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# Minimal Flavor Violation, Seesaw, and R-parity

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*Based on: E. Nikolidakis, C.S., arXiv:0710.3129*

- Outline

  *A - Motivation and generalities*

  *B - Minimal Flavor Violation: spurions and expansions*

  *C - Phenomenological consequences*

  *D - Conclusion*

# Motivation and generalities

## • A few generalities about the MSSM

*Supersymmetry*: Unify matter (fermions) and interactions (bosons).

*MSSM*: the simplest (phenomenologically viable) realization of supersymmetry

*Characteristics*:
- Doubling of matter & gauge degrees of freedom,
- Specific Higgs sector (2HDM - type II),
- Few free parameters in its supersymmetric sector.

*Flavor-symmetry*: three generations of (s)quarks and (s)leptons (exact replicas) with identical gauge interactions → Invariance under:

*Chivukula,Georgi '87*

$$G_f = U(3)^5 = \underbrace{SU(3)_Q \times SU(3)_U \times SU(3)_D}_{G_q} \times \underbrace{SU(3)_L \times SU(3)_E}_{G_\ell} \times G_1$$

$$Q \xrightarrow{G_f} g_Q Q,\ \ U \xrightarrow{G_f} U g_U^\dagger,\ \ D \xrightarrow{G_f} D g_D^\dagger,\ \ L \xrightarrow{G_f} g_L L,\ \ E \xrightarrow{G_f} E g_E^\dagger$$

*Superpotential breaks* $G_f$: it sets (but does not explain) the masses and mixings

$$\mathcal{W}_{RPC} = U^I \mathbf{Y}_u^{IJ} (Q^J H_u) - D^I \mathbf{Y}_d^{IJ} (Q^J H_d) - E^I \mathbf{Y}_\ell^{IJ} (L^J H_d) + \mu (H_u H_d)$$

All (s)quark/(s)lepton masses/interactions in terms of the *Yukawas* (alignment).

## • The flavor-related puzzles

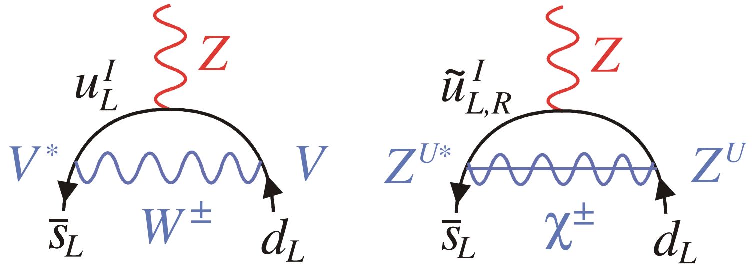
*1. Squark misalignment and FCNC* due to soft SUSY-breaking terms:

$$\mathcal{L}_{soft}^{RPC} \ni -\tilde{Q}^{\dagger}\mathbf{m}_{Q}^{2}\tilde{Q} - \tilde{U}\mathbf{m}_{U}^{2}\tilde{U}^{\dagger} - \tilde{D}\mathbf{m}_{D}^{2}\tilde{D}^{\dagger} - \tilde{U}\mathbf{A}_{u}(\tilde{Q}H_{u}) + \tilde{D}\mathbf{A}_{d}(\tilde{Q}H_{d}) + \ldots$$

Constraints: $\Delta M_{Bs,Bd}, b \to s\gamma, B \to \psi K,$
$B_{s,d} \to \ell^{+}\ell^{-}, B \to X\ell^{+}\ell^{-}, \varepsilon_{K}, K \to \pi\nu\bar{\nu}, \ldots$

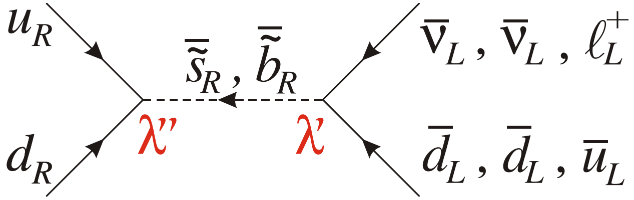
MFV to enforce sufficient alignment.

*Hall,Randall '90, D'Ambrosio,Giudice,Isidori,Strumia '02*



*2. R-parity and proton decay*: $\tau_{p^+} > 10^{30}\ years \Rightarrow |\lambda'\lambda''| \le 10^{-27}$ ?

$$\mathcal{W}_{RPV} = \lambda^{IJK}(L^{I}L^{J})E^{K} + \lambda'^{IJK}(L^{I}Q^{J})D^{K}$$
$$+\lambda''^{IJK}U^{I}D^{J}D^{K} + \mu'^{I}(L^{I}H_{d})$$



Forbidden by R-parity → sparticle pair production, LSP and dark matter,...

*Farrar,Fayet '78*

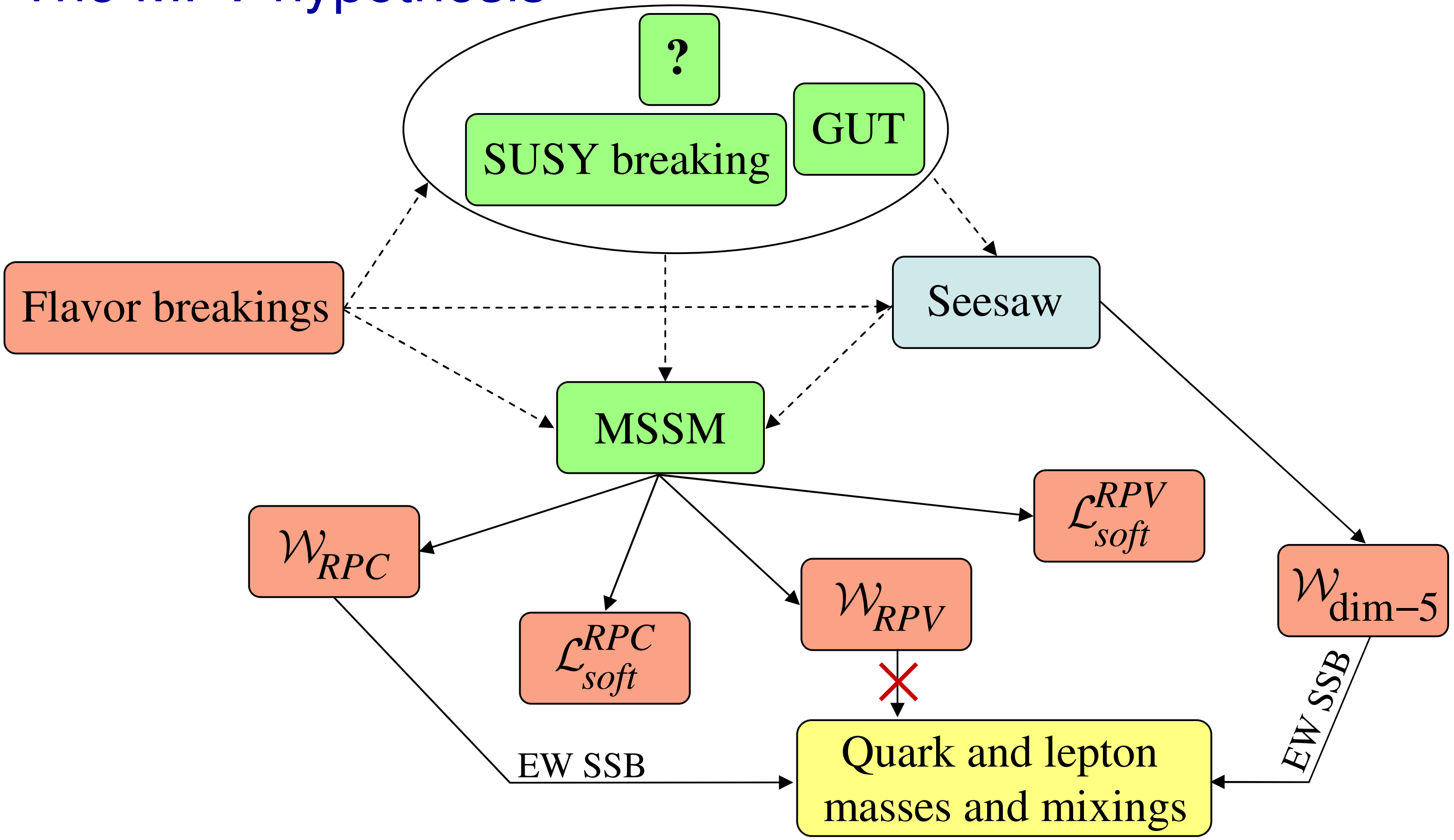
*3. Seesaw and neutrino masses*: In the MSSM, no $\nu_R$ and $\nu_L$ massless.

Seesaw from heavy right-handed neutrinos: $\mathcal{W}_{N} = N^{I}\mathbf{M}^{IJ}N^{J} + N^{I}\mathbf{Y}_{\nu}^{IJ}(L^{J}H_{u})$

$$\to (\mathbf{Y}_{\nu}^{T}\mathbf{M}^{-1}\mathbf{Y}_{\nu})^{IJ}(L^{I}H_{u})(L^{J}H_{u}) \to v_{u}^{2}\nu_{L}^{I}(\mathbf{Y}_{\nu}^{T}\mathbf{M}^{-1}\mathbf{Y}_{\nu})^{IJ}\nu_{L}^{J}$$

*Minkowski '77, Gell-Mann,Ramond,Slansky '79, Yanagida '79, Glashow '79,...*

- The MFV hypothesis



*Assume*: - Simple (though unknown) origin for the flavor symmetry breaking.
- Percolates down to the lowest level → Relations between flavor-breakings.

*Bottom-up approach*: start from the experimentally known flavor-structures.
*Symmetry principle*: elementary sources of flavor-breaking treated as spurions.

*Goal: To show that when MFV is enforced, R-parity is no longer needed.*

# MFV spurions and expansions

- Construction of the MFV expansions

*Spurions and invariance:* Out of a *minimal set of spurions*, i.e. breakings in definite directions in flavor-space, parametrize all the *MSSM flavor-breaking couplings as formally invariant under* $G_f$, up to some *flavor* $U(1)$'s which are a priori broken.

*Hall,Randall '90, D'Ambrosio,Giudice,Isidori,Strumia '02*

1. *Which spurions to choose*: Minimal set able to induce the known flavor-structures.

Yukawas: $\mathbf{Y}_u \xrightarrow{G_f} g_U \mathbf{Y}_u g_Q^\dagger,\ \mathbf{Y}_d \xrightarrow{G_f} g_D \mathbf{Y}_d g_Q^\dagger,\ \mathbf{Y}_\ell \xrightarrow{G_f} g_E \mathbf{Y}_\ell g_L^\dagger \quad \Rightarrow \mathcal{W}_{RPC}$ invariant.

Neutrino: $\mathbf{Y}_\nu \xrightarrow{G_f \times U(3)_N} g_N \mathbf{Y}_\nu g_L^\dagger$

$U(3)_N$ singlets: $\begin{cases} \boldsymbol{\Upsilon}_\nu \equiv v_u \mathbf{Y}_\nu^T \mathbf{M}^{-1} \mathbf{Y}_\nu \xrightarrow{G_f} g_L^* \boldsymbol{\Upsilon}_\nu g_L^\dagger, \\ \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \xrightarrow{G_f} g_L \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu g_L^\dagger \end{cases}$

For simplicity, $\mathbf{M} \equiv M_R \mathbf{1}$,

with $M_R \sim 10^{12-14}\,\mathrm{GeV}$, the seesaw scale.

*Cirigliano,Grinstein,Isidori,Wise '05*

| | $G_q$ <br> $U(3)_{Q,U,D}$ | $G_\ell$ <br> $U(3)_{L,E}$ |
|---|---|---|
| $\mathbf{Y}_u$ | $(\bar{3},3,1)$ | $(1,1)$ |
| $\mathbf{Y}_d$ | $(\bar{3},1,3)$ | $(1,1)$ |
| $\mathbf{Y}_\ell$ | $(1,1,1)$ | $(\bar{3},3)$ |
| $\boldsymbol{\Upsilon}_\nu$ | $(1,1,1)$ | $(\bar{6},1)$ |
| $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ | $(1,1,1)$ | $(8,1)$ |

Background: $\mathbf{Y}_u = \dfrac{\mathbf{m}_u}{v_u} V,\ \mathbf{Y}_d = \dfrac{\mathbf{m}_d}{v_d},\ \mathbf{Y}_\ell = \dfrac{\mathbf{m}_\ell}{v_d},\ \boldsymbol{\Upsilon}_\nu = U^* \dfrac{\mathbf{m}_\nu}{v_u} U^\dagger,\ \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \overset{CP}{=} \dfrac{M_R}{v_u} \boldsymbol{\Upsilon}_\nu$

# MFV expansion

*2. Invariants*: contract spurions and fields using the invariant tensors $\delta^{IJ}$, $\varepsilon^{IJK}$

$$Q^{\dagger I}(\mathrm{m}_Q^2)^{IJ}Q^J \;\Rightarrow\; \mathrm{m}_Q^2 \overset{G_f}{\rightarrow} g_Q \mathrm{m}_Q^2 g_Q^\dagger \;\Rightarrow\; \mathrm{m}_Q^2 = m_0^2(a_0 1 + a_1 \mathrm{Y}_u^\dagger \mathrm{Y}_u + a_2 \mathrm{Y}_d^\dagger \mathrm{Y}_d + \ldots)$$

→ Where $a_i \sim O(1)$ MFV coefficents,
→ The CKM matrix remains the only source of flavor-breaking.

Sometimes large (but always *finite*!) number of possible terms.

(*Cayley-Hamilton Theorem* + *Third generation dominance for* $u,d,\ell$)

$$\mathrm{m}_Q^2 = m_0^2[\mathrm{R}_q]_{h.c.}, \quad \mathrm{R}_q = 1,\ \mathrm{Y}_u^\dagger \mathrm{Y}_u,\ \mathrm{Y}_d^\dagger \mathrm{Y}_d,\ \mathrm{Y}_u^\dagger \mathrm{Y}_u \mathrm{Y}_d^\dagger \mathrm{Y}_d,\ \mathrm{Y}_d^\dagger \mathrm{Y}_d \mathrm{Y}_u^\dagger \mathrm{Y}_u \sim \mathbf{8}_Q$$

Similar for leptons, with nine terms for $\mathrm{R}_\ell \sim \mathbf{8}_L$
└──→ Useful for LFV effects. *Borzumati,Masiero '86*

*3. At least one $\varepsilon$-tensor for RPV terms* → invariance only under $G_q \times G_\ell = SU(3)^5$

$$\lambda''^{IJK} = \varepsilon^{LJK}(\mathrm{Y}_u \mathrm{Y}_d^\dagger)^{IL} \qquad \Rightarrow \lambda''^{IJK} U^I D^J D^K \rightarrow \det(g_D^\dagger)\lambda''^{IJK} U^I D^J D^K$$

$$\lambda''^{IJK} = \varepsilon^{IMN}(\mathrm{Y}_d \mathrm{Y}_u^\dagger)^{JM}(\mathrm{Y}_d \mathrm{Y}_u^\dagger)^{KN} \qquad \Rightarrow \lambda''^{IJK} U^I D^J D^K \rightarrow \det(g_U^\dagger)\lambda''^{IJK} U^I D^J D^K$$

$$\lambda''^{IJK} = \varepsilon^{LMN}\mathrm{Y}_u^{IL}\mathrm{Y}_d^{JM}\mathrm{Y}_d^{KN} \qquad \Rightarrow \lambda''^{IJK} U^I D^J D^K \rightarrow \det(g_Q^\dagger)\lambda''^{IJK} U^I D^J D^K$$

Flavor-directions in which baryon and lepton-numbers are violated are free.
In other words: some $U(1)$'s can still be enforced (but not the five of them).

# MFV expansion

| Structures and MFV terms | | Scaling | Breaking |
|---|---|---|---|
| $\mu_1'^{I}$ | $\mu\overline{\Upsilon}_\nu^I, \quad \overline{\Upsilon}_\nu^I \equiv \varepsilon^{QMJ}(\mathbf{R}_\ell \Upsilon_\nu^\dagger \mathbf{R}_\ell^T)^{QM}\mathbf{R}_\ell^{JI}$ | $\tan^2\beta$ | $U(1)_L$ |
| $\lambda_1^{IJK}$ | $\overline{\Upsilon}_\nu^I(\mathbf{Y}_\ell \mathbf{R}_\ell)^{KJ}$ | $\tan^3\beta$ | $U(1)_L$ |
| $\lambda_2^{IJK}$ | $\varepsilon^{LMN}\mathbf{R}_\ell^{LI}(\mathbf{Y}_\ell \mathbf{R}_\ell \Upsilon_\nu^\dagger \mathbf{R}_\ell^T)^{KM}\mathbf{R}_\ell^{NJ}$ | $\tan\beta$ | $U(1)_L$ |
| $\lambda_3^{IJK}$ | $\overline{\Upsilon}_\nu^I \varepsilon^{LMN}\varepsilon^{ABC}\mathbf{R}_e^{KA}(\mathbf{R}_\ell \mathbf{Y}_\ell^\dagger)^{LB}(\mathbf{R}_\ell \mathbf{Y}_\ell^\dagger)^{MC}\mathbf{R}_\ell^{NJ}, \ldots$ | $\tan^4\beta$ | $U(1)_{L,E}$ |
| $\lambda_1'^{IJK}$ | $\overline{\Upsilon}_\nu^I(\mathbf{Y}_d \mathbf{R}_q)^{KJ}$ | $\tan^3\beta$ | $U(1)_L$ |
| $\lambda_2'^{IJK}$ | $\overline{\Upsilon}_\nu^I \varepsilon^{LMN}\varepsilon^{ABC}\mathbf{R}_d^{KA}(\mathbf{R}_q \mathbf{Y}_d^\dagger)^{LB}(\mathbf{R}_q \mathbf{Y}_d^\dagger)^{MC}\mathbf{R}_q^{NJ}$ | $\tan^4\beta$ | $U(1)_{L,D,Q}$ |
| $\lambda_1''^{IJK}$ | $\varepsilon^{LMN}(\mathbf{Y}_u \mathbf{R}_q \mathbf{Y}_d^\dagger)^{IL}\mathbf{R}_d^{JM}\mathbf{R}_d^{KN}$ | $\tan\beta$ | $U(1)_D$ |
| $\lambda_2''^{IJK}$ | $\varepsilon^{LMN}\mathbf{R}_u^{IL}(\mathbf{Y}_d \mathbf{R}_q \mathbf{Y}_u^\dagger)^{JM}(\mathbf{Y}_d \mathbf{R}_q \mathbf{Y}_u^\dagger)^{KN}$ | $\tan^2\beta$ | $U(1)_U$ |
| $\lambda_3''^{IJK}$ | $\varepsilon^{LMN}(\mathbf{Y}_u \mathbf{R}_q)^{IL}(\mathbf{Y}_d \mathbf{R}_q)^{JM}(\mathbf{Y}_d \mathbf{R}_q)^{KN}$ | $\tan^2\beta$ | $U(1)_Q$ |
| $\lambda_4''^{IJK}$ | $\varepsilon^{LMN}\varepsilon^{ABC}\varepsilon^{DEF}(\mathbf{R}_q \mathbf{Y}_d^\dagger)^{LD}(\mathbf{R}_q \mathbf{Y}_u^\dagger)^{MA}(\mathbf{R}_q \mathbf{Y}_u^\dagger)^{NB}\mathbf{R}_u^{IC}\mathbf{R}_d^{JE}\mathbf{R}_d^{KF}$ | $\tan\beta$ | $U(1)_{Q,U,D}$ |

*Only the minimal set of spurions is needed!*

Spurion $\Upsilon_\nu \sim (\overline{6},1)_{G_\ell}$ needed for $\Delta L = 1$ couplings → suppressed by neutrino masses.

→ $\Delta L = 1$ forbidden when $m_\nu = 0$.

# MFV expansion

| Structures and MFV terms | | Scaling | Breaking |
|---|---|---|---|
| $\mu_1'^{I}$ | $\mu\overline{\Upsilon}_\nu^I,\quad \overline{\Upsilon}_\nu^I \equiv \varepsilon^{QMJ}(\mathrm{R}_\ell \Upsilon_\nu^\dagger \mathrm{R}_\ell^T)^{QM}\mathrm{R}_\ell^{JI}$ | $\tan^2\beta$ | $U(1)_L$ |
| $\lambda_1^{IJK}$ | $\overline{\Upsilon}_\nu^I(\mathrm{Y}_\ell \mathrm{R}_\ell)^{KJ}$ | $\tan^3\beta$ | $U(1)_L$ |
| $\lambda_2^{IJK}$ | $\varepsilon^{LMN}\mathrm{R}_\ell^{LI}(\mathrm{Y}_\ell \mathrm{R}_\ell \Upsilon_\nu^\dagger \mathrm{R}_\ell^T)^{KM}\mathrm{R}_\ell^{NJ}$ | $\tan\beta$ | $U(1)_L$ |
| $\lambda_3^{IJK}$ | $\overline{\Upsilon}_\nu^I\varepsilon^{LMN}\varepsilon^{ABC}\mathrm{R}_e^{KA}(\mathrm{R}_\ell \mathrm{Y}_\ell^\dagger)^{LB}(\mathrm{R}_\ell \mathrm{Y}_\ell^\dagger)^{MC}\mathrm{R}_\ell^{NJ},\ldots$ | $\tan^4\beta$ | $U(1)_{L,E}$ |
| $\lambda_1'^{IJK}$ | $\overline{\Upsilon}_\nu^I(\mathrm{Y}_d \mathrm{R}_q)^{KJ}$ | $\tan^3\beta$ | $U(1)_L$ |
| $\lambda_2'^{IJK}$ | $\overline{\Upsilon}_\nu^I\varepsilon^{LMN}\varepsilon^{ABC}\mathrm{R}_d^{KA}(\mathrm{R}_q \mathrm{Y}_d^\dagger)^{LB}(\mathrm{R}_q \mathrm{Y}_d^\dagger)^{MC}\mathrm{R}_q^{NJ}$ | $\tan^4\beta$ | $U(1)_{L,D,Q}$ |
| $\lambda_1''^{IJK}$ | $\varepsilon^{LMN}(\mathrm{Y}_u \mathrm{R}_q \mathrm{Y}_d^\dagger)^{IL}\mathrm{R}_d^{JM}\mathrm{R}_d^{KN}$ | $\tan\beta$ | $U(1)_D$ |
| $\lambda_2''^{IJK}$ | $\varepsilon^{LMN}\mathrm{R}_u^{IL}(\mathrm{Y}_d \mathrm{R}_q \mathrm{Y}_u^\dagger)^{JM}(\mathrm{Y}_d \mathrm{R}_q \mathrm{Y}_u^\dagger)^{KN}$ | $\tan^2\beta$ | $U(1)_U$ |
| $\lambda_3''^{IJK}$ | $\varepsilon^{LMN}(\mathrm{Y}_u \mathrm{R}_q)^{IL}(\mathrm{Y}_d \mathrm{R}_q)^{JM}(\mathrm{Y}_d \mathrm{R}_q)^{KN}$ | $\tan^2\beta$ | $U(1)_Q$ |
| $\lambda_4''^{IJK}$ | $\varepsilon^{LMN}\varepsilon^{ABC}\varepsilon^{DEF}(\mathrm{R}_q \mathrm{Y}_d^\dagger)^{LD}(\mathrm{R}_q \mathrm{Y}_u^\dagger)^{MA}(\mathrm{R}_q \mathrm{Y}_u^\dagger)^{NB}\mathrm{R}_u^{IC}\mathrm{R}_d^{JE}\mathrm{R}_d^{KF}$ | $\tan\beta$ | $U(1)_{Q,U,D}$ |

- All $\Delta L = 1$ couplings further suppressed by lepton-mass factors ( $\Upsilon_\nu$ symmetric).

$$\varepsilon^{QMI}(\Upsilon_\nu^\dagger)^{QM} \equiv 0 \quad \Rightarrow \quad \overline{\Upsilon}_\nu^I = \varepsilon^{QMI}(\mathrm{Y}_\ell^\dagger \mathrm{Y}_\ell \Upsilon_\nu^\dagger)^{QM} + \ldots$$

- All couplings scale at least linearly with $\tan\beta$.

# MFV expansion

| Structures and MFV terms | | Scaling | Breaking |
|---|---|---|---|
| $\mu_1^{\prime I}$ | $\mu\bar{\Upsilon}_\nu^I,\ \ \bar{\Upsilon}_\nu^I \equiv \varepsilon^{QMJ}(\mathrm{R}_\ell \Upsilon_\nu^\dagger \mathrm{R}_\ell^T)^{QM}\mathrm{R}_\ell^{JI}$ | $\tan^2\beta$ | $U(1)_L$ |
| $\lambda_1^{IJK}$ | $\bar{\Upsilon}_\nu^I(\mathrm{Y}_\ell \mathrm{R}_\ell)^{KJ}$ | $\tan^3\beta$ | $U(1)_L$ |
| $\lambda_2^{IJK}$ | $\varepsilon^{LMN}\mathrm{R}_\ell^{LI}(\mathrm{Y}_\ell \mathrm{R}_\ell \Upsilon_\nu^\dagger \mathrm{R}_\ell^T)^{KM}\mathrm{R}_\ell^{NJ}$ | $\tan\beta$ | $U(1)_L$ |
| $\lambda_3^{IJK}$ | $\bar{\Upsilon}_\nu^I \varepsilon^{LMN}\varepsilon^{ABC}\mathrm{R}_e^{KA}(\mathrm{R}_\ell \mathrm{Y}_\ell^\dagger)^{LB}(\mathrm{R}_\ell \mathrm{Y}_\ell^\dagger)^{MC}\mathrm{R}_\ell^{NJ},\ldots$ | $\tan^4\beta$ | $U(1)_{L,E}$ |
| $\lambda_1^{\prime IJK}$ | $\bar{\Upsilon}_\nu^I(\mathrm{Y}_d \mathrm{R}_q)^{KJ}$ | $\tan^3\beta$ | $U(1)_L$ |
| $\lambda_2^{\prime IJK}$ | $\bar{\Upsilon}_\nu^I \varepsilon^{LMN}\varepsilon^{ABC}\mathrm{R}_d^{KA}(\mathrm{R}_q \mathrm{Y}_d^\dagger)^{LB}(\mathrm{R}_q \mathrm{Y}_d^\dagger)^{MC}\mathrm{R}_q^{NJ}$ | $\tan^4\beta$ | $U(1)_{L,D,Q}$ |
| $\lambda_1^{\prime\prime IJK}$ | $\varepsilon^{LMN}(\mathrm{Y}_u \mathrm{R}_q \mathrm{Y}_d^\dagger)^{IL}\mathrm{R}_d^{JM}\mathrm{R}_d^{KN}$ | $\tan\beta$ | $U(1)_D$ |
| $\lambda_2^{\prime\prime IJK}$ | $\varepsilon^{LMN}\mathrm{R}_u^{IL}(\mathrm{Y}_d \mathrm{R}_q \mathrm{Y}_u^\dagger)^{JM}(\mathrm{Y}_d \mathrm{R}_q \mathrm{Y}_u^\dagger)^{KN}$ | $\tan^2\beta$ | $U(1)_U$ |
| $\lambda_3^{\prime\prime IJK}$ | $\varepsilon^{LMN}(\mathrm{Y}_u \mathrm{R}_q)^{IL}(\mathrm{Y}_d \mathrm{R}_q)^{JM}(\mathrm{Y}_d \mathrm{R}_q)^{KN}$ | $\tan^2\beta$ | $U(1)_Q$ |
| $\lambda_4^{\prime\prime IJK}$ | $\varepsilon^{LMN}\varepsilon^{ABC}\varepsilon^{DEF}(\mathrm{R}_q \mathrm{Y}_d^\dagger)^{LD}(\mathrm{R}_q \mathrm{Y}_u^\dagger)^{MA}(\mathrm{R}_q \mathrm{Y}_u^\dagger)^{NB}\mathrm{R}_u^{IC}\mathrm{R}_d^{JE}\mathrm{R}_d^{KF}$ | $\tan\beta$ | $U(1)_{Q,U,D}$ |

- The invariance under some $U(1)$'s can be enforced.
  → Does not forbid any structure, but suppresses them by $\det(\mathrm{Y}_{u,d,\ell})$ factors.

- Similar expansions for RPV soft-breaking terms (up to their normalization).

## MFV expansion

| Structures and MFV terms | | Scaling | Breaking |
|---|---|---|---|
| $\mu_1'^I$ | $\mu\bar{\Upsilon}_\nu^I,\quad \bar{\Upsilon}_\nu^I \equiv \varepsilon^{QMJ}(\mathrm{R}_\ell \Upsilon_\nu^\dagger \mathrm{R}_\ell^T)^{QM}\mathrm{R}_\ell^{JI}$ | $\tan^2\beta$ | $U(1)_L$ |
| $\lambda_1^{IJK}$ | $\bar{\Upsilon}_\nu^I(\mathrm{Y}_\ell \mathrm{R}_\ell)^{KJ}$ | $\tan^3\beta$ | $U(1)_L$ |
| $\lambda_2^{IJK}$ | $\varepsilon^{LMN}\mathrm{R}_\ell^{LI}(\mathrm{Y}_\ell \mathrm{R}_\ell \Upsilon_\nu^\dagger \mathrm{R}_\ell^T)^{KM}\mathrm{R}_\ell^{NJ}$ | $\tan\beta$ | $U(1)_L$ |
| $\lambda_3^{IJK}$ | $\bar{\Upsilon}_\nu^I \varepsilon^{LMN}\varepsilon^{ABC}\mathrm{R}_e^{KA}(\mathrm{R}_\ell \mathrm{Y}_\ell^\dagger)^{LB}(\mathrm{R}_\ell \mathrm{Y}_\ell^\dagger)^{MC}\mathrm{R}_\ell^{NJ},\ldots$ | $\tan^4\beta$ | $U(1)_{L,E}$ |
| $\lambda_1'^{IJK}$ | $\bar{\Upsilon}_\nu^I(\mathrm{Y}_d \mathrm{R}_q)^{KJ}$ | $\tan^3\beta$ | $U(1)_L$ |
| $\lambda_2'^{IJK}$ | $\bar{\Upsilon}_\nu^I \varepsilon^{LMN}\varepsilon^{ABC}\mathrm{R}_d^{KA}(\mathrm{R}_q \mathrm{Y}_d^\dagger)^{LB}(\mathrm{R}_q \mathrm{Y}_d^\dagger)^{MC}\mathrm{R}_q^{NJ}$ | $\tan^4\beta$ | $U(1)_{L,D,Q}$ |
| $\lambda_1''^{IJK}$ | $\varepsilon^{LMN}(\mathrm{Y}_u \mathrm{R}_q \mathrm{Y}_d^\dagger)^{IL}\mathrm{R}_d^{JM}\mathrm{R}_d^{KN}$ | $\tan\beta$ | $U(1)_D$ |
| $\lambda_2''^{IJK}$ | $\varepsilon^{LMN}\mathrm{R}_u^{IL}(\mathrm{Y}_d \mathrm{R}_q \mathrm{Y}_u^\dagger)^{JM}(\mathrm{Y}_d \mathrm{R}_q \mathrm{Y}_u^\dagger)^{KN}$ | $\tan^2\beta$ | $U(1)_U$ |
| $\lambda_3''^{IJK}$ | $\varepsilon^{LMN}(\mathrm{Y}_u \mathrm{R}_q)^{IL}(\mathrm{Y}_d \mathrm{R}_q)^{JM}(\mathrm{Y}_d \mathrm{R}_q)^{KN}$ | $\tan^2\beta$ | $U(1)_Q$ |
| $\lambda_4''^{IJK}$ | $\varepsilon^{LMN}\varepsilon^{ABC}\varepsilon^{DEF}(\mathrm{R}_q \mathrm{Y}_d^\dagger)^{LD}(\mathrm{R}_q \mathrm{Y}_u^\dagger)^{MA}(\mathrm{R}_q \mathrm{Y}_u^\dagger)^{NB}\mathrm{R}_u^{IC}\mathrm{R}_d^{JE}\mathrm{R}_d^{KF}$ | $\tan\beta$ | $U(1)_{Q,U,D}$ |

- Basis-dependent separation RPV–RPC ($H_d \leftrightarrow L^I$), *but MFV expansions are stable*.

Near-alignment: $\mu'^I = \mu\bar{\Upsilon}_\nu^I, b'^I = b\bar{\Upsilon}_\nu^I, m_{Ld}^I = m_{H_d}^2\bar{\Upsilon}_\nu^I \;\Rightarrow \langle\tilde{\nu}^I\rangle = v_d\bar{\Upsilon}_\nu^I$

$\Rightarrow$ Physical RPV corrections to $m_\nu \sim O(m_\nu^2)$.

*Hall,Suzuki '84*
*Banks et al '95*
*Davidson,Ellis '97*
*Grossman,Haber '99, ...*

# Phenomenological consequences

## • Numerical estimates: Preliminaries

*Antisymmetry and suppression:* Besides the proportionality to *neutrino masses*, the antisymmetric $\varepsilon$-tensors imply that all RPV couplings are proportional to *light-fermion masses*, hence significantly suppressed.

Dependence is quite involved, and different from the naive helicity-suppression:

$$\lambda'^{IJK} \neq O(m_e^I m_u^J m_d^K),\ \ \lambda''^{IJK} \neq O(m_u^I m_d^J m_d^K),\ \ldots$$

*Reduced basis:* Not all operators of equal size, some can always be neglected.

- However: the reduced basis, with the minimal number of operators, strongly depends on $\tan\beta$, lightest neutrino mass $m_\nu$, and the seesaw scale $M_R$.
- For $\tan\beta < 20$, $M_R < 2\cdot 10^{13}\,GeV$, $m_\nu > 0.05\,eV$: only $10$ to $20$ dominant terms.

*Numerical estimates* for the maximal order of magnitudes in four extreme scenarios:

$\mathrm{Case\,I:}\quad \tan\beta = 5 \quad M_R = 10^{12}\,GeV \quad m_\nu = 0.5\,eV$

$\mathrm{Case\,II:}\quad \tan\beta = 50 \quad M_R = 10^{12}\,GeV \quad m_\nu = 0.5\,eV$

$\mathrm{Case\,III:}\quad \tan\beta = 5 \quad M_R = 2\cdot 10^{14}\,GeV \quad m_\nu = 0\,eV$

$\mathrm{Case\,IV:}\quad \tan\beta = 50 \quad M_R = 2\cdot 10^{14}\,GeV \quad m_\nu = 0\,eV$

Cases II & IV: maximize $H_d$ Yukawa couplings $\mathrm{Y}_d$ and $\mathrm{Y}_\ell$,

Cases III & IV: maximize neutrino spurions $\Upsilon_\nu$ and $\mathrm{Y}_\nu^\dagger \mathrm{Y}_\nu$.

- ## Freezing the spurions

*Our goal*: Leading, order-of-magnitude estimates in terms of $\tan\beta, M_R, m_\nu$, assuming MFV coefficients are not larger than about one.

1. *Quark and charged lepton* Yukawas fixed from the PDG masses and CKM:

$$\mathrm{Y}_u = \frac{\mathrm{m}_u}{v_u} V,\ \mathrm{Y}_d = \frac{\mathrm{m}_d}{v_u}\tan\beta,\ \mathrm{Y}_\ell = \frac{\mathrm{m}_\ell}{v_u}\tan\beta$$

2. *Neutrino masses*: set $\theta_{13} \approx 0, \theta_{atm} \approx 45^\circ$ and $\tan\theta_\odot \approx 2/3$ such that

$$\Upsilon_\nu = U^* \frac{\mathrm{m}_\nu}{v_u} U^\dagger \approx \frac{1}{v_u}\left( m_\nu 1 + \frac{\Delta m_{21}}{3}\begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix} + \frac{\Delta m_{31}}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}\right)$$

- As $m_\nu \equiv m_{\nu 1}$ increases, $\Upsilon_\nu$ gets more diagonal (normal spectrum for larger effects) for fixed $\Delta m_\odot^2 \approx 8\cdot 10^{-5}\,\mathrm{eV}^2, \Delta m_{atm}^2 \approx 2.6\cdot 10^{-3}\,\mathrm{eV}^2$.
- For most operators, the piece $\sim 1$ drops out since $\bar{\Upsilon}_\nu^I = \varepsilon^{QMI}(\mathrm{Y}_\ell^\dagger \mathrm{Y}_\ell \Upsilon_\nu^\dagger)^{QM} + \ldots$

3. *Neutrino octet spurion* fixed in the CP-limit: $\mathrm{Y}_\nu^\dagger \mathrm{Y}_\nu \overset{CP}{=} \frac{M_R}{v_u}\Upsilon_\nu$

Perturbative bound: $\mathrm{Y}_\nu^\dagger \mathrm{Y}_\nu \leq 1 \rightarrow \frac{\max[m_\nu, \Delta m_{31}]}{1\,\mathrm{eV}} \frac{M_R}{10^{13}\,\mathrm{GeV}} \leq 3$

- Order-of-magnitude estimates

*Example 1*. Auxiliary neutrino spurion $\overline{\Upsilon}_\nu^I \equiv \varepsilon^{QMJ}(\mathrm{R}_\ell \Upsilon_\nu^\dagger \mathrm{R}_\ell^T)^{QM}\mathrm{R}_\ell^{JI}$:

large $\tan\beta$

$$\overline{\Upsilon}_\nu^I = \begin{pmatrix}17\\19\\21\end{pmatrix}, \begin{pmatrix}15\\17\\19\end{pmatrix}, \begin{pmatrix}16\\17\\18\end{pmatrix}, \begin{pmatrix}14\\15\\16\end{pmatrix},$$

large $M_R$, small $m_\nu$

- Tunes $\mu'(H_u L)$ and $\lambda'(LQ)D$,
- Similar behavior for $\lambda(LL)E$,
- Inverted hierarchy,
- Very suppressed ($M_R = 10^{12-14}$ GeV),
- Quadratic in $\tan\beta$,
- $M_R \nearrow$ and/or $m_\nu \searrow$: hierarchies softened.

*Example 2*. Baryonic couplings $\lambda''^{IJK} U^I D^J D^K$:

| Structure | $\lambda''_1$ | $\lambda''_2$ | $\lambda''_3$ | $\lambda''_{4,5}$ |
|---|---|---|---|---|
| Broken $U(1)$ | $U(1)_D$ | $U(1)_U$ | $U(1)_Q$ | $U(1)_{U,D,Q}$ |
| Scaling | $\tan\beta$ | $\tan^2\beta$ | $\tan^2\beta$ | $\tan\beta$ |
| $\tan\beta = 5$ | $\begin{pmatrix}8&8&8\\4&6&5\\1&6&4\end{pmatrix}$ | $\begin{pmatrix}11&6&7\\12&9&9\\13&12&13\end{pmatrix}$ | $\begin{pmatrix}13&8&10\\10&6&7\\6&5&6\end{pmatrix}$ | $\begin{pmatrix}5&5&5\\7&9&7\\7&12&10\end{pmatrix}$ |
| $\tan\beta = 50$ | $\begin{pmatrix}7&7&7\\3&5&4\\0&5&3\end{pmatrix}$ | $\begin{pmatrix}9&4&5\\10&7&7\\11&10&11\end{pmatrix}$ | $\begin{pmatrix}11&6&8\\8&4&5\\4&3&4\end{pmatrix}$ | $\begin{pmatrix}4&4&4\\6&8&6\\6&11&9\end{pmatrix}$ |

*Notation*: $x \equiv O(10^{-x})$

$X^{IJ} \equiv \lambda''^{I(JK)}$,

$(JK) = 12, 23, 31$

# • Bounds on RPV couplings

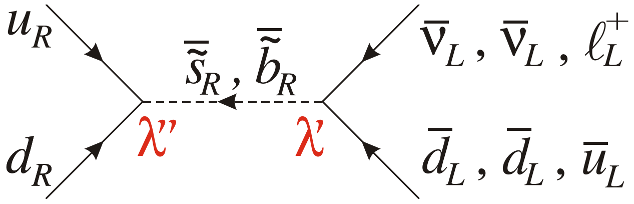
Barbier et al. '05

*1. Bounds from $\Delta L = 1$, $\Delta B = 0$ processes:*

All easily satisfied, because $\Delta L = 1$ couplings $\mu', \lambda, \lambda' < O(10^{-13})$.

*2. Bounds from $\Delta B = 1$ nucleon decays:* $p, n \to \pi\nu, \pi\ell, K\nu, K\ell, \ldots$

Tree-level:



Very constraining (see next slide).

Loop-level:

$$\left|\lambda'_{IJK}\lambda''_{I'J'K'}\right| < O(10^{-9} - 10^{-11})$$

Automatic since $\lambda' < O(10^{-13})$.

*3. Bounds from neutron oscillations:*

Tree-level (very approximative):

$$\left|\lambda''_{11I}\right| < (10^{-8} - 10^{-7})\,\frac{10^8\,s}{\tau_{osc}}\left(\frac{\tilde{m}}{100\,\mathrm{GeV}}\right)^{5/2},$$

Loop-level:

$$\left|\lambda''_{312}\right| < [10^{-3}, 10^{-2}]\left(\frac{200\,\mathrm{MeV}}{m_s}\right),$$

for $\tilde{m}_q \sim [100\,\mathrm{GeV}, 200\,\mathrm{GeV}]$.

Not very constraining for squark masses above a few hundred GeV.

## 4. MFV predictions and bounds on $\Delta B = 1$ nucleon decays

| Approximate bounds | $(I, J = 1, 2, 3, M = 1, 2)$ | I | | | II | | | III | | | IV | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | A | B | C | A | B | C | A | B | C | A | B | C |
| $\lvert\lambda'_{JMI}\lambda''^{*}_{11I}, \lambda'_{M1I}\lambda''^{*}_{12I}\rvert$ | $< 10^{-27}\,\tilde{d}_R^I$ | 24 | 25 | 28 | 20 | 20 | 23 | 23 | 24 | 27 | 18 | 19 | 22 |
| $\lvert\lambda'_{IJM}\lambda''^{*}_{11J}\rvert$ | $< 10^{-27}\,\tilde{d}_L^J(\delta_J^D)^{-1}$ | 24 | 28 | 31 | 20 | 23 | 25 | 23 | 27 | 29 | 19 | 21 | 24 |
| $\lvert\lambda'_{MJ1}\lambda''^{*}_{J12}\rvert$ | $< 10^{-26}\,\tilde{u}_L^J(\delta_J^U)^{-1}$ | 23 | 29 | 29 | 18 | 23 | 23 | 21 | 27 | 27 | 16 | 22 | 22 |
| $\lvert\lambda_{212,322}\lambda''^{*}_{112}\rvert$ | $< 10^{-20}(\tilde{m} \sim 1\,\mathrm{TeV})$ | 21 | 27 | 30 | 19 | 23 | 26 | 20 | 26 | 28 | 18 | 22 | 25 |
| $\lvert\lambda_{133,323}\lambda''^{*}_{112}\rvert$ | $< 10^{-21}(\tilde{m} \sim 1\,\mathrm{TeV})$ | 20 | 26 | 28 | 18 | 22 | 25 | 19 | 25 | 27 | 17 | 21 | 24 |
| $\lvert\lambda''_{112}\mu'_I / \mu\rvert$ | $< 10^{-23}\,\tilde{u}_R$ | 22 | 27 | 30 | 19 | 23 | 26 | 20 | 26 | 29 | 17 | 21 | 24 |
| $\lvert\lambda''_{312}\mu'_I / \mu\rvert$ | $< 10^{-16}\,\tilde{d}_R$ | 18 | 23 | 23 | 14 | 18 | 18 | 16 | 22 | 22 | 13 | 17 | 17 |

$\tilde{q} \equiv m_{\tilde{q}}^2 / (100\,\mathrm{GeV})^2$

$\delta_J^X \equiv (m_X^2)_{LR}^{JJ} / (m_X^2)_R^J$

Case II,IV : large $\tan\beta$

Case III, IV : $\begin{cases} \text{large } M_R \\ \text{small } m_\nu \end{cases}$

A : $SU(3)^5$

B : $SU(3)^5 \times U(1)_D \times U(1)_E$

C : $SU(3)^5 \times U(1)_D \times U(1)_E \times U(1)_U$

*Conservative:* - Light-quark masses at 2 GeV, on-shell lepton masses,

- MFV coefficients of $O(1)$, while $O(\lambda)$ or $O(g^2/4\pi)$ equally natural,
- No GIM-like interferences for a given mechanism, and no cancellations among possible mechanisms for a given final state.

- Consequences

*1. FCNC and LFV:* - Moderate $\tan\beta$ preferred → suppresses Higgs FCNC.
- Large $m_\nu$ preferred → suppresses LFV effects.
- Alternative: if $U(1)_L$ imposed, additional factor $\det(\mathbf{Y}_\ell) < 10^{-6}$.

*2. $\Delta B = 1$ effects at low-energy:* squarks as di-quark currents

$$\frac{\left|\lambda''_{IJK}\lambda''^{*}_{LMN}\right|}{\tilde{m}_q^2} \leq 10^{-8}\,\mathrm{GeV}^{-2}\,\frac{(100\,\mathrm{GeV})^2}{\tilde{m}_q^2} \quad \leftrightarrow \quad G_F \sim 10^{-5}\,\mathrm{GeV}^{-2}$$

Largest for the stop exchange, when $\tan\beta$ not large so that $U(1)_D$ can be broken.

$$\left|\lambda''_{312}\lambda''^{*}_{331}\right| \sim 10^{-4} - 10^{-5}, \left|\lambda''_{312}\lambda''^{*}_{323}\right| \sim 10^{-5} - 10^{-6}$$

But, typically small w.r.t. SM contributions, and challenging hadronic uncertainties.

*3. $\Delta B = 1$ effects at colliders:* RPV implies drastic changes for the phenomenology

Accessibility of the signals strongly depends on $\left|\lambda''_{312}\right| \sim 10^{-1} - 10^{-5}$

- *LSP decays*, maybe in the detector, and needs not be colorless and neutral.
- *Single stop* resonant production $pp \to \tilde{t}$, single gluino production $\tilde{t} \to t\tilde{g}$.
- *Top production* from down squark decay $(\tilde{d}, \tilde{s}, \tilde{b}) \to t + (d, s, b)$.
- …

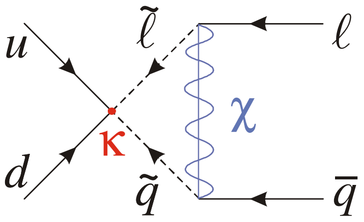
# A few thoughts about possible extensions

*1. Other seesaw types:* Stability of MFV predictions, at least to a large extent.

Seesaw → Majorana mass for $\nu_L$: $\Upsilon_\nu \sim (\bar{6},1)$
Seesaw → Other unsuppressed spurions (here $\mathrm{Y}_\nu^\dagger \mathrm{Y}_\nu \sim (8,1)$)

MFV stability implies that no unsuppressed spurion can transform as $(\bar{6},1)$.

*2. Dim-5 RPC operators:* MFV separately suppresses $\Delta L = 1$ and $\Delta B = 1$ effects,



$$\mathcal{W}_{\mathrm{dim}-5} \ni \frac{\kappa_1^{IJKL}}{\Lambda_{\Delta L=1}}(Q^I Q^J)(Q^K L^L) + \frac{\kappa_2^{IJKL}}{\Lambda_{\Delta L=1}}(D^I U^J U^K)E^L$$

*Ibanez,Ross '92*

Loop-level $\kappa_1$, $\kappa_2$ should be subleading compared to tree-level $\lambda\times\lambda''$, $\lambda'\times\lambda''$.

*3. GUT flavor groups:* leptons and quarks flavor-groups not necessarily factorized.

Example: $G_f = U(3)_{\bar{5}} \times U(3)_{10}$ : $\mathrm{Y}_{\bar{5}} \sim (\bar{3},\bar{3}), \mathrm{Y}_{10} \sim (1,\bar{6})$

*Cirigliano,Grinstein, Isidori,Wise '05*

Seesaw spurions not required for $\mathcal{W}_{RPV} = \Lambda^{IJK}\bar{5}^I\bar{5}^J 10^K + \ldots$

*But:* - Antisymmetric $\varepsilon$-tensors still needed → some suppression remains,
- Flavor-group much smaller, with $U(1)_D \sim U(1)_{\bar{5}}$, $U(1)_{U,E} \sim U(1)_{10}$
→ $\mathcal{W}_{RPV}$ may then only arise after $SU(5)$ is broken.

# Conclusion

# Conclusion

*The MFV hypothesis under the $U(3)^5$ MSSM flavor-group simultaneously:*

- accounts for FCNC suppression (squark-quark alignment)
- suppresses the proton decay width down to acceptable levels.

The *very long proton lifetime* is then seen a direct consequence of:

- Hierarchy in the fermion masses and CKM matrix,
- Smallness of the neutrino masses.

*MSSM R-parity looses its main appeal* (already undermined by dim-5 ops.)

But, may have to be replaced by some flavor U(1)'s.

*Some consequences:*

- Indirect constraints on $m_\nu$, $\tan\beta$, and the seesaw scale (for *FCNC* and *LFV*),
- *Proton decay* presumably close to its current experimental bounds,
- *Colliders*: MFV can tell us where to expect significant SUSY signals,
  In particular: single stop production or top production.

*Extensions:* Consequences still to be investigated. In particular:

- In *GUT's/SUSY-breaking*: ensuring the emergence of MFV is
  a powerful method towards satisfying low-energy constraints.
- For *cosmology*, MSSM-LSP not stable, and baryon number violated?

# Backup

## *Backup 1. Expansions for RPC soft-breaking terms*

Cayley-Hamilton: $\mathrm{A}^3 - \langle\mathrm{A}\rangle\mathrm{A}^2 + \frac{1}{2}\mathrm{A}(\langle\mathrm{A}\rangle^2 - \langle\mathrm{A}^2\rangle) - \frac{1}{3}\langle\mathrm{A}^3\rangle + \frac{1}{2}\langle\mathrm{A}\rangle\langle\mathrm{A}^2\rangle - \frac{1}{6}\langle\mathrm{A}\rangle^3 = 0$

Third generation dominance: $(\mathrm{Y}^\dagger_{u,d,\ell}\mathrm{Y}_{u,d,\ell})^2 = y^2_{t,b,\tau}\mathrm{Y}^\dagger_{u,d,\ell}\mathrm{Y}_{u,d,\ell}$

Octets: $\mathrm{X}_i \equiv \mathrm{Y}_i^\dagger\mathrm{Y}_i\ ,\ \ \mathrm{R}_\ell = 1,\ \mathrm{X}_\ell,\ \mathrm{X}_\nu,\ \mathrm{X}_\ell\mathrm{X}_\nu,\ \mathrm{X}_\nu\mathrm{X}_\ell,\ \mathrm{X}_\nu^2,\ \mathrm{X}_\ell\mathrm{X}_\nu^2,\ \mathrm{X}_\nu^2\mathrm{X}_\ell,\ \mathrm{X}_\nu^2\mathrm{X}_\ell\mathrm{X}_\nu\ ,$

$\mathrm{R}_q = 1,\ \mathrm{X}_u,\ \mathrm{X}_d,\ \mathrm{X}_u\mathrm{X}_d,\ \mathrm{X}_d\mathrm{X}_u\ ,\ \ \mathrm{R}_{u,d} = 1, \mathrm{Y}_{u,d}\mathrm{R}_q\mathrm{Y}^\dagger_{u,d}\ ,\ \ \mathrm{R}_e = 1, \mathrm{Y}_\ell\mathrm{R}_\ell\mathrm{Y}^\dagger_\ell$

RPC-terms: $\mathrm{m}^2_Q = m_0^2[\mathrm{R}_q]_{h.c.},\ \mathrm{m}^2_U = m_0^2[\mathrm{R}_u]_{h.c.},\ \mathrm{m}^2_D = m_0^2[\mathrm{R}_d]_{h.c.},$

$\mathrm{m}^2_L = m_0^2[\mathrm{R}_\ell]_{h.c.},\ \mathrm{m}^2_E = m_0^2[\mathrm{R}_e]_{h.c.}$

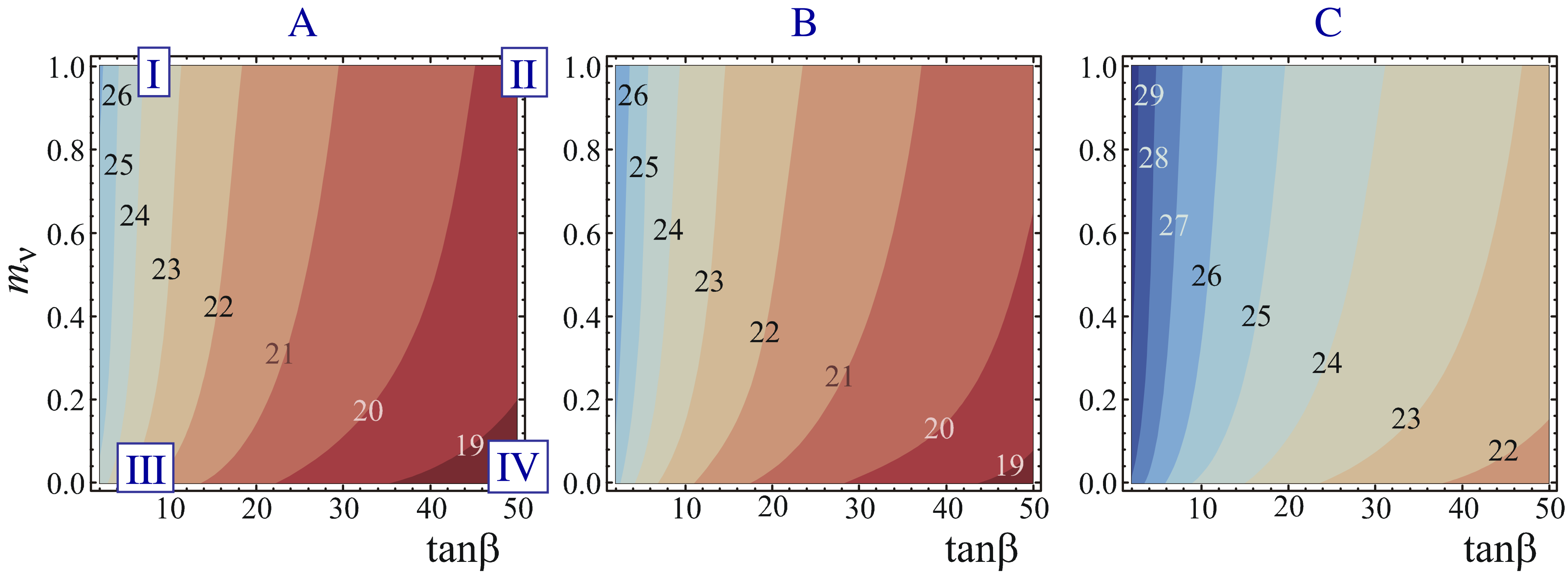
$$\mathrm{A}_u^{IJ} = A_0((\mathrm{Y}_u\mathrm{R}_q)^{IJ} + \varepsilon^{LMN}\varepsilon^{ABC}\mathrm{R}_u^{KA}(\mathrm{R}_q\mathrm{Y}_u^\dagger)^{LB}(\mathrm{R}_q\mathrm{Y}_u^\dagger)^{MC}\mathrm{R}_q^{NJ})$$

$$\mathrm{A}_d^{IJ} = A_0((\mathrm{Y}_d\mathrm{R}_q)^{IJ} + \varepsilon^{LMN}\varepsilon^{ABC}\mathrm{R}_d^{KA}(\mathrm{R}_q\mathrm{Y}_d^\dagger)^{LB}(\mathrm{R}_q\mathrm{Y}_d^\dagger)^{MC}\mathrm{R}_q^{NJ})$$

$$\mathrm{A}_\ell^{IJ} = A_0((\mathrm{Y}_\ell\mathrm{R}_\ell)^{IJ} + \varepsilon^{LMN}\varepsilon^{ABC}\mathrm{R}_e^{KA}(\mathrm{R}_\ell\mathrm{Y}_\ell^\dagger)^{LB}(\mathrm{R}_\ell\mathrm{Y}_\ell^\dagger)^{MC}\mathrm{R}_\ell^{NJ})$$

- Sum with $O(1)$ MFV coefficents,
- For trilinear terms, the $\varepsilon$-structures are new, though small but for 11 mixing,
- LFV effects tuned by $\mathrm{Y}_\nu^\dagger\mathrm{Y}_\nu$, and only a finite number of terms. *Borzumati,Masiero '86*
- Suppression of the FCNC's analyzed in *Isidori et al. '06 / Altmannshofer,Buras,Guadagnoli '07*

# *Backup 2. Behavior of the bound $|\lambda'_{M1I}\lambda''^{*}_{12I}|$ (others are similar)*



I : $\tan\beta = 5$, $M_R = 10^{12}\,GeV$, $m_\nu = 0.5\,eV$,
II : $\tan\beta = 50$, $M_R = 10^{12}\,GeV$, $m_\nu = 0.5\,eV$,
III : $\tan\beta = 5$, $M_R = 2\cdot 10^{14}\,GeV$, $m_\nu = 0\,eV$,
IV : $\tan\beta = 50$, $M_R = 2\cdot 10^{14}\,GeV$, $m_\nu = 0\,eV$,

A : $SU(3)^5$
B : $SU(3)^5 \times U(1)_D \times U(1)_E$
C : $SU(3)^5 \times U(1)_D \times U(1)_E \times U(1)_U$

$(x \equiv O(10^{-x}))$