Light pseudoscalar mesons in $2+1$ flavor QCD

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All results are preliminary

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Motivation

Goal: calculate hadronic observables on the lattice, relevant for fundamental quark property determination with controlled extrapolations to the physical limit of QCD:

$$M_\pi \to 135\,\mathrm{MeV}, \qquad a \to 0, \qquad L \to \infty$$

Pseudo-Goldstone boson (PGB) masses and decay constants give access to:

- Fundamental parameters: m_{ud} and m_s
- Flavor mixing parameters: $\pi, K \to \mu\bar{\nu}$ allows precise determination of $|V_{us}/V_{ud}|$ *given a precise calculation of* F_K/F_π

  $\Rightarrow$ important check of $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$ and universality
- Properties of QCD vacuum: $\langle \bar{q}q \rangle$ and F
- Higher order couplings of chiral Lagrangian: $(2L_6 - L_4)$, $(2L_8 - L_5)$, L_4, L_5 ...

Our two approaches

In both cases: $N_f = 2 + 1$ tree-level, $O(a)$-improved Wilson seas (break $SU(3)_A$)

1. **"Unitary" simulations:** valence quarks are discretized in the same way as the sea quarks
2. **"Mixed-action" simulations:** valence quarks are chirally symmetric overlap (Ginsparg-Wilson) fermions

Why use a mixed action approach?

- $+$ Recent algorithmic (multiple time-scale integration, Hasenbusch acceleration, RHMC, DDHMC ...) (Sexton & Weingarten '92, Hasenbusch '01, Clark et al '06, Lüscher '05, Urbach et al '06, ...) and hardware advances

  $\Rightarrow N_f = 2+1$ QCD with e.g. $M_\pi^{\rm lat} \sim 190\,\mathrm{MeV}$, $a \sim 0.09\,\mathrm{fm}$ and $L \sim 4.2\,\mathrm{fm}$ becoming accessible to Wilson fermions

  $\Rightarrow$ near-continuum chiral p-regime w/out conceptual pbs of staggered fermions
- $+$ Overlap inversions are numerically feasible on these backgrounds

  $\Rightarrow$ full χS (in valence sector) w/out cost of dynamical overlap fermions

  $\Rightarrow$ simplified renormalization

  $\Rightarrow$ full $O(a)$ improvement w/ only NP $O(a)$-improved Wilson sea action
- $+$ To extrapolate to physical and chiral limits in a model independent-way $\longrightarrow$ finite-volume (FV) mixed action (MA) PQχPT (Sharpe '90 '92, Bernard & Golterman '92 '94, Sharpe & Shoresh '00 '01, Sharpe & Singleton '98, Aoki '03, Bär et al '03 '04, Sharpe '06, ...)
- $-$ Discretization induced unitarity violations, but should be able to describe low energy manifestations with MA PQχPT (Golterman et al '05)

Finite-volume mixed action PQχPT

- An effective theory in finite volume for the PGBs of χSB which includes discretization errors (Sharpe & Singleton '98). Expansion in:
  - $(M_{PGB}/4\pi F_\pi)^2 \sim 0.03 \div 0.2$
  - $(p/4\pi F_\pi)^2 \sim (1/2LF_\pi)^2 \sim 0.06$
  - $\alpha_s a\Lambda_{\rm QCD} \sim 0.06 \longleftarrow$ we use tree-level $O(a)$-improved Wilson seas
- Take here $(M_{PGB}/4\pi F_\pi)^2 \sim (p/4\pi F_\pi)^2 \sim \alpha_s a\Lambda_{\rm QCD}$

  $\longrightarrow$ p-regime and above phase transitions (Aoki or 1st order)
- Allow for $O(a^2)$ unitarity violations
- Allow sea and valence quarks to have different masses (Sharpe '90 '92, Bernard & Golterman '92 '94, Sharpe & Shoresh '00 '01)

  $\Rightarrow$ in continuum (or w/ GW quarks), can consider

$$G_c \equiv [SU(N_f+N_v|N_v)_L \otimes SU(N_f+N_v|N_v)_R] \otimes U(1)_{L+R}$$

$$\longrightarrow SU(N_f+N_v|N_v)_{L+R} \otimes U(1)_{L+R}$$

Inclusion of discretization errors at NLO

(Sharpe & Singleton '98, Aoki '03, Bär et al '03 '04, Sharpe '06, Chen et al '07)

- Executive summary:
  - construct Symanzik effective action of Wilson fermions at $O(a^2)$ (Symanzik '75 '83, Sheikholeslami & Wohlert '85)
  - for discretization operators which break G_c
    $\longrightarrow$ additional spurions $\sim a,\ a^2$
  - construct χ-Lagrangian using spurions in all possible ways consistent with G_c and power counting
  - operators which preserve G_c contribute to LO and NLO continuum LECs at NNLO and NNNLO and $O(4)$-breaking operators at NNNLO
- Upshot of analysis:
  - W-on-W:
    $\longrightarrow$ $8+1$ coupling constants of $O(ap^2, a^2)$
  - GW-on-W:
    $\longrightarrow$ 1 extra LEC of $O(a^2)$

Unitarity violations: the a_0 propagator

(Golterman et al '05, Chen et al '07)

Assume light sea (ℓ) and valence (v) are tuned such that

$$M_{vv} = M_{\ell\ell} \doteq M_\pi$$

Then, MA PQχPT at LO gives ($m_1 = m_2 \doteq m_v$)

$$C_{a_0}(t) \equiv a^3 \sum_{\vec{x}} \langle \bar{q}_2 q_1(\vec{x}, t) \bar{q}_1 q_2(0) \rangle$$

$$\xrightarrow{t\to+\infty} \frac{B^2}{L^3} \left\{ C_{K\bar{K}}(t) + \frac{2}{3} C_{\pi\eta}(t) - 2\frac{a^2\Delta}{M_\pi^2} (M_\pi t + 1)\, C_{\pi\pi}(t) \right\}$$

$\Rightarrow$ in a_0 channel $O(a^2)$ unitarity violations are LO, only vanish in continuum limit and are exponentially and polynomially enhanced in t

PQ result also has $m_{val} - m_{sea}$ unitarity violations

Charged PGB masses at NLO in finite volume Ω

$$
\begin{aligned}
(M_{12}^2)_\Omega^{\rm NLO} \quad = \quad & (m_1+m_2)B\Big\{1+\frac{1}{(4\pi F)^2}\Big[\text{PQ-logs}(\mu, M_{11}, M_{22}, M_{\ell\ell}, M_{ss}) \\
& +(2\alpha_6-\alpha_4)(\mu)(2M_{\ell\ell}^2+M_{ss}^2)+(2\alpha_8-\alpha_5)(\mu)\,M_{12}^2 \\
& +a\beta_M+a^2\Delta\times\text{UV-logs}(\mu, M_{11}, M_{22})+a^2\gamma_M(\mu)+\text{FV}\Big]\Big\}
\end{aligned}
$$

with $\alpha_i(\mu)\equiv 8(4\pi)^2 L_i(\mu)$

Continuum or GW-on-GW

- $m_1\,, m_2$: Lagrangian masses
- $\Delta=\gamma_M=0=\beta_M$

W-on-W

- $m_1\,, m_2$: NLO, AWI masses
- $\beta_M=O(\Lambda_{\rm QCD}^3)$ for W, $O(\alpha_s\Lambda_{\rm QCD}^3)$ for TL $O(a)$–W, 0 for NP $O(a)$–W
- $\Delta=\gamma_M=0$

. . . and their decay constants

GW-on-W

- $m_1\,, m_2$: GW Lagrangian masses
- $\beta_M = O(\Lambda^3_{\rm QCD})$ for W, $O(\alpha_s\Lambda^3_{\rm QCD})$ for TL $O(a)$–W, 0 for NP $O(a)$–W
- $\Delta\,, \gamma_M = O(\Lambda^4_{\rm QCD})$

  $\Rightarrow$ MA unitarity violations for $a \neq 0$

Charged PGB decay constants at NLO in Ω

$$\begin{aligned}(F_{12})^{\rm NLO}_{\Omega} \quad &= \quad F\Big\{1 + \tfrac{1}{2(4\pi F)^2}\Big[\text{PQ-logs}(\mu, M_{11}, M_{22}, M_{\ell\ell}, M_{ss}) \\ &\qquad + \alpha_4(\mu)(2M^2_{\ell\ell} + M^2_{ss}) + \alpha_5(\mu)\, M^2_{12} \\ &\qquad + a\beta_F + a^2\Delta \times \text{UV-logs}(M_{11}, M_{22}) + a^2\gamma_F + \text{FV}\Big]\Big\}\end{aligned}$$

Same three cases here as for masses, but in GW-on-W case, MA unitarity violations $\propto a^2\Delta$ are $SU(3)_{\rm val}$-breaking and do not depend on μ

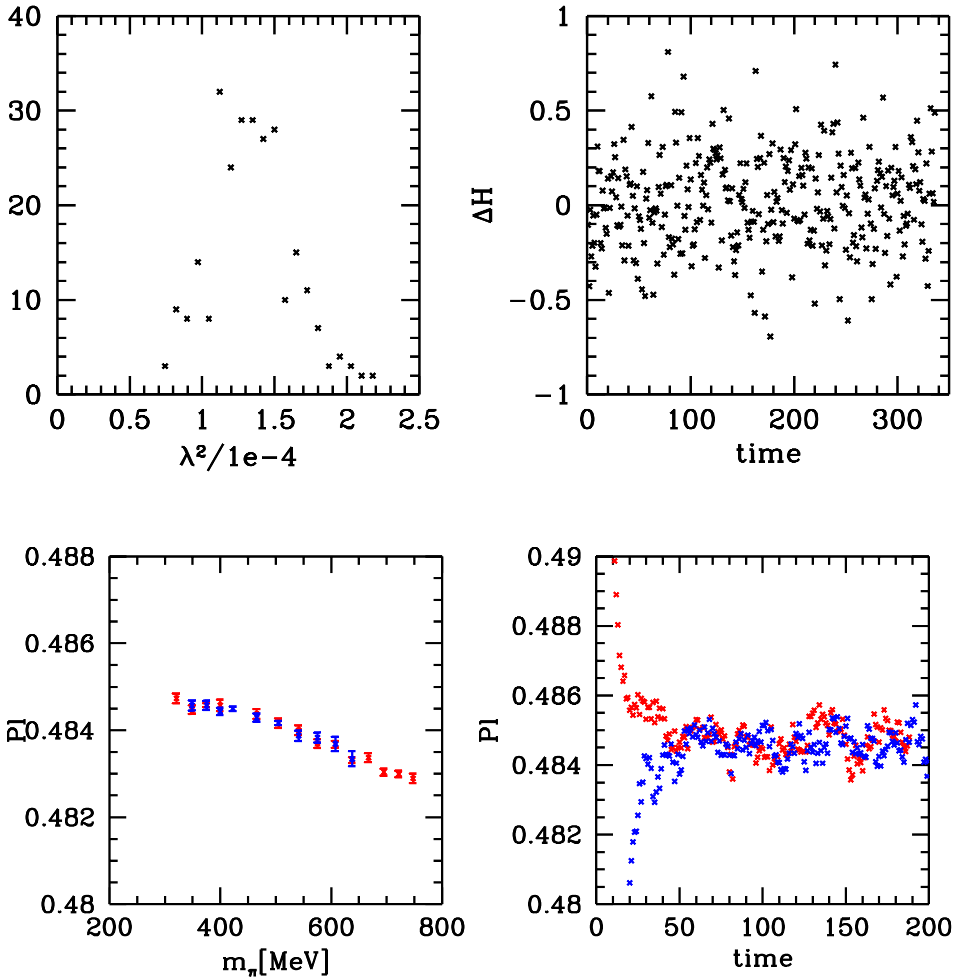
Simulation ingredients

- Gauge action: tree-level Symanzik improved
- Sea quarks: smeared-link, tree-level $O(a)$-improved Wilson fermions
- Valence quarks: same as sea (“unitary”) or smeared-link overlap fermions (“mixed-action”)
- Algorithm: Rational HMC with even-odd preconditioning, multiple time-scale Omelyan integration and Hasenbusch acceleration (Clark et al ’06, Sexton & Weingarten ’92, Omelyan et al ’03, Hasenbusch ’01, Urbach et al ’06)
- Renormalization: non-perturbative à la Rome-Southampton
- Parameters:
  - $a \sim 0.09\,\mathrm{fm}$
  - $M_\pi^{\mathrm{lat}} \sim 190, 300, 410, 490, 570\,\mathrm{MeV}$ with $M_\pi^{\mathrm{lat}} L \gtrsim 4$
  - Overlap roughly matched with Wilson
  - m_s^{lat} such that $M_K^{\mathrm{lat}} \simeq 1.07 M_K$ and 2 valence m_s^{lat} at $190, 300\,\mathrm{MeV}$
  - 34 configs at $190\,\mathrm{MeV}$, 68 at $300\,\mathrm{MeV}$ and $O(100)$ at other points

Calculations performed on BG/L’s at FZ Jülich and on clusters at the University of Wuppertal and CPT Marseille

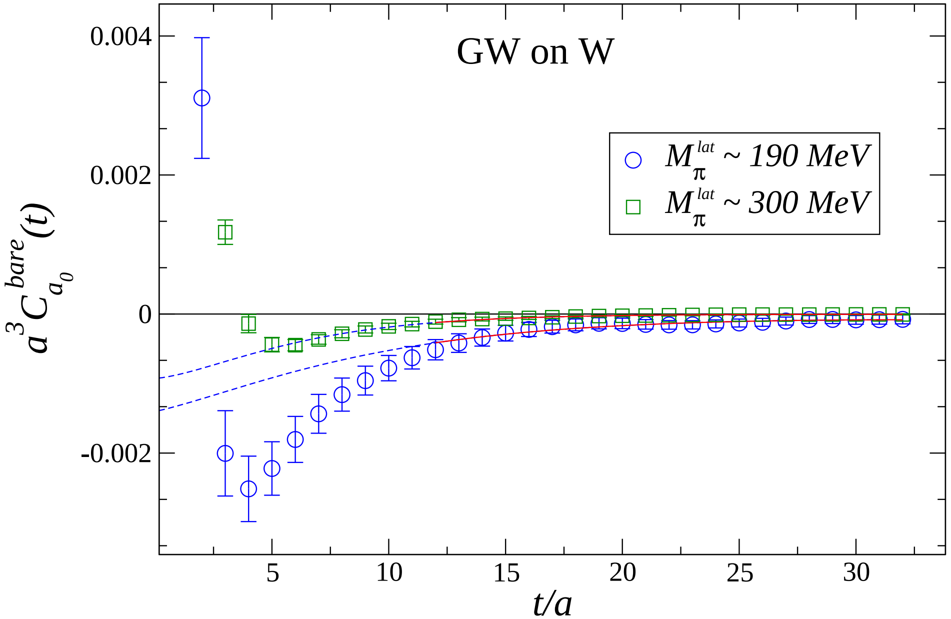
No metastabilities and stable algorithm

e.g. $a \sim 0.15\,\mathrm{fm}$, $\Omega/a^4 = 16^3 \times 32$ and $M_\pi^{\mathrm{lat}} \simeq 300\,\mathrm{MeV}$ (difficult simulation according to $\sqrt{\langle(\lambda_{min} - \bar{\lambda}_{min})^2\rangle} \simeq a/\sqrt{\Omega}$ criterion (Del Debbio et al '05))



Unitarity violations in the a_0 propagator (preliminary)

1 parameter ($a^4\Delta$) fit of scalar-isovector propagators to chiral expression for $C_{a_0}(t)$ at $M_\pi^{\rm lat} \sim 190\,\mathrm{MeV}$ and $300\,\mathrm{MeV}$

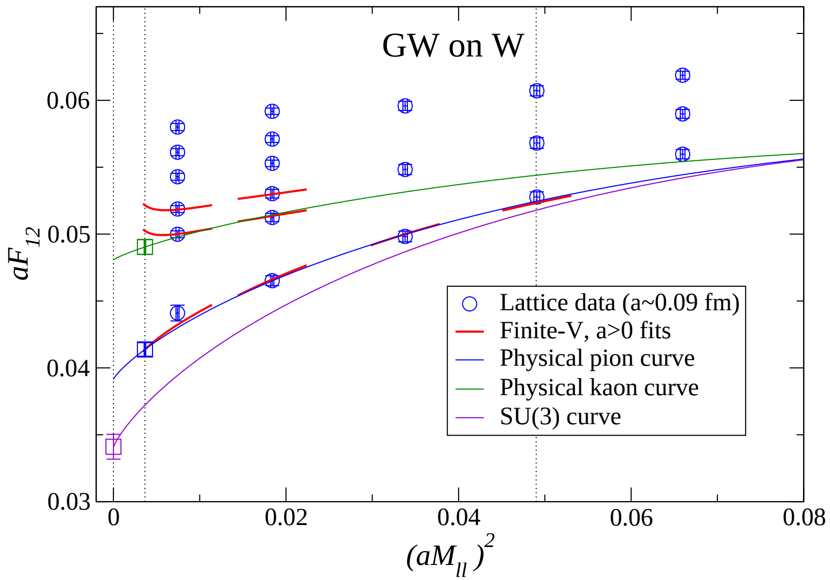


- Find $a^4\Delta = 0.015(6)$ and $0.024(10)$, i.e. compatible

$\Rightarrow$ $a\sqrt{\Delta} \sim 0.27\,\mathrm{GeV}$ and $0.35\,\mathrm{GeV}$, which compete with meson masses in chiral expressions

Preliminary fit to the PGB decay constants

- aF_{12} obtained using AWI $\longrightarrow$ no renormalization needed thanks to valence χS
- Fit 8 points with $M_\pi^{\text{lat}} \leq 500\,\text{MeV}$ and $M_K^{\text{lat}} \leq 590\,\text{MeV}$ to NLO expression with FV corrections and unitarity violations constrained with a_0 prior, $a^4\Delta = 0.024(10)$

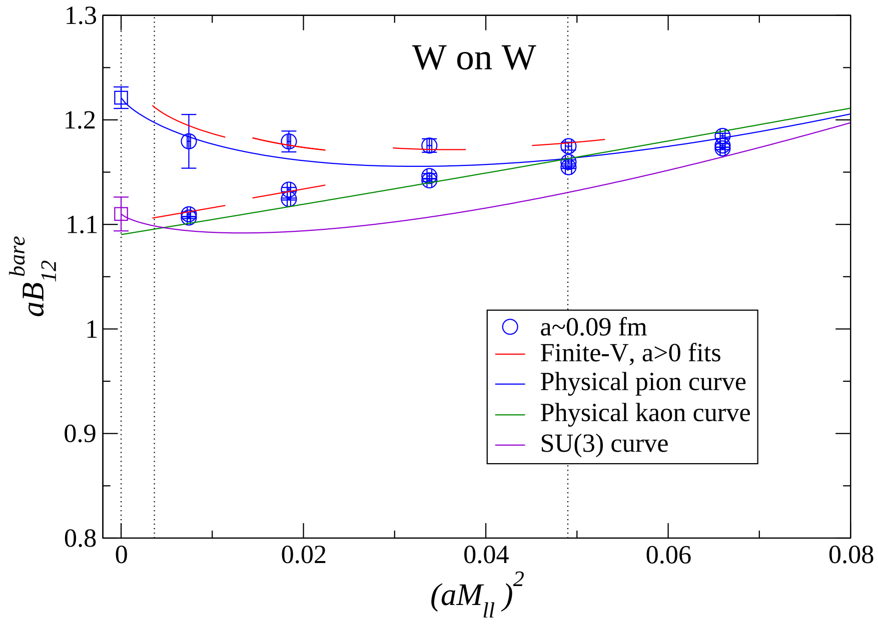


- Good χ^2/dof and find $a^4\Delta = 0.025(8)$
- Get a from self-consistent extrapolation to physical point

Preliminary fit to the W-on-W PGB masses

- Unitary theory
- Fit 6 points with $M_\pi^{\rm lat} \leq 500\,{\rm MeV}$ and $M_K^{\rm lat} \leq 590\,{\rm MeV}$ are fitted to NLO expression with FV corrections for

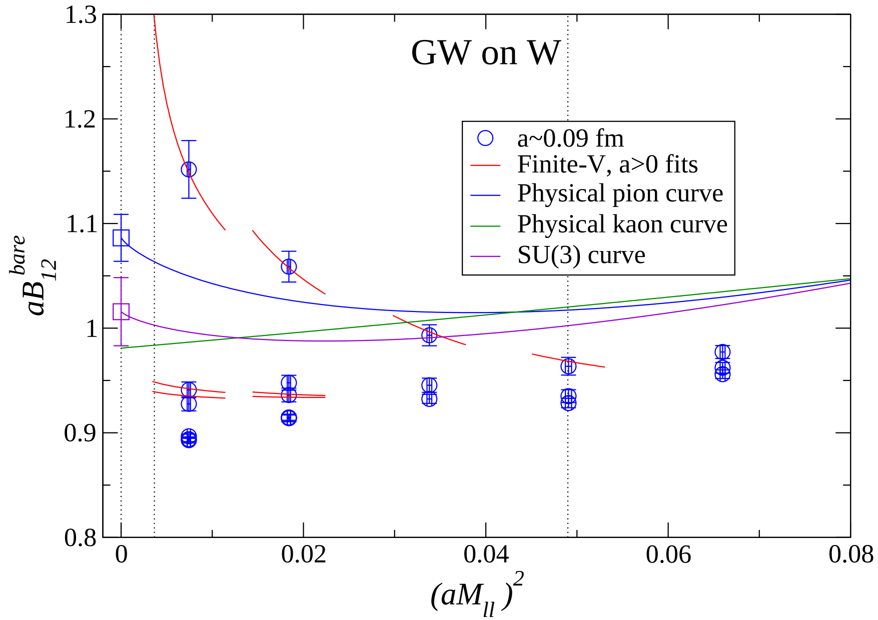
$$aB_{12}^{\rm bare} \equiv (aM_{12})^2/(am_1 + am_2)_{\rm AWI}^{\rm bare}$$



- Good χ^2/dof

Preliminary fit to the GW-on-W PGB masses

- Substantial deviation from behavior of W-on-W results and features not explainable with continuum PQχPT
- Fit 8 points with $M_\pi^{\rm lat} \le 500\,\rm MeV$ and $M_K^{\rm lat} \le 590\,\rm MeV$ to NLO expression with FV corrections and unitarity violations constrained with a_0 prior, $a^4\Delta = 0.024(10)$



- Good χ^2/dof and find $a^4\Delta = 0.020(6)$
- Physical results consistent with W-on-W, but residual discretization errors in overall scale of condensates and quark masses may be significant

Indicative PGB decay constant and mass fit results

Errors are statistical only $(M_\pi = 135\,\mathrm{MeV},\ M_K = 494\,\mathrm{MeV})$

qty	GW-on-W	W-on-W	MILC '07
a_{F_π} [fm]	0.088(1)		
F_K/F_π	1.185(7)		$1.197(3)^{+6}_{-13}$
$F_\pi/F_{N_f=2}$	1.056(1)		$1.052(3)^{+6}_{-3}$
$F_{N_f=2}/F_{N_f=3}$	1.15(2)		$1.15(5)^{+13}_{-3}$
$\alpha_4(M_\eta)$	0.7(1)		$0.5(4)^{+4}_{-1}$
$\alpha_5(M_\eta)$	2.9(1)		$2.8(3)^{+3}_{-1}$
$(2\alpha_6-\alpha_4)(M_\eta)$	0.20(4)	0.29(2)	$0.5(1)^{+3}_{-4}$
$(2\alpha_8-\alpha_5)(M_\eta)$	-0.62(13)	-0.71(3)	-0.1(1)(1)
m_s/m_{ud}	28.0(6)	28.3(1)	27.2(1)(3)(0)(0)
$\langle\bar{q}q\rangle_{N_f=2}/\langle\bar{q}q\rangle_{N_f=3}$	1.41(5)	1.45(5)	$1.52(17)^{+38}_{-15}$
quantities still requiring renormalization			
$m_{ud}^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})$ [MeV]	$3.8(1)(??)/Z_S$	$3.41(5)/Z_S$	3.2(0)(1)(2)(0)
$m_s^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})$ [MeV]	$107(3)(??)/Z_S$	$96(1)/Z_S$	88(0)(3)(4)(0)
$-\langle\bar{q}q\rangle^{\overline{\mathrm{MS}}}_{N_f=3}(2\,\mathrm{GeV})$ [MeV^3]	$Z_S\times[236(3)(??)]^3$	$Z_S\times[243(2)]^3$	$[242(9)^{+5}_{-17}(4)]^3$
$-\langle\bar{q}q\rangle^{\overline{\mathrm{MS}}}_{N_f=2}(2\,\mathrm{GeV})$ [MeV^3]	$Z_S\times[265(2)(??)]^3$	$Z_S\times[275(1)]^3$	$[278(1)^{+2}_{-3}(5)]^3$

Conclusion

- PGB masses and decay constants provide access to many important quantities, e.g. light quark masses, CKM matrix elements, vacuum properties and chiral Lagrangian LECs
- We are actively pursuing lattice calculations with $2+1$ dynamical flavors of tree-level improved Wilson sea quarks close to the physical QCD point
- Preliminary results for PGB masses and decay constants composed of either tree-level improved Wilson valence quarks (“unitary” simulations) or chirally symmetric overlap valence quarks (“mixed-action” simulations) were presented
- Fits of the valence and sea-quark mass-dependence of these results to NLO expressions in finite-volume MA PQχPT expressions were performed
- In the MA case, the unitarity violations predicted by MA PQχPT appear to provide a consistent description of the unphysical features in our results
- More detailed analyses and data other lattice spacings are needed to determine the extent to which we can reach the physical point in a model-independent way
- Weak matrix elements and non-perturbative renormalization are also being studied