Light glueball spectrum from the AdS/CFT correspondence

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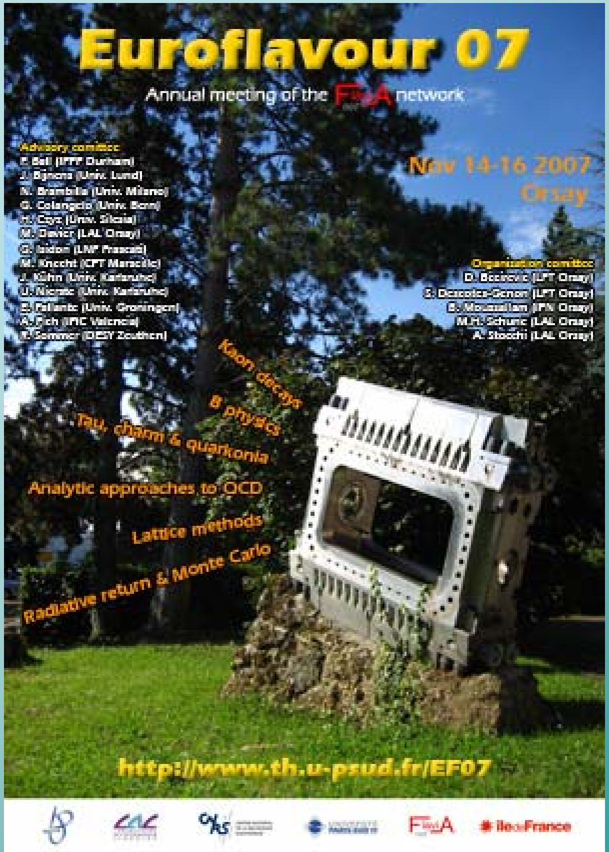


In collaboration with P. Colangelo, F. De Fazio, S. Nicotri

Orsay 14/11/2007
Euroflavour '07

AdS/CFT correspondence provides a new way to address Physics at strong coupling

- Hadronic spectrum:
  - vector ρ meson (Erlich et al. ‘05)
  - glueballs (Pietro Colangelo, Fulvia De Fazio, F.J., Stefano Nicotri ‘07)

↳ **the subject of this talk**

- Hadronic (ρ, π) form factors (Brodsky, de Teramond, Radyushkin ‘07)
- Q$\overline{\text{Q}}$ potential (Andreev, Zakharov ‘07)
- Gluon condensate (Andreev, Zhakarov ‘07) :

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle = (0.010 \pm 0.0023)\ \mathrm{GeV}^4$$

- U(1)$_A$ sector of QCD/ η’ mass (Katz, Schwartz, Schäfer ‘07)
- Low Energy Constants, χSB (Da Rold, Pomarol ‘05)
- Deep Inelastic Scattering (Braga ’07)
- Heavy ion collisions/QGP :
  - strongly coupled plasma features
  - confinement/deconfinement transition

  (Rajagopal, Shuryak, Iancu, Mueller ‘07)

Maldacena's AdS/CFT duality conjecture ('98)

IIB (oriented closed) superstring theory ⟺ $\mathcal{N}$ = 4 Superconformal YM theory SU(N_c)

in $AdS_5 \times S^5$ ⟺ in the boundary ∂AdS_5 (z → 0)

Anti de Sitter space × compact space

Holographic spacetime / bulk
(<u>no physical</u> extra dimensions) :

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2) + ds^2_{S^5}$$

R : AdS typical size

l_s : string typical size

$$\boxed{\begin{aligned} g_{string} &= g^2_{YM} \\ \frac{R^4}{l_s^4} &= 4\pi g^2_{YM} N_c \end{aligned}}$$

't Hooft coupling : $\lambda \equiv g^2_{YM} N_c$

- classical limit : **g_{string} → 0**
- supergravity limit : l_s << R

⟺

- large N_c (λ fixed) : g_{YM} → 0
- large 't Hooft limit : **λ >> 1**

$$\boxed{e^{iS_5^{eff}[X(x^\mu,z)]} = \left\langle e^{i\int d^4x\, X_0(x^\mu)\mathcal{O}(x^\mu)} \right\rangle_{CFT}}$$

Weakly - coupled effective theory
in a warped higher dim. space

Strongly - coupled gauge theory

Classical bulk field $X(x^\mu, z)$ $\xrightarrow[\text{Z} \to 0]{}$ Boundary value $X_0(x^\mu)$: Source for $\mathcal{O}(x^\mu)$

Scale invariance and its breaking (or what is z ?)

⇒ AdS/CFT provides 2 languages for deriving correlator functions !

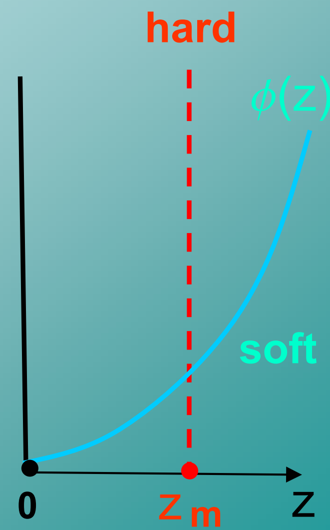
- $ds^2 = \frac{R^2}{z^2}(dx^2 + dz^2)$ inv. / scale transf. mapped into the 5th holographic coord. z

$$x \to \lambda x \longrightarrow z \to \lambda z$$

- AdS modes in z : extension of the hadron wave functions into the 5th holo. coord. z
- different values of z : different scales at which the hadron is examined **:**

- ∂AdS_5 (z → 0) i.e. q → ∞ **:** UV regime

- max. separation of quarks (~ x^2) → max. value of z at IR boundary **:**



Hard wall approx. (Polchinski, Strassler ‘01) : $0 < z \leq z_m \sim 1/\Lambda_{QCD}$

⇒ quadratic Regge trajectories: $m_n^2 \propto n^2$

Soft wall approx. (Karch et al ‘06) : background dilaton $\phi(z) = a^2 z^2$

⇒ linear Regge trajectories for vector mesons : $m_n^2 \propto n$

⇒ (a, z_m) ***break*** conformal inv. of CFT : introduction of ***QCD scale*** Λ_{QCD}

- AdS/CFT : String-like theories → QCD-like gauge theories (up-down approach)
- AdS/QCD : QCD properties → weakly-coupled effective theory in a warped higher dim. space (***bottom-up*** approach)

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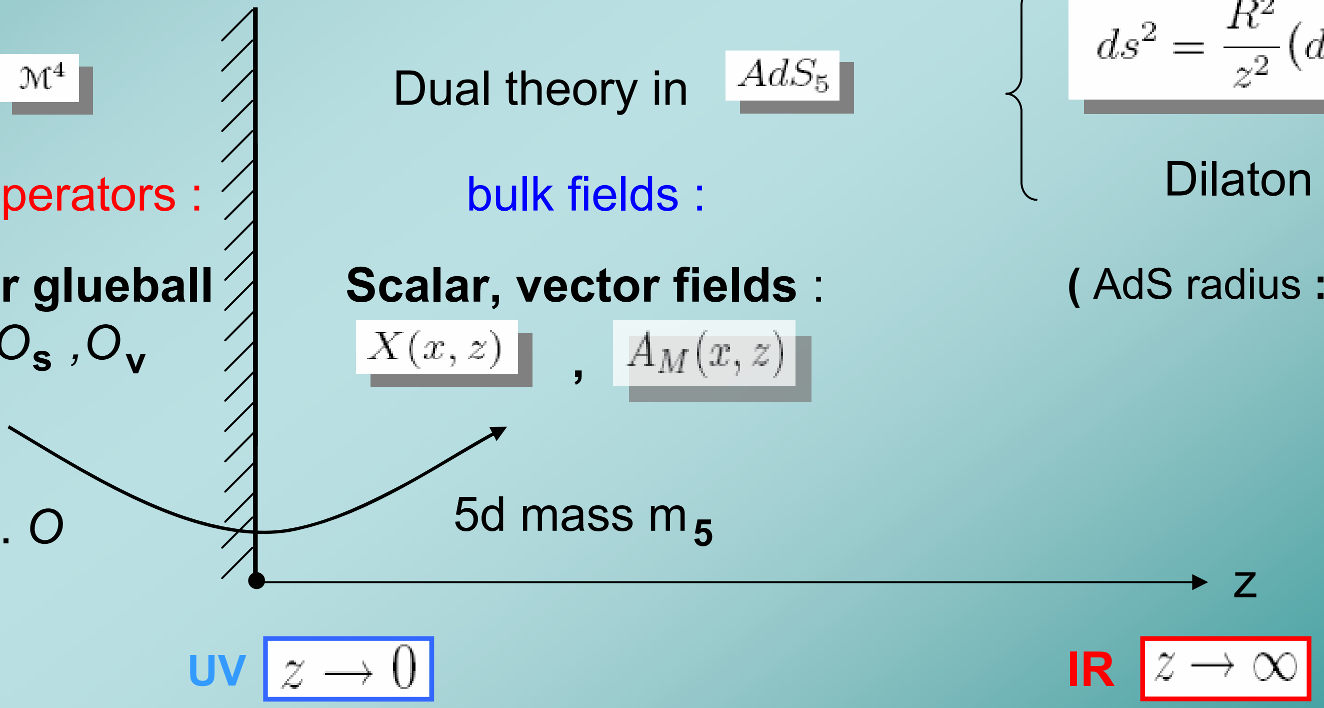
AdS/QCD Model of light glueballs (scalar, vector)

Glueballs : Bound-states of gluons well defined in large N_c limit



QCD in $\mathcal{M}^4$

Boundary operators :

Scalar, vector glueball operators O_s , O_v

dim. O

Dual theory in AdS_5

bulk fields :

Scalar, vector fields : $X(x,z)$, $A_M(x,z)$

5d mass m_5

z

UV $z \to 0$

IR $z \to \infty$

$$ds^2 = \frac{R^2}{z^2}(dx^2 + dz^2)$$

Dilaton : $\phi(z)$

(AdS radius : R=1)

- **Scalar bulk field :**

$$S_5^{eff} = -\frac{1}{2}\int d^5x\sqrt{-g}e^{-\phi(z)}g^{MN}(\partial_M X)(\partial_N X)$$

- **Vector bulk field :**

$$S_5^{eff} = -\frac{1}{2}\int d^5x\sqrt{-g}e^{-\phi(z)}\left[\frac{1}{2}g^{MN}g^{ST}F_{MS}F_{NT} + m_{AdS}^2\, g^{ST}A_S A_T\right]$$

5-dim. bulk

Dilaton $\phi(z) = a^2z^2$

Bulk field mass

$F_{MS} = \partial_M A_S - \partial_S A_M$

⇓

- **Broken** AdS isometries/conformal sym. (**energy scale [a]=1**)
- **Regge behaviour** of the mass spectrum

- (Classical) eq. of motion :

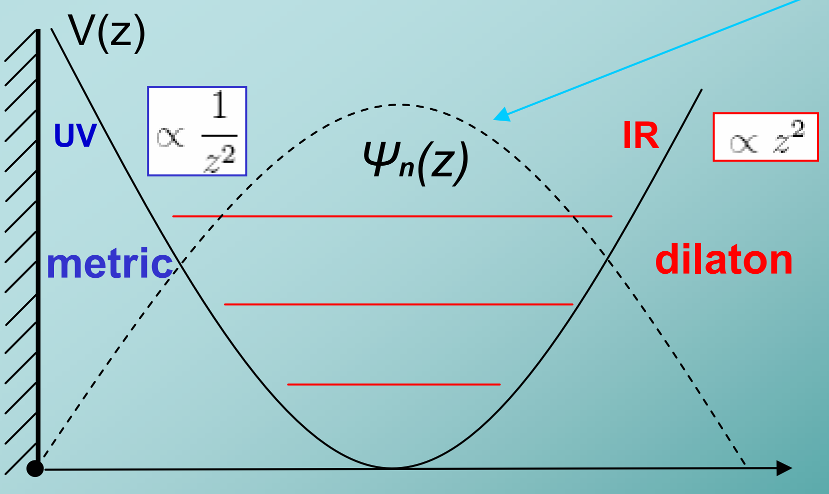
$$\partial_N\left(\sqrt{-g}e^{-\phi}F^{NM}\right) - \sqrt{-g}e^{-\phi}m_5^2A^M = 0$$

- Bulk field decomposition (mode) :

$$A_\mu(x,z) = \epsilon_\mu e^{iq\cdot x}\psi(z)$$

holo. wave function

M_n^2



QCD

Effective theory

- Schrödinger eq. : $-\psi'' + V(z)\psi = m_n^2\psi(z)$ with $V(z) = \underbrace{a^4 z^2}_{} + \underbrace{\frac{4m_5^2 + (c+2)c}{4z^2}}_{} + \underbrace{(c-1)a^2}_{}$

$$\begin{cases} c = 1 : & A_M(x,z) \\ c = 3 : & X(x,z) \end{cases}$$

dilaton $\phi(z) = a^2 z^2$

(**IR** : z → ∞)

metric $g_{MN} = \frac{1}{z^2}\eta_{MN}$

(**UV** : z → 0)

- **Mass spectrum :** $m_n^2 = \left(4n + 1 + c + \sqrt{(c+1)^2 + 4m_5^2}\right)a^2$

- **Holo. wave function :** (regular)

$$\psi_n(z) = A_n\, e^{-a^2z^2/2} z^{g(c,m_5^2)+1/2}\, \underbrace{{}_1F_1\left(-n, g(c,m_5^2)+1, a^2z^2\right)}_{\text{polynomial}} \longrightarrow 0 \begin{cases} z \to \infty \\ z \to 0 \end{cases}$$

- Spectrum given by a simple 1d. QM Shrödinger-like equation to resolve !

- AdS/CFT provides another language with tractable computations for non-perturbative Physics!

- Scalar glueball : $\boxed{m_n^2 = \left(4n+8\right)a^2}$
- Vector glueball : $\boxed{m_n^2 = \left(4n+12\right)a^2}$

⇨ **Regge behaviour (n >>1) :** $\boxed{m_n^2 \propto n}$ (dilaton $\phi(z) = a^2 z^2$)

$\neq$
- up-down approach
- Hard wall $0 < z \leq z_m$ (z_m : IR brane)

$\left.\right\}$ $m_n^2 \propto n^2$ (KK spectrum)

⇨ Ground states : $\boxed{m_{G_1}^2 - m_{G_0}^2 = m_\rho^2}$ (a=m_ρ /2 (Karch et al.'06))

AdS/QCD		QCDSR			Lattice QCD	
PLB 652:73-78,2007		**Dominguez, Paver ('86)**	**Narison (hep-ph/9612457)**	**Hang, Zhang (hep-ph/9801214)**	**Morningstar (hep-lat/9901004)**	**Meyer (hep-lat/0508002)**
m_{G_0}	1.096	< 1	1.5 (0.2)	1.580(150)	1.730(50)(80)	1.475(30)(65)
m_{G_1}	1.342				3.850(50)(190)	3.240(330)(150)

too light (?)

Let us see if this picture can be improved with further deviations of conformality

⇨ Background corrections (but still close to AdS$_5$)

Modification of the background (Karch et al. '06)

modification of the **dilaton**

$$\begin{aligned} \phi(z) &= a^2 z^2 + \lambda a z \\ g_{MN} &= \frac{1}{z^2}\eta_{MN} \end{aligned}$$

IR subleading
(z → ∞)

$$m_n^2 = m_{n,(0)}^2 + \lambda m_{n,(1)}^2$$

modification of the **metric**

$$\begin{aligned} \phi(z) &= a^2 z^2 \\ g_{MN} &= \frac{e^{-\lambda a z}}{z^2}\eta_{MN} \end{aligned}$$

UV subleading
(z → 0)

Mass splitting

- **dilaton :** $m_{G_1}^2 - m_{G_0}^2 = \left(4 - \frac{3\sqrt{\pi}}{128}\lambda\right)a^2$
- **metric :** $m_{G_1}^2 - m_{G_0}^2 = \left(4 - \frac{1899\sqrt{\pi}}{128}\lambda\right)a^2$

$\lambda < 0$

Mass splitting increases

Maximun effect : **metric**

Conclusion

AdS/CFT provides a new way to address Physics at strong coupling

⇨ Scalar and vector glueball mass spectra Ground states : $\boxed{m_{G_1}^2 - m_{G_0}^2 = m_\rho^2}$

⇨ **Perturbation** of the background (metric/dilaton)

- **dilaton** modification: $\boxed{m_{G_1}^2 - m_{G_0}^2 = \left(4 - \frac{3\sqrt{\pi}}{128}\lambda\right)a^2}$
- **metric** modification : $\boxed{m_{G_1}^2 - m_{G_0}^2 = \left(4 - \frac{1899\sqrt{\pi}}{128}\lambda\right)a^2}$

max. mass splitting ($\lambda < 0$) ↗

AdS/QCD at its very beginning :

- fashionable at present (bad reason to investigate it)

up-down approach (quantitative predictions difficult)

⇩

- there is the strong hope to identify the ***Dual Theory of QCD*** ⇨ predictions (at low energy !)

⇧

bottom-up approach

Backup Slides

More about the operator/field correspondence

- Bulk field A(x,z) : p-form (totally antisymmetric tensor with p indexes)

  0-form : ϕ (scalar)

  1-form : A_M (vector)

  2-form : $A_{[M,N]}$ (strength field F_{MN})

- eq. of motion $\mathfrak{D}A(x^M) = 0$ ⟸ mass term $m_{AdS}^2 A(x^M)$

Bulk

p, m$_{AdS}$

- Superconformal gauge theory : conformal group invariant

m$_{AdS}$

m$_{AdS}$ **m$_{AdS}$**

Scale transf. : $x^\mu \rightarrow \lambda x^\mu$

Field $A_0(x^\mu) \rightarrow \lambda^{-\tilde{\Delta}} A_0(x^\mu)$

Operator $O(x^\mu) \rightarrow \lambda^{-\Delta} O(x^\mu)$

$\Delta, \tilde{\Delta}$: scaling dim. = canonical dim.
(without anomalous dim.)

$$\langle e^{i\int d^4x A_0(x) O(x)} \rangle_{CFT} \rightarrow \langle e^{i\int d^4x \lambda^{4} \lambda^{-\tilde{\Delta}} A_0(x) \lambda^{-\Delta} O(x)} \rangle_{CFT}$$

$$4 - \tilde{\Delta} - \Delta = 0$$ or $$\tilde{\Delta} = 4 - \Delta$$

Boundary

4, Δ

$$R^2 m_{AdS}^2 = (\Delta - p)(\Delta + p - 4)$$

AdS ⟷ CFT

Holographic space : Bulk AdS_5 ⟷ Our spacetime $\mathcal{M}^4$

String theory
- weakly coupled
- classical

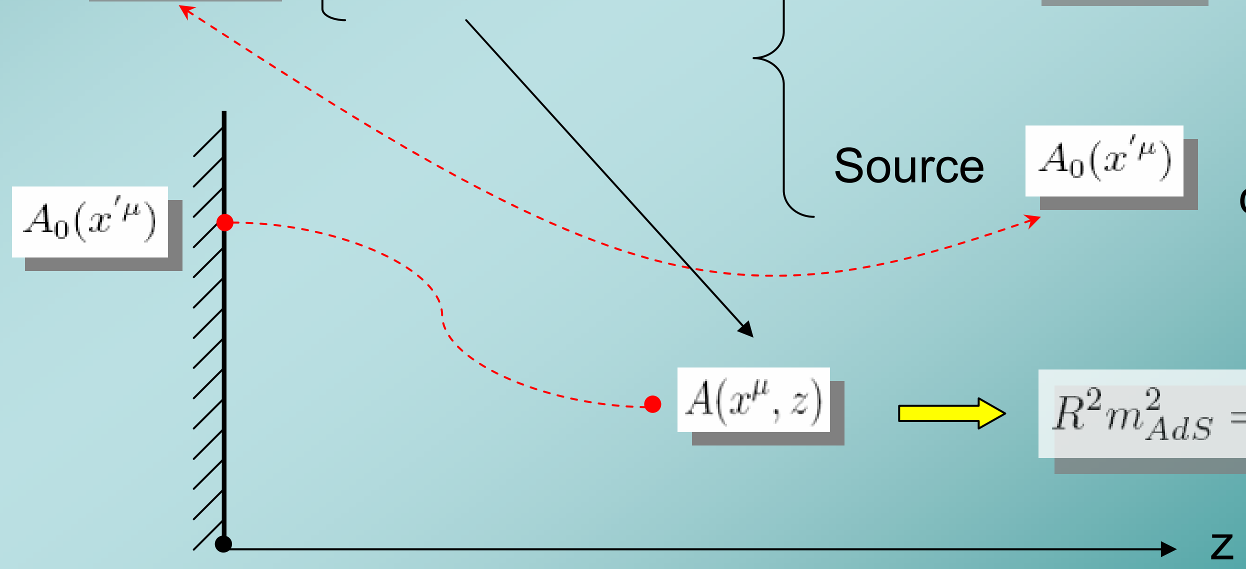
⟷ SU(N)
- strongly coupled λ
- SUSY
- conformal

Bulk field $A(x^\mu, z)$
- p-form
- massive

⟷

Operator $O(x^\mu)$ scaling dimension Δ

Source $A_0(x'^\mu)$ scaling dimension $\tilde{\Delta}$



$A(x^\mu, z)$ → $R^2 m^2_{AdS} = (\Delta - p)(\Delta + p - 4)$

AdS/QCD Correspondence (Witten '98)

QCD SU(3)c

- ~~squarks, gluinos~~ → ~~SUSY~~
- massive quarks
- renormalisation scale μ

massive quarks, renormalisation scale μ → ~~CFT~~

Holographic dual theory of QCD ?

How modifying AdS/CFT towards AdS/QCD ?

QCD could be nearly conformal (**UV**) (Brodsky '02; Alkofer et al. '04)

QCD could have **IR** fixed point

Dimensionless renormalized Green function : $G^{(n)}(p, m, \lambda, \mu)$

Renormalization :

$\mu \to \bar{\mu}(t) = e^t \mu$

- effective coupling $\lambda \to \bar{\lambda}(t, \lambda)$ with $\frac{d\bar{\lambda}(t)}{dt} = \beta(\bar{\lambda})$
- effective mass $m \to \bar{m}(t, m)$ with $\frac{d \ln \bar{m}(t)}{dt} = \gamma_m(\bar{\lambda})$

Homogeneous RGE :

$$(\mu\frac{\partial}{\partial\mu} + \beta(\lambda)\frac{\partial}{\partial\lambda} + \gamma_m(\lambda)m\frac{\partial}{\partial m})G^{(n)} = 0$$

Scale transf. :

$$G^{(n)}(p,m,\lambda,\mu) \rightarrow G^{(n)}(e^t p,\underbrace{m,}_{0}\lambda,\mu) = G^{(n)}(p,\underbrace{\bar{\lambda}(t),}_{\lambda}\underbrace{e^{-t}\bar{m}(t),}_{0}\mu)$$

Chiral limit m=0 : λ(t) breaks scale invariance

Classical theory or fixed point : β=0 and λ(t) = λ = const.

scale invariant theory

Chiral QCD : m=0

IR fixed point λ* : β(λ*)=0

QCD nearly conformal invariant

AdS/CFT

AdS/QCD

Effective bulk field action

$$S_5^{eff}[A(x^M)]$$

Deformation of the geometry

$$ds^2 = e^{2A(z)}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2)$$

AdS/QCD spectrum of ρ meson (Son et al. '05)

QCD in $\mathcal{M}^4$

Chiral symmetry

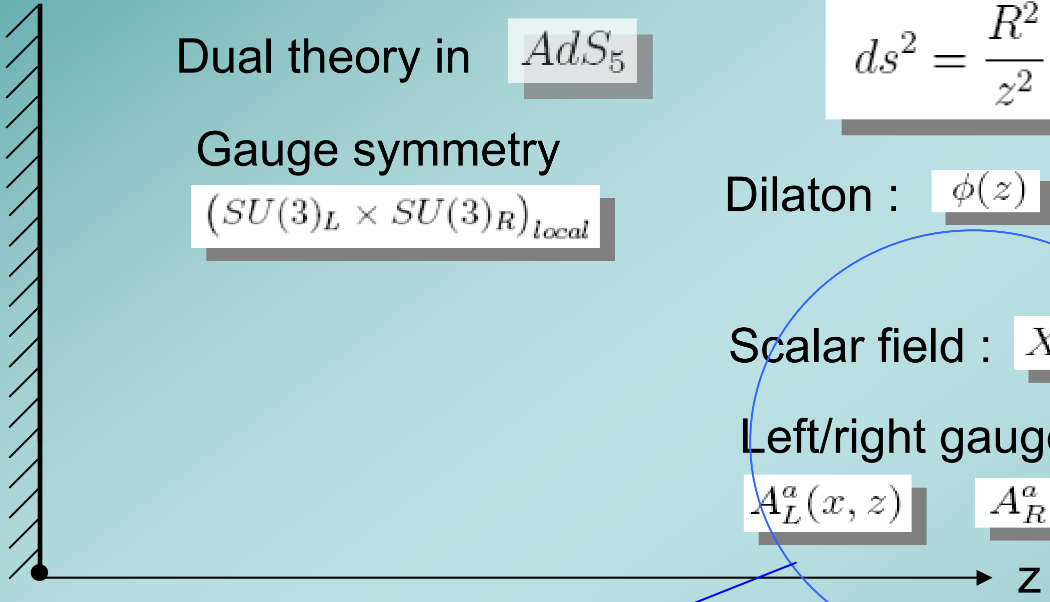
$(SU(3)_L \times SU(3)_R)_{global}$

Condensate : $\bar{q}q(x)$

Left/right currents :

$j^a_L(x)$ $j^a_R(x)$

operators



Dual theory in AdS_5

Gauge symmetry

$(SU(3)_L \times SU(3)_R)_{local}$

$$ds^2 = \frac{R^2}{z^2}(dx^2 + dz^2)$$

Dilaton : $\phi(z)$

Scalar field : $X(x,z)$

Left/right gauge fields :

$A^a_L(x,z)$ $A^a_R(x,z)$

bulk fields

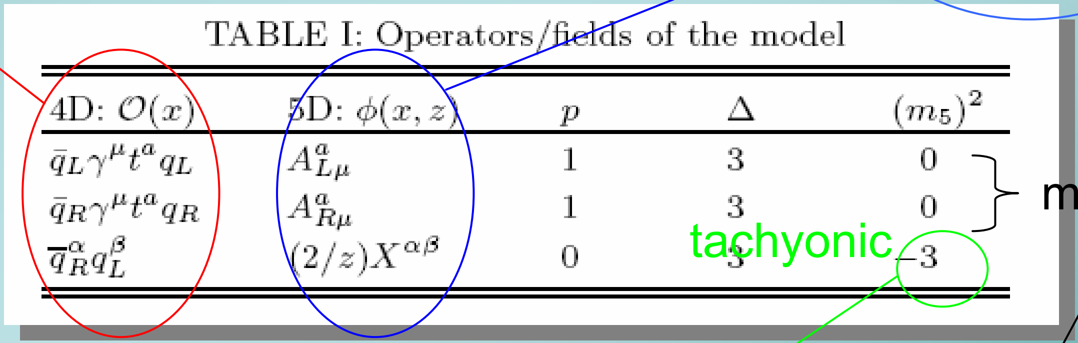


TABLE I: Operators/fields of the model

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	p	Δ	$(m_5)^2$
$\bar{q}_L\gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R\gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

massless

tachyonic

$$S_5^{eff} = \int d^5x\sqrt{-g}e^{-\phi}\left[|DX|^2 + m_5^2 X^2 + \frac{1}{4g_5^2}Tr\left(F_L^2 + F_R^2\right)\right]$$

(Classical) eq. of motion : $\partial_M\left(\sqrt{-g}e^{-\phi}\left[\partial^M V^N - \partial^N V^M\right]\right) = 0$

ρ meson vector field : $V = \frac{A_R + A_L}{2}$ ⇒ $V_\mu(x,z) = \underbrace{\epsilon_\mu e^{iq\cdot x}}_{\text{plane wave}}\underbrace{\psi(z)}_{\text{holo. wave function}}$

Schrödinger eq. : $-\psi'' + V(z)\psi = m_n^2\psi(z)$

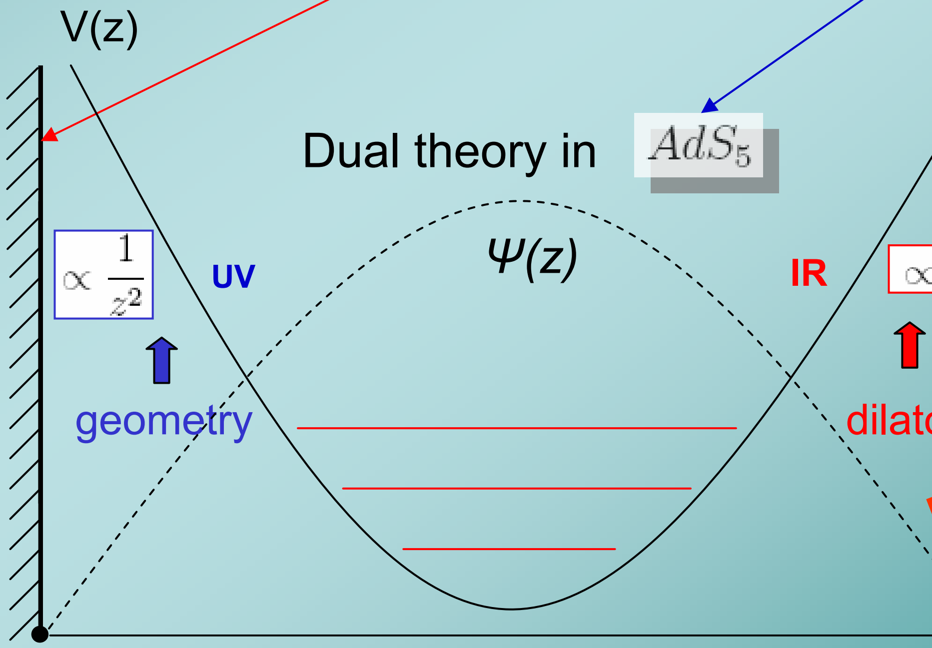
Regge behaviour : $m_n^2 \propto n$ ⇒ connection dilaton/geometry

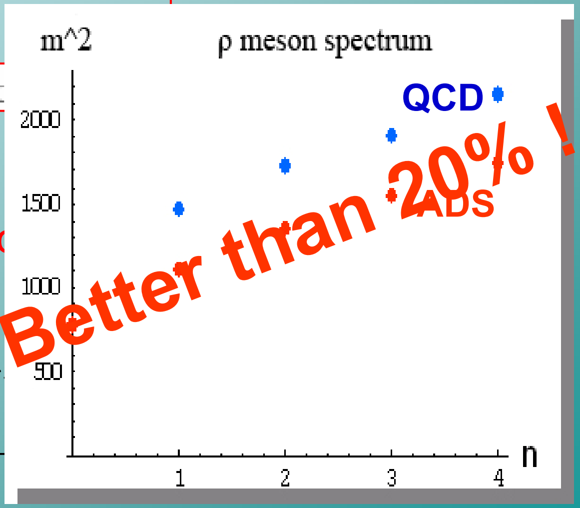
$$\left\{\begin{array}{l} \phi - A \xrightarrow{z\to 0} -\ln(\frac{z}{R}) \\ \phi - A \xrightarrow{z\to\infty} \frac{z^2}{R^2} \end{array}\right.$$

$M_n^2 = 4n+4$

QCD in $\mathcal{M}^4$

Dual theory in AdS_5





Better than 20% !

Holographic Models of mesons

I) Top-to-bottom approach :

M-theory/Supertring Theory
Bulk : $AdS_5 \times S^5$

→

QCD-like Gauge theory
Boundary : ∂AdS_4

II) Bottom-up approach (**AdS/QCD**) :

(String-inspired) dual theory :
$S_5^{eff}[A(x^M)]$

Holographic spacetime :
distorted AdS_5

←

Properties of QCD SU(3)**c :**

- ~~squarks, gluinos~~ → ~~SUSY~~
- massive quarks
- renormalisation scale μ

→ ~~CFT~~

Minkowski $\mathcal{M}^4$

Confinement, Chiral symmetry breaking, masses, decay constants, form factors, etc…

Glueball Spectroscopy

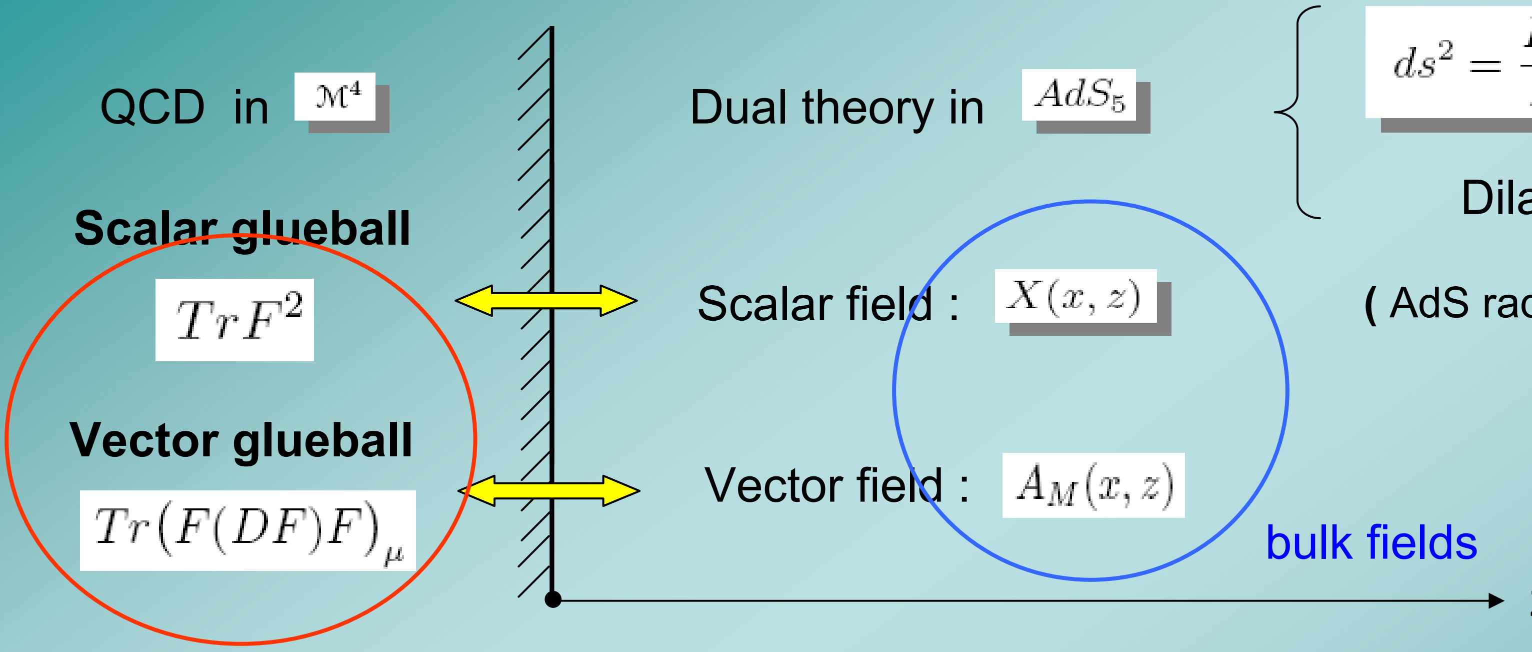
(Colangelo, de Fazio, Nicotri, F.J. '07)

- **Scalar 0^{++} and vector 1^{--} mass spectrum**
  (pseudoscalar 0^{-+}, hybrid mesons)

- **Dual theory of QCD (if exists…)**

AdS/QCD Model of light glueballs (scalar, vector)

Glueballs : Bound-states of gluons (gg...)



QCD in $\mathcal{M}^4$

Dual theory in AdS_5

$$ds^2 = \frac{R^2}{z^2}(dx^2 + dz^2)$$

Dilaton : $\phi(z)$

(AdS radius : R=1)

Scalar glueball

TrF^2

Vector glueball

$Tr\big(F(DF)F\big)_\mu$

Scalar field : $X(x,z)$

Vector field : $A_M(x,z)$

bulk fields

z

boundary operators

Operators / fields of the model

4D : $\mathcal{O}(x)$	5D : $\phi(x,z)$	p	Δ	m_{Ads}^2	
TrF^2	$X(x,z)$	0	4	0	massless
$Tr\big(F(DF)F\big)_\mu$	$A_M(x,z)$	1	7	24	massive

	J^{PC}	boundary			bulk		
Scalar glueball	0^{++}	TrF^2	(Δ=4)	⇒	$X(x,z)$	(p=0)	$m_5^2 = 0$
Vector glueball	1^{--}	$Tr(F(DF)F)_\mu$	(Δ=7)	⇒	$A_M(x,z)$	(p=1)	$m_5^2 = 24$

AdS/CFT

$$A(x^M) = \int_{M^4} d^4x' K(x^M, x'^\mu) A_0(x'^\mu)$$

$$m_5^2 = (\Delta - p)(\Delta + p - 4)$$

⇒ AdS/QCD

$$A(x^M) \overset{?}{\longleftrightarrow} A_0(x^\mu)$$

$$m_5^2 = m_{AdS}^2$$

- **Scalar bulk field :**

$$S_5^{eff} = -\frac{1}{2}\int d^5x\sqrt{-g}e^{-\phi(z)}g^{MN}(\partial_M X)(\partial_N X)$$

- **Vector bulk field :**

$$S_5^{eff} = -\frac{1}{2}\int d^5x\sqrt{-g}e^{-\phi(z)}\left[\frac{1}{2}g^{MN}g^{ST}F_{MS}F_{NT} + m_{AdS}^2 g^{ST}A_S A_T\right]$$

5-dim. bulk

Dilaton $\phi(z) = a^2 z^2$

Bulk field mass

$$F_{MS} = \partial_M A_S - \partial_S A_M$$

⇓

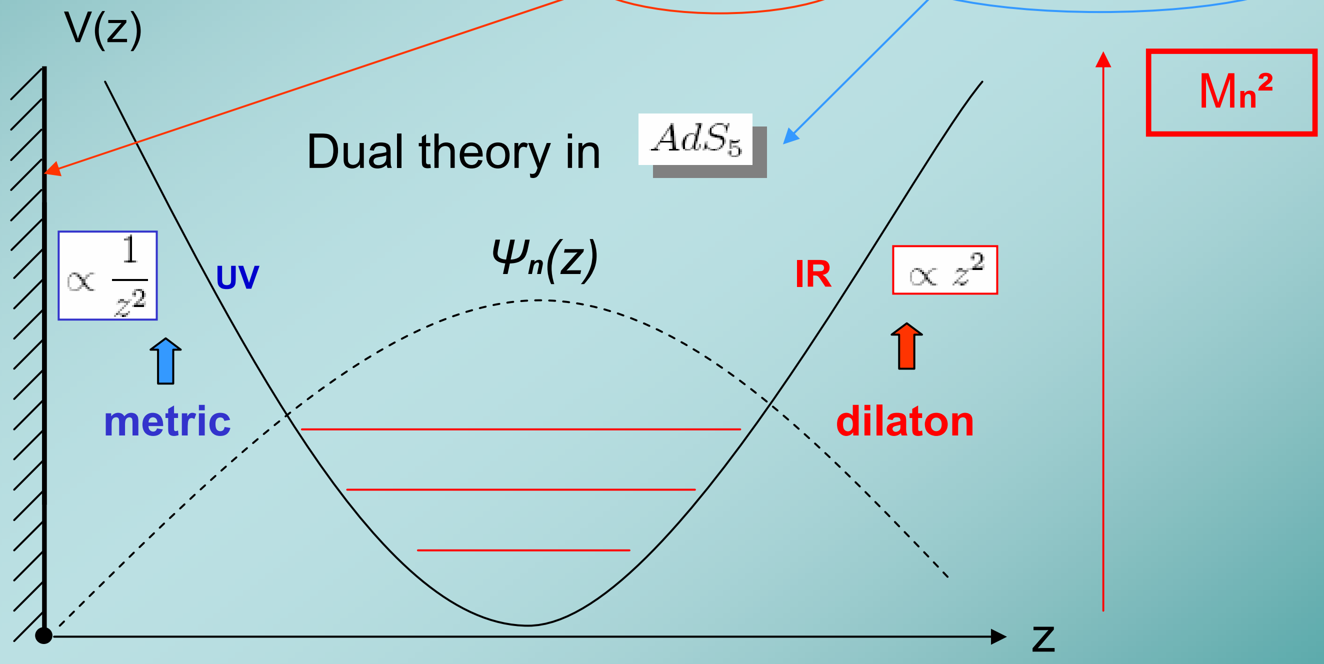
- Broken AdS isometries/conformal sym. (energy scale [a]=1)
- Regge behaviour of the mass spectrum

- (Classical) eq. of motion : $$\partial_N\left(\sqrt{-g}e^{-\phi}F^{NM}\right)-\sqrt{-g}e^{-\phi}m_5^2A^M=0$$

- Bulk field decomposition (mode) : $$A_\mu(x,z)=\underbrace{\epsilon_\mu e^{iq\cdot x}}_{\text{plane wave}}\underbrace{\psi(z)}_{\text{holo. wave function}}$$

QCD in $\mathcal{M}^4$

Dual theory in AdS_5



- Schrödinger eq. : $$-\psi''+V(z)\psi=m_n^2\psi(z)$$ with $$V(z)=\underbrace{a^4z^2}_{}+\underbrace{\frac{4m_5^2+(c+2)c}{4z^2}}_{}+\underbrace{(c-1)a^2}_{}$$

$\left\{\begin{array}{l}\text{c = 1 : } A_M(x,z)\\ \text{c = 3 : } X(x,z)\end{array}\right.$

dilaton $\phi(z)=a^2z^2$

metric $g_{MN}=\frac{1}{z^2}\eta_{MN}$

(**IR** : z → ∞)

(**UV** : z → 0)

- **Mass spectrum :** $m_n^2 = \left(4n + 1 + c + \sqrt{(c+1)^2 + 4m_5^2}\,\right)a^2$

- **Holo. wave function :** $\psi_n(z) = A_n\, e^{-a^2z^2/2} z^{g(c,m_5^2)+1/2}\, {}_1F_1\left(-n, g(c, m_5^2) + 1, a^2z^2\right)$ $\longrightarrow 0 \begin{cases} z \to \infty \\ z \to 0 \end{cases}$

$$g(m_5, c) = \sqrt{m_5^2 + \frac{(c+1)^2}{4}}$$

Kummer confluent hypergeometric function (-n < 0 : polynomial)

		Scalar glueball	**Vector glueball**	**Vector ρ meson** (Son et al. '05)
J^{PC} :		0^{++}	1^{--}	1^{--}
Boundary		TrF^2	$Tr\big(F(DF)F\big)_\mu$	$j_L^a(x)$ $j_R^a(x)$
		(Δ=4)	(Δ=7)	(Δ=3)
Bulk		$X(x,z)$	$A_M(x,z)$	$A_L^a(x,z)$ $A_R^a(x,z)$
		(p=0)	(p=1)	(p=0)
		$m_5^2 = 0$	$m_5^2 = 24$	$m_5^2 = 0$
Spectra		$\boxed{m_n^2 = \left(4n+8\right)a^2}$	$\boxed{m_n^2 = \left(4n+12\right)a^2}$	$\boxed{m_n^2 = \left(4n+4\right)a^2}$

Perturbed background

Background :

- AdS dual spacetime : $ds^2 = e^{2A(z)}\eta_{MN}ds^M dx^N = \frac{1}{z^2}(dx^2 + dz^2)$
- Dilaton : $\phi(z) = a^2 z^2$

Regge behaviour : $m_n^2 \propto n$ ⇨ connection dilaton/metric

- z → 0 : asymptotic AdS $\phi - A \xrightarrow{z\to 0} -\ln(z)$
- z → ∞ : harmonic-like potential $\phi - A \xrightarrow{z\to\infty} z^2$
- Higher spin meson spectrum $A(z) \not\ni z^{2+\beta}$ β > 0

Perturbation :

$\phi - A \sim z^\alpha$

0 ≤ α < 2

α = 1

Decay constants of glueballs

Operator/field correspondence : $e^{iS_5^{eff}[X(x,z)]} = \langle e^{i\int d^4x X_0(x)\mathcal{O}(x)}\rangle_{CFT}$

2-points correlator function $\Pi(q^2)$ ⟹ Decay constant $f_n = \langle 0|\mathcal{O}(0)|n\rangle$

$$\boxed{\Pi_{QCD}(q^2) = \Pi_{AdS}(q^2)}$$

- **QCD :** $\Pi_{QCD}(q^2) \equiv i\int d^4x\, e^{iq.x}\langle 0|T[\mathcal{O}(x)\mathcal{O}(0)]|0\rangle$

Completeness in the 2 chronological order : $\Pi_{QCD}(q^2) = \Sigma_n \frac{f_n^2}{q^2+m_n^2}$

- **AdS :** $\Pi_{AdS}(q^2) = \left(\underline{\tilde{X}(q,z)}, \partial_z\tilde{X}(q,z)\right)\Big|_{z\to 0}$ ⟹ Bulk-to-boundary propagator

Fourier transf. of X(x,z)

Bulk-to-boundary propagator (massless scalar bulk field) :

$$X(x,z) = \int_{M^4} d^4x' \underbrace{K(x,z;x',0)} X_0(x')$$

Boundary translation invariance : $K(x-x';z,0) \xrightarrow{z\to 0} \delta^4(x-x')$

$\tilde{X}(q,z) = \tilde{K}(q,z)\tilde{X}_0(q)$ **with** $\tilde{K}(q,z) \xrightarrow{z\to 0} 1$ (massless scalar)

⇒ $$\boxed{\Pi_{AdS}(q^2) = \tilde{K}(q,z)\left(\frac{e^{-\phi(z)}}{z^3}\right)\partial_z \tilde{K}(q,z)\Big|_{z\to 0}}$$

- $q^2 = -m_n^2$ normalizable bulk mode $\tilde{K}_n(z)$ ⇒ dual to particle states

  z → 0 $\boxed{\tilde{K}_n(z) \sim A_n\, z^4}$

- $q^2 > 0$ non-normalizable bulk mode $\tilde{K}(q,z)$ ⇒ dual to currents (virtuality)

  (deep inelastic limit : $q^2 \to \infty$)

  z → 0 $\boxed{\tilde{K}(q,z) \sim 1}$

eq. of motion : $\mathcal{D}\tilde{K}_n(z) = \left[\partial_z\left(\frac{e^{-\phi}}{z^3}\partial_z\right) + m_n^2\frac{e^{-\phi}}{z^3}\right]\tilde{K}_n(z) = 0$ $\quad q^2 = -m_n^2$

Sturm-Liouville operator

completeness

Green's function : $\mathcal{D}G(q^2; z, z') = -\delta(z - z')$

$$G(q^2; z, z') = \Sigma_n \frac{\tilde{K}_n(z)\tilde{K}_n(z')}{q^2 + m_n^2}$$

Green's theorem : $\tilde{K}(q, z) = \tilde{K}(q, z')\left(\frac{e^{-\phi(z')}}{z'^3}\right)\partial_{z'}G(q^2, z', z)\Big|_{z'\to 0}$

$$\boxed{\Pi_{AdS}(q^2) = \Sigma_n \frac{1}{q^2 + m_n^2}\left[\underbrace{\tilde{K}(q, z)}_{\mathbf{1}}\underbrace{\frac{e^{-\phi(z)}}{z^3}}_{\mathbf{1/\,z^3}}\underbrace{\partial_z\tilde{K}_n(z)}_{\mathbf{4A_n\,z^3}}\right]^2\Big|_{z\to 0}}$$

$$\boxed{f_n = 4A_n \sim \sqrt{8(n+1)(n+2)}}$$

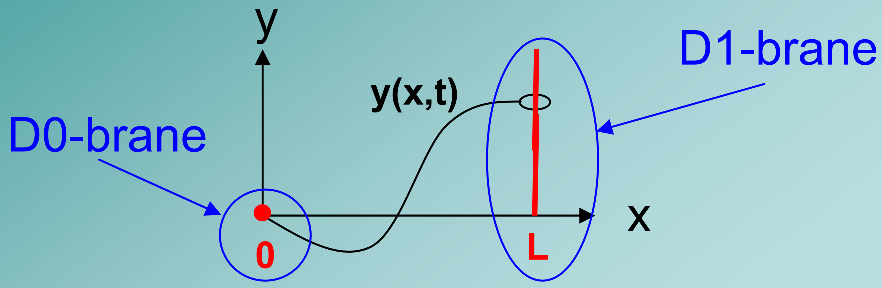
Heavy-light meson spectrum (Evans et al. ’06)

$Q\bar{q}$ mesons $\begin{cases} D = c\bar{q} \\ B = b\bar{q} \end{cases}$ (q=u,d,s) ⇨ D(irichlet) p-brane model of spacetime :

- p spatial-dim. object
- (p+1)-dim. spacetime

⇩

D3-brane in 4-dim. Spacetime : $\mathcal{M}^4$?

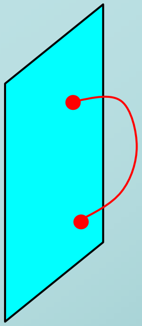


$$\underbrace{y(0,t)=0 \qquad y'(L,t)=0}$$

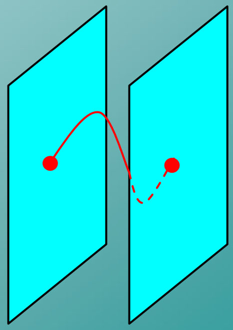
Dp-branes : boundary conditions ⇨ Open string endpoints attached to Dp-branes

Open string spectrum

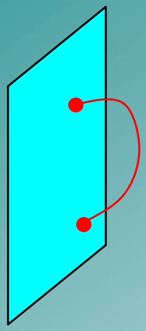
D3-brane :



D3-D3-branes :



D3-brane :

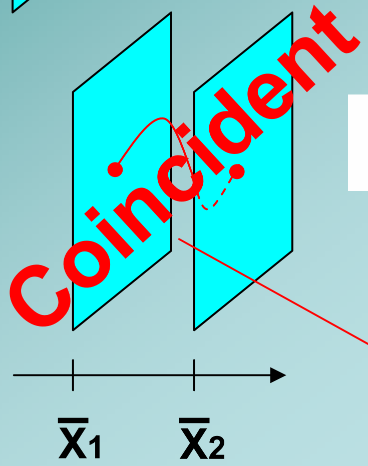


$$M^2 = \frac{1}{\alpha'}(N-1)$$

⇒ 1 massless vector
(tachyon, massless scalars)

(harm. osc. E=ħω(N+1/2))

D3-D3-branes :



$$M^2 = \underbrace{\frac{1}{\alpha'}(N-1)}_{\text{quantum osc.}} + \underbrace{[T_0(\bar{x}_2-\bar{x}_1)]^2}_{\text{classical energy of the stretched string}}$$

: (energy/length) x (length)

⇒ 1 massive vector
(tachyon, massive scalars)

$$M^2 = [T_0(\bar{x}_2-\bar{x}_1)]^2$$

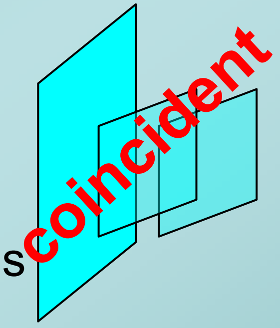
$\bar{x}_1 \longrightarrow \bar{x}_2$ ⇒

1 massless vector

$$M^2 = 0$$

Standard Model
(QCD)

$\mathcal{M}^4$ 3 D3-branes



3 x 3 massless vectors : 9 gauge fields : SU(3) x U(1)
in (3+1) spacetime

↓

SU(3)c

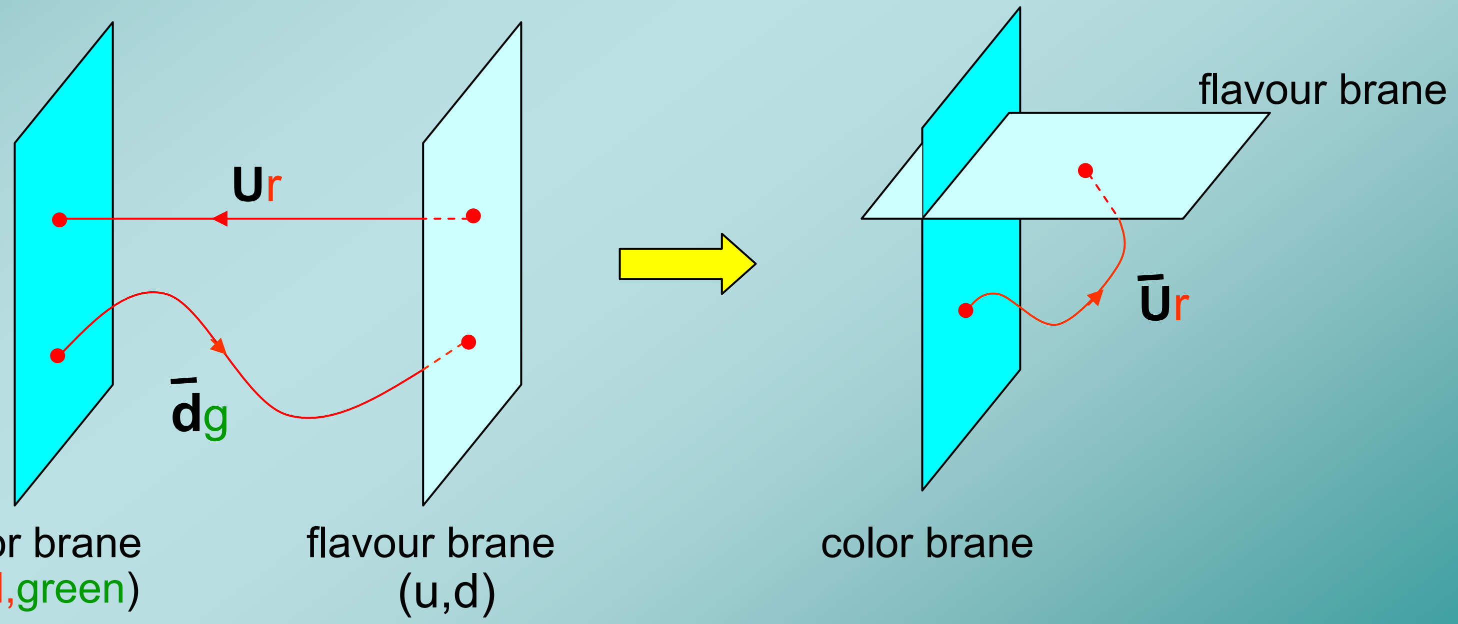
- N superposed Dp-branes → Gauge theory SU(N) in (p+1) spacetime

3 D3-branes → SU(3) in (3+1) spacetime

↓ Boundary of the bulk

↓ $\mathcal{M}^4$

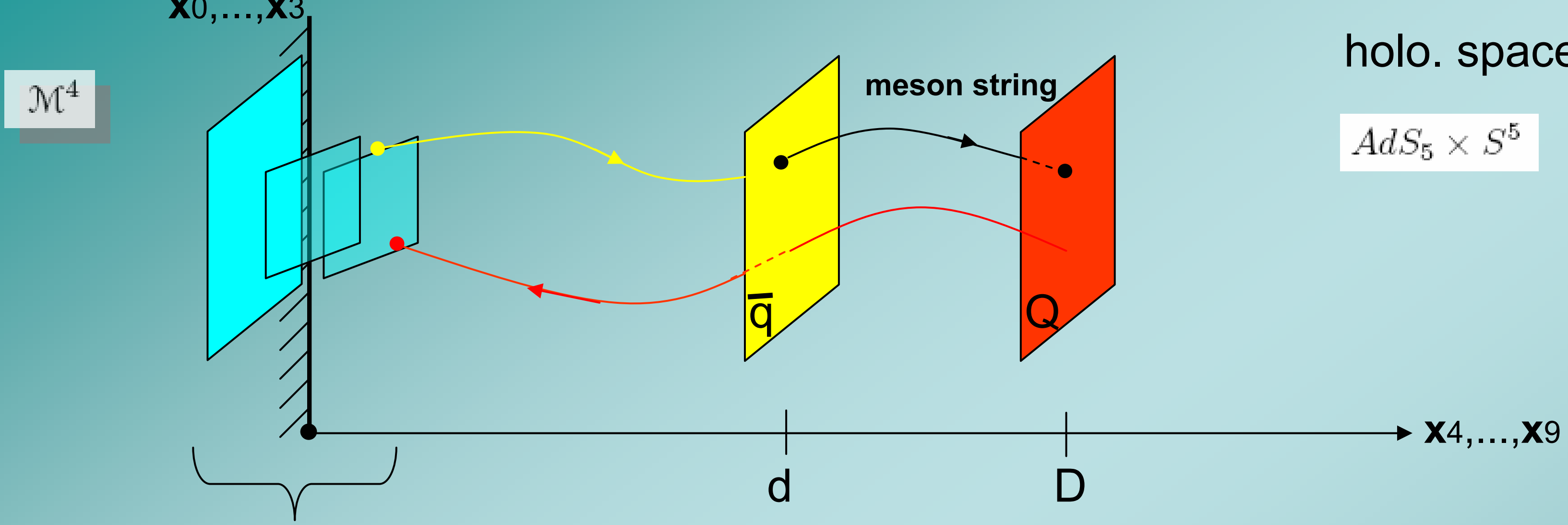
- Gluons : open strings with the 2 endpoints attached on the 3 (colored) D3-branes
- Quarks : open strings with
  - 1 endpoint attached on the 3 (colored) D3-brane
  - 1 endpoint attached to a flavour Dp-brane (D7-brane)



↳ Massive quarks $M^2 = \left[T_0(\bar{x}_2 - \bar{x}_1)\right]^2$

↳ Massless (chiral) quarks

D3-D7-brane model of heavy-light mesons



holo. spacetime

$AdS_5 \times S^5$

3 D3-baryonic
branes (r,b,g)

SU(3) : QCD

2 D7-flavour branes
(u,d,s) and (c,b)

D7-D3 open string spectrum :

$$M^2 = \frac{1}{\alpha'}\left(N-1+\frac{1}{4}\right) + \left[T_0(\bar{x}_2-\bar{x}_1)\right]^2$$

semi-classical string limit ⟶ D>>d (B meson)

Heavy-light meson spectrum :

$$M^2 = \left[T_0(D-d)\right]^2$$

M_ρ = 770 MeV : d

M_Υ = 9.4 GeV : D

B meson : **M_B** = 6529 MeV (5279 MeV)

better than 20% !

AdS/CFT Correspondence (Maldacena '98)

Supergravity limit of M-theory/Superstring Theory in $AdS_5 \times S^5$ ⟷ Large N limit of Superconformal SU(N) gauge theory in ∂AdS_4

$$\underbrace{\text{Anti de Sitter space} \times \text{compact space}}_{\text{Holographic spacetime / bulk}}$$

Holographic spacetime / bulk ⟷ Minkowski spacetime $\mathcal{M}^4$

(no physical extra dimensions)

Anti-de Sitter AdS_5 (d=5) : $ds^2 = g_{MN} dx^M dx^N$ (M,N=0,1,2,3,4)
(-,+,+,+,+)

- Solution of vacuum Einstein equation :

$$R_{MN} - \frac{1}{2} g_{MN} R = \frac{1}{2} g_{MN} \Lambda$$

cosmological constant $\Lambda > 0$

- Isometry group SO(2,4)
(preserves distances, ~ SO(1,3))

on the boundary →

Conformal SO(2,4) group acting on $\mathcal{M}^4$

Supergravity limit of M-theory/ Superstring Theory in AdS_5 ⟺ Large N limit of Superconformal SU(N) gauge theory in $\mathcal{M}^4$

Parameter correspondence

(g_s, α') ⟺ (g_{YM}, N)

String coupling

$\sqrt{\alpha'} \equiv \ell_s$ String length

YM coupling

Gauge group $SU(N)$

$$g_s = g_{YM}^2$$

$$\frac{R^4}{(\alpha')^2} = \underbrace{4\pi N g_{YM}^2}$$

R : AdS radius (AdS typical size)

't Hooft coupling $\lambda \equiv N g_{YM}^2$

't Hooft limit
- λ fixed but large
- N >> 1

$g_{YM}^2 = \frac{\lambda}{N}$ << 1 ⟹ $g_s \ll 1$ perturbative

$R \gg \ell_s$

strong coupling λ

supergravity

Classical
Perturbative } string theory in AdS_5 ⟺ Strongly coupled gauge theory in $\mathcal{M}^4$

Symmetry correspondence

Local (gauged) symmetry ⟺ Global symmetry

Ex. : chiral sym. $(SU(3)_L \times SU(3)_R)_{local}$ $(SU(3)_L \times SU(3)_R)_{global}$

Operator/field correspondence

(Witten ’98, Gubser, Klebanov, Polyakov ‘98)

Bulk field (p-form) $A(x^M)$ ⟺ Operator (scaling dim. **Δ**)

$$e^{iS_5^{eff}[A(x^M)]} = \langle e^{i\int d^4x A_0(x) O(x)} \rangle_{CFT}$$

Bulk field $A(x^M)$ → Source field $A_0(x^\mu)$ of operator $O(x^\mu)$

boundary coord.
(μ,ν=0,1,2,3)

Bulk-to-boundary propagator :

$$A(x^M) = \int_{M^4} d^4x' K(x^M, x'^\mu) A_0(x'^\mu)$$

AdS mass of the bulk field :

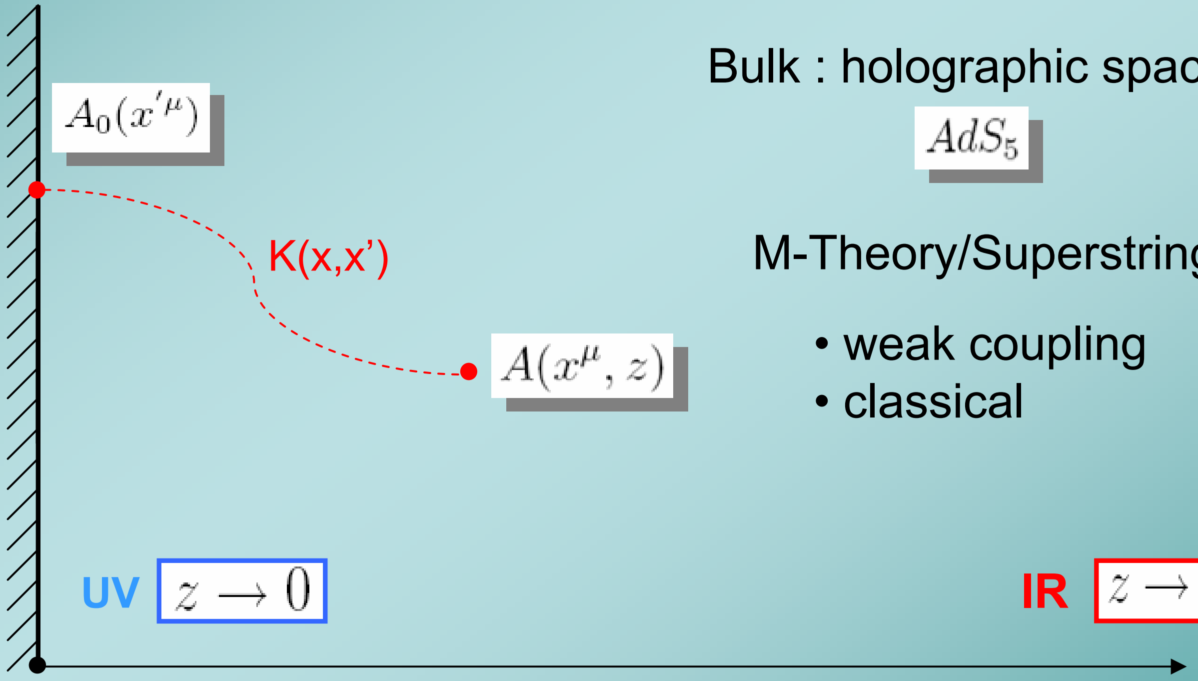
$$R^2 m_{AdS}^2 = (\Delta - p)(\Delta + p - 4)$$

∂AdS_4

Conformally flat metric : $g_{MN} = e^{2A(x)}\eta_{MN}$

$A(x) = -\ln\frac{z}{R}$ where $z \equiv x^4$

Holographic spacetime : $ds^2 = g_{MN}dx^M dx^N = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2)$



Our spacetime

$\mathcal{M}^4$

SU(N)

- strong coupling
- SUSY
- conformal

$A_0(x'^\mu)$

K(x,x')

$A(x^\mu, z)$

Bulk : holographic spacetime

AdS_5

M-Theory/Superstring

- weak coupling
- classical

UV $z \to 0$

IR $z \to \infty$

z holographic coordinate

↓

Energy scale

$(x^\mu, z) = (0, 0)$