Bounds in $\pi\,\pi$ scattering from dispersion relations

Constraining $\bar{l}_1$ & $\bar{l}_2$ and L_1, L_2 & L_3

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Outline

1. Motivations
2. $SU(2)$
   - $\pi\,\pi$ scattering
   - Fixed t dispersion relations & positivity conditions
   - Bounds on chiral LECs and the Linear Sigma Model
   - Equivalence with `Pennington & Portoles`
3. $SU(3)$
   - $SU(3)_V$ limit
   - Symmetry breaking
   - Results
4. Conclusions

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$$T(a\,b \to c\,d) \;=\; A(s,t)\,\delta^{ab}\delta^{cd} + A(t,s)\,\delta^{ac}\delta^{bd} + A(u,t)\delta^{ad}\delta^{bc}$$

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Then we can write

$$T^I(s,t) = C_u^{II'}\,T^{I'}(u,t)\,, \quad C_u^{II'}C_u^{I'J} = \delta_{IJ}\,, \quad C_u = \frac{1}{6}\begin{pmatrix} 2 & -6 & 10 \\ -2 & 3 & 5 \\ 2 & 3 & 1 \end{pmatrix},$$

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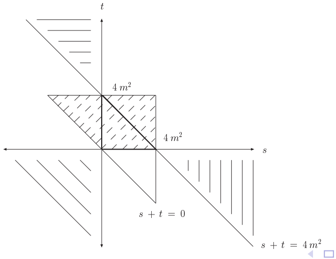

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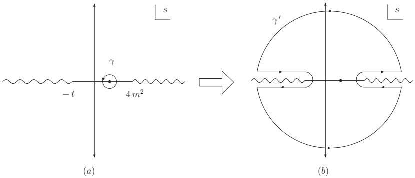

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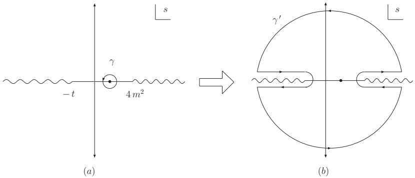

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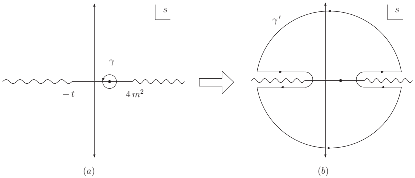

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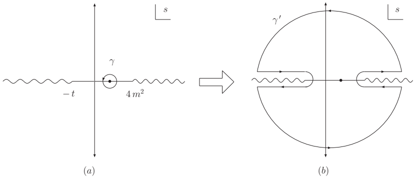

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For $s+t \geq 0$ $s \leq 4\,m_\pi^2$ both denominators ≥ 0 in the integral path

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Positivity conditions :

Inside the region $\mathcal{A} \equiv \{s \leq 4\,m_\pi^2\,,\, 0 \leq t \leq 4\,m^2 \;\&\; s+t \geq 0\}$

$$\frac{\mathrm{d}^2}{\mathrm{d}s^2}T\left(\pi^0\pi^0 \to \pi^0\pi^0\right)[(s,t)\in\mathcal{A}] \geq 0\,, \qquad \frac{\mathrm{d}^2}{\mathrm{d}s^2}T\left(\pi^+\pi^+ \to \pi^+\pi^+\right)[(s,t)\in\mathcal{A}] \geq 0\,,$$

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FlaviA net

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Bounds on chiral LECs

In the region $\mathcal{A}$ we can apply χPT at $\mathcal{O}(p^4)$ to obtain $[\frac{d^2}{ds^2}\mathcal{O}(p^2) = 0]$

$$\sum_{i=1}^{2} \alpha_{ji}\,\bar{l}_i \,-\, f_j[(s,t)\in\mathcal{A}] \,\geq\, 0 \quad\Longrightarrow\quad \sum_{i=1}^{2} \alpha_{ji}\,\bar{l}_i \,\geq\, f_j[(s,t)\in\mathcal{A}]\big|_{\max}$$

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Process	LECs combination	Bound	Experimental value
$\pi^0\pi^0 \to \pi^0\pi^0$	$\bar{l}_1 + 2\,\bar{l}_2$ [1,2]	$\geq \frac{157}{40} = 3.925$	8.2 ± 0.6
$\pi^+\pi^0 \to \pi^+\pi^0$	$\bar{l}_2$ [1,2]	$\geq \frac{27}{20} = 1.350$	4.3 ± 0.1
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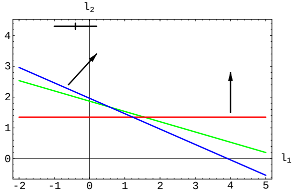

Is the Linear Sigma Model consistent?

♦ Functional integration of σ particle $\Longrightarrow$ $\bar{l}_1$ and $\bar{l}_2$ in LSM : (at one–loop)

$$\bar{l}_1 = \frac{24\,\pi^2}{g} + \log\left(\frac{m_\sigma}{m_\pi}\right) - \frac{35}{6}\,, \quad \bar{l}_2 = \log\left(\frac{m_\sigma}{m_\pi}\right) - \frac{11}{6}$$

Gasser et al '84

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- Plugging into the second bound $\log\left(\frac{m_\sigma}{m}\right) \geq \frac{191}{60}$ violated for $m_\sigma \lesssim 24\,m_\pi$!!!

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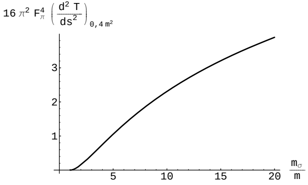

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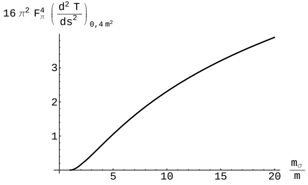

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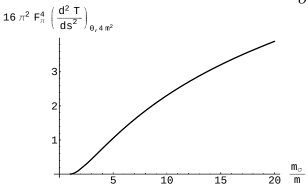

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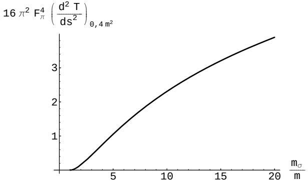

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2. $SU(2)$
   - $\pi\,\pi$ scattering
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   - Bounds on chiral LECs and the Linear Sigma Model
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Scattering lengths

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Outline

$SU(3)_V$ limit (LECs independent of m_q)

Octet–to–octet scattering

Missmatch Clebsch-Gordan ↔ tensor analysis

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but...need to reconsider the positivity conditions

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$$P_\ell \left[1 + \frac{s\,t}{(s + m^2 - M^2)^2 - 4\, m^2\, s} \right] \geq 0 \quad \text{in the two integrals}$$

FlaviA net

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$$P_\ell\left[1 + \frac{s\,t}{(s+m^2-M^2)^2 - 4\,m^2\,s}\right] \geq 0 \quad \text{in the two integrals}$$

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$$\frac{d^2}{ds^2} T\left(\pi^+\pi^+ \to \pi^+\pi^+\right)[(s,t) \in \mathcal{A}] \geq 0\,, \qquad \frac{d^2}{ds^2} T(\pi^0\pi^0 \to \pi^0\pi^0)[(s,t) \in \mathcal{A}] \geq 0\,,$$

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Outline

1. Motivations
2. $SU(2)$
   - $\pi\,\pi$ scattering
   - Fixed t dispersion relations & positivity conditions
   - Bounds on chiral LECs and the Linear Sigma Model
   - Equivalence with Pennington & Portoles
3. $SU(3)$
   - $SU(3)_V$ limit
   - Symmetry breaking
   - Results
4. Conclusions

Results

	$10^3 \times L_1^r(\mu)$	$10^3 \times L_2^r(\mu)$	$10^3 \times L_3$
$\mu = m_\rho$	0.43 ± 0.12	0.43 ± 0.12	-2.35 ± 0.37
$\mu = m_K$	0.69 ± 0.12	1.26 ± 0.12	-2.35 ± 0.37
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Process	$10^3\, \alpha_i L^i(\mu)$	$\mu = m_\pi$	$\mu = m_K$	$m_\pi = m_K$	$m_\pi \neq m_K$
$\pi^0\pi^0$	$2L_1^r(\mu) + 2L_2^r(\mu) + L_3$	6.20 ± 0.5	1.6 ± 0.5	≥ 2.27	$\geq$ 2.28
$\pi^+\pi^0$	$L_2^r(\mu)$	2.81 ± 0.12	1.26 ± 0.12	≥ 0.75	$\geq$ 0.95
$\pi^+\pi^+$	$2L_1^r(\mu) + 3L_2^r(\mu) + L_3$	9.0 ± 0.6	2.8 ± 0.6	≥ 3.32	$\geq$ 3.91
$K\,\eta$	$12L_2^r(\mu) + L_3$	31.4 ± 1.5	12.8 ± 1.5	≥ 8.6	-
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The present accuracy of the experimental determinations for L_1, L_2 and L_3 is not enough to discern whether $SU(3)$ χPT at $\mathcal{O}(p^4)$ satisfies the axiomatic principles

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