π^0 decays:
overview and recent studies

Karol Kampf
PSI, Switzerland & IPNP, Charles University, Czech Republic
karol.kampf∈psi.ch

Orsay, November 16, 2007

in collaboration with M. Knecht, B. Moussallam, J. Novotný

Outline:

- Motivation
- Decay modes:
  - Dalitz decay
  - Double dalitz
  - $\pi^0 \to e^+e^-$
  - $\pi^0 \to \gamma\gamma$
- Conclusion

PDG 2007 about π^0

π^0 DECAY MODES

For decay limits to particles which are not established, see the appropriate Search sections (A^0 (axion) and Other Light Boson (X^0) Searches, etc.).

	Mode		Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	2γ		(98.798 ± 0.032) %	S=1.1
Γ_2	$e^+ e^- \gamma$		(1.198 ± 0.032) %	S=1.1
Γ_3	γ positronium		$(1.82 \pm 0.29) \times 10^{-9}$	
Γ_4	$e^+ e^+ e^- e^-$		$(3.14 \pm 0.30) \times 10^{-5}$	
Γ_5	$e^+ e^-$		$(6.46 \pm 0.33) \times 10^{-8}$	
Γ_6	4γ		$< 2 \times 10^{-8}$	CL=90%
Γ_7	$\nu \overline{\nu}$		[a] $< 2.7 \times 10^{-7}$	CL=90%
Γ_8	$\nu_e \overline{\nu}_e$		$< 1.7 \times 10^{-6}$	CL=90%
Γ_9	$\nu_\mu \overline{\nu}_\mu$		$< 1.6 \times 10^{-6}$	CL=90%
Γ_{10}	$\nu_\tau \overline{\nu}_\tau$		$< 2.1 \times 10^{-6}$	CL=90%
Γ_{11}	$\gamma \nu \overline{\nu}$		$< 6 \times 10^{-4}$	CL=90%
Charge conjugation (*C*) or Lepton Family number (*LF*) violating modes				
Γ_{12}	3γ	*C*	$< 3.1 \times 10^{-8}$	CL=90%
Γ_{13}	$\mu^+ e^-$	*LF*	$< 3.8 \times 10^{-10}$	CL=90%
Γ_{14}	$\mu^- e^+$	*LF*	$< 3.4 \times 10^{-9}$	CL=90%
Γ_{15}	$\mu^+ e^- + \mu^- e^+$	*LF*	$< 1.72 \times 10^{-8}$	CL=90%

Motivation

these processes enable to study

- Chiral Perturbation Theory at N^{i}LO
- QCD constraints
- Large N_C methods
- electromagnetic contributions

and (last but not least) can be (hopefully) directly confronted by existing experiments.

They are also important 'indirectly' in many other processes ($g-2$, normalization of decays of kaons, understand the background etc.).

Technically, one can use the results (with different masses) also for other particles e.g. for η physics (η-(double)Dalitz, $\eta \to e^+e^-$, $\mu^+\mu^-$, etc.).

Dalitz decay

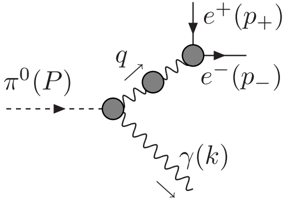
[K.K., Knecht, Novotný '06]

history

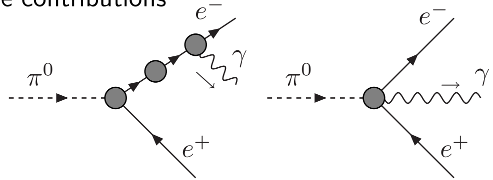
- First calculated by [Dalitz '51].
- Radiative corrections studied by [Joseph '60], [Lautrup, Smith '71], [Mikaelian, Smith '72]
- and during the 1980s by Tupper, Grose, Samuel, Lambin, Pestieau, Roberts...

Dalitz decay: Anatomy of the amplitude

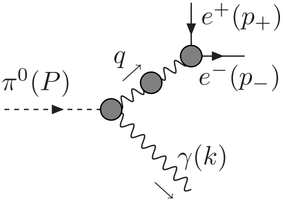
- one-photon reducible graphs: electron-positron pair is produced by a single photon (Dalitz pair)



- one-photon irreducible graphs
  - one-fermion reducible topologies
  - one-photon irreducible contributions



Dalitz decay: One-photon reducible diagrams



$$\Gamma_\mu^{1\gamma R}(p_+,p_-,k) = \mathrm{i}e^2 \varepsilon_\mu{}^{\nu\alpha\beta} q_\alpha k_\beta \, \mathcal{F}_{\pi^0\gamma\gamma^*}(q^2) \, \mathrm{i}D^T_{\nu\rho}(q)(-\mathrm{i}e)\Lambda^\rho \qquad (1)$$

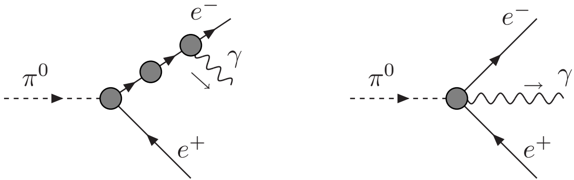
$\mathcal{F}_{\pi^0\gamma\gamma^*}(q^2)$ is related to the doubly off-shell form factor $\mathcal{A}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)$, defined as

$$\int d^4x \, e^{il\cdot x} \langle 0|T(j^\mu(x) j^\nu(0)|\pi^0(P)\rangle =$$

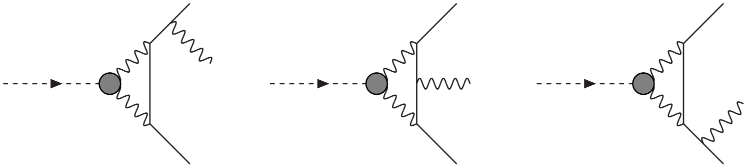
$$= -\mathrm{i}\varepsilon^{\mu\nu\alpha\beta} l_\alpha P_\beta \, \mathcal{A}_{\pi^0\gamma^*\gamma^*}(l^2, (P-l)^2) \qquad (2)$$

Dalitz decay: One-photon irreducible diagrams

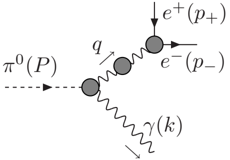
One-fermion reducible and one-particle irreducible graphs:



represent (<u>together</u>) a transverse subset. They both start at order $\mathcal{O}(e^5)$:



Dalitz decay: Leading order



The leading order amplitude corresponds to the $\mathcal{O}(e^3)$ one-photon reducible contribution with the form factor $\mathcal{A}_{\pi^0\gamma^*\gamma^*}(l^2,(P-l)^2)$ reduced to its expression for a pointlike pion, i.e. a constant, $\mathcal{A}^{LO}_{\pi^0\gamma^*\gamma^*} = -N_C/12\pi^2 F_\pi$, fixed by the chiral anomaly. Partial decay rates read

$$\frac{d\Gamma^{LO}}{dx dy} = \frac{\alpha^3}{(4\pi)^4}\frac{M_{\pi^0}}{F_\pi^2}\frac{(1-x)^3}{x^2}\left[M_{\pi^0}^2 x(1+y^2)+4m^2\right],$$
$$\frac{d\Gamma^{LO}}{dx} = \frac{\alpha^3}{(4\pi)^4}\frac{8}{3}\frac{M_{\pi^0}}{F_\pi^2}\frac{(1-x)^3}{x^2}(xM_{\pi^0}^2+2m^2). \qquad (3)$$

Dalitz decay: Next-to-leading order corrections

These corrections will be described as

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}x\mathrm{d}y} = \delta(x,y)\,\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x\mathrm{d}y}, \qquad \frac{\mathrm{d}\Gamma}{\mathrm{d}x} = \delta(x)\,\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x}. \tag{4}$$

One can define a slope parameter a_π

$$\mathcal{F}_{\pi^0\gamma\gamma^*}(q^2) \;=\; \mathcal{F}_{\pi^0\gamma\gamma^*}(0)\left[1 \,+\, a_\pi\,\frac{q^2}{M_{\pi^0}^2} \,+\cdots\right], \tag{5}$$

which can be obtained from

$$\frac{\mathrm{d}\Gamma^{exp}}{\mathrm{d}x} \,-\, \delta_{QED}(x)\,\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x} \;=\; \frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x}\,[1+2x\,a_\pi]. \tag{6}$$

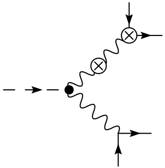
Dalitz decay: results

- Our work provides a detailed analysis of NLO radiative corrections to the Dalitz decay amplitude.
- The off-shell pion-photon transition form factor was included:
  this requires a treatment of non perturbative strong interaction effects
- The one-photon irreducible contributions, which had been usually neglected, were included.
  We have shown that, although these contributions are negligible as far as the corrections to the total decay rate are concerned, they are however sizeable in regions of the Dalitz plot which are relevant for the determination of the slope parameter a_π of the pion-photon transition form factor.
- Our prediction for the slope parameter $a_\pi = 0.029 \pm 0.005$ is in good agreement with the determinations obtained from the (model dependent) extrapolation of the CELLO and CLEO data.
  Unfortunately, the experimental error bars on the latest values of a_π extracted from the Dalitz decay are still too large
- Hopefully, future experiments will improve the situation in this respect.
- some existing data could partially test our results (NA48, PrimEx (?))

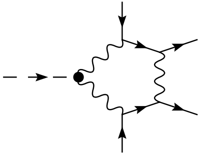
Double Dalitz decay

collaboration with M. Knecht, J. Novotný

It seems natural to convert the on-shell photon to the other Dalitz pair and obtain immediately Double Dalitz decay. This is true for LO:



However, for higher orders we have new topologies [Barker et al. '03]:



We are recalculating these results and try to put them together with our parameters introduced in the context of $\pi^0 \to e^+e^-\gamma$.

Our motivation: possible verification in the Dirac experiment

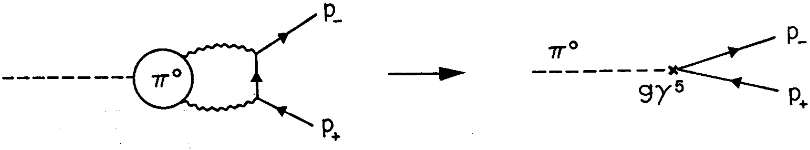
$\pi^0 \to e^+e^-$ (Drell decay)

history

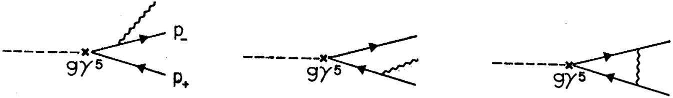
- first studied in [S. Drell '59]
- radiative corrections: [L. Bergström '83]
- further teor. studies: e.g. [Gomez Dumm, Pich '98], [Knecht, et al. '99]
- recent experiment: KTeV E799-II [Abouzaid '07]
- and its confrontation with theory: [Dorokhov, Ivanov '07]

$\pi^0 \to e^+e^-$

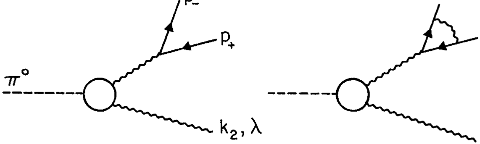
Bergström analysis: pictures taken from [Bergström '83]



simplifications: invariant mass of the lepton pair, structure of the vertex, bremsstrahlung of the internal lepton are neglected



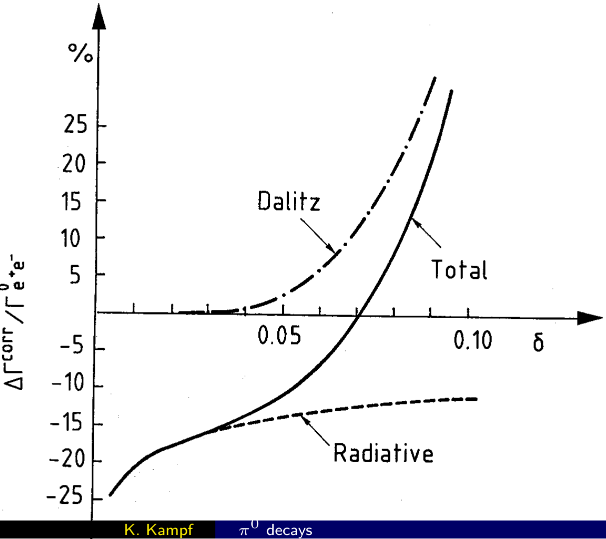
+ interference with Dalitz:



$\pi^0 \to e^+e^-$

Bergström analysis: pictures taken from [Bergström '83]

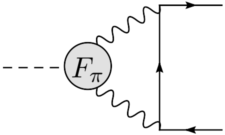
Results of the radiative corrections as a function of the acceptance for the m_{ee}



$\pi^0 \to e^+e^-$

normalization to the two-photon BR

$$R \equiv \frac{\mathrm{Br}(\pi^0 \to e^+e^-)}{\mathrm{Br}(\pi^0 \to \gamma\gamma)} = 2\Big(\frac{\alpha m_e}{\pi M_\pi}\Big)^2 \sqrt{1 - 4\frac{m^2}{s}}|\mathcal{A}|^2$$

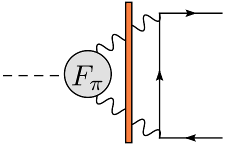
$\mathcal{A} \sim$ 

, where $F_\pi(0,0) = 1$

$\pi^0 \to e^+e^-$

normalization to the two-photon BR

$$R \equiv \frac{\mathrm{Br}(\pi^0 \to e^+e^-)}{\mathrm{Br}(\pi^0 \to \gamma\gamma)} = 2\left(\frac{\alpha m_e}{\pi M_\pi}\right)^2 \sqrt{1 - 4\frac{m^2}{s}} |\mathcal{A}|^2$$

$\mathcal{A} \sim$  , where $F_\pi(0,0) = 1$

Imaginary part is model independent

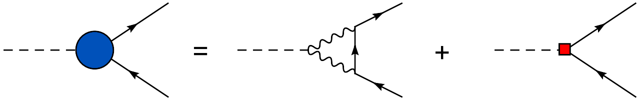
$$\mathrm{Im}\,\mathcal{A} = \frac{\pi}{2\beta_e} \log \frac{1-\beta_e}{1+\beta_e}$$

and implies the so-called unitary bound (neglecting radiative corrections):

$$R \geq R^{\mathrm{unit}} = 4.75 \times 10^{-8}$$

$\pi^0 \to e^+e^-$

Real part can be parameterized using the dispersive techniques. The unknown subtraction constant can be expressed by means of the low-energy constant $\chi(\mu)$ which describes the direct interactions of π^0 with lepton pair to LO. For details see e.g. [Dorokhov, Ivanov '07]. Schematically we have (cf. also [Knecht, Peris, Perrottet and de Rafael '99]):



$$\mathcal{A} = \frac{N_C}{3}\left[-\frac{5}{2} + \frac{3}{2}\log\frac{m^2}{\mu^2} + C\right] + \chi(\mu)$$

The different techniques can be used in order to extract this constant:

	CLEO+OPE	QCDsr	gVMD	QM	NχQM	NQM	Experiment
$-3\log(m/\mu) - \chi(\mu)$	21.9 ± 0.3	21.7 ± 0.1	21.9	23.4 ± 0.5	22.1 ± 0.5	24.5	18.6 ± 0.9
$B\left(\pi^0 \to e^+e^-\right) \times 10^8$	6.23 ± 0.09	6.21 ± 0.05	6.2	5.8 ± 0.2	6.1 ± 0.2	5.38	7.49 ± 0.38

(Table taken from [Dorokhov and Ivanov '07])

$\pi^0 \to e^+e^-$

Are the radiative corrections under control?
Apparently they play the important role in $\pi^0 \to e^+e^-$ analysis;
extrapolating the radiative tail one finds [KTeV in Abouzaid et al. '07]:

$$\mathrm{Br}^{x_D>0.95}(\pi^0 \to e^+e^-) \times 10^8 = 6.5 \pm 0.4 \longrightarrow \mathrm{Br}^{\text{no rad.}} = 7.5 \pm 0.4$$

How is this extrapolation by $12\,\%$ and $3.4\,\%$ given by Bergström approximation reliable?

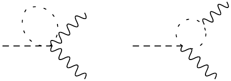
$\Rightarrow$ we must calculate the whole radiative correction.

$\Rightarrow$ this means the full two-loop calculation

Loop strategy: toy example

Demonstration of the basic recipe on one-loop correction to $\pi \to \gamma\gamma$.

1. Diagrams to be calculated

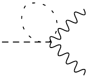
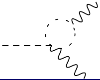


2. To find a superior topology with # prop $\geq$ # ind. scalars (so called auxiliary diagram method), e.g.:

3. Calculate this topology for general powers of propagators (λ_i)

$$I_{\lambda_1\lambda_2\lambda_3} = \frac{i\pi^{d/2}}{\Gamma(\lambda_1)\Gamma(\lambda_2)} \int \frac{dz}{2\pi i} \frac{\Gamma(\lambda_2+z)\Gamma(-z)\Gamma(-\lambda_1-\lambda_2-2\lambda_3-2\epsilon+4-z)\Gamma(\lambda_1+\lambda_2+\lambda_3+\epsilon-2+z)\Gamma(-\lambda_2-\lambda_3-\epsilon+2-z)}{\Gamma(-\lambda_1-\lambda_2-\lambda_3-2\epsilon+4-z)\Gamma(-\lambda_3-\epsilon+2)(m^2)^{\lambda_1+\lambda_2+\lambda_3+\epsilon-2}}$$

4. Express the topologies (with scalars in numerator) by this formula

 $= I_{100}$  $\sim I_{110}, I_{1-10}, \ldots$

Loop strategy: toy example

Demonstration of the basic recipe on one-loop correction to $\pi \to \gamma\gamma$.

This procedure is algorithmically very simple (=we can use computers) but has some drawbacks.

One has to be careful in calculating some specific values of λs. E.g. in our example we have $\Gamma(\lambda_2)$ in denominator, but this doesn't mean that I_{100} is zero (one has to take properly the limit $\lambda_2 \to 0$ on $I_{1\lambda_2 0}$).

The number of terms to take care of can be enormous – it is good to find some relations among them; for this we can use the IBP equations:

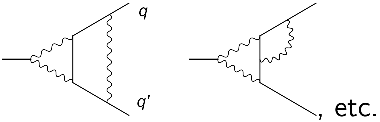
$$\int d^d k \frac{\partial}{\partial k^\mu} \frac{(k^\mu, p_i^\mu, \ldots)}{\ldots} = 0 \tag{7}$$

to obtained the so-called master integrals.

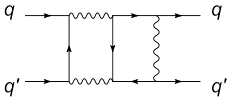
Back to $\pi^0 \to e^+e^-$, preliminary 2-loop study

collaboration with M. Knecht, J. Novotný

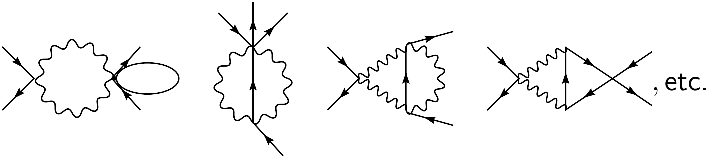
All radiative corrections to $\pi \to e^+e^-$



can be obtained 'simply' from the 'double-box' topology



Number of scalar integrals is about 200. We have used IBP to obtain about 20 master integrals to be calculated, for example:

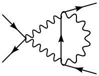


$\pi^0 \to e^+e^-$, preliminary 2-loop study

To summarize: all complicated spinor-, tensor-, two-loop integrals were reduced, so far, to the reasonable number of scalar neat (without numerator) two-loop graphs. Some of them can be calculated trivially ("loop $\times$ loop"), some of them can be found in the literature, some of them, however, we have to study little bit deeper. This work is in progress now:

we try to use different methods and perform the crosschecks. We employ (for a review cf. [Smirnov: Feynman Integral Calculus]): Mellin-Barnes representation [see D.Greynat talk], differential method (in general too complicated, however suitable for 3-body processes; enables to classify subclasses of diagrams), "infrared simplification" (if we are sure of infrared safeness we can trivially put $m_e = 0$ in some of these master integrals).

Example: all methods agree up to ϵ^0 on the following diagram



$$\approx \frac{1}{2\epsilon^2} + \frac{\frac{5}{2} + i\pi - \ln\frac{M^2}{\mu^2}}{\epsilon} - \frac{11\pi^2}{12} + \frac{19}{2} + 5i\pi + \ln^2\frac{M^2}{\mu^2} - (5+2i\pi)\ln(\frac{M^2}{\mu^2}) + O(\epsilon^1, m_e/M)$$

[I am grateful to B.Jantzen for helping me with Mellin-Barnes representation]; for full result see also [Bonciani et al. '04]

Last but not least: $\pi^0 \to \gamma\gamma$

π^0 is the lightest hadron $\Rightarrow$ primary decay mode is $\pi^0 \to \gamma\gamma$
The rate can be predicted exactly in chiral limit by QCD axial anomaly:

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{m_{\pi^0}^3}{64\pi}\left(\frac{\alpha N_C}{3\pi F_\pi}\right)^2 = 7.73\,eV$$

At NLO for $N_f = 2$, corrections are hidden in replacement $F_\pi \to F_{\pi^0}$ and $O(p^6)$ LECs [Donoghue et al. '85] [Bijnens et al. '88]
Three-flavour case can get us further in this studying, mainly by means of π^0, η, η' mixing, resulting to [Goity, Bernstein, Holstein '02]:

$$\Gamma^{\text{NLO}} = 8.1 \pm 0.08\,\text{eV}$$

or by studying in detail the electromagnetic corrections in $SU(2)$ [Ananth., Moussallam '02]:

$$\Gamma^{\text{NLO}} = 8.06 \pm 0.02 \pm 0.06\,\text{eV}$$

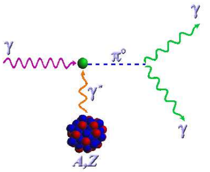
(errors mainly due to the uncertainty in R and F_π).
Quite recently another study based on dispersion relations, QCD sum rules, using only the value $\Gamma(\eta \to \gamma\gamma)$ gives [Ioffe, Oganesian '07]:

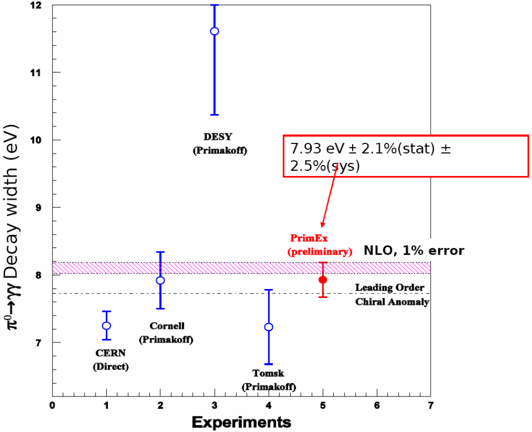
$$\Gamma^{\text{NLO}} = 7.93 \pm 0.11\,\text{eV}$$

$\pi^0 \to \gamma\gamma$

One can imagine three kinds of measurements:

- direct (time of flight)
- photon collisions
- Primakoff effect (i.e. photopion production in the Coulomb field of nucleus [Primakoff '51])





[PrimEx group April '07]

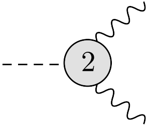
$\pi^0 \to \gamma\gamma$ at NNLO, preliminary results

work in collaboration with B.Moussallam

In the isospin limit we can expect that numerically relevant could be only double logarithms with m_π. Defining

$$\Lambda \equiv \frac{1}{(4\pi)^2}\frac{1}{d-4}\,, \quad L \equiv \frac{1}{(4\pi)^2}\ln\frac{m_\pi^2}{\mu^2}$$

all two-loop diagrams can be written schematically



$$= x(d)(c\mu)^{2(d-4)}\left[\Lambda^2 + L\Lambda + \tfrac{1}{4}L^2\right].$$

Using Weinberg consistency relation [Colangelo '95], [Bijnens et al. 98] we can set x_0 calculating only one-loop diagrams with one divergent NLO LEC

$\pi^0 \to \gamma\gamma$ at NNLO, prelir

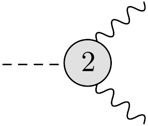
work in collaboration with B.M

In the isospin limit we can expe ... ly

double logarithms with m_π. De

$$\Lambda \equiv \frac{1}{(4\pi)^2}\frac{1}{d\,-}$$

all two-loop diagrams can be w



$= x(d$

table 3 in [Bijnens, Girlanda, Talavera '01]

monomial (o_i^W)	i 2-flavour	$384\pi^2 F^2 \eta_i^{(2)}$
$\epsilon^{\mu\nu\alpha\beta}\langle\chi_+ [f_{-\mu\nu}, u_\alpha u_\beta]\rangle$	1	0
$\epsilon^{\mu\nu\alpha\beta}\langle\chi_- \{f_{+\mu\nu}, u_\alpha u_\beta\}\rangle$	2	0
$i\epsilon^{\mu\nu\alpha\beta}\langle\chi_- f_{+\mu\nu} f_{+\alpha\beta}\rangle$	3	0
$i\epsilon^{\mu\nu\alpha\beta}\langle\chi_- f_{-\mu\nu} f_{-\alpha\beta}\rangle$	4	0
$i\epsilon^{\mu\nu\alpha\beta}\langle\chi_+ [f_{+\mu\nu}, f_{-\alpha\beta}]\rangle$	5	0
$\epsilon^{\mu\nu\alpha\beta}\langle f_{+\mu\nu}\rangle\langle\chi_- u_\alpha u_\beta\rangle$	6	3
$i\epsilon^{\mu\nu\alpha\beta}\langle f_{+\mu\nu}\rangle\langle f_{+\alpha\beta}\chi_-\rangle$	7	3
$i\epsilon^{\mu\nu\alpha\beta}\langle f_{+\mu\nu}\rangle\langle f_{+\alpha\beta}\rangle\langle\chi_-\rangle$	8	$-\frac{3}{2}$
$i\epsilon^{\mu\nu\alpha\beta}\langle f_{+\gamma\mu}\rangle\langle h_{\gamma\nu} u_\alpha u_\beta\rangle$	9	6
$i\epsilon^{\mu\nu\alpha\beta}\langle f_{+\gamma\mu}\rangle\langle f_{-\gamma\nu} u_\alpha u_\beta\rangle$	10	-18
$\epsilon^{\mu\nu\alpha\beta}\langle f_{+\mu\nu}\rangle\langle f_{+\gamma\alpha} h_{\gamma\beta}\rangle$	11	12
$\epsilon^{\mu\nu\alpha\beta}\langle f_{+\mu\nu}\rangle\langle f_{+\gamma\alpha} f_{-\gamma\beta}\rangle$	12	0
$\epsilon^{\mu\nu\alpha\beta}\langle\nabla_\gamma f_{+\gamma\mu}\rangle\langle f_{+\nu\alpha} u_\beta\rangle$	13	-12

Using Weinberg consistency relation [Colangelo '95], [Bijnens et al. 98] we can set x_0 calculating only one-loop diagrams with one divergent NLO LEC

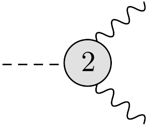
$\pi^0 \to \gamma\gamma$ at NNLO, preliminary results

work in collaboration with B.Moussallam

In the isospin limit we can expect that numerically relevant could be only double logarithms with m_π. Defining

$$\Lambda \equiv \frac{1}{(4\pi)^2}\frac{1}{d-4}\,, \quad L \equiv \frac{1}{(4\pi)^2}\ln\frac{m_\pi^2}{\mu^2}$$

all two-loop diagrams can be written schematically



$$= x(d)(c\mu)^{2(d-4)}\left[\Lambda^2 + L\Lambda + \tfrac{1}{4}L^2\right].$$

Using Weinberg consistency relation [Colangelo '95], [Bijnens et al. 98] we can set x_0 calculating only one-loop diagrams with one divergent NLO LEC [Bijnens, Girlanda, Talavera '01]. Our preliminary result is:

$$\Gamma = \frac{m_\pi^3}{64\pi}\left(\frac{\alpha}{F_\pi \pi}\right)^2\left(1 - \frac{1}{6}\frac{m_\pi^4}{F_\pi^4}L^2\right)^2 \quad \to \text{effect smaller than } 0.1\,\%$$

Conclusion

The 4 most important (in BR) decay modes of π^0 were discussed. The ordering in this talk reflects the ordering of my study in time

- $\pi \to e^+e^-\gamma$
  published in [KK, Knecht, Novotný '06]
- $\pi \to e^+e^-e^+e^-$
  only preliminary first study
- $\pi \to e^+e^-$
  classification of diagrams finished, painstaking work is now in progress
- $\pi \to \gamma\gamma$
  important process, so far we have calculated double-logarithm correction which is negligible. This process, however, deserves the full two loop calculation...