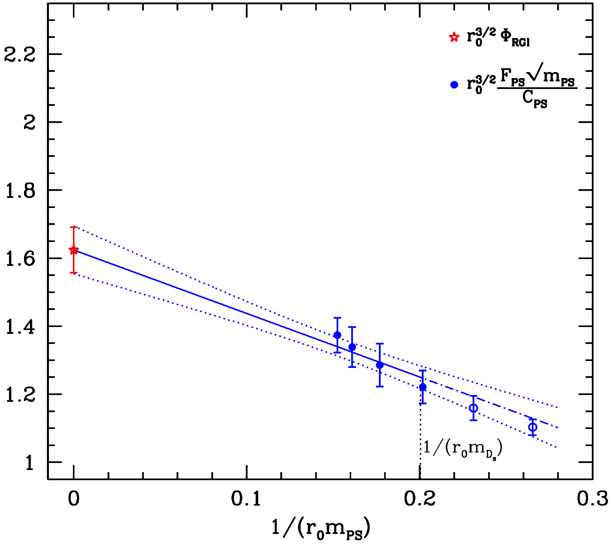
Mass dependence of the heavy-strange decay constant



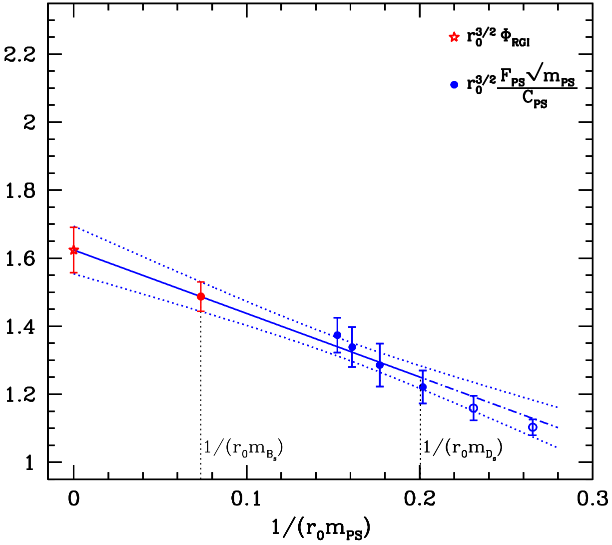
Mass dependence of the heavy-strange decay constant



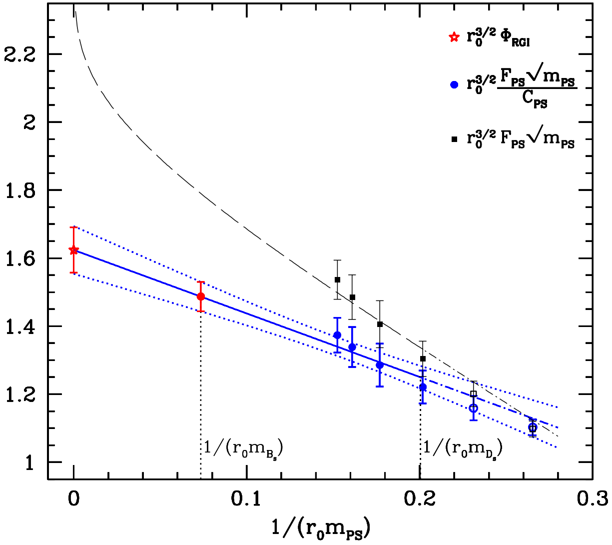
Mass dependence of the heavy-strange decay constant



Mass dependence of the heavy-strange decay constant



Mass dependence of the heavy-strange decay constant



Conclusions

The decay constant in HQET smoothly connects to the continuum data on F_{PS} in the QCD charm quark regime via *linear interpolation in* $1/m$, once

1. renormalization $[\rightarrow\ (\mathrm{NP})\ Z_{RGI}\,]$
2. and matching $[\rightarrow\ C_{PS}\,]$

of the static effective theory result are accounted for with good precision

Conclusions

The decay constant in HQET smoothly connects to the continuum data on F_{PS} in the QCD charm quark regime via *linear interpolation in* $1/m$, once

1. renormalization $[\to \text{(NP)}\, Z_{RGI}]$
2. and matching $[\to C_{PS}]$

of the static effective theory result are accounted for with good precision

- Fit of the mass dependence to the form suggested by the HQET expansion

$$r_0^{3/2} \frac{F_{PS}\sqrt{m_{PS}}}{C_{PS}(M/\Lambda_{\overline{MS}})} = A\left(1 + \frac{B}{r_0 m_{PS}}\right) \qquad m_{PS} \gtrsim m_{D_s}$$

$\Rightarrow$ (effective resp. phenomenological) slope $B/(r_0 m_{PS}) \approx -0.5\,\text{GeV}/m_{PS}$

- Accurate knowledge of the HQET current's anomalous dimension is crucial: C_{PS} carries $\approx 50\%$ of the m–dependence of $F_{PS}\sqrt{m_{PS}}$ for $m_{PS} \gtrsim m_{D_s}$ (!)

Conclusions

The decay constant in HQET smoothly connects to the continuum data on F_{PS} in the QCD charm quark regime via *linear interpolation in* $1/m$, once

1. renormalization $[\rightarrow \text{(NP)}\; Z_{RGI}]$
2. and matching $[\rightarrow C_{PS}]$

of the static effective theory result are accounted for with good precision

- Fit of the mass dependence to the form suggested by the HQET expansion

$$r_0^{3/2}\,\frac{F_{PS}\sqrt{m_{PS}}}{C_{PS}\,(M/\Lambda_{\overline{MS}})} = A\left(1+\frac{B}{r_0 m_{PS}}\right) \qquad m_{PS} \gtrsim m_{D_s}$$

$\Rightarrow$ (effective resp. phenomenological) slope $B/(r_0 m_{PS}) \approx -0.5\,\text{GeV}/m_{PS}$

- Accurate knowledge of the HQET current's anomalous dimension is crucial: C_{PS} carries $\approx 50\%$ of the m–dependence of $F_{PS}\sqrt{m_{PS}}$ for $m_{PS} \gtrsim m_{D_s}$ (!)

- The (safe) interpolation yields for the quenched B_s–meson decay constant:

$$r_0^{3/2}\,F_{B_s}\sqrt{m_{B_s}}\,/C_{PS}(M_b/\Lambda_{\overline{MS}}) = 1.487(43) \quad \Rightarrow \quad F_{B_s} = 193(6)\,\text{MeV}$$

Conclusions

The decay constant in HQET smoothly connects to the continuum data on F_{PS} in the QCD charm quark regime via *linear interpolation in* $1/m$, once

1. renormalization $[\rightarrow \text{(NP)}\, Z_{RGI}]$
2. and matching $[\rightarrow C_{PS}]$

of the static effective theory result are accounted for with good precision

Cautionary remark

Natural to calculate the $1/m$–term directly in HQET, but then one faces the general problem in determining power corrections in QCD:

With the l–loop AD and $(l-1)$–loop C_{match}, C_{PS} has an error of

$$\Delta(C_{PS}) \propto \left[\bar{g}^2(m_b)\right]^l \sim \left\{\frac{1}{2b_0 \ln(m_b/\Lambda_{QCD})}\right\}^l \overset{m_b\to\infty}{\gg} \frac{\Lambda_{QCD}}{m_b}$$

⇒ As for $m \gg 1$ unknown pert. corrections in C_{PS} dominate over the NP $1/m$–term, a *true* HQET expansion requires a *NP matching of HQET and QCD, resp. a NP estimate of* C_{PS} [H. & Sommer, 2004]

[Blossier, Della Morte, Garron, Sommer & Papinutto, 2006 & 2007]