WHAT IS RESONANCE SATURATION ?

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Based on work done in collaboration with

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Introduction

- $$\mathcal{L}_\chi = \underbrace{\frac{f_\pi^2}{4}\,\mathrm{Tr}\left(D_\mu U D^\mu U^\dagger\right)}_{\mathcal{O}(p^2)} + \underbrace{\overbrace{L_1}^{LEC}\,\mathrm{Tr}\left(D_\mu U D^\mu U^\dagger\right)\mathrm{Tr}\left(D_\mu U D^\mu U^\dagger\right) + \ldots}_{\mathcal{O}(p^4)+\mathcal{O}(p^6)+\ldots}$$

- Proliferation of LECs: $\left\{\begin{array}{ll} \mathcal{O}(p^4) \to & \sim 10^1 \\ \mathcal{O}(p^6) \to & \sim 10^2 \\ \mathcal{O}(p^8) \to & \sim 10^3 \text{ ?} \\ & \vdots \end{array}\right.$

- LECs are indispensable to make predictions.

- $\mathbb{PHYSICS}\,(LECs) = \mathbb{PHYSICS}\,(short\ distances)$.

HOW ?

Resonance Saturation *(aka VMD + extensions)*

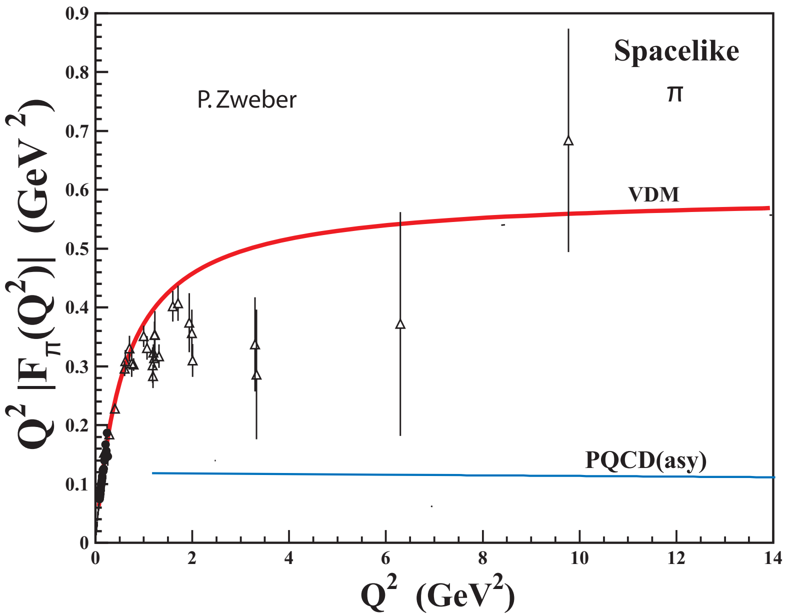
$$< \pi(p')|V_\mu|\pi(p) >= F(Q^2)\,(p'-p)_\mu \quad , \quad -Q^2 = (p'-p)^2$$

$$F(Q^2) = 1 - Q^2 \sum_R^{\infty} \frac{C_R^2}{Q^2 + M_R^2} \qquad (\mathrm{N_c} \to \infty, \quad \text{one subtr.}, \quad \text{meromorphic})$$

$$\approx 16\pi F_\pi^2 \frac{\alpha_s(Q^2)}{Q^2} + \ldots \qquad (Q^2 \text{ large, PQCD})$$

$$\approx 1 - 2L_9 \frac{Q^2}{F_\pi^2} + \ldots \qquad (Q^2 \text{ small})$$

$$\simeq \frac{M_V^2}{Q^2 + M_V^2} \qquad (MHA) \quad , \qquad L_9 \equiv \frac{1}{2}\frac{F_\pi^2}{M_V^2} \sim 7 \times 10^{-3} = L_9(M_\rho)$$



Sakurai '69
Ecker et al. '89
Donoghue et al. '89
Moussallam '97

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Knecht, de Rafael '98
Perrottet, de Rafael, S.P. '98

$$F(Q^2) = 1 - Q^2 \sum_R^{\infty} \frac{C_R^2}{Q^2 + M_R^2}$$

$$\overset{(\star)}{\approx} \frac{M_V^2}{Q^2 + M_V^2}$$

- What is this approximation $(\star)$? (1,2,....∞)
- Where in the complex Q^2 plane does $(\star)$ converge ?
- How are the poles/residues of the approx. $(\star)$ related to the physical counterparts ?

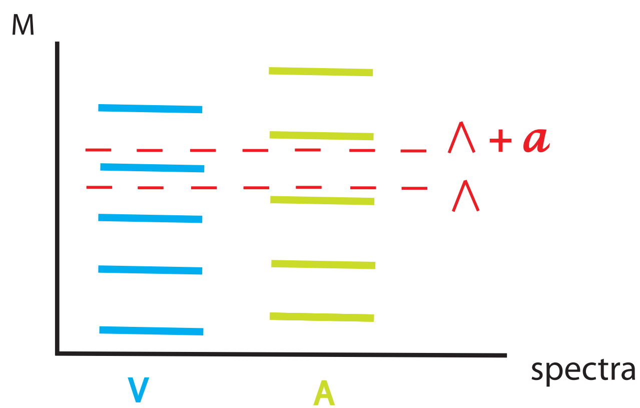
High Energy: WSRs

$< VV - AA >$ with regulator cutoff Λ:

$$Q^2\Pi_{LR}(Q^2) = \lim_{\Lambda\to\infty}\left\{-F_0^2 - Q^2\sum_A^{N_A(\Lambda)}\frac{F_A^2}{Q^2+M_A^2} + Q^2\sum_V^{N_V(\Lambda)}\frac{F_V^2}{Q^2+M_V^2}\right\}$$

$$\sum_V^{N_V(\Lambda)}\frac{F_V^2}{Q^2+M_V^2} \sim \sum_A^{N_A(\Lambda)}\frac{F_A^2}{Q^2+M_A^2} \sim \log\frac{\Lambda^2}{Q^2} + \mathcal{O}(\frac{1}{\Lambda^2}) \quad , \quad (\Lambda^2 \gg Q^2 \to \infty)$$

Universality $\Rightarrow \Pi_{V,A}(Q^2)$ invariant under $\Lambda \to \Lambda + a \quad , \quad a \ll \Lambda$



$$N_V(\Lambda) \quad \to \quad N_V(\Lambda + a) = N_V(\Lambda) + 1$$
$$N_A(\Lambda) \quad \to \quad N_A(\Lambda + a) = N_A(\Lambda)$$

High Energy: WSRs *(and II)*

$$Q^2\Pi_{LR}(Q^2)|_{Q^2\to\infty} \sim \lim_{\Lambda\to\infty} \left(\underbrace{-F_0^2 - \sum_A^{N_A(\Lambda)} F_A^2 + \sum_V^{N_V(\Lambda)} F_V^2}_{WSR1} \right.$$

$$\left. - \frac{1}{Q^2} \underbrace{\left[\sum_A^{N_A(\Lambda)} F_A^2 M_A^2 - \sum_V^{N_V(\Lambda)} F_V^2 M_V^2 \right]}_{WSR2} + \underbrace{\mathcal{O}(\frac{1}{Q^4})}_{\log Q^2 ?} \right)$$

$WSRs$ not invariant under $N_A \to N_A$ and $N_V \to N_V + 1$ (Golterman, S.P. '03)

Regge $\Longrightarrow$ $M^2_{V_n,A_n} \sim n$, $F^2_{V_n,A_n} \sim F^2$ (parton model O.K.)

$$WSR1 \sim \lim_{\Lambda\to\infty} \left[\sum_A^{N_A(\Lambda)} 1 - \sum_V^{N_V(\Lambda)} 1 \right] = \ ??$$

- Physical masses and decay constants do not obey WSRs.

Low Energy: L_8

$$< SS - PP >: \quad \Pi'_{S-P}(Q^2) = \lim_{\Lambda\to\infty} \int_0^{\Lambda^2} \frac{dt}{t+Q^2} \left(\rho_S(t) - \rho'_P(t)\right)$$

$$L_8 \sim \lim_{\Lambda, N_c\to\infty} \int_0^{\Lambda^2} \frac{dt}{t} \left(\rho_S(t) - \rho'_P(t)\right) = \lim_{\Lambda\to\infty} \int_0^{\Lambda^2} \frac{dt}{t} \lim_{N_c\to\infty} \left(\rho_S(t) - \rho'_P(t)\right)$$

$$\sim \lim_{\Lambda\to\infty} \left(\sum_n^{N_S(\Lambda)} \frac{F_S^2(n)}{M_S^2(n)} - \sum_n^{N_P(\Lambda)} \frac{F_P^2(n)}{M_P^2(n)} \right)$$

Regge $\Rightarrow M^2_{S,P}(n) \sim n$ and $F^2_{S,P}(n) \sim M^2_{S,P}(n)$ (parton model O.K.)

$$\therefore L_8 \sim \lim_{\Lambda\to\infty} \left(\sum_n^{N_S(\Lambda)} 1 - \sum_n^{N_P(\Lambda)} 1 \right) = ??$$

- no decoupling at large resonance masses.
- physical masses and decay constants do not obey L_8 sum rule.

$$L_8 \sim \lim_{Q^2\to\infty} Q^2 \left\{ \lim_{\substack{N_S, N_P\to\infty \\ M_S^2(N_S)/M_P^2(N_P)\to 1}} \sum_n^{N_S} \frac{F_S^2}{M_S^2(M_S^2+Q^2)} - (S\to P) \right\}$$ (Golterman, Cata, S.P.'06)

What is resonance saturation ? $(N_c \to \infty)$

- It's a Pade Approximant to a meromorphic function.

  ► $F(Q^2) \approx \frac{M_V^2}{Q^2+M_V^2}$ is the PA $P_1^0(Q^2)$ to $F(Q^2)$.

- $N_{A,V}$ resonances in $Q^2\Pi_{LR}(Q^2) = -F_0^2 - Q^2 \sum_A^{N_A} \frac{F_A^2}{Q^2+M_A^2} + (A \to V)$

  $\Longrightarrow P_N^N(Q^2)$ with $N = N_A + N_V$

  $\oplus\ 1/Q^4$ fall-off $\Longrightarrow P_N^{N-2}(Q^2)$

  ► PA's parameters do obey WSRs.

- N resonances in $\Pi'_{S-P}(Q^2) \Longrightarrow P_N^{N-1}(Q^2)$.

  ► PA's parameters do obey L_8 sum rule.

∴ Begin to see the tip of the iceberg:

Parameters (residues + poles) of Pade Approx.	$\neq$	Decay constants and masses of Green's functions

Pade Approximants

(Physics $\Leftrightarrow z \equiv Q^2$)

Let $\quad G(z)|_{z\to 0} \approx G_0 + G_1\, z + G_2\, z^2 + G_3\, z^3 + ...$

Define rational function $P_N^M(z)$ such that

$$P_N^M(z) \equiv \frac{Q_M(z)}{R_N(z)} \approx G_0 + G_1 z + G_2 z^2 + ... + G_{M+N} z^{M+N} + \mathcal{O}(z^{M+N+1})$$

If $G(z) \sim 1/z^K$, choose $P_{M+K}^M(z)$.

(Pommerenke '73)

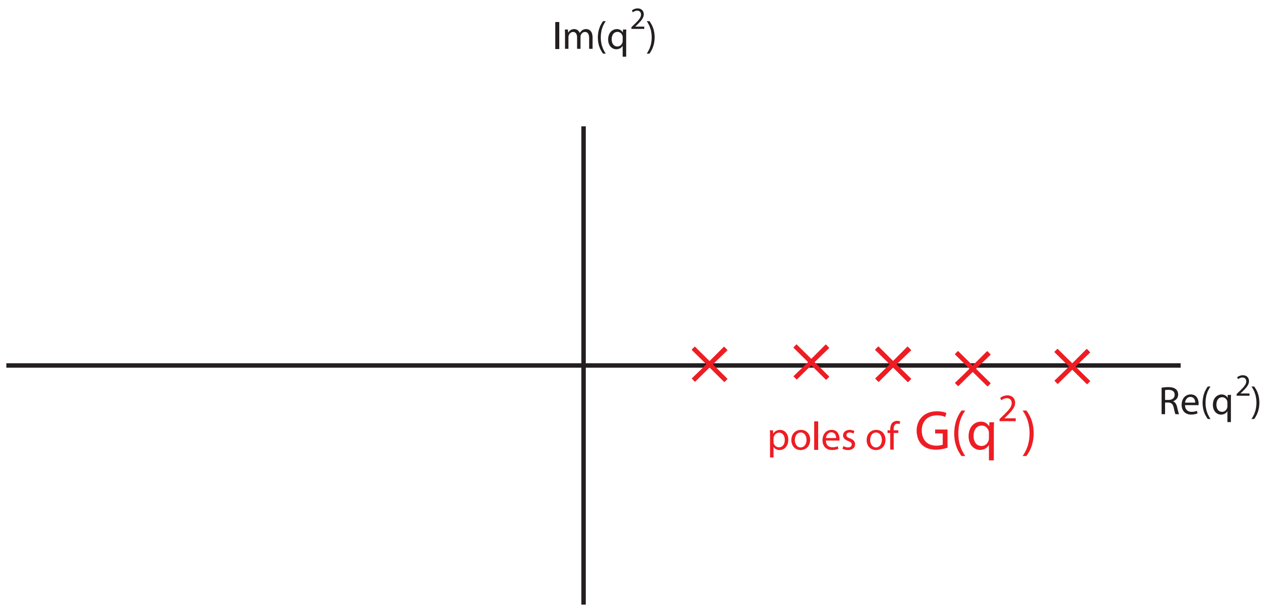
Convergence Theorem

Let $G(z)$ be meromorphic and analytic at the origin. Then,

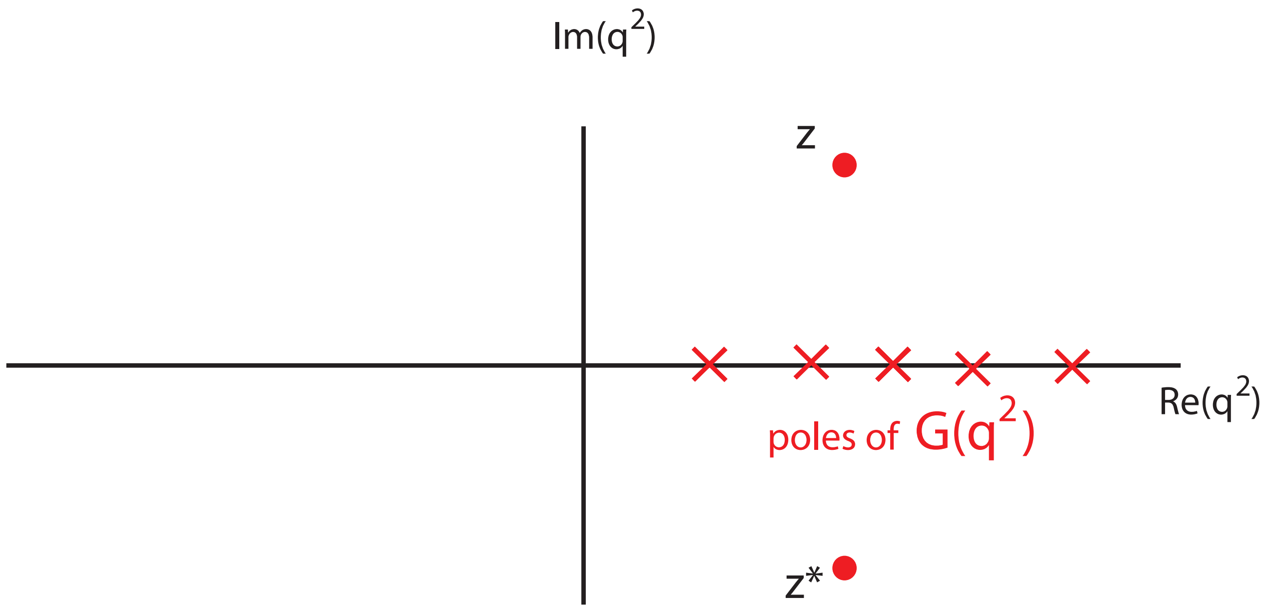
$$\lim_{M\to\infty} P_{M+K}^M(z) = G(z)$$

for $z \in$ compact set in $\mathbb{C}$, except on isolated points.

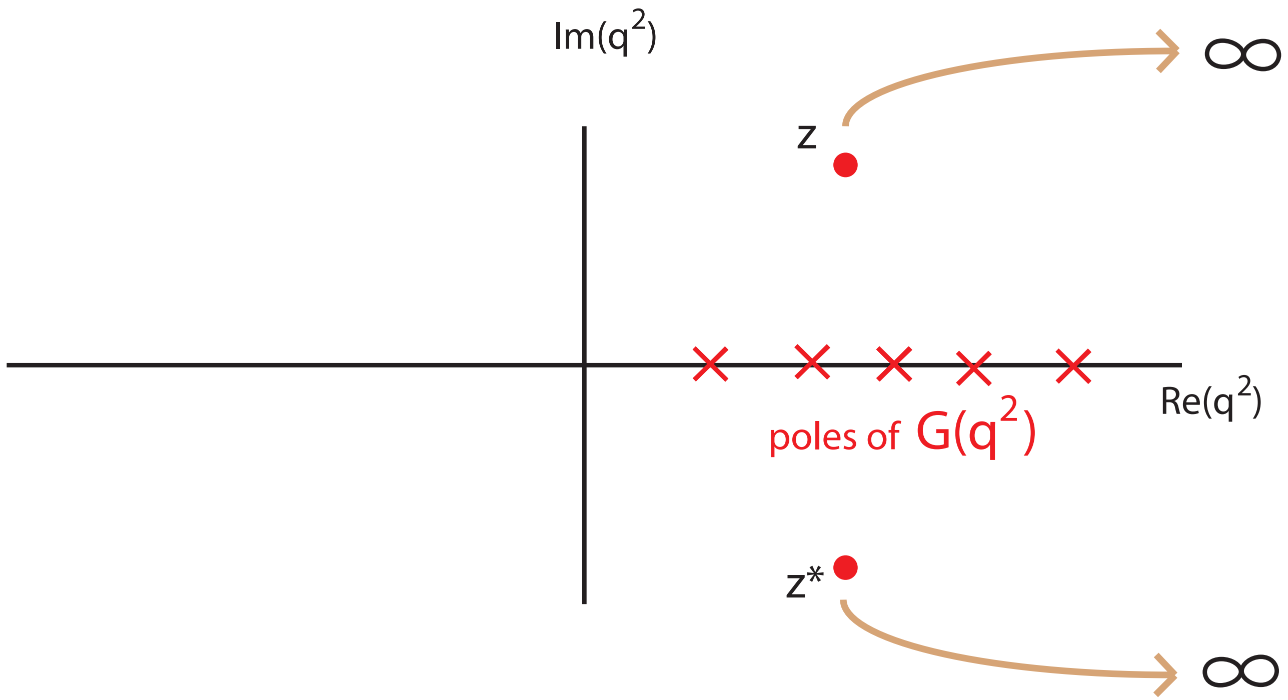
Convergence Map



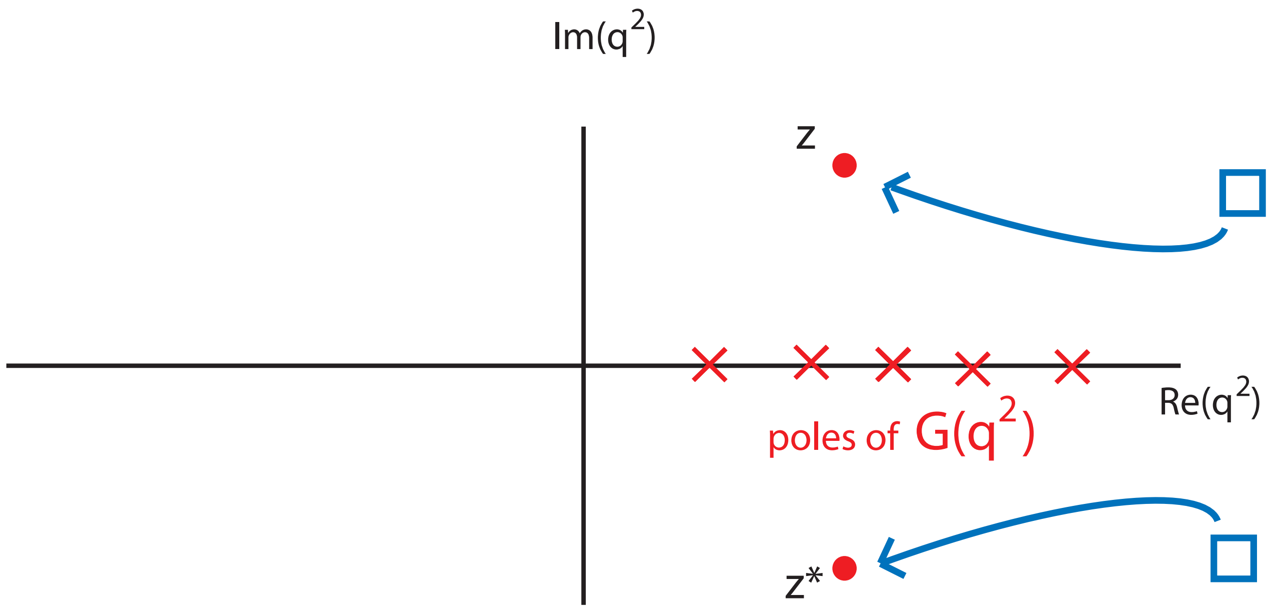
Convergence Map



Convergence Map

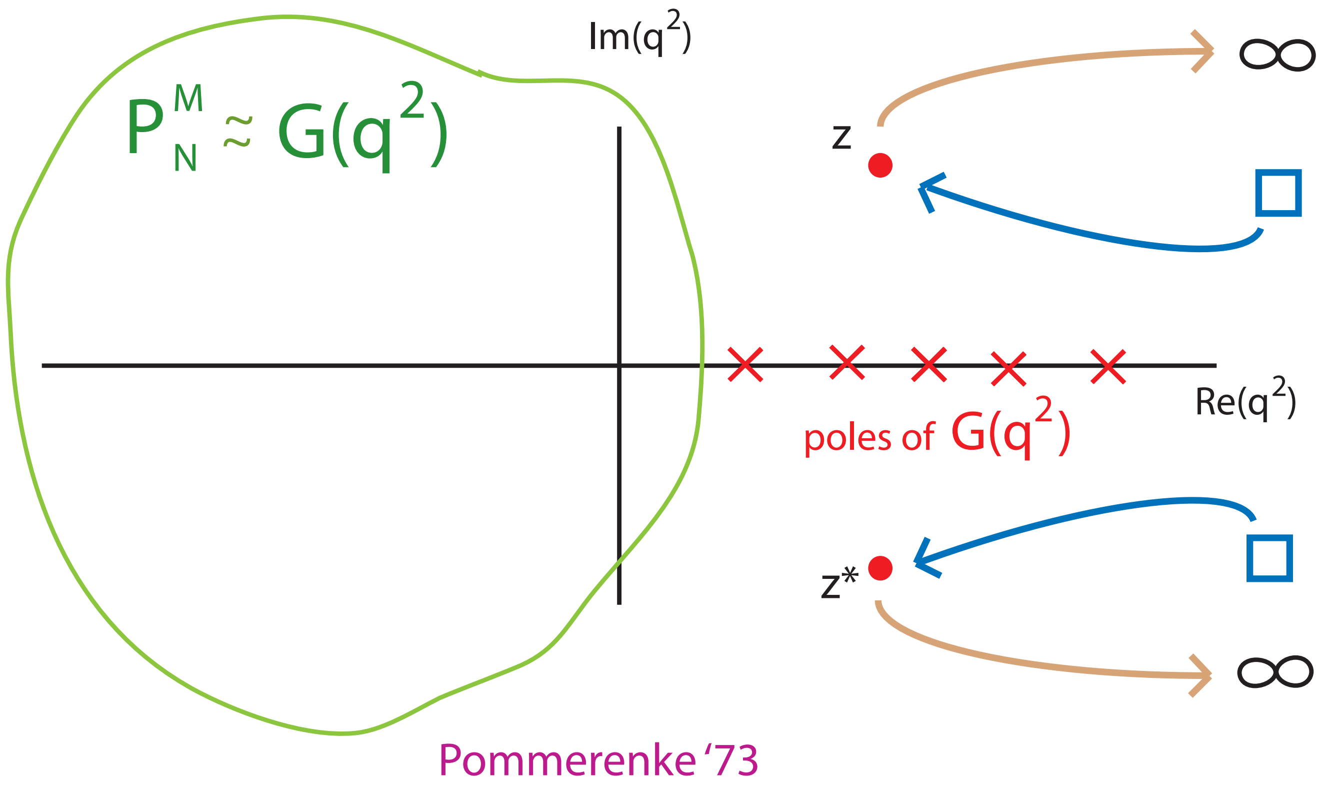


Convergence Map



$\blacksquare$ = pole U zero ≡ ``defect´´

Convergence Map



= pole U zero ≡ ``defect´´

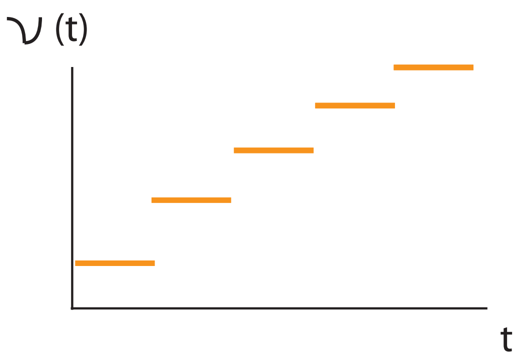
Simple case: Stieltjes

$$G(Q^2) = \int_{M^2}^{\infty} \frac{d\nu(t)}{t+Q^2} = \sum_{n=0}^{\infty} G_n \, Q^{2n}$$

with $\nu(t)$ a bounded non-decreasing function $\not G_n = (-1)^n \int_{M^2}^{\infty} \frac{d\nu(t)}{t^n}$.

E.g.: Vac. Polarization $\Pi_V(Q^2)$ (with <u>one</u> subtraction).

$$d\nu(t) = dt \sum_{R=0}^{\infty} \frac{F_R^2}{t} \, \delta(t - M_R^2) \quad , \quad M_0 = M_\rho , \quad \text{etc...} \quad G(Q^2) = \frac{\Pi_V(0) - \Pi_V(Q^2)}{Q^2}$$

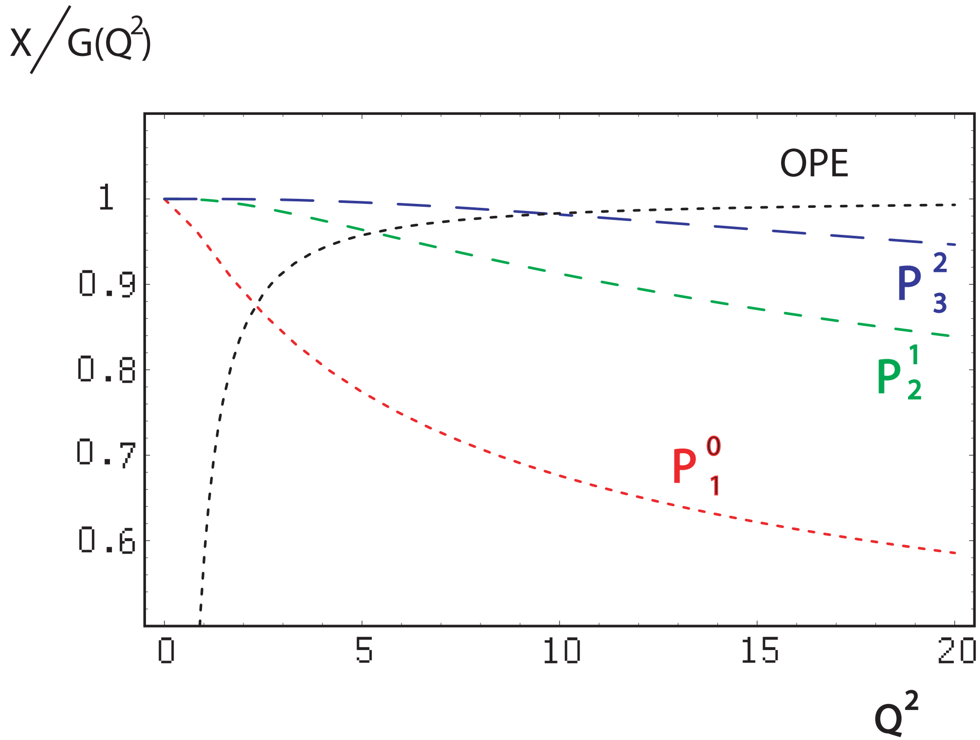


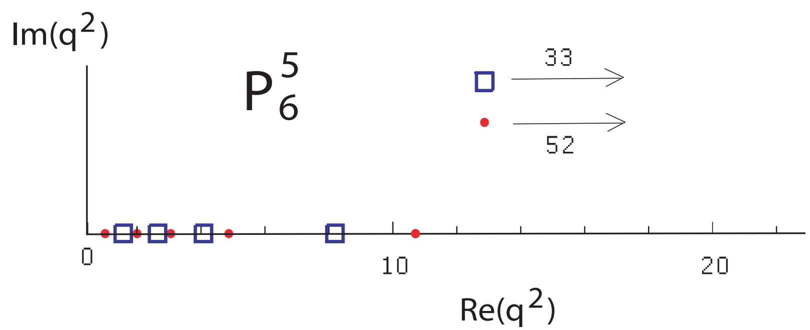
Simple case: Stieltjes (II)

E.g.: $F_n = 1\ ,\ M_n^2 = n\ ,\ n = 1, 2, 3, \ldots \Rightarrow \quad G(Q^2) = \frac{\psi(Q^2+1)+\gamma}{Q^2}\ ,\ \psi(z) = \frac{d}{dz}\log\Gamma(z)$

$$G(Q^2)|_{Q^2\to 0} \approx \sum_{p=0}^{\infty} \zeta(p+2)\,(-Q^2)^p \quad , \quad i.e. \quad R_{\text{conv.}} = 1$$

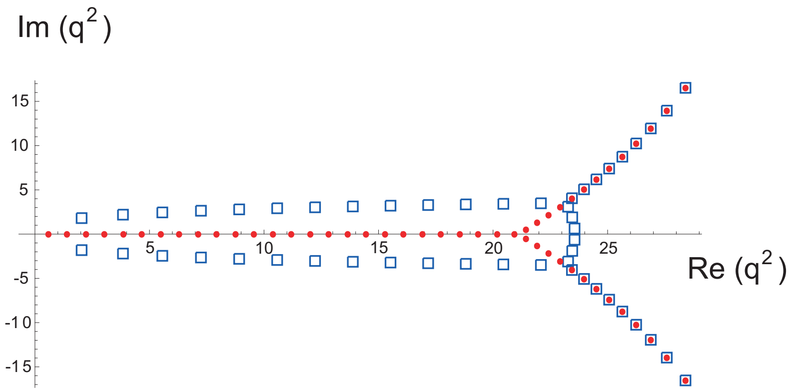
$$G(Q^2)|_{Q^2\to\infty} \approx \frac{\log Q^2 + \gamma}{Q^2} + \frac{1}{2Q^4} - \sum_{p=1}^{\infty} \frac{B_{2p}}{2p(Q^2)^{2p+1}}$$





Main Properties of PAs

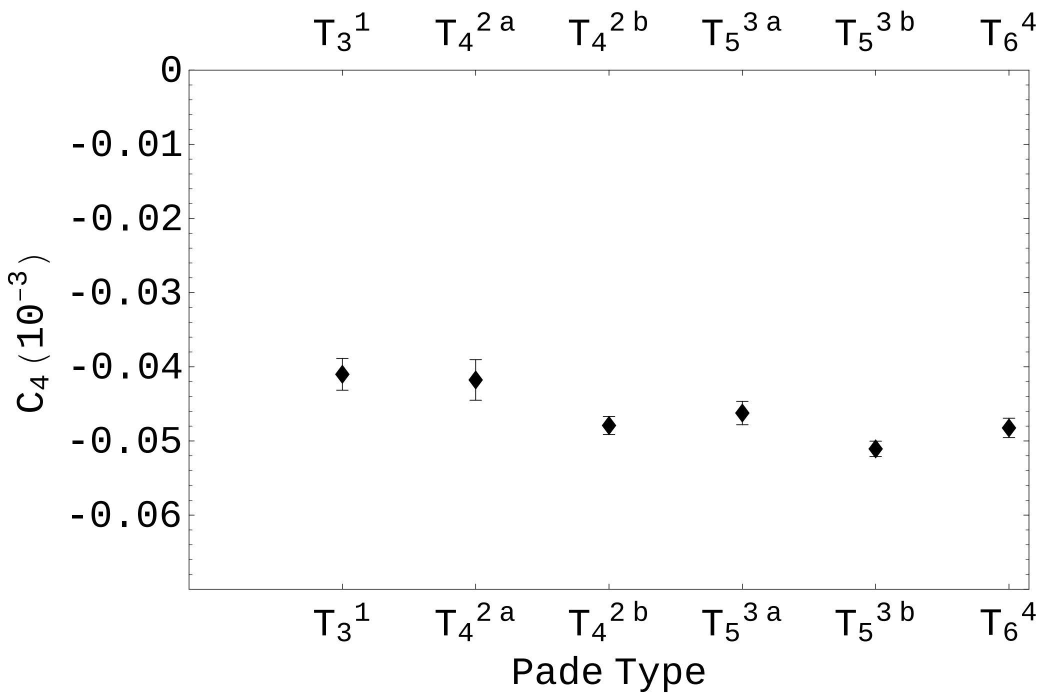
- In general, a PA may have complex poles (but it's O.K. !).
  - E.g. P_2^0 to $Q^2\Pi_{LR}$ has $M_{V,A}$ complex. (Masjuan,S.P., '07)
- Pommerenke's Thm $\Rightarrow \exists$ convergence in complex Q^2 plane, away from singularities.
  - Residues and poles of PAs are not physical.
- PAs approximate original function at the expense of altering residues and/or poles hierarchically (more the farther away from the origin).
- E.g.: P_{52}^{50} approximation to Regge-like model of $Q^2\Pi_{LR}(Q^2)$



P. Masjuan, S.P., JHEP05 (2007) 040.

- Pade-Type Approximants are PAs with poles predetermined at the physical masses. Then, residues pay full price i.e. last residue considered, completely off (recall previous •).

- E.g.: QCD, Pade-Type prediction for $O(p^6)$ contribution to $< VV - AA >$. Physical masses from PDG.



Masjuan, S.P., in preparation.

Conclusions and Outlook

- Resonance saturation at large-N_c can be understood from the theory of Pade Approximants to meromorphic functions.

  (Rate of convergence ?)

- Chiral expansion allows to construct rational approx. at finite Q^2 in region free of poles.

- At the last poles, the approximation is unreliable:

  ▶ Last Residues/poles in rational approx. not physical.

  E.g., form factors not to be extracted from rational approx. to 3-point functions (Bijnens et al. '03).

- Parameters in a Lagrangian of large-N_c QCD with a finite number of resonances do not in general coincide with physical decay constants and masses.

  (Lagrangian of the rational approx.?)