Light quark masses and pseudo-scalar decay constants from Nf=2 LQCD with twisted mass fermions

V. Lubicz

for the European Twisted Mass Collaboration

Results for:

- Light quark masses: ms, ms/mud
- Pseudoscalar decay constant: fk, fk/fπ

  → determination of Vus

FlaviA net

EuroFlavour '07

14-16 November 2007

Univ. Paris-Sud 11, Orsay

European Twisted Mass Collaboration

- Cyprus (Nicosia)
- France (Orsay, Grenoble)
- Italy (Rome I,II,III, Trento)
- Spain (Valencia)
- Switzerland (Zurich)
- United Kingdom (Liverpool)
- Germany (Berlin/Zeuthen, Hamburg, Münster)

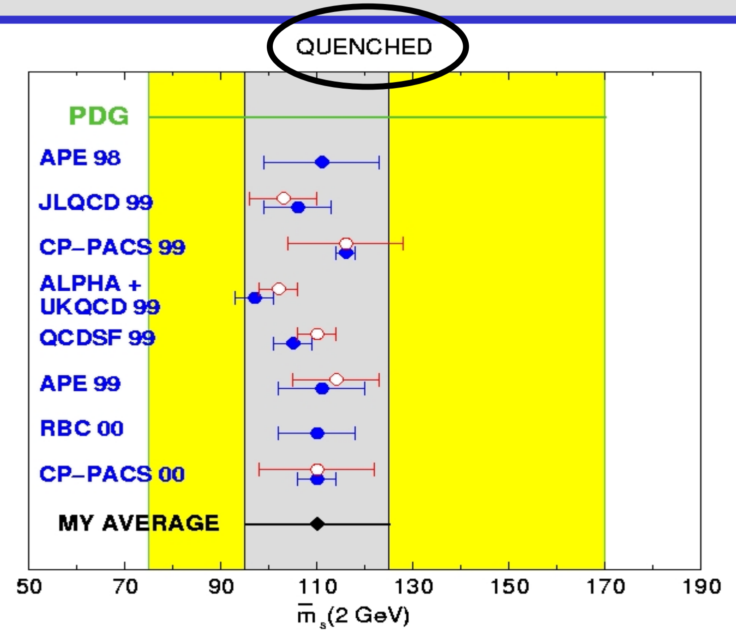
- Simulations with Nf=2 twisted mass quarks at maximal twist
- $300\ \mathrm{MeV} \lesssim M_{PS} \lesssim 600\ \mathrm{MeV}$
- $a \approx \{0.10, 0.09, 0.07\}$ fm

Overview of lattice results for ms

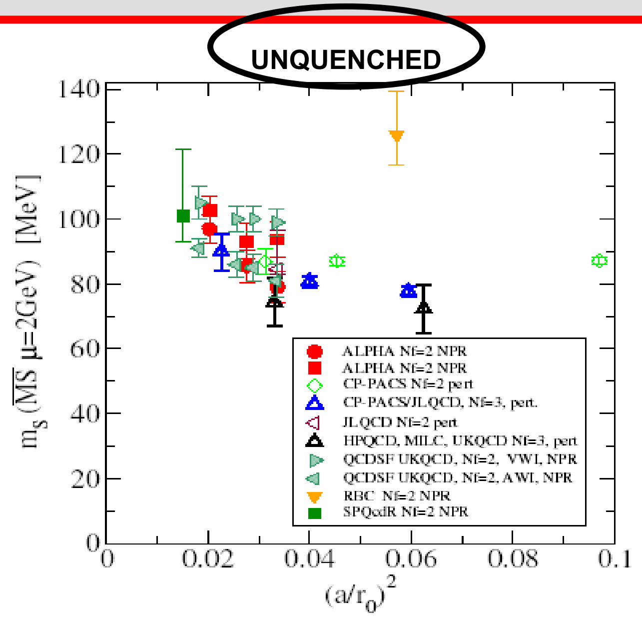
1994: First lattice calculation of quark masses with NLO accuracy
[C.Allton et al.,NPB431,1994]

2000: Typical quenched average: $\bar{m}_s\left(2\ \mathrm{GeV}\right) \simeq 110 \pm 15\ \mathrm{MeV}$

2001-… : Unquenched calculations. First results: $\bar{m}_s = 70 - 120\ \mathrm{MeV}$



V.L., Lattice 2000

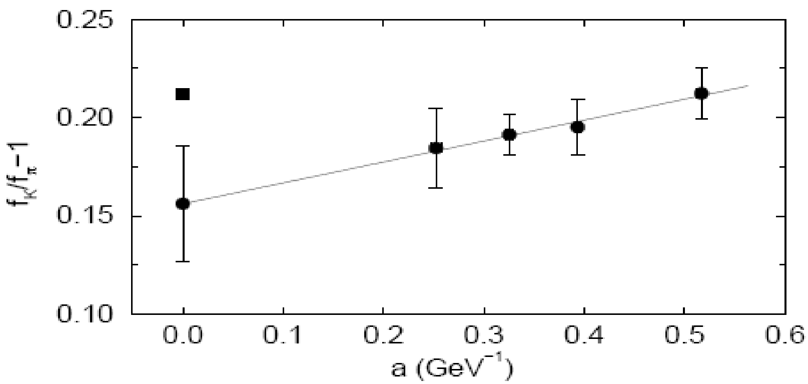


T.Izubuchi, Lattice 2005

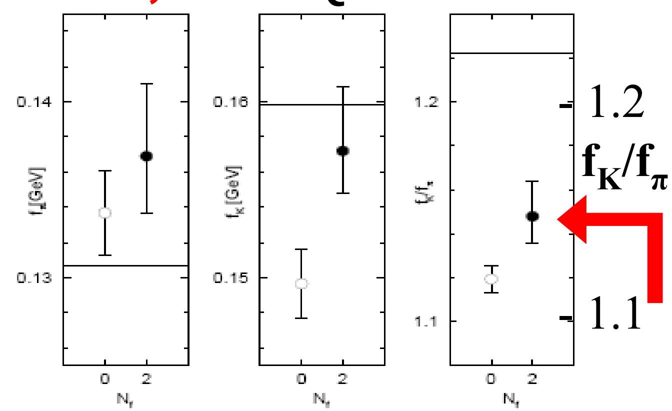
Lattice results for f_K

Leptonic (f_K) and semileptonic ($f_+^{K\pi}$) kaon decays are the simplest processes from which Vus can be determined

Nf=0 CP-PACS 2000



Nf=0,2 JLQCD 2002



- Quenched results typically indicated $f_K/f_\pi - 1 \simeq 0.15$, i.e. 25% smaller than the experimental value
- Similar results obtained by the first unquenched calculations
- A common feature of these calculations: $M_\pi \gtrsim 500$ MeV

DESY 07-148, FTUV-07-2709, IFIC/07-57,
MS-TP-07-23, RM3-TH/07-11, ROM2F/2007/16,
SFB/CPP-07-58, TUM-HEP-676/07

Light quark masses and pseudoscalar decay constants from $N_f = 2$ Lattice QCD with twisted mass fermions

European Twisted Mass Collaboration

B. Blossier[a], Ph. Boucaud[b], P. Dimopoulos[c], F. Farchioni[d], R. Frezzotti[c], V. Gimenez[e], G. Herdoiza[c], K. Jansen[a], V. Lubicz[f], C. Michael[g], D. Palao[e], M. Papinutto[h], A. Shindler[a], S. Simula[f], C. Tarantino[i], C. Urbach[g], U. Wenger[j]

In collaboration with

Simulation details

- Gauge action: Tree-level Symanzik improved
- Fermionic action: Nf=2 twisted mass at maximal twist
- Gauge coupling: β=3.9, $a \simeq 0.087$ fm, $1/a \simeq 2.3$ GeV
- Volume/a^4: $24^3 \times 48$ ➡ $L \approx 2$ fm
- # of independent gauge configurations: 240 (for each μ_{sea})
- Values of sea quark masses:

$$a\mu_{sea}=\{0.0040, 0.0064, 0.0085, 0,0100, 0.0150\}$$

➡ $$\mu_{sea} \in \left[\sim \frac{1}{6} m_s^{phys}, \sim \frac{2}{3} m_s^{phys}\right] \quad , \quad M_\pi^2 \in [\sim 300 \text{ MeV}, \sim 600 \text{ MeV}]$$

- Values of valence quark masses:

$$a\mu_{1,2}=\{a\mu_{sea}\} + \{0.0220, 0,0270, 0.0320\}$$

➡ $$\mu_{1,2} \simeq m_s^{phys} \quad , \quad M_K^2 \in [\sim 550 \text{ MeV}, \sim 750 \text{ MeV}]$$

Maximally twisted Lattice QCD

$$S = a^4 \sum_x \bar{\psi}_x \left[\gamma \cdot \tilde{\nabla} + \mu - i\gamma_5 \tau_3 \left(-a \frac{r}{2} \nabla^* \cdot \nabla + M_{cr} \right) \right] \psi_x$$

- Automatic O(a)-improvement obtained by tuning a single parameter (Mcr)

Frezzotti, Rossi, 2003

- The renormalization pattern is significantly simplified.

For the present calculation:

$$\bar{m}(\mu_R) = Z_m \, \mu(a)$$

Only multiplicative renormalization

$$f_{PS} = (\mu_1 + \mu_2) \frac{\left| \langle 0 | P^{\pm}(0) | P \rangle \right|}{M_{PS}^2}$$

No renormalization required

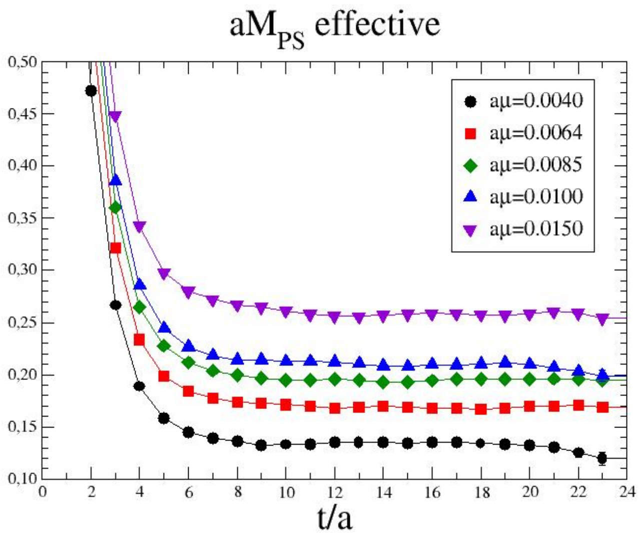
- Isospin and parity are broken at $O(a^2)$. W.r.t. the standard Wilson action, the symmetry is rotated: $SU(2)^{oblique} = \{ Q_1^A, Q_2^A, Q_3^V \}$

The strategy

- $M_{PS}^{\pm}$ and $f_{PS}^{\pm}$ are computed for several (150) combinations of sea and valence quark masses (partially quenched setup for kaons)

- We study the dependence of $M_{PS}^{\pm}$ and $f_{PS}^{\pm}$ on $\{\mu_s, \mu_1, \mu_2\}$, extrapolate to m_{ud} and interpolate to m_s

- We fix:
  - am_{ud} from $[M_\pi/f_\pi]^{exp}$
  - am_s from $[M_K/M_\pi]^{exp}$
  - a from $[f_\pi]^{exp}$



(All-to-all propagators evaluated using a stochastic method)

C.McNeile and C.Michael, 07

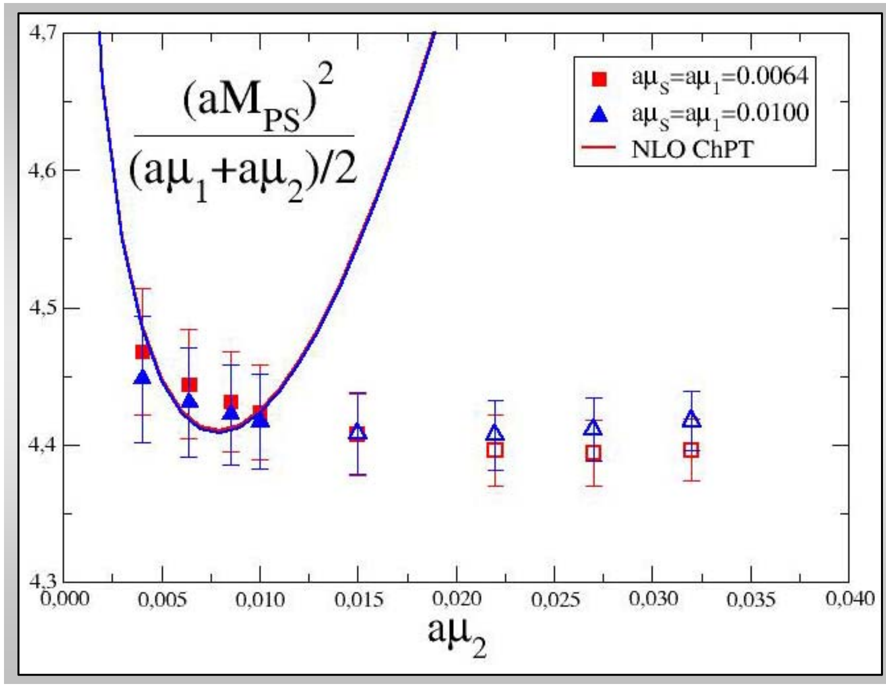
We predict:

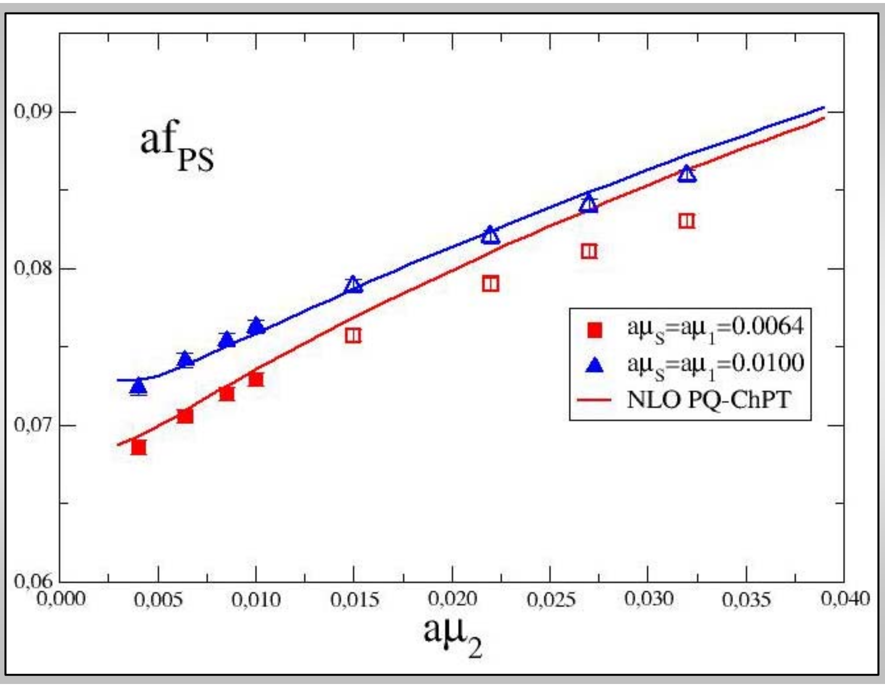
m_{ud}, m_s, f_K/f_π

Chiral extrapolations

In order to determine the physical properties of K mesons we need to investigate the quark mass dependence over a large range of masses, from m_s down to m_{ud} ($\simeq m_s/6$ in the simulation)

We find that NLO (PQ)ChPT describes the lattice data for M_{PS} and f_{PS} only up of masses $m_q \simeq m_s/2$ ($M_{PS} \simeq$ 500 MeV)





We considered 2 functional forms for the quark mass dependence:

- 1) NLO PQChPT [S. Sharpe '97] + local NNLO contributions:

$$M_{PS}^2(\mu_S,\mu_1,\mu_2) = B_0(\mu_1+\mu_2)\cdot\left[1+\frac{\xi_1(\xi_S-\xi_1)\ln 2\xi_1}{(\xi_2-\xi_1)}-\frac{\xi_2(\xi_S-\xi_2)\ln 2\xi_2}{(\xi_2-\xi_1)}+ +a_V\,\xi_{12}+a_S\,\xi_S+a_{VV}\,\xi_{12}^2+a_{SS}\,\xi_S^2+a_{VS}\,\xi_{12}\,\xi_S+a_{VD}\,\xi_{D12}^2\right],$$

$$f_{PS}(\mu_S,\mu_1,\mu_2) = f\cdot\left[1-\xi_{1S}\ln 2\xi_{1S}-\xi_{2S}\ln 2\xi_{2S}+\frac{\xi_1\xi_2-\xi_S\xi_{12}}{2(\xi_2-\xi_1)}\ln\left(\frac{\xi_1}{\xi_2}\right)+ +(b_V+1/2)\,\xi_{12}+(b_S-1/2)\,\xi_S+b_{VV}\,\xi_{12}^2+b_{SS}\,\xi_S^2+b_{VS}\,\xi_{12}\,\xi_S+b_{VD}\,\xi_{D12}^2\right],$$

with $\xi_i = 2B_0\mu_i/(4\pi f)^2$, $\xi_{ij} = B_0(\mu_i+\mu_j)/(4\pi f)^2$, $\xi_{Dij} = B_0(\mu_i-\mu_j)/(4\pi f)^2$

NNLO chiral logs [J. Bijnens, T.A. Lahde '05] not included because they involve a large number of LECs

- 2) Polynomial dependence

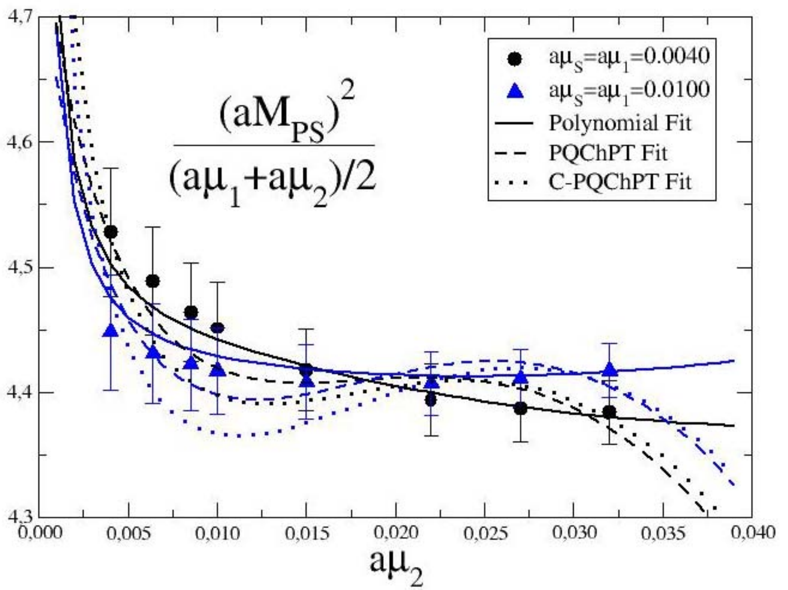
absent in the polynomial case

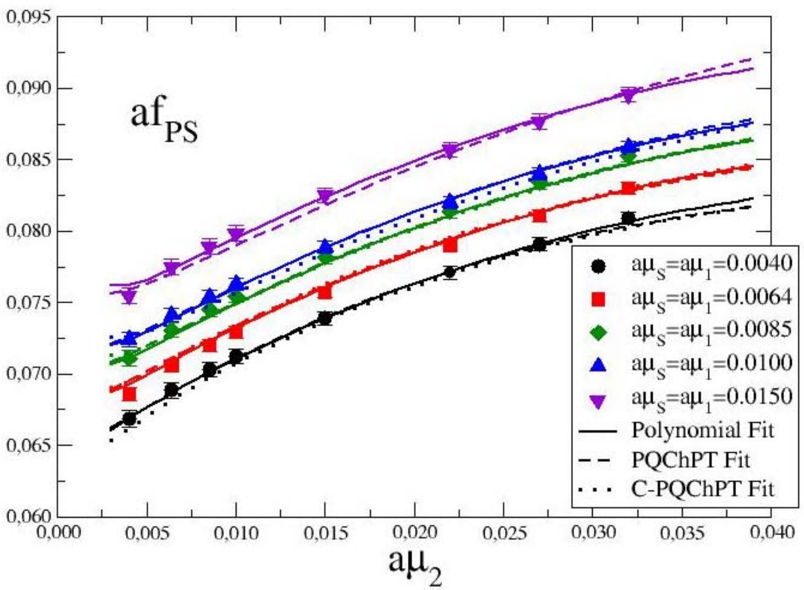
3 FITS FOR THE CHIRAL EXTRAPOLATION:

- Polynomial
- PQChPT
- "Constrained" PQChPT(*)

(*) In order to describe more accurately the pion sector, the LO and NLO LECs are determined from a fit on the unitary points at the 4 lightest quark masses, $m \leq m_s/2$

All fits provide a good χ^2





Partially quenched chiral logs

PQChPT is affected by divergent chiral logs:

$$\mu_S \log \mu_{1,2} \to \infty, \text{ for } \mu_{1,2} \to 0 \text{ at fixed } \mu_S$$

The divergence is not expected to affect the extrapolation to the physical point, since the sea and the light valence quark masses are degenerate in this case.

In order to verify this assumption we have performed the fits on two different sets of data:

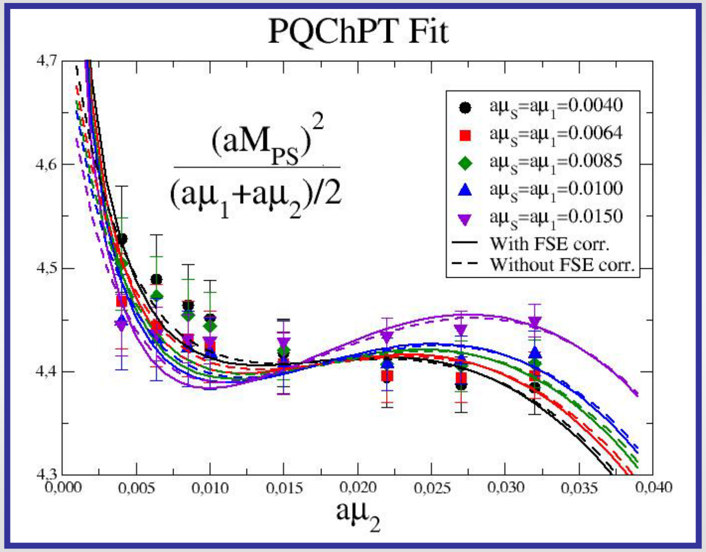
- All 150 combinations of quark masses
- Only the 30 combinations which satisfy $\mu_2 \geq \mu_1 = \mu_S$, i.e. which are safe from potentially divergent chiral logs

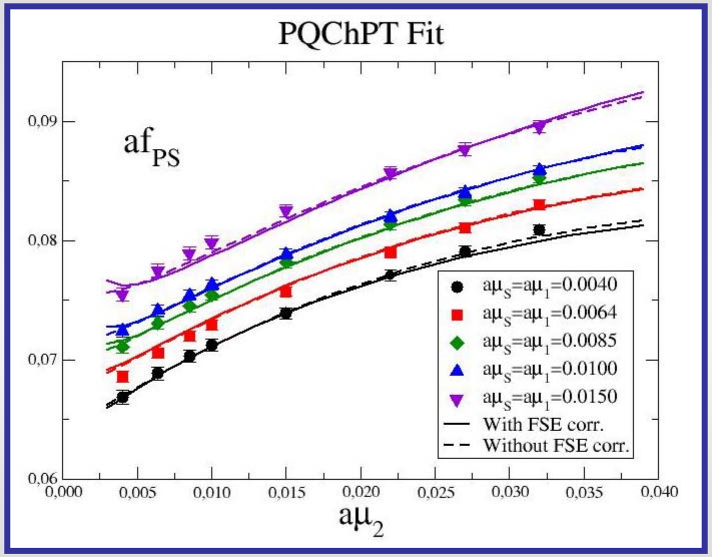
The results from the two sets of data are in perfect agreement and we conservatively consider only those obtained from the reduced set which have larger errors

Finite size effects

Our simulation is at a single volume: $L \approx 2.1$ fm , $M_{PS}L \geq 3.2$

In the fits based on PQChPT, we take FSE into account by including FS corrections predicted by NLO PQChPT
[D. Becirevic, G. Villadoro '04]





The estimated corrections are $\Delta m_{PS} \leq 0.6\%$ and $\Delta f_{PS} \leq 2.5\%$

Discretization effects

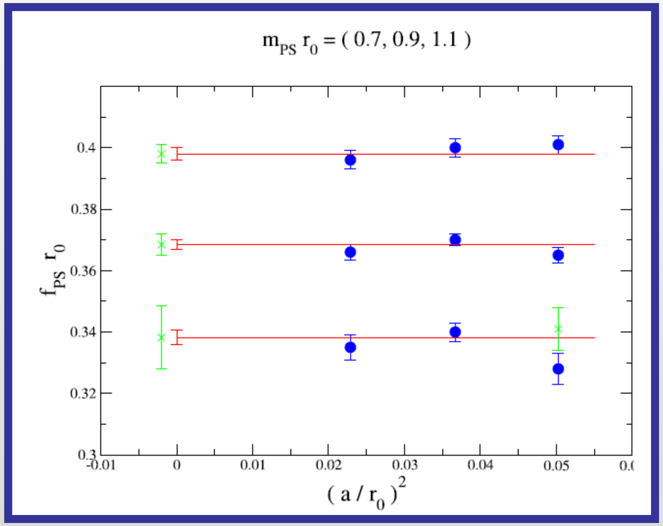
Discretization effects can be analyzed by performing the **Symanzik expansion**: [R.Frezzotti, G.C.Rossi, LATT07]

$$\left(m_{PS}^{\pm}\right)^2 = m_\pi^2 + O\left(a^2 m_\pi^2, a^4\right)$$

$$f_{PS}^{\pm} = f_\pi + a^2 \kappa_\pi + O\left(a^2 m_\pi^2, a^4\right)$$

- In these quantities finite cutoff effects are **parametrically small** and like in a chiral invariant (Ginsparg-Wilson) lattice formulation. They are **NNLO** in **lattice ChPT**.

- In **lattice ChPT** these finite cutoff effects enter at **NNLO**.



For the neutral pion mass

$$\left(m_{PS}^{0}\right)^2 = m_\pi^2 + a^2 \zeta_\pi + O\left(a^2 m_\pi^2, a^4\right)$$

where $\zeta_\pi \sim \left|\langle 0 | P^3 | \pi^0 \rangle\right|^2 \sim 20 - 25 \Lambda_{QCD}^4$

RESULTS

From the 3 fits we obtain (in physical units):

Fit	Polynomial	PQChPT	C-PQChPT
$m_{ud}^{\overline{\rm MS}}$ (MeV)	4.07(9)(33)	3.82(15)(25)	3.74(13)(21)
$m_s^{\overline{\rm MS}}$ (MeV)	109(2)(9)	107(3)(7)	102(3)(6)
m_s/m_{ud}	26.7(2)(0)	27.9(2)(0)	27.4(3)(0)
f_K (MeV)	158.7(11)(89)	160.2(15)(54)	161.8(10)(0)
f_K/f_π	1.214(8)(0)	1.225(11)(0)	1.238(7)(0)

and we quote:

$$m_{ud}^{\overline{\rm MS}}(2\ {\rm GeV}) = 3.85 \pm 0.12 \pm 0.40\ {\rm MeV} \quad , \quad m_s^{\overline{\rm MS}}(2\ {\rm GeV}) = 105 \pm 3 \pm 9\ {\rm MeV}$$

$$m_s/m_{ud} = 27.3 \pm 0.3 \pm 1.2\,,$$

$$f_K = 161.7 \pm 1.2 \pm 3.1\ {\rm MeV} \quad , \quad f_K/f_\pi = 1.227 \pm 0.009 \pm 0.024\,.$$

Non perturbative renormalization

A crucial ingredient in our determination of quark masses is the non perturbative determination of the quark mass renormalization constant.

We used the RI-MOM non perturbative method and obtained:

$$\overline{m}(\mu_R) = Z_P^{-1}\,\mu(a) \qquad Z_P^{RI-MOM}(\mu_R = 1/a) = 0.39(1)(2)$$

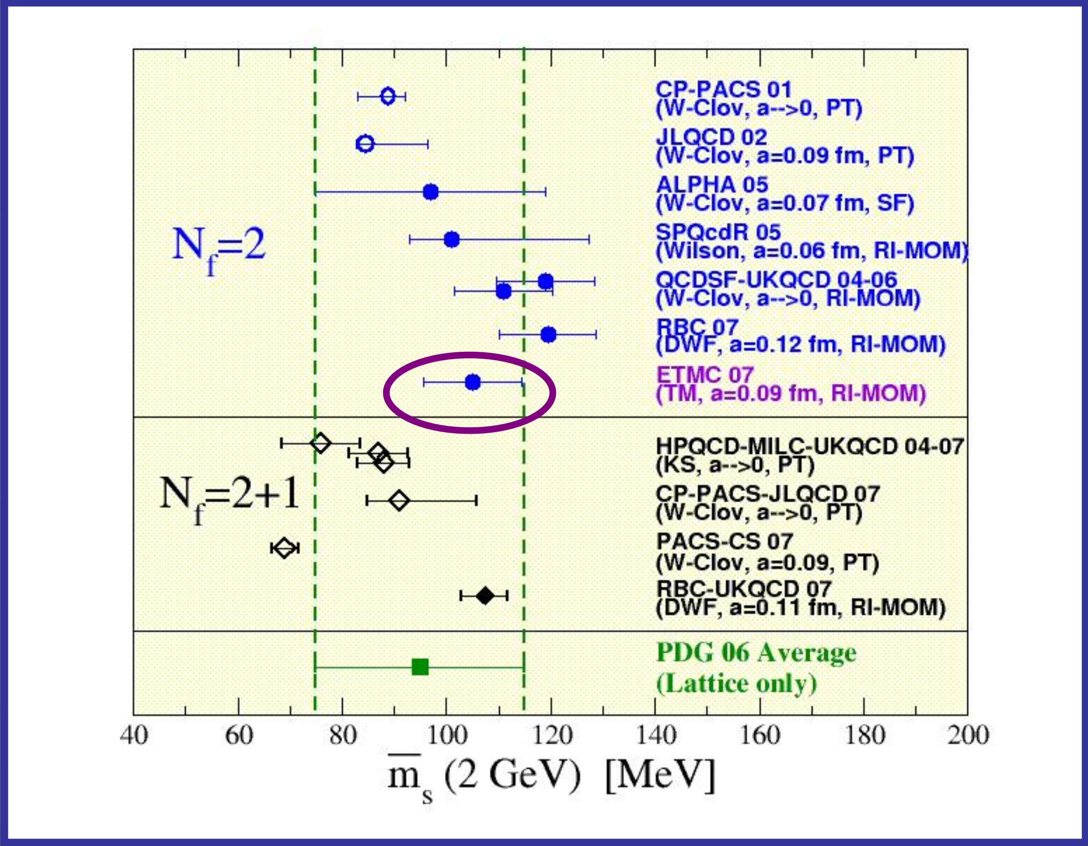
Had we used the **perturbative estimate** of Zm we would have obtained:

$$m_s^{\overline{MS}}(2\ \mathrm{GeV}) = 72 \pm 2 \pm 9\ \mathrm{MeV} \qquad (Z_m)_{PERT.}$$

rather than:

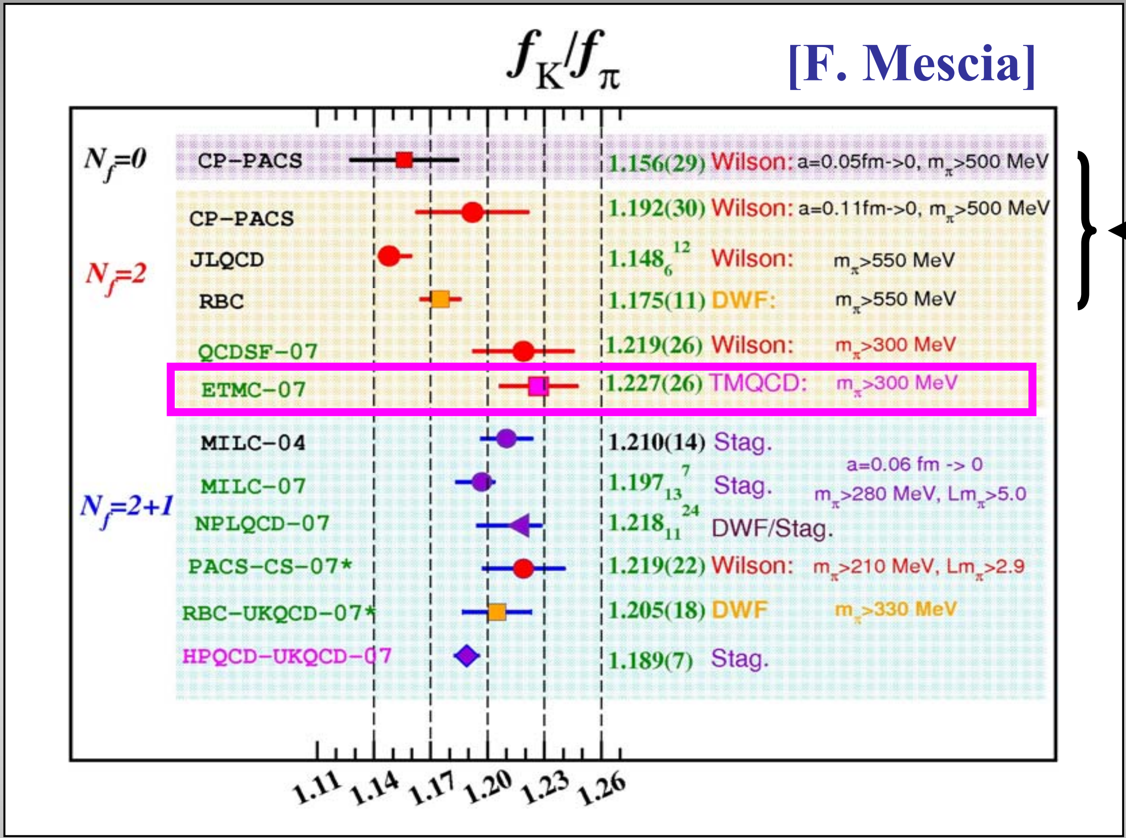
$$m_s^{\overline{MS}}(2\ \mathrm{GeV}) = 105 \pm 3 \pm 9\ \mathrm{MeV} \qquad (Z_m)_{NON-PERT.}$$

SUMMARY OF UNQUENCHED RESULTS FOR ms



(*) Empty symbols: perturbative renormalization

SUMMARY OF RESULTS FOR fk/fπ



"Heavy" quark masses

From $\Gamma\left(K \to \mu\bar{\nu}_\mu(\gamma)\right)/\Gamma\left(\pi \to \mu\bar{\nu}_\mu(\gamma)\right)$, $|V_{ud}|$ from nuclear β-decays and our estimate of f_K/f_π we get:

$|V_{us}|_{Kl2}=0.2192(5)(47)$ ⟷ $|V_{us}|_{Kl3}=0.2255(19)$

Backup slides

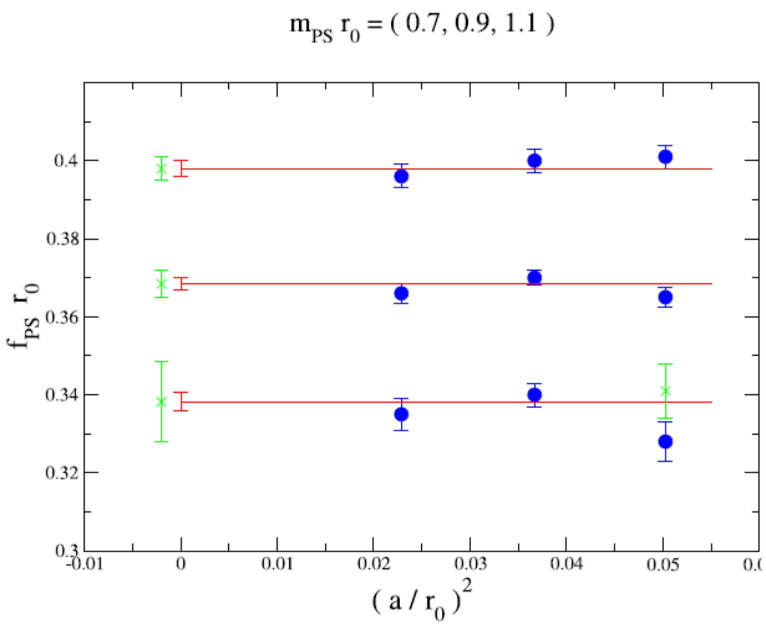
RESULTS FOR THE FIT PARAMETERS

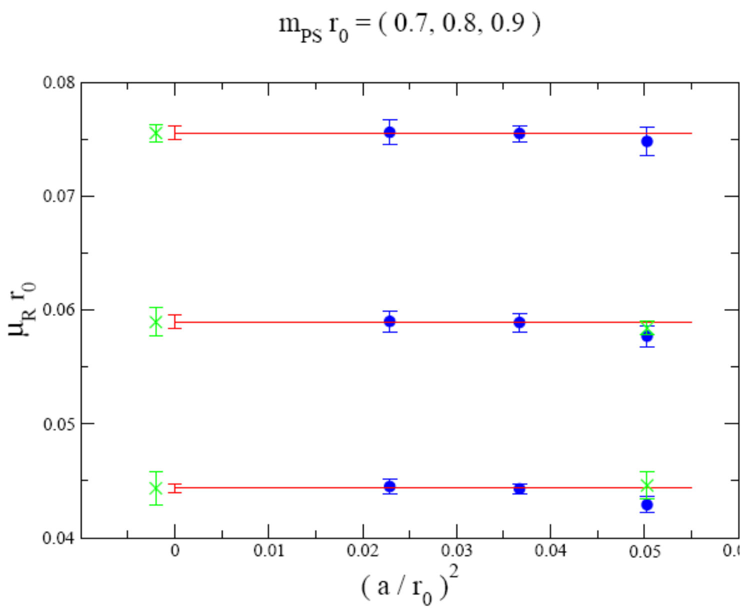
	All data			Only $\mu_2 \geq \mu_1 = \mu_S$		
Fit	Polynomial	PQChPT	C-PQChPT	Polynomial	PQChPT	C-PQChPT
$2aB_0$	4.59(3)	4.79(6)	4.82(10)	4.55(6)	4.86(12)	4.82(10)
af	0.0607(6)	0.0577(6)	0.0552(12)	0.0606(9)	0.0574(14)	0.0552(12)
a_V	-0.63(7)	2.37(10)	2.15(18)	-0.52(16)	1.91(15)	2.15(18)
a_S	0.0	-1.44(10)	-1.35(12)	0.0	-1.04(37)	-1.35(12)
b_V	2.66(4)	0.68(5)	0.86(8)	2.56(13)	0.49(12)	0.75(8)
b_S	0.86(13)	-1.22(15)	-0.25(23)	1.03(15)	-0.94(34)	-0.13(24)
a_{VV}	2.6(2)	-9.3(3)	-8.3(6)	2.3(5)	-7.8(18)	-5.8(7)
a_{VS}	0.0	7.6(4)	6.9(3)	0.0	6.0(38)	0.0
a_{SS}	0.0	0.0	0.0	0.0	0.0	5.9(7)
a_{VD}	-0.6(1)	-3.8(2)	-3.2(3)	-0.9(6)	-2.6(21)	-5.1(4)
b_{VV}	-4.0(2)	1.2(2)	0.9(1)	-4.1(8)	0.0	2.3(5)
b_{VS}	0.0	6.0(6)	3.7(12)	0.0	7.1(21)	0.0
b_{SS}	0.0	0.0	-5.3(14)	0.0	0.0	-2.0(6)
b_{VD}	-3.7(2)	-3.8(2)	-3.0(3)	-2.6(6)	0.0	-3.1(6)
χ^2/d.o.f.	0.38	1.34	1.11	0.28	0.40	0.78

RESULTS FOR THE PHYSICAL QUANTITIES IN LATTICE UNITS

	All data			Only $\mu_2 \geq \mu_1 = \mu_S$		
Fit	Polynomial	PQChPT	C-PQChPT	Polynomial	PQChPT	C-PQChPT
$am_{ud} \cdot 10^3$	0.90(2)	0.86(2)	0.79(4)	0.91(3)	0.84(5)	0.79(4)
am_s	0.0243(5)	0.0235(5)	0.0218(10)	0.0243(7)	0.0234(12)	0.0217(10)
m_s/m_{ud}	26.9(1)	27.4(2)	27.5(3)	26.7(2)	27.9(2)	27.4(3)
aM_π	0.0642(6)	0.0632(6)	0.0610(12)	0.0642(9)	0.0629(14)	0.0610(12)
aM_K	0.235(2)	0.232(2)	0.224(4)	0.235(3)	0.231(5)	0.224(4)
af_π	0.0622(6)	0.0612(6)	0.0591(11)	0.0622(8)	0.0609(13)	0.0591(11)
af_K	0.0756(7)	0.0744(7)	0.0730(11)	0.0755(8)	0.0747(11)	0.0731(12)
f_K/f_π	1.216(3)	1.215(4)	1.236(8)	1.214(8)	1.225(11)	1.238(7)

SCALING TESTS





ETM Collaboration,
P. Dimopoulos et al,
PoS **LAT2007**