Non-perturbative HQET on the lattice at O(1/M)

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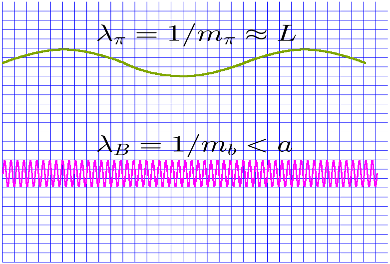
Lattice HQET

Why HQET on the lattice ? The reason is mainly practical:

- finite volume effects are mainly triggered by the light degrees of freedom. The usual requirement is $m_{PS}L > 4$ and m_{PS} is typically around the kaon mass in real lattice simulations $\Rightarrow L \simeq 2\ \mathrm{fm}$.
- cutoff effects are tuned by the heavy quark mass.
  $a << 1/m_b \simeq 0.03\ \mathrm{fm}$.

$\Rightarrow L/a \simeq 100$ is needed to have those systematics under control !!
Integrating out the heavy quark mass in this case is useful !!



The b quark mass and the B_s meson decay constant in HQET

ALPHA Collaboration

In collaboration with N. Garron, M. Papinutto, R. Sommer and B. Blossier

Why do we like HQET[Eichten and Hill, '89] ?

- Theoretically very sound
- Can be treated non-perturbatively including renormalization (and O(1/M)) [Heitger and Sommer, 2003]
- Subleading corrections can be computed systematically or estimated by combining with relativistic quarks around the charm
- The continuum limit is well defined and can be reached numerically [ALPHA, 2003]
- Unquenching can be included now
- Can be used together with other methods, eg the Rome II method [Guazzini, Sommer and Tantalo, 2007]

still it might be a little involved ....

A bit of notation

Field content: ψ_h s.t. $P_+\psi_h = \psi_h$ with $P_+ = \frac{1+\gamma_0}{2}$

$$S_{HQET} = a^4 \sum_x \left\{ \bar{\psi}_h(D_0 + \delta m)\psi_h + \omega_{spin}\bar{\psi}_h(-\sigma\mathbf{B})\psi_h + \omega_{kin}\bar{\psi}_h\left(-\frac{1}{2}\mathbf{D}^2\right)\psi_h \right.$$

- 3 parameters (we'll get rid of one through spin-average) to be set in order to reproduce QCD up to $O(1/m_b^2)$.
- ω_{spin} and ω_{kin} formally $O(1/m_b)$.
- Renormalization and matching !
- The two steps could be performed separately. In particular at *leading order in* $1/m_b$ matching can be done in perturbation theory. Here we are interested in $1/m_b$ corrections and do the two things at the same time and non-perturbatively.

$1/m_b$ corrections

take for example

$$m_{B^*}^2 - m_B^2 = C_{mag}(m_b/\Lambda_{\mathrm{QCD}})\ \langle B|\bar{\psi}_h \sigma \mathbf{B} \psi_h|B^*\rangle + \mathrm{O}(1/m_b)$$

$C_{mag}(m_b/\Lambda_{\mathrm{QCD}})$ has a perturbative expanison. At order $n-1$ the truncation error:

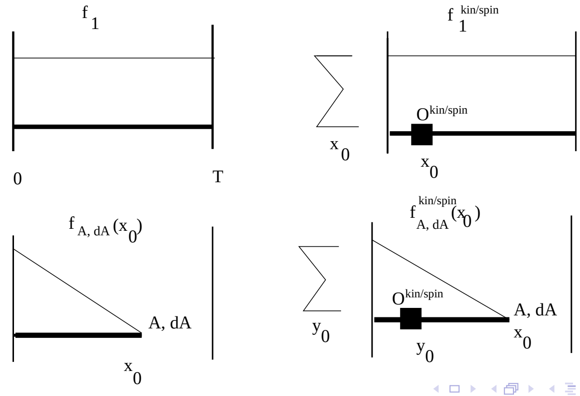
$$\simeq \alpha(m_b)^n \simeq \left\{ \frac{1}{2b_0 \ln(m_b/\Lambda_{\mathrm{QCD}})} \right\}^n >> \frac{\Lambda_{\mathrm{QCD}}}{m_b} \quad \text{as } m_b \to \infty$$

- The perturbative corrections to the leading term are much larger than the power corrections !!
- Put everything on the lattice, including matching ("Wilson") coefficients.

We don't include the next to leading terms of the $1/m_b$ expansion in the action, the theory would be non renormalizable. We treat them as insertions into correlation functions and consider the static action only.

$$e^{-(S_{rel}+S_{HQET})} = e^{-(S_{rel}+S_{stat})} \times [1 - a^4 \sum_x \mathcal{L}^{(1)}(x, \omega_{spin}, \omega_{kin}) + \ldots]$$

and $S_{stat} = a^4 \sum_x \bar{\psi}_h(x) D_0^{HYP} \psi_h(x)$ [spin-flavor symmetric]

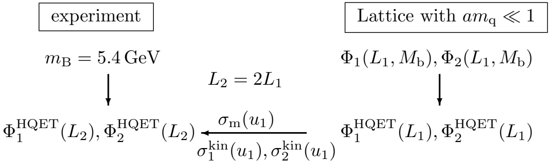


Overview of the approach

- We will use a finite volume scheme (Schrödinger functional). The volume L_1 should be small enough to simulate relativistic b-quarks ($a \ll 1/m_b$) but also such that $\frac{1}{L_1 m_b} \simeq \frac{\Lambda_{\rm QCD}}{m_b}$ (in the end $L_1 \simeq 0.4$ fm.)
- Considering spin-averaged quantities, we are left with two coefficients. Strategy: define two (sensible) quantities Φ_k and require (in small volume)

$$\Phi_k^{HQET} = \Phi_k^{QCD} \quad k = 1, 2$$

- Evolve these quantities in the effective theory to large volumes. There the B-meson mass expressed in terms of Φ_k and large volume HQET quantities can be used to fix the b-quark mass.



The Schrödinger functional

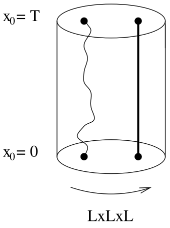
QCD in a finite volume $L^3 \times T$ with periodic boundary conditions in space and Dirichlet boundary condition is time.

Periodicity up to a phase θ: $\psi(x + L\hat{k}) = e^{i\theta}\psi(x)$

- QCD correlations (b-quark boundary field: ζ_b)

$$f_1 \propto \sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z}} \langle \bar{\zeta}'_l(\mathbf{u})\gamma_5\zeta'_b(\mathbf{v})\ \bar{\zeta}_b(\mathbf{y})\gamma_5\zeta_l(\mathbf{z})\rangle \quad \mathrm{PS}$$

$$k_1 \propto \sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z},k} \langle \bar{\zeta}'_l(\mathbf{u})\gamma_k\zeta'_b(\mathbf{v})\ \bar{\zeta}_b(\mathbf{y})\gamma_k\zeta_l(\mathbf{z})\rangle \quad \mathrm{V}$$



- in HQET (b-quark boundary field: ζ_h) up to $O(1/m_b^2)$. [$m_{bare} = \delta m + m_b$]

$$(f_1)_R = Z_{\zeta_h}^2 Z_{\zeta}^2 e^{-m_{bare}T}\left\{ f_1^{stat} + \omega_{kin} f_1^{kin} + \omega_{spin} f_1^{spin} \right\}$$

$$(k_1)_R = Z_{\zeta_h}^2 Z_{\zeta}^2 e^{-m_{bare}T}\left\{ f_1^{stat} + \omega_{kin} f_1^{kin} - \frac{1}{3}\omega_{spin} f_1^{spin} \right\}$$

Main advantage of these SF correlators: no $1/m_b$ terms in the HQET expansion of the boundary fields.

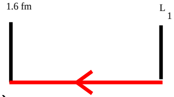
QCD HQET

$$\Phi_1(L,M) = \ln\left(\frac{f_1(\theta)}{f_1(\theta')}\right) + 3\ln\left(\frac{k_1(\theta)}{k_1(\theta')}\right) - R_1^{stat} = \omega_{kin} R_1^{kin}(\theta,\theta')$$

$$\Phi_2(L,M) = L\Gamma_1 = -L\tilde{\partial}_T \ln f_1^{av} = L[m_{bare} + \Gamma_1^{stat} + \omega_{kin}\Gamma_1^{kin}]$$

$$f_1^{av} = (f_1 k_1^3)^{1/4}$$

- For the lattice spacings (ie β values) used here (L_1) we could determine ω_{kin} and m_{bare} as functions of M (implicitly we'll do that).
- but we don't get the relation among M and m_B to fix the b-quark mass.
- To this end we first evolve the Φ_k to larger volumes



$$\Phi_k(L_2,M,\beta) = \sum_j \Sigma_{kj}(L_1,\beta)\Phi_j(L_1,M,\beta)$$

the SSF Σ_{kj} are defined in HQET and have a continuum limit. Notice $\Phi_k(L_2,M)$ still depend on M !!

- The parameters (β) used in L_2 can be used in large volumes. There we take a phenomenological quantity to fix M_b

$$m_{B^{av}} = E^{stat} + \omega_{kin} E^{kin} + m_{bare} \, ,$$

$$E^{stat} = \lim_{L\to\infty} \Gamma_1^{stat} \, , \quad E^{kin} = -\frac{1}{2}\langle B| \sum_z \bar{\psi}_h \mathbf{D}^2 \psi_h(0,z)|B\rangle$$

rewriting ω_{kin} and m_{bare} in terms of $\Phi_k(L_2, M) = \Sigma_{kj}\Phi_j(L_1, M)$ we get a relation between $m_{B^{av}}$ and M which we solve for M_b.

- The relation involves quantities, which have a continuum limit either in QCD or HQET. The bare parameters have disappeared (implicit above).

The static piece

In this case only 1 parameter (δm, linearly divergent) to be eliminated

1: ∞ volume. Experimental input m_B:

$$m_B = E^{stat} + m_{bare}$$

m_{bare} is a counter-term in the action $\rightarrow$ independent from L

2: Small volume (L_1) matching condition

$$\Gamma_1(M_b, L_1) = \Gamma_1^{stat}(L_1) + m_{bare}$$

M_b obtained by inserting 2 in 1:

$$m_B = [E^{stat} - \Gamma_1^{stat}(L_1)] + \Gamma_1(M_b, L_1)$$

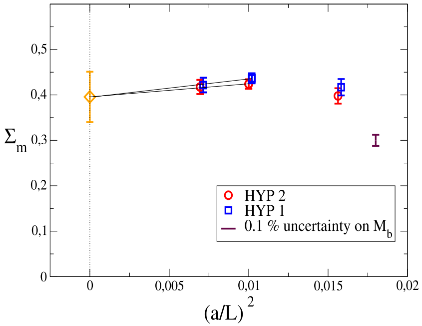
Each piece has a well defined continuum limit in HQET or QCD.

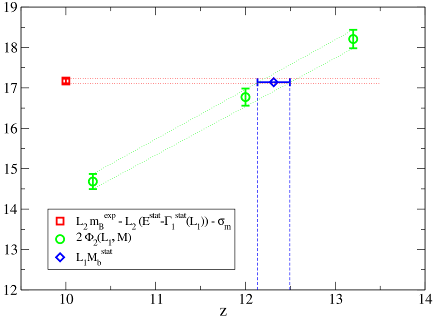
The difference between ∞ volume ($\simeq 1.6$ fm.) and L_1 ($\simeq 0.4$ fm.) is too large to simulate at the same bare parameters and a fine a.

We insert one SSF evolution

$$2L_1 m_B - 2L_1[E^{stat} - \Gamma_1^{stat}(2L_1)] - \sigma_m(L_1) = 2L_1\Gamma_1(M_b, L_1)$$

$$\sigma_m(L_1) = 2L_1(\Gamma_1^{stat}(2L_1) - \Gamma_1^{stat}(L_1))$$





$$M_b^{stat} = 6.806(79)\ GeV$$

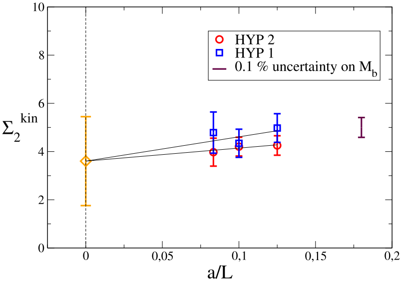
We also get

$$S = \frac{1}{L_1}\frac{\partial \Phi_2(L_1, M)}{\partial M} = 0.61(5)$$

Similarly for NLO corrections

$$m_{B^{av}} = m_B^{stat} + m_B^1$$

with m_B^1 in terms of E^{kin}, SSF and $\Phi_k(L_1, M_B^{stat})$, eg (notice a/L now):



and $M_B^1 = -\frac{m_B^1}{S}$

In the $\overline{\mathrm{MS}}$ scheme at the scale m_b

$$m_b(m_b) = 4.347(48)\ GeV$$

- We tried (12) different matching conditions. All results agree, indicating very small $1/m_b^2$ corrections. [ALPHA, JHEP 0701:007,2007]

θ_0	$r_0\, M_{\rm b}^{(0)}$	$r_0\, M_{\rm b} = r_0\,(M_{\rm b}^{(0)} + M_{\rm b}^{(1a)} + M_{\rm b}^{(1b)})$		
		$\theta_1 = 0$ $\theta_2 = 1/2$	$\theta_1 = 1/2$ $\theta_2 = 1$	$\theta_1 = 1$ $\theta_2 = 0$
0	17.25(20)	17.12(22)	17.12(22)	17.12(22)
0	17.05(25)	17.25(28)	17.23(27)	17.24(27)
1/2	17.01(22)	17.23(28)	17.21(27)	17.22(28)
1	16.78(28)	17.17(32)	17.14(30)	17.15(30)

The B_s meson decay constant

Operators have an expansion in $1/m_b$ too. The a-expansion and the heavy quark expansion are not independent on the lattice, they are expansions in the dimension of the operators ($1/m_b$ terms must be introduced together with O(a) improvement terms of the static action).

$$A_0^{HQET} = Z_A^{HQET}\left(A_0^{stat} + (O(a) + O(1/m_b)) \times c_A^{HQET} A_0^{(1)}\right) \ ,$$

$$A_0^{(1)}(x) = (\bar{\psi}_l(x)\gamma_j D_j)\psi_h(x)$$

In our notation Z_A^{HQET} includes the matching coefficient.

For the decay constant 4 Φ_i's are needed in the small volume matching to QCD. The SSF also becomes a 4×4 matrix [ALPHA, Lattice 2007]

- (Quenched) Results from different matching conditions:

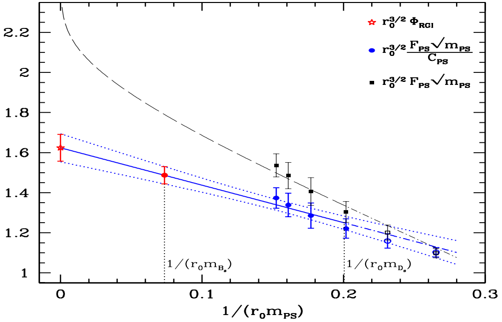
Preliminary !!

θ_0	$F_{B_s}^{\mathrm{stat}}$ [MeV]	$F_{B_s}^{\mathrm{stat}} + F_{B_s}^{(1)}$[MeV]		
		$\theta_1 = 0$ $\theta_2 = 0.5$	$\theta_1 = 0.5$ $\theta_2 = 1$	$\theta_1 = 1$ $\theta_2 = 0$
0	224 ± 5	185 ± 21	186 ± 22	189 ± 22
0.5	220 ± 5	185 ± 21	187 ± 22	189 ± 22
1	209 ± 5	184 ± 21	185 ± 21	188 ± 22

Same pattern as for m_{b}. Results are perfectly consistent after the inclusion of the $1/m_{\mathrm{b}}$ terms. More than suggested by the errors, as eg

$$F_{\mathrm{B_s}}^{\mathrm{stat}+(1)}(\theta_0 = 0, \theta_1 = 1, \theta_2 = 0) - F_{\mathrm{B_s}}^{\mathrm{stat}+(1)}(\theta_0 = 1, \theta_1 = 0, \theta_2 = 0.5) = 4 \pm 2 \ \mathrm{MeV} \, .$$

Comparison with other determinations beyond the static approximation



F_{B_s} =193(6) MeV [ALPHA, arXiv:0710.2201 [hep-lat] and talk tomorrow by J. Heitger]

Rome II SSF method with static constraints: $F_{B_s} = 191(6)$ MeV

[Guazzini, Sommer and Tantalo, arXiv:0710.2229 [hep-lat]]

Conclusions

1. To keep the pace with forth-coming experiments and really help in the quest for New Physics, lattice results in Heavy Flavor Physics must aim at high precision.
2. To this end all the systematics must be kept under control. Unquenching, renormalization, continuum limit, chiral extrapolations, each of them can easily have a $5-10\%$ uncertainty associated.
3. I've given an example how this can be done non-perturbatively while discussing the b-quark mass in HQET and the $B_{\rm s}$ meson decay constant. Almost done, it was quenched. Unquenching is ongoing [ALPHA, Lattice 2007 and JHEP 0702:079,2007 for $F_{\rm B_s}^{\rm stat}$]
   1. The approach can be extended to other quantities, eg B_B and the form factors for semileptonic decays.
   2. The approach can be extended to other formulations, eg for the non-perturbative determinations of the parameters in Fermilab-like actions [N. Christ, Li and Lin, 2006].

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