V_{ub} and $B \to \pi$ form factors from light-cone sum rules revisited

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LPT Orsay

paper in preparation

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Euroflavour07 , 14.11.07

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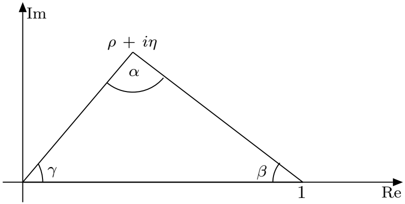
Unitarity-Triangle

- Cabibbo-Kobayashi-Maskawa-Matrix:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \qquad V\,V^\dagger = V^\dagger\,V = 1$$



$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$

$$1 + \frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} + \frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cd}} = 0$$

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- Cabibbo-Kobayashi-Maskawa-Matrix:

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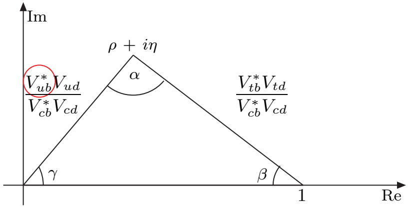

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$

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- V_{ub} main topic of this talk

Most recent $|V_{ub}|$-determinations

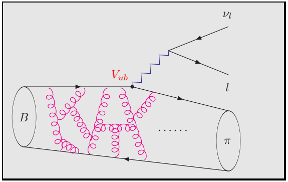
inclusive determinations		
Method	$\|V_{ub}\| \times 10^{-3}$	ref.
LLR	$4.40 \pm 0.30 \pm 0.41 \pm 0.23$	Golubev, Skovpen, Lüth '07
LNP	$4.28 \pm 0.29 \pm 0.29 \pm 0.26 \pm 0.28$	
kinetic scheme	$3.87 \pm 0.26 \begin{array}{l} +0.27 \\ -0.33 \end{array}$	Gambino, Giordano Ossola, Uraltsev '07
	$4.44 \pm 0.32 \begin{array}{l} +0.36 \\ -0.54 \end{array}$	
	$4.05 \pm 0.23 \begin{array}{l} +0.28 \\ -0.51 \end{array}$	
ACM	$3.69 \pm 0.13 \pm 0.31$	Aglietti, Di Lodovico, Ferrera, Ricciardi '07

exclusive determinations		
Method	$\|V_{ub}\| \times 10^{-3}$	ref.
Lattice	$3.78 \pm 0.25 \pm 0.52$	Fermilab/MILC '05
Lattice	$3.55 \pm 0.25 \pm 0.50$	HPQCD '07
comb.	$3.47 \pm 0.29 \pm 0.03$	Flynn, Nieves '07
LCSR	$3.5 \pm 0.4 \pm 0.1$	Ball '06

Exclusive decays sensitive to $|V_{ub}|$

Channel	BR$\times 10^4$	hadronic input	theory
$B^- \to \tau^- \bar{\nu}_\tau$	$1.79^{+0.56+0.39}_{-0.49-0.46}$ [Belle] < 1.7(90% CL) [BaBar]	f_B	Lattice ($2\oplus 1$) QCD SR
$\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l$	$1.39 \pm 0.06 \pm 0.06$ [HFAG] q^2-shape [BaBar]	FF $f^+_{B\pi}(q^2)$	Lattice ($2 \oplus 1$), SCET, LCSR
$\bar{B}^0 \to \rho^+ l^- \bar{\nu}_l$	$2.14 \pm 0.21 \pm 0.51 \pm 0.28$ [BaBar] $2.17 \pm 0.54 \pm 0.31 \pm 0.08$[Belle]	three FF's	Lattice (quench.) LCSR
$B^- \to l^- \bar{\nu}_l \gamma$	< 0.05 (90% CL) [BaBar]	two FF's	QCDF,LCSR
$B^- \to \pi^- \pi^0$	0.057 ± 0.004 [HFAG]	hadr. ampl.	input: $B \to \pi$ FF QCDF, SCET

$B \to \pi l \nu$ decay



- hadronic matrix element needed

$$\langle \pi(p) | b\, \gamma_\mu\, \bar{u} | B(p+q) \rangle \;=\; (2p\,+\,q)_\mu\, f^+_{B\pi}(q^2) \,+\, q_\mu\, f^-_{B\pi}(q^2)$$

- lepton spectrum $m_l = 0$:

$$\frac{d\Gamma}{dq^2}(B \to \pi^- l^+ \nu_l) \;=\; \frac{G_F^2\, |V_{ub}|^2}{192\, \pi^3\, m_B^3}\, \lambda^{3/2}(q^2)\, |f^+_{B\pi}(q^2)|^2$$

$$\lambda(q^2) = (m_B^2 + m_\pi^2 - q^2)^2 \,-\, 4 m_B^2 m_\pi^2 \qquad 0 \le q^2 \le (m_B - m_\pi)^2 \approx 26.4\,\mathrm{GeV}^2$$

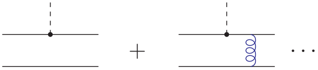
$B \to \pi$ form factor

- m_b-scaling:

$$f^+_{B\pi}(q^2 \approx 0) \sim m_b^{-3/2} \qquad f^+_{B\pi}(q^2 = q^2_{max}) \sim \sqrt{m_b} \quad \text{(Isgur-Wise-limit)}$$

▷ hard and soft formfactor

$$f^+_{B\pi}(q^2) = f^{s+}_{B\pi}(q^2) + f^{h+}_{B\pi}(q^2)$$



- symmetries:

$$f^{s+}_{B\pi}(0) = f^{s0}_{B\pi}(0) = f^{sT}_{B\pi}(0) \cdots$$

[Charles et al.][Beneke, Feldmann]

- analytic function of q^2

$$f^+_{B\pi}(q^2) = \frac{f_{B^*} g_{B^* B\pi}}{m^2_{B^*} - q^2} + \frac{1}{\pi} \int\limits_{(m_B+m_\pi)^2}^{\infty} ds \frac{\mathrm{Im} f^+_{B\pi}(s)}{s - q^2}$$

Parametrizations

- simple parametrizations:

$$f^+(q^2) = \frac{f^+(0)}{1 - \frac{q^2}{m_{B*}^2}} + \frac{r\,\frac{q^2}{m_{B*}^2}}{1 - \alpha\frac{q^2}{m_{B*}^2}}$$

[Becirevic, Kaidalov][Ball, Zwicky]

$$f^+(q^2) = \frac{f^+(0)\left(1 - \delta\frac{q^2}{m_{B*}^2}\right)}{\left(1 - \frac{q^2}{m_{B*}^2}\right)\left(1 - [\alpha + \delta(1 - \alpha)]\,\frac{q^2}{m_{B*}^2}\right)}$$

[Hill]

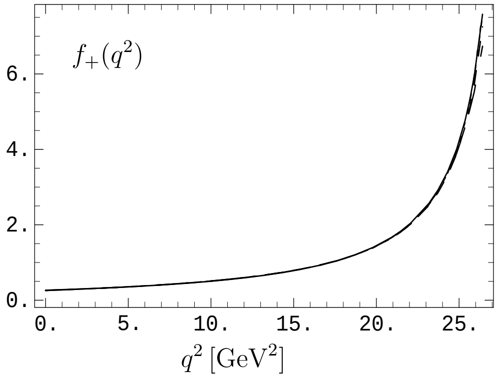
- more elaborate parametrizations based on unitarity:

$$f^+(q^2) = \frac{1}{P(q^2)\Phi(q^2, t_0)} \sum_{k=0}^{\infty} a_k \left[z(q^2, t_0)\right]^k \qquad \sum_{k=0}^{\infty} a_k^2 \leq 1$$

[Boyd, Grinstein, Lebed]

- Omnes representation [Albertus, Flynn, Hernandez, Nieves, Verde-Velasco]

Fit to experimental spectrum



- taken from " V_{ub} from UTangles and $B \to \pi l \nu$" [Ball 06]
- fit of five different parametrizations, yielding

$$|V_{ub}\, f^+_{B\pi}(0)| = (0.91 \pm [0.06]_{shape} \pm [0.03]_{BR}) \times 10^{-3}$$

- remaining task normalization of $f^+_{B\pi}(q^2)$ at single point

Method

correlation function

$$F(q^2) = i \int d^4x\, e^{iqx} \langle 0 \,|\, T\{ j_\mu(x)\, j_5^\dagger(0) \} \,|\, \pi(p) \rangle$$

hadronic dispersion relation

$$F^{(had)}(q^2) = \frac{2m_B^2 f_B f_{B\pi}^+(q^2)}{(m_B^2 - (p+q)^2)} + \int_{s_0^h}^{\infty} ds \frac{\rho(s)}{s - (p+q)^2}$$

operator-product-expansion

$$F^{(OPE)}(q^2) = \frac{1}{\pi} \int_{m_b^2}^{\infty} ds \frac{\mathrm{Im}\, F^{(OPE)}(s)}{s - (p+q)^2}$$

quark-hadron-duality

$$\int_{s_0^h}^{\infty} ds \frac{\rho(s)}{s - (p+q)^2} \approx \frac{1}{\pi} \int_{s_0^B}^{\infty} ds \frac{\mathrm{Im}\, F^{(OPE)}(s)}{s - (p+q)^2}$$

$$\frac{2m_B^2 f_B f_{B\pi}^+(q^2)}{(m_B^2 - (p+q)^2)} = \frac{1}{\pi} \int_{m_b^2}^{s_0^B} ds \frac{\mathrm{Im}\, F^{(OPE)}(s)}{s - (p+q)^2}$$

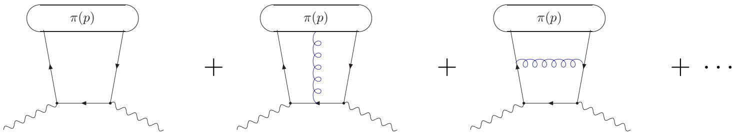
Borel-transformation

sum rule

$$f_{B\pi}^+(q^2) = \frac{e^{m_B^2/M^2}}{2m_B^2 f_B} \frac{1}{\pi} \int_{m_b^2}^{s_0^B} ds\, \mathrm{Im}\, F^{(OPE)}(s)\, e^{-s/M^2}$$

Calculational details

- OPE:



- universal input: nonlocal operators, expansion near the light cone in twist components

$$\langle \pi(p)|\bar{u}(x_1)\Gamma d(x_2)|0\rangle \sim \varphi_\pi^{(t)}(u) \qquad \text{twist } t = 2, 3, 4, \ldots$$

$$\langle \pi(p)|\bar{u}(x_1) G_{\mu\nu}(x_2)\Gamma d(x_3)|0\rangle \sim \Phi_{3\pi}^{(t)}(u) \qquad \text{twist } t = 3, 4, \ldots$$

- general form: convolution

$$T_0^{(t)} \otimes \varphi_\pi^{(t)} + \frac{C_F\,\alpha_s}{4\pi} T_1^{(t)} \otimes \varphi_\pi^{(t)} + \cdots$$

- included two-particle contributions up to twist-4, three-particle to twist-4 and α_s-corrections to twist-2, twist-3

Numerics (Inputs)

- universal inputs:
  - α_s in full QCD, $\overline{MS}$-mass $m_b(m_b) = 4.164 \pm 0.025\,\text{GeV}$ [Kühn, Steinhauser, Sturm]
  - distribution amplitudes $\varphi_\pi^{(t)}(u)$, $\Phi_{3\pi}^{(t)}(\alpha_i)$
    $\Rightarrow$ twist-2 Gegenbauer moments a_2^π, $a_4^\pi, \ldots$
    $\Rightarrow$ higher twist parameters $f_{3\pi}$, $\omega_{3\pi}$, δ_π, $\epsilon_\pi, \ldots$ [Ball, Braun, Lenz 06]
    $\Rightarrow$ normalization f_π, $\mu_\pi = \frac{m_\pi^2}{m_u+m_d}, \ldots$ [PDG]
  - f_B taken from two-point sum rule in $\overline{MS}$-scheme to $O(\alpha_s)$-accuracy
    $\Rightarrow$ condensates $\langle q\bar{q}\rangle$, $\langle G^2\rangle, \ldots$ [Jamin, Lange 01]

$$\langle q\bar{q}\rangle(1\,\text{GeV}) = -\frac{1}{2} f_\pi^2\, \mu_\pi(1\,\text{GeV})$$

GMOR-relation

- sum rule parameters:
  - parameters from LCSR s_0^B, M
  - parameters from f_B sum rule $\bar{s}_0^B$, $\bar{M}$

scale dependence in general taken at one-loop order

Numerics (Procedure)

1. M^2, $\bar{M}^2$ constrained via validity of local or light-cone expansion and smallness ($\leq 30\%$) of continuum contribution
2. $\mu = \mu_r = \mu_m = \mu_f$ constrained by α_s expansion of correlation function
3. s_0^B, $\bar{s}_0^B$ fixed by m_B^2 sum rule

$$m_B^2 = -\frac{d}{d\frac{1}{M^2}} \log \left(\frac{1}{\pi} \int_{m_b^2}^{s_0^B} ds\, e^{-\frac{s}{M^2}} \, \mathrm{Im}\, F^{(OPE)}(s) \right)$$

4. a_2^π, a_4^π fitting form factor shape from sum rule to experimental spectrum

Results

- well known sum rule calculation [Ball, Zwicky 05]

$$f^+_{B\pi}(0) = 0.258 \pm 0.031$$

- new result (preliminary)

$$f^+_{B\pi}(0) = 0.28 \pm 0.04$$

largest uncertainty by light quark masses via GMOR relation

- slightly larger, but...

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switching to pole mass

- our central values of input parameters nearly identical to set 4 by [Ball, Zwicky 05]
- their result for set 4

$$f_{B\pi}^{+}(0) = 0.274$$

V_{ub} from exclusive decays, reprise

- Taking [Ball 06]

$$|V_{ub}\, f^{+}_{B\pi}(0)| = (0.91 \pm [0.06]_{shape} \pm [0.03]_{BR}) \times 10^{-3}$$

V_{ub} from exclusive decays, reprise

- Taking [Ball 06]

$$|V_{ub}\, f^{+}_{B\pi}(0)| = (0.91 \pm [0.06]_{shape} \pm [0.03]_{BR}) \times 10^{-3}$$

- we get a new result for $|V_{ub}|$

exclusive determinations		
Method	$\lvert V_{ub}\rvert \times 10^{-3}$	ref.
Lattice	$3.78 \pm 0.25 \pm 0.52$	Fermilab/MILC '05
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LCSR	$3.5 \pm 0.4 \pm 0.1$	Ball '06
LCSR	$3.3 \left[\begin{array}{c} +0.5 \\ -0.4 \end{array} \right]_{th} \pm [0.2]_{shape} \pm [0.1]_{BR}$	this work preliminary