Improving the theoretical status of $\pi(K) \to e\bar{\nu}_e[\gamma]$

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1. Motivation

i) $R^{(P)}_{e/\mu} \equiv \Gamma(P \to e\bar{\nu}_e[\gamma])/\Gamma(P \to \mu\bar{\nu}_\mu[\gamma])$ $(P = \pi, K)$ helicity suppressed

sensitive probe af SM extensions

attention in SUSY

ii) Effects from weak-scale new physics $(\Delta R_{e/\mu})/R_{e/\mu} \sim 10^{-4} - 10^{-2}$

iii) Ongoing experimental searches

$$(\Delta R^{(\pi)}_{e/\mu})/R^{(\pi)}_{e/\mu} \lesssim 5 \times 10^{-4}$$

$$(\Delta R^{(K)}_{e/\mu})/R^{(K)}_{e/\mu} \lesssim 3 \times 10^{-3}$$

iv) Strong dynamics cancels out partially in the ratio

SM can reach this precision

Required $O(e^2 p^4)$ corrections

present predictions:

$R^{(\pi)}_{e/\mu} = (1.2352 \pm 0.0005) \times 10^{-4}$ *

$R^{(\pi)}_{e/\mu} = (1.2354 \pm 0.0002) \times 10^{-4}$ **

$R^{(K)}_{e/\mu} = (2.472 \pm 0.001) \times 10^{-5}$ **

* Marciano and Sirlin '93
** Finkemeier '96

2. Overview

i) Our observable

$$R_{e/\mu}^{(\pi)} = \frac{\Gamma\left(\pi^+ \to e^+\nu_e(\gamma)\right)}{\Gamma\left(\pi^+ \to \mu^+\nu_\mu(\gamma)\right)}$$

ii) Framework: ChPT with photons and leptons *

iii) LO result [$O(p^2)$]

$$\Gamma^{(0)}(\pi \to \ell\nu) = \frac{G_F^2 |V_{ud}|^2 F_\pi^2}{4\pi} m_\pi \, m_\ell^2 \left(1 - \frac{m_\ell^2}{m_\pi^2}\right)^2 \longrightarrow R_{e/\mu}^{(0),(\pi)} = \frac{m_e^2}{m_\mu^2}\left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2$$

iv) NLO result

$$\Gamma(\pi \to \ell\nu[\gamma]) = \Gamma^{(0)}(\pi \to \ell\nu) \times \left[1 + 2\,\mathrm{Re}\left(r_\ell^{e^2p^2} + r_\ell^{e^2p^4}\right) + \delta_\ell^{e^2p^2} + \delta_\ell^{e^2p^4}\right]$$

$$R_{e/\mu}^{(\pi)} = R_{e/\mu}^{(0),(\pi)} \left[1 + \Delta_{e^2p^2}^{(\pi)} + \Delta_{e^2p^4}^{(\pi)} + \cdots\right]$$

$$\Delta_{e^2p^{2n}}^{(\pi)} = 2\,\mathrm{Re}\left(r_e^{e^2p^{2n}} - r_\mu^{e^2p^{2n}}\right) + \left(\delta_e^{e^2p^{2n}} - \delta_\mu^{e^2p^{2n}}\right)$$

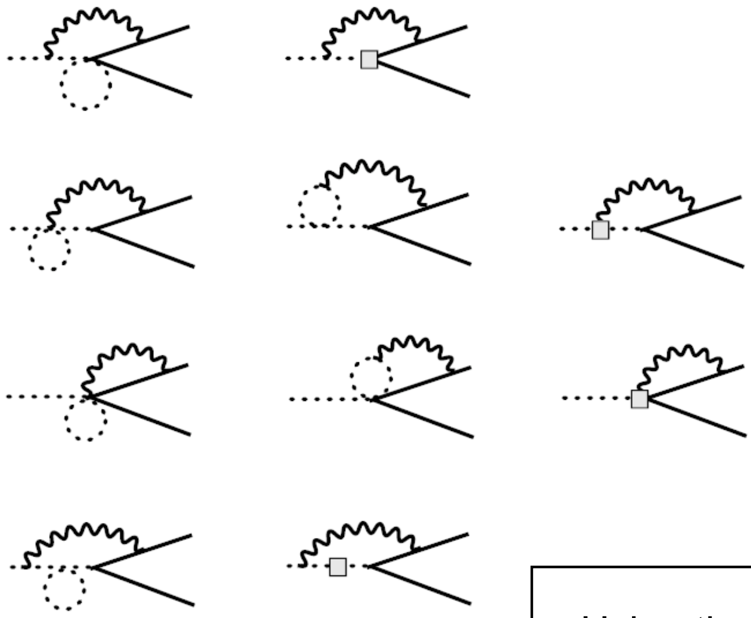
already calculated* ($\Delta_{e^2p^2}^{(\pi)}$)

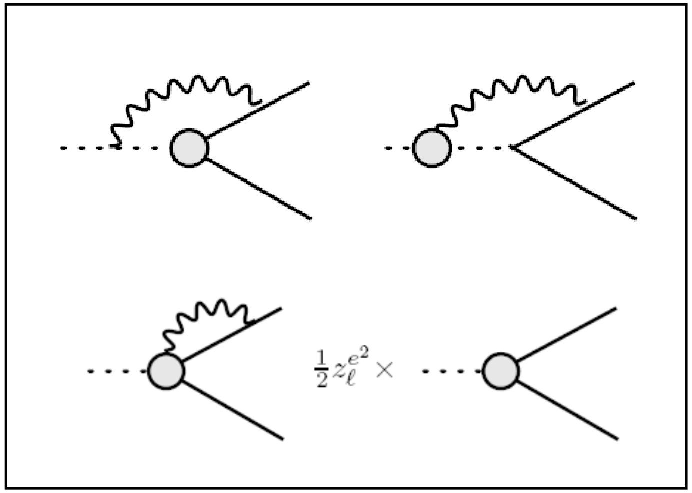
our aim ($\Delta_{e^2p^4}^{(\pi)}$)

* Knecht et al. '00

3. Virtual photon corrections

i) Feynman diagrams in which the photon connect to the lepton





- Using the physical $O(p^4)$ mas[redacted] $^2p^2)$
- Using a set of “effective” one-loop [redacted]d
  to $O(p^4)$ in d-dimenions

Genuine two-loop calculation with the one-loop subdivergences already subtracted

Virtual photon corrections (continuation)

ii) Non-1PI contributions

$$T_\ell^{e^2p^4}\Big|_{\text{non}-1PI} = \frac{1}{2} z_\ell^{e^2} \left(\frac{F_\pi^{(4)}}{F} - 1 \right) \times T_\ell^{p^2} ,$$

iii) 1PI contributions

$$T_\ell^{e^2p^4}\Big|_{1PI} = 2G_F V_{ud}^* e^2 F \int \frac{d^dq}{(2\pi)^d} \frac{\bar{u}_L(p_\nu)\gamma^\nu \left[-(\not p_\ell - \not q) + m_\ell\right] \gamma^\mu v(p_\ell)}{[q^2 - 2q \cdot p_\ell + i\epsilon] \, [q^2 - m_\gamma^2 + i\epsilon]} T_{\mu\nu}^{V-A}(p,q)$$
$$+ \left(m_\pi^2\big|_{p^4} - m_\pi^2\big|_{p^2} \right) \frac{\partial}{\partial m_\pi^2} T_\ell^{e^2p^2} ,$$

$$T_{\mu\nu}^{V-A} = \frac{1}{\sqrt{2}F} \int dx \; e^{iqx + iWy} \, \langle 0 | T(J_\mu^{EM}(x) \, (V_\nu - A_\nu)(y) | \pi^+(p) \rangle , \quad *$$

$$\left(T^{V-A}\right)^{\mu\nu}(p,q) = iV_1 \, \epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta + \left[\frac{(2p-q)^\mu (p-q)^\nu}{2p \cdot q - q^2} + g^{\mu\nu} \right] \left(\frac{F_\pi^{(4)}}{F} - 1 \right)$$
$$-A_1 \left(q \cdot p g^{\mu\nu} - p^\mu q^\nu \right) - (A_2 - A_1) \left(q^2 g^{\mu\nu} - q^\mu q^\nu \right)$$
$$+ \left[\frac{(2p-q)^\mu (p-q)^\nu}{2p \cdot q - q^2} - \frac{q^\mu (p-q)^\nu}{q^2} \right] \left(F_V^{\pi\pi}(q^2) - 1 \right)$$
$$-A_3 \left[q \cdot p \left(q^\mu p^\nu - q^\mu q^\nu \right) + q^2 \left(p^\mu q^\nu - p^\mu p^\nu \right) \right]$$

what we have to calculate

iv) Final contributions

$$T_\ell^{e^2p^4} = T_{V_1} + T_{A_1} + T_{A_2} + T_{F_V} + \left(\frac{F_\pi^{(4)}}{F} - 1 \right) T_\ell^{e^2p^2} + \left(m_\pi^2\big|_{p^4} - m_\pi^2\big|_{p^2} \right) \frac{\partial}{\partial m_\pi^2} T_\ell^{e^2p^2}$$

* Bijnens et al. '93

Virtual photon corrections (continuation)

vi) Form factors in d-dimensions

$$V_1 = -\frac{N_C}{24\pi^2 F^2}$$

$$A_1 = -\frac{4\,(L_9 + L_{10})}{F^2}$$

$$A_2 = -2\frac{\left(F_V^{\pi\pi}(q^2) - 1\right)}{q^2}$$

vii) Results*

$$F_V^{\pi\pi}(q^2) = 1 + 2\,H_{\pi\pi}(q^2) + H_{KK}(q^2)$$

$$r_\ell^{e^2p^4}\Big|_{V_1} = e^2\frac{(\mu c)^{4w}}{(4\pi)^4}\,\frac{1}{w}\,\frac{8N_C}{9}\frac{m_\ell^2}{F^2} + \frac{\alpha}{4\pi}V_1\left[\frac{m_\pi^2}{3}\ell_\ell + m_\ell^2\left(\frac{5}{9} - \frac{4}{3}\ell_\ell\right)\right]$$

$$r_\ell^{e^2p^4}\Big|_{A_1} = -e^2\frac{(\mu c)^{4w}}{(4\pi)^4}\,\frac{1}{w}\,\frac{28}{3}\left(\bar{L}_9 + \bar{L}_{10}\right)\frac{m_\ell^2}{F^2} + \frac{\alpha}{4\pi}A_1\left[-\frac{m_\pi^2}{3}\ell_\ell + m_\ell^2\left(\frac{13}{9} + \frac{7}{3}\ell_\ell\right)\right]$$

$$r_\ell^{e^2p^4}\Big|_{A_2} = e^2\frac{(\mu c)^{4w}}{(4\pi)^4}\left[\frac{1}{w^2} + \frac{1}{w}\left(\frac{3}{2} + 16\bar{L}_9\right)\right]\frac{m_\ell^2}{F^2}$$
$$+\frac{\alpha}{4\pi}\frac{m_\ell^2}{(4\pi F)^2}\left\{\left[\frac{13}{9} + 8\,\bar{L}_9 + \frac{16}{9}\left(\ell_\pi + \frac{1}{2}\ell_K\right) + \frac{2}{3}\left(\ell_\pi^2 + \frac{1}{2}\ell_K^2\right)\right]\right.$$
$$\left. +4\left(4\bar{L}_9 - \frac{1}{6} - \frac{1}{3}\ell_\pi - \frac{1}{6}\ell_K\right)\ell_\ell - \frac{8}{9}\log(z_\ell\sqrt{\tilde{z}_\ell}) + f_1(z_\ell) + \frac{1}{2}\,f_1(\tilde{z}_\ell)\right\}$$

$$r_\ell^{e^2p^4}\Big|_{F_V} = -e^2\frac{(\mu c)^{4w}}{(4\pi)^4}\left[\frac{1}{4\,w^2} + \frac{1}{w}\left(\frac{5}{12} + 4\bar{L}_9\right)\right]\frac{m_\ell^2}{F^2}$$
$$+\frac{\alpha}{4\pi}\frac{m_\ell^2}{(4\pi F)^2}\left\{\left[-\frac{19}{36} - \left(\frac{1}{2} + 4\bar{L}_9\right)\ell_\pi + \frac{1}{6}\ell_\pi^2 + \frac{1}{6}\ell_\pi\ell_K - \frac{1}{3}\ell_K - \frac{1}{12}\ell_K^2\right]\right.$$
$$\left. +\frac{z_\ell}{1 - z_\ell}\log z_\ell\left(4\bar{L}_9 - \frac{1}{6} - \frac{1}{3}\ell_\pi - \frac{1}{6}\ell_K\right) + f_2(z_\ell) + f_3(\tilde{z}_\ell, \tilde{z}_\pi)\right\}.$$

* Bijnens et al. ‘97
* Gasser et al. ‘99

4. Matching

i) Amplitude UV finite ⟶ local counterterm

$$r_\ell^{e^2p^4}\Big|_{CT} = e^2\,\frac{m_\ell^2}{F^2}\,\frac{(\mu c)^{4w}}{(4\pi)^4}\left[\frac{d_2}{w^2} + \frac{d_1^{(0)} + d_1^{(L)}(\mu)}{w} + r_{CT}(\mu)\right]$$

ii) $r_{CT}(\mu)$ needs to be estimated

truncated large-N_C QCD

$$T_\ell^{e^2p^4,CT} = \int \frac{d^dq}{(2\pi)^d}\,K(q,p,p_e)\,\Pi_{QCD_\infty}(q^2,W^2)\Big|_{e^2p^4} - \int \frac{d^dq}{(2\pi)^d}\,K(q,p,p_e)\,\Pi^{p^4}_{ChPT_\infty}(q^2,W^2)\,.$$

iii) Meromorphic approximation of form factors* ⟵ OPE

* Moussalam '97
* Knecht et al. '01
* Cirigliano et al. '04 '06

5. Real photon corrections

i) Radiative decay

$$\pi^+(p) \to \ell^+(p_\ell)\nu(p_\nu)\gamma(q)$$

Inner Bremssttrahlung (IB) $O(ep)$

Structure dependent (SD) $O(ep^3)$

ii) Amplitude*

$$T_\ell^{\rm rad} = i\,2eG_F F_\pi V_{ud}^*\,\epsilon_\mu^*(q)\Big(B^\mu - H^{\mu\nu}\,l_\nu\Big)$$

$$B^\mu = m_\ell\,\bar{u}_L(p_\nu)\left[\frac{2p^\mu}{2p\cdot q} - \frac{2p_\ell^\mu + \not{q}\gamma^\mu}{2p_\ell\cdot q}\right]v(p_\ell)$$

$$H^{\mu\nu} = iV_1\epsilon^{\mu\nu\alpha\beta}q_\alpha p_\beta - A_1\Big(q\cdot(p-q)\,g^{\mu\nu} - (p-q)^\mu\,q^\nu\Big)$$

$$l_\nu = \bar{u}_L(p_\nu)\gamma_\nu v(p_\ell)\,,$$

iii) Results

IB contribution → $\delta_\ell^{e^2p^2}$ **

Interference between IB and SD → $\delta_\ell^{e^2p^4}$

SD contribution → $\delta_\ell^{e^2p^6}$

* Bijnens et al. '93
** Kinoshita '59

6. Phenomenology

i) Observable

$$R_{e/\mu}^{(P)} \equiv \Gamma(P \to e\bar{\nu}_e[\gamma])/\Gamma(P \to \mu\bar{\nu}_\mu[\gamma]) \ (P = \pi, K)$$

ii) Perturbative organization

$$R_{e/\mu}^{(P)} = R_{e/\mu}^{(0),(P)} \left[1 + \Delta_{e^2p^2}^{(P)} + \Delta_{e^2p^4}^{(P)} + \Delta_{e^2p^6}^{(P)} + \dots\right]\left[1 + \Delta_{LL}\right]$$

$$\Delta_{e^2p^4}^{(P)} = \frac{\alpha}{\pi}\frac{m_\mu^2}{m_\rho^2}\left(c_2^{(P)} \log\frac{m_\rho^2}{m_\mu^2} + c_3^{(P)} + c_4^{(P)}(m_\mu/m_P)\right) + \frac{\alpha}{\pi}\frac{m_P^2}{m_\rho^2}\tilde{c}_2^{(P)} \log\frac{m_\mu^2}{m_e^2}$$

iii) Result

	$(P = \pi)$	$(P = K)$
$\tilde{c}_2^{(P)}$	0	$(7.84 \pm 0.07_\gamma) \times 10^{-2}$
$c_2^{(P)}$	$5.2 \pm 0.4_{L_9} \pm 0.01_\gamma$	$4.3 \pm 0.4_{L_9} \pm 0.01_\gamma$
$c_3^{(P)}$	$-10.5 \pm 2.3_{\rm m} \pm 0.53_{L_9}$	$-4.73 \pm 2.3_{\rm m} \pm 0.28_{L_9}$
$c_4^{(P)}(m_\mu)$	$1.69 \pm 0.07_{L_9}$	$0.22 \pm 0.01_{L_9}$

	$(P = \pi)$	$(P = K)$
$\Delta_{e^2p^2}^{(P)}$ (%)	-3.929	-3.786
$\Delta_{e^2p^4}^{(P)}$ (%)	0.053 ± 0.011	0.135 ± 0.011
$\Delta_{e^2p^6}^{(P)}$ (%)	0.073	
Δ_{LL} (%)	0.055	0.055

Matching uncertainty estimated in a conservative way:
100% of the local contribution + residual scale dependence

Phenomenology (continuation)

$$R_{e/\mu}^{(\pi)} = (1.2352 \pm 0.0001) \times 10^{-4}$$
$$R_{e/\mu}^{(K)} = (2.477 \pm 0.001) \times 10^{-5} \ .$$

	$10^4 \cdot R_{e/\mu}^{(\pi)}$	$10^5 \cdot R_{e/\mu}^{(K)}$
This work	1.2352 ± 0.0001	2.477 ± 0.001
Marciano et al.	1.2352 ± 0.0005	
Finkemeier	1.2354 ± 0.0002	2.472 ± 0.001

7. The individual $\pi(K) \to \ell\bar{\nu}_\ell$ modes

LO result

Universal short-distance EW correction

Kinoshita' 56

$$\Gamma_{P_{\ell 2[\gamma]}} = \Gamma^{(0)} \times \left\{1 + \frac{2\alpha}{\pi}\log\frac{m_Z}{m_\rho}\right\} \times \left\{1 + \frac{\alpha}{\pi} F(m_\ell^2/m_P^2)\right\} \times \left\{1 - \frac{\alpha}{\pi}\left[\frac{3}{2}\log\frac{m_\rho}{m_P} + c_1^{(P)} + \frac{m_\ell^2}{m_\rho^2}\left(c_2^{(P)}\log\frac{m_\rho^2}{m_\ell^2} + c_3^{(P)} + c_4^{(P)}(m_\ell/m_P)\right) - \frac{m_P^2}{m_\rho^2}\tilde{c}_2^{(P)}\log\frac{m_\rho^2}{m_\ell^2}\right]\right\},$$

Studied at $O(e^2p^2)$:
Given in Knecht et al. '00
Estimated in Descotes-Genon et al. '05

This work at $O(e^2p^4)$

The only remaining task:
$c_1^{(P)}$ at $O(e^2p^4)$

Future work