$1/N_c$ and $1/n$ preasymptotic effects in Current-Current correlators

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Motivation

What information can one obtain about hadronic (non-perturbative) physics from the the operator product expansion?

Let us first set the discussion for the vector-vector correlator

$$(q^\mu q^\nu - g^{\mu\nu})\Pi_V(q^2) = i\int d^4x e^{iqx}\langle \mathrm{vac}|J_V^\mu(x)J_V^\nu(0)|\mathrm{vac}\rangle$$

$J_V^\mu = \sum_f Q_f\bar{\psi}_f\gamma^\mu\psi_f$, and its Adler function ($Q^2 = -q^2$)

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We now consider the large N_c limit

$$\mathcal{A}_{hadr.}(Q^2) = Q^2\sum_{n=0}^{\infty}\frac{F_V^2(n)}{(Q^2+M_V^2(n))^2}$$

$$\mathcal{A}_{OPE}(Q^2) = \sum_f Q_f^2\left[\frac{4}{3}\frac{N_c}{16\pi^2}\left(1+\frac{3}{8}N_c\frac{\alpha_{\mathcal{A}}(Q^2)}{\pi}\right)\right.$$
$$\left.+\frac{C(\alpha_s(Q^2))}{Q^4}\beta(\alpha_s(\nu))\langle vac|G^2(\nu)|vac\rangle + \mathcal{O}\left(1/Q^6\right)\right]$$

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We quantify this discussion in a (Regge) model in the large N_c limit.

$$M_n^2 = Bn$$

and address the following question:
What information can we get from the combined use of the OPE of $\Pi(Q^2)$ and the mass spectrum if we go beyond the parton model and consider $1/n$ corrections to the mass spectrum?

Input:
1) Mass Spectrum (for large n)

$$M_V^2(n) = B_V n + A_V + \frac{C_V}{n} + \cdots$$

2) $\mathcal{A}_{OPE}$

Output: $F_V^2(n)$
For the decay constants, we will have a double expansion in $1/n$ and $1/\ln n$.

$$F_V^2(n) = \sum_{s=0}^{\infty} F_{V,s}^2(n)\frac{1}{n^s} = F_{V,0}^2(n) + \frac{F_{V,1}^2(n)}{n} + \frac{F_{V,2}^2(n)}{n^2} + \cdots ,$$

where the coefficients $F_{V,s}^2(n)$ have a logarithmic dependence on n:

$$F_{V,s}^2(n) = \sum_{r=0}^{\infty} C_{V,s}^{(r)}(n)\frac{1}{\ln^r n} .$$

Note that in this case we also have an expansion in $1/\ln n$.

Using the Euler-MacLaurin asymptotic expansion ($\Lambda_{\rm QCD} \ll n^* \Lambda_{\rm QCD} \ll Q$)

$$\mathcal{A}(Q^2) = Q^2 \sum_{n=0}^{n^*} \frac{F_V^2(n)}{(Q^2 + M_V^2(n))^2} + Q^2 \sum_{n=n^*}^{\infty} \frac{F_V^2(n)}{(Q^2 + M_V^2(n))^2} \,.$$

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$$\mathcal{A}(Q^2) = Q^2 \int_0^\infty dn \frac{F_V^2(n)}{(Q^2+M_V^2(n))^2} + \left[\sum_{n=0}^{n^*-1} \frac{Q^2 F_V^2(n)}{(Q^2+M_V^2(n))^2} - \int_0^{n^*} \frac{Q^2 F_V^2(n)}{(Q^2+M_V^2(n))^2}\right]$$

$$+\frac{Q^2}{2}\frac{F_V^2(n^*)}{(Q^2+M_V^2(n^*))^2} + Q^2\sum_{k=1}^{\infty}(-1)^k \frac{|B_{2k}|}{(2k)!}\frac{d^{(2k-1)}}{dn^{(2k-1)}}\frac{F_V^2(n)}{(Q^2+M_V^2(n))^2}\Bigg|_{n=n^*},$$

Using the Euler-MacLaurin asymptotic expansion ($\Lambda_{\rm QCD} \ll n^* \Lambda_{\rm QCD} \ll Q$)

$$\mathcal{A}(Q^2) \simeq Q^2 \int_0^\infty dn \frac{F_V^2(n)}{(Q^2 + M_V^2(n))^2} + \cdots$$

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$$\int_{n^*}^\infty dn \frac{1}{n^s} \frac{1}{(Q^2 + nB)^{2+r}} \ln^t n$$

$$r \to \frac{1}{Q^{2r}} \qquad s \to \frac{1}{Q^{2s}} \left(\ln^s Q^2 + \cdots \right) \qquad t \to \ln^s Q^2 + \cdots$$

We get a systematic method to get corrections to the decay constant for a given spectrum.

$$\frac{F^2_{V,0}(n)}{B_V} = \frac{1}{\pi}\mathrm{Im}\Pi_V^{pert.}(B_V n)\,.$$

or ($\tilde{n} = nB_V/\Lambda_{\overline{\mathrm{MS}}}$)

$$F^2_{V,LO}(n) = B_V \frac{4}{3}\frac{N_c}{16\pi^2}\sum_f Q_f^2 \left\{ 1 + \frac{9}{22}\frac{1}{\ln \tilde{n}} + \frac{1}{\ln^2 \tilde{n}}\left[-\frac{459}{1331}\ln\ln\tilde{n} + \frac{144}{121}\left(\frac{243}{128} - \frac{11}{8}\zeta(3)\right)\right] + \cdots + \mathcal{O}\left(\frac{1}{\ln^4 n}\right)\right\}.$$

$$\frac{F^2_{V,1}(n)}{n} = \frac{A_V}{B_V}\frac{d}{dn}F^2_{V,0}(n)$$

$$F^2_{V,2}(n) = -C_V \frac{4}{3}\frac{N_c}{16\pi^2}\sum_f Q_f^2 \left\{ 1 + \frac{3}{8\pi}N_c\alpha_s(nB_V) + \left[\frac{287 - 176\,\zeta(3)}{128\pi^2} - \frac{11 A_V^2}{64\pi^2 B_V C_V} - \frac{35}{88}\frac{\beta(\alpha_s(\nu))\langle vac|G^2(\nu)|vac\rangle}{B_V C_V N_c^2}\right] N_c^2\alpha_s^2(nB_V) + \mathcal{O}\left(\alpha_s^3(nB_V)\right)\right\}\,.$$

Other currents

Axial-vector: $B_V, A_V, C_V \to B_A, A_A, C_A$
One can not fix $B_V = B_A$ from the OPE alone (Golterman-Peris for the OPE in the parton model approximation)
Scalar/Pseudo-scalar:

$$F^2_{(X),0}(n) = \frac{B_X^2}{8\pi^2} N_c \left[\left(\frac{\alpha_s(nB_X)}{\alpha_s(\mu^2)} \right)^{\bar{\gamma}_0} \frac{c(\alpha_s(nB_X))}{c(\alpha_s(\mu^2))} \right]^2 \times \left(1 + r_1 \frac{\alpha_s(nB_X)}{\pi} + r_2 \frac{\alpha_s^2(nB_X)}{\pi^2} + r_3 \frac{\alpha_s^3(nB_X)}{\pi^3} \right) ,$$

$$F^2_{(X),1}(n) = \frac{A_X}{B_X} \frac{d}{dn} \left(n F^2_{(X),0}(n) \right) .$$

$$F^2_{(X),2}(n) = \frac{n}{B_X} \frac{d}{dn} \left(\frac{1}{2} A_X F^2_{(X),1}(n) + C_X F^2_{(X),0}(n) \right) + \frac{9N_c}{88} \frac{N_c \alpha_s(B_X n)}{\pi} \left(1 - \frac{11}{12} \frac{N_c \alpha_s(B_X n)}{\pi} \mathrm{Log}\left(\frac{B_X}{\mu^2} \right) \right) \frac{\beta(\alpha_s(\nu)) \langle vac | G^2(\nu) | vac \rangle}{N_c^2} .$$

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$$
\begin{aligned}
B_V(\mathrm{I}) &= 1.525\,\mathrm{GeV}^2\,, \quad A_V(\mathrm{I}) = -1.038\,\mathrm{GeV}^2\,, \quad C_V(\mathrm{I}) = 0.123\,\mathrm{GeV}^2\,,\\
B_V(\mathrm{II}) &= 1.128\,\mathrm{GeV}^2\,, \quad A_V(\mathrm{II}) = 0.353\,\mathrm{GeV}^2\,, \quad C_V(\mathrm{II}) = -0.885\,\mathrm{GeV}^2\,,\\
B_A &= 1.278\,\mathrm{GeV}^2\,, \quad A_A = -0.100\,\mathrm{GeV}^2\,, \quad C_A = 0.349\,\mathrm{GeV}^2\,.
\end{aligned}
$$

	$n=1$	$n=2$	$n=3$	$n=4$
$M_\rho(\mathrm{I})$	$781(775.5 \pm 0.4)$	$1440(1459 \pm 11)$	$1892(1870 \pm 20)$	$2257(2265 \pm 40)$
$M_\rho(\mathrm{II})$	$772(775.5 \pm 0.4)$	$1472(1459 \pm 11)$	$1855(1870 \pm 20)$	$2155(2149 \pm 17)$
M_{a_1}	$1236(1230 \pm 40)$	$1622(1647 \pm 22)$	$1962(1930^{+30}_{-70})$	$2258(2270^{+55}_{-40})$
$F_V(\mathrm{I})$	$156(156 \pm 1)$	155	154	153
$F_V(\mathrm{II})$	$185(156 \pm 1)$	147	139	135
F_A	$123(122 \pm 24)$	137	139	139

Table: *We give the experimental values of the masses (in MeV) and electromagnetic decay constants (when available) for vector and axial vector particles (within parenthesis), compared with the values obtained from the fit. For the vector states we consider two possible Regge trajectories that we label* I *and* II *respectively.*

$$B_S = 0.456\,\mathrm{GeV}^2 \qquad A_S = 1.262\,\mathrm{GeV}^2 \qquad C_S = -0.746\,\mathrm{GeV}^2$$
$$B_P = 1.040\,\mathrm{GeV}^2 \qquad A_P = 1.589\,\mathrm{GeV}^2 \qquad C_P = -0.926\,\mathrm{GeV}^2\,.$$

	$n=1$	$n=2$	$n=3$	$n=4$
M_{f_0}	$986(980 \pm 10)$	$1342(1370)$	$1544(1507 \pm 5)$	$1703(1718 \pm 6)$
M_π	$1305(1300 \pm 100)$	$1791(1812 \pm 14)$	$2098(2070 \pm 35)$	$2349(2360 \pm 30)$
G_S	16585.5	209	112.5	95
G_P	3049	282	224	202

Table: *We give the experimental values of the masses (in MeV) for scalar and pseudoscalar particles (within parenthesis), compared with the values obtained from the fit. We take $\alpha_s(1\,\mathrm{GeV}) = 0.5$ and $\beta\langle G^2\rangle = -(352\,\mathrm{MeV})^4$. The values of G_s and G_P depend on the factorization scale. We have taken $\mu^2 = B_S$ and $\mu^2 = B_P$ for the scalars and pseudoscalars respectively.*

We can check the method in the 't Hooft model (QCD in two dimensions in the large N_c limit)

$$\mathcal{A}_X^{hadr.} = Q^2 \sum_{n_X \cdots}^{\infty} \frac{F_X^2(n)}{(M_n^2 + Q^2)^2}$$

Input:

$$M^2(n) = \pi^2 \beta^2 n - 2\beta^2 \ln n + \cdots$$

$$\mathcal{A}_X^{pert.} = \frac{1}{2\pi}\left(1 - \frac{\beta^2}{Q^2} + D_X \frac{m\langle\bar{\psi}\psi\rangle}{Q^2}\right) + \mathcal{O}\left(m^2, \frac{1}{Q^4}\right) \,,$$

where $D_S = 1$ and $D_P = -3$.
Output

$$F^2(n) = \pi\beta^2 - \frac{2\beta^2}{\pi n} + \cdots \,.$$

$$M_n^2\phi_n(x) = \hat{P}^2\phi_n(x) \equiv \left(\frac{m_R^2}{x} + \frac{m_R^2}{1-x}\right)\phi_n(x) - \beta^2\int_0^1 dy\phi_n(y)\mathrm{P}\frac{1}{(y-x)^2},$$

$$F_S(n) = \frac{m}{2\sqrt{\pi}}\int_0^1 dx\phi_n(x)\left(\frac{1}{x} - \frac{1}{1-x}\right) = \frac{m}{\sqrt{\pi}}\int_0^1 dx\frac{\phi_n(x)}{x} \text{ for } n \text{ odd}$$

and zero otherwise. In particular this implies that in the sum the ground state does not contribute. For the pseudoscalar we have

$$F_P(n) = \frac{m}{2\sqrt{\pi}}\int_0^1 dx\phi_n(x)\left(\frac{1}{x} + \frac{1}{1-x}\right) = \frac{m}{\sqrt{\pi}}\int_0^1 dx\frac{\phi_n(x)}{x} \text{ for } n \text{ even},$$

$$\phi_n(x) = c_n x^{\beta_i}(1 + o(x))$$

$$\lim_{m_i\to 0} F(n) = \frac{\pi}{\sqrt{3}}c_n\beta$$

$c_0 = 1$ but c_n for large n?

Boundary-layer equation

$$\phi(\xi) = \frac{m_R^2}{\xi}\phi(\xi) - \beta^2 \int_0^\infty d\xi' \phi(\xi') \mathrm{P} \frac{1}{(\xi' - \xi)^2} \,,$$

where

$$\phi(\xi) \equiv \lim_{n\to\infty} \phi_n(\xi/M_n^2) \,.$$

In the large n limit one can obtain expressions for $F^2(n)$ (Brower et al.):

$$\int_0^\infty d\xi \frac{\phi(\xi)}{\xi} = \pi \frac{\beta}{m} \,, \qquad \int_0^\infty d\xi \phi(\xi) = \beta\pi m \,.$$

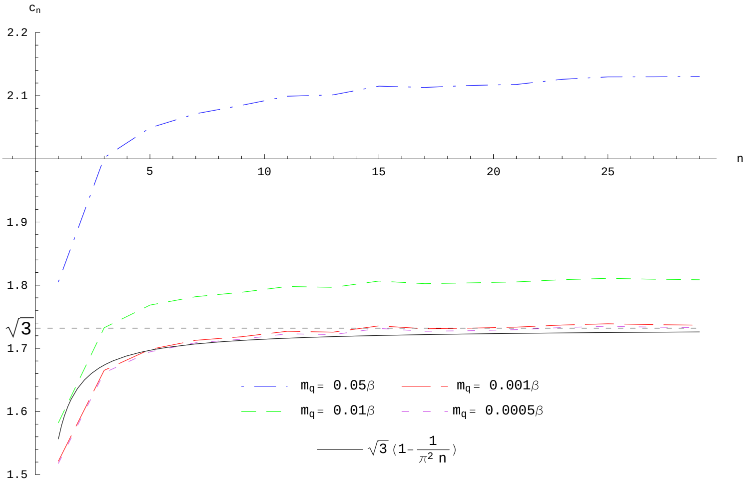
Therefore, for large values of n we obtain

$$c_n = \sqrt{3} + O(1/n) \,.$$

Preasymptotic effects in $1/n$

We expect

$$c_n = \sqrt{3}\left(1 - \frac{1}{\pi^2 n} + O(1/n^2)\right) .$$



Motivation (finite N_c)

Can we describe $R(q^2)$ with perturbation theory? In principle perturbation theory only applies to Euclidean quantities.

Euclidean

$$\Pi_V(-q^2) \sim \ln Q^2$$

Minkwoski cut

$$Im\Pi_V(q^2) \sim R(q^2) \sim \sum_n F_n^2 \delta(q^2 - M_n^2)$$

This does not look like perturbation theory ...

$$Im\Pi_V^{pert.}(q^2) \sim constant$$

Large N_c → example of maximal duality violation (Shifman)

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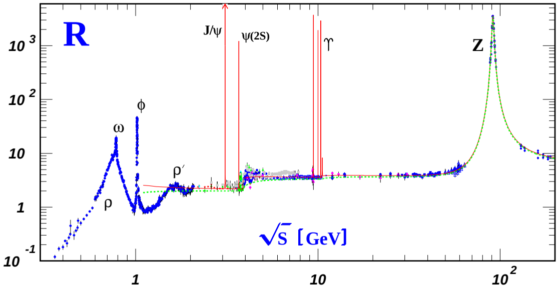
Large $N_c \rightarrow$ example of maximal duality violation (Shifman)

$N_c \neq \infty$ ($N_c = 3$). We expect that "in some way" we will have smoothing to the perturbative curve.
Actually this is what we see from experiment (though actually not that easy to quantify the "in some way" agreement) but this is not a "proof" that one can do perturbation theory in the Minkowski cut.



Therefore, from the mathematical point of view, one should
1) Proof that one can do perturbation theory in the Minkowski cut at finite N_c up to terms that vanishes when $q^2 \to \infty$
2) Quantify the error associated to doing the OPE in the Minkowski cut. What is left? What is the difference between OPE and the full theoretical result?

$1/N_c$ corrections

$$\Pi_V(Q^2) = \sum_{n=1}^{\infty} \frac{F_V^2(n)}{\left(\frac{Q^2}{\Lambda^2}\right)^{1-\frac{B}{\pi N_c}} \Lambda^2 + M_V^2(n)} .$$

For $F_V^2(n)$ =constant and $M_V^2(n) = B_V n$, we recover the model of Shifman et al.. In this case the duality-violating effects are exponentially suppressed.

$$\Pi_V(Q^2) = \Pi_{V,OPE}(Q^2) + \text{exponential suppressed terms} \qquad \arg\left(\frac{Q^2}{\Lambda^2}\right)^{1-\frac{B}{\pi N_c}} < \pi$$

In particular

$$\arg\left(\frac{-q^2}{\Lambda^2}\right)^{1-\frac{B}{\pi N_c}} < \pi$$

Therefore,

$$Im\Pi_V(-q^2) = Im\Pi_{V,OPE}(-q^2) + \text{exponential suppressed terms}$$

Does this model survives the inclusion of the perturbative logs of Q?
In order to do so we allow $F_V^2(n)$ to be n-dependent.

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In the large N_c limit

$$\frac{F_{V,0}^{2,\infty}}{\Lambda^2} = \mathcal{A}_0 \left(1 + r_1^\infty \frac{\alpha_M^\infty(B_V n)}{\pi}\right)$$

If we write

$$\frac{F_{V,0}^2}{\Lambda^2} = \frac{\mathcal{A}_0}{1-\frac{B}{\pi N_c}} \left(1 + r_1 \frac{\alpha_M(B_V n)}{\pi}\right) + \frac{B}{\pi N_c}\frac{\delta F_{V,0}^2}{\Lambda^2}$$

with $\delta F^2 \sim \alpha_s^2(B_v) \ln n$ or

$$\frac{F_{V,0}^2}{\Lambda^2} = \frac{\mathcal{A}_0}{1-\frac{B}{\pi N_c}} \left[\left(1 + r_1 \frac{\alpha_M(B_V n^{1+\frac{B}{\pi N_c}})}{\pi}\right) + \frac{B}{\pi N_c}\frac{\delta \tilde{F}_{V,0}^2}{\Lambda^2}\right]$$

with

$$\frac{\delta\tilde{F}^2}{\Lambda^2} = -\frac{1}{24}\beta_0^2\frac{r_1}{\pi}\alpha_s(B_V)^3 \; .$$

We still find that the the duality-violating effects are exponentially suppressed.

Conclusions

We have addressed the following questions:
1) What information can we get from the combined use of the OPE of $\Pi(Q^2)$ and the mass spectrum
a) If we go beyond the parton model and consider $1/n$ corrections to the mass spectrum?
b) If we go beyond the large N_c limit and consider $1/N_c$ corrections?

Using the OPE (going beyond the parton model) and the mass spectrum we can fix the decay constant as a logarithmically modulated $1/n$ expansion.
We have done so for vector, axial-vector, scalar, pseudoscalar currents.
We have also performed the same calculation in the 't Hooft model and we have found consistency.

The model of Shifman et al. for the current-current correlator at finite N_c can be improved by the incorporation of the perturbative $\ln Q^2$. Even after the inclussion of these logarithms the violating duality effects are exponentially suppressed in this model.
We find at finite N_c or a large N_c that we can not fix $B_V = B_A$ from the OPE alone.

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