Matching the 2 loop ChPT with the dispersive representation of the Kπ scalar form factor

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Outline

1. Introduction and Motivation
2. How to determine $F_K/F_{\pi,}$ $f_+(0)$, C_{12} and C_{34} ? Standard Model and beyond
3. ChPT at 2 loops in the isospin limit.
4. Dispersive Representation of the $K\pi$ scalar form factor.
5. Matching.
6. Results and Conclusion.

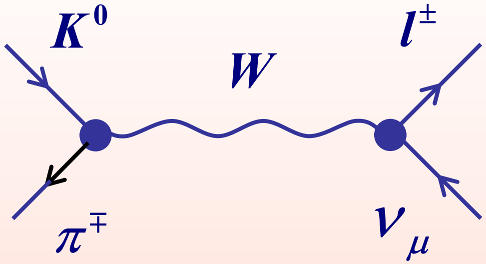
1. Introduction and Motivation

- Determination
  - F_K/F_π
  - $f_+(0)$
  - C_{12} and C_{34} 2 $\mathcal{O}(p^6)$ LECs which enters K_{l3} decays

    ⇒ λ_0 and Δ_{CT}

- Why ?
  - LECs play an important role in ChPT calculations. enter different processes. Ex: C_{12} into $\eta \rightarrow 3\pi$ (cf Bijnens'Talk) **[Bijnens&Ghorbani'07]**
  - Tests of the Standard Model in K_{l3} decays:
    - Extraction of Vus from K Decay Rates
    - Callan-Treiman Theorem.

1.1 Definition



- K_{l3} decays $K^0 \rightarrow \pi^\mp l^\pm \nu_\mu$

- The hadronic element :

$$\langle \pi^-(p_\pi) | \bar{s}\gamma_\mu u | K^0(p_K) \rangle = f_+(t)(p_K + p_\pi)_\mu + f_-(t)(p_K - p_\pi)_\mu$$

→ $f_+(t)$, $f_-(t)$: form factors

→ $t = q^2 = (p_\mu + p_{\nu_\mu})^2 = (p_K - p_\pi)^2$

- We consider : - the vector form factor $f_+(t)$.
  - the scalar form factor $f_s(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$

→ Normalization : $\bar{f}_+(t) = \frac{f_+(t)}{f_+(0)}, \ \bar{f}_+(0) = 1$ and $\bar{f}_0(t) = \frac{f_s(t)}{f_+(0)}, \ \bar{f}_0(0) = 1$

1.2 Extraction of Vus

- Decay rate formula

$$\frac{\Gamma_{K^{+/0}l3}}{\tau_{K^{+/0}}} = \frac{C_K^2 G_F^2 m_{K^{+/0}}^5}{192\pi^3} S_{EW}\left(1+2\Delta^{EM}_{K^{+/0}l}\right)\left|f_+^{K^{+/0}}(0)V_{us}\right|^2 I^l_{K^{+/0}} \qquad l=(e,\mu)$$

Kaon life time

Radiative Corrections

$$I^l_{K^{+/0}} = \int dt \frac{1}{m^8_{K^{+/0}}}\lambda^{3/2} F\left(t, \overline{f}_+(t), \overline{f}_0(t)\right)$$

→ $\boxed{f_+(0)\left|V_{us}\right|}$

- Knowledge of $f_+(0)$ → extraction of Vus and test of the unitarity of the first line of V_{CKM}

$$\boxed{\left|V_{ud}\right|^2 + \left|V_{us}\right|^2 + \left|V_{ub}\right|^2 \overset{?}{=} 1}$$

$0^+ \rightarrow 0^+$ β decays Kl3 decays Negligible (B decays)

- If ≠1 , new physics: 4 generations, RHCs…

1.3 Callan-Treiman Theorem

- $\bar{f}_0(\Delta_{K\pi}) \equiv C$: Scalar form factor at the CT point
- CT theorem :

$$\bar{f}_0(\Delta_{K\pi}) = \frac{F_K}{F_\pi}\frac{1}{f_+(0)} + \Delta_{CT}$$

$$\Delta_{K\pi} = m_K^2 - m_\pi^2$$

- Knowing $\frac{F_K}{F_\pi},\ f_+(0),\ \Delta_{CT}$ allows to test the SM couplings

- We write as **[Bernard et al'06]**

$$C \equiv f(\Delta_{K\pi}) = \frac{F_K \left|\mathcal{A}_{eff}^{us}\right|}{F_\pi \left|\mathcal{A}_{eff}^{ud}\right|} \frac{1}{f_+(0)\left|\mathcal{V}_{eff}^{us}\right|} \left|\mathcal{V}_{eff}^{ud}\right| \frac{\left|\mathcal{A}_{eff}^{ud}\right|\left|\mathcal{V}_{eff}^{us}\right|}{\left|\mathcal{A}_{eff}^{us}\right|\left|\mathcal{V}_{eff}^{ud}\right|} + \Delta_{CT}$$

$$\frac{\Gamma(K \to \mu\nu(\gamma))}{\Gamma(\pi \to \mu\nu(\gamma))}$$

$$\Gamma(K \to \pi e\nu)$$

$0^+ \to 0^+$ β decays

$$C = B_{\exp} R + \Delta_{CT} \quad \Longleftrightarrow \quad \ln C = \ln B_{\exp} + \ln R + \frac{\Delta_{CT}}{B_{\exp}}$$

$B_{\exp} = 1.2446(41)$ and $\ln C = 0.2188(35) + \Delta\varepsilon$

- SM case $\mathcal{V}_{eff} = -\mathcal{A}_{eff} = V_{CKM}$ $\Rightarrow$ $R = 1$ and $\Delta\varepsilon = \Delta_{CT}$
- If $\mathcal{V}_{eff} \neq \mathcal{A}_{eff}$ then $R \neq 1$ and $\Delta\varepsilon = \Delta\varepsilon|_{bSM} + \Delta_{CT}$
  Test of right-handed couplings of quarks to W.

1.4 How to determine F_K/F_{π}, $f_+(0)$, C_{12} and C_{34} ?

Some examples:

- C_{12} and C_{34}:
  - by resonance models and RChPT **[Cirigliano et al, '05]** → $f_+(0)$
  - Matching between 2 loop and dispersive parametrization taking input values for F_K/F_{π} and $f_+(0)$ **[Jamin et al '04]**

- F_K/F_{π}, $f_+(0)$
  - Lattice: many groups and methods: for example **[Mescia, HEP'07] (see discussion)**
  - $f_+(0)$, quark model **[Leutwyler&Roos'84']**

- Here matching between the 2 loop calculation and a dispersive parametrization for $f_0(t)$ **from [Bernard et al '06]**
  - Similar to Jamin et al but use $\widehat{F_K}/\widehat{F_{\pi}}$ and $\widehat{f_+(0)}$. and parametrize our ignorance of these QCD quantities.

2. F_K,F_π, $f_+(0)$… in the SM and beyond

- The charged current Lagrangien:

$$\boxed{\mathcal{L}_{CC} = \tilde{g}\left[\mathbf{1}_\mu + \frac{1}{2}\bar{U}\left(\mathcal{V}_{eff}\gamma_\mu + \mathcal{A}_{eff}\gamma_\mu\gamma_5\right)D\right]W^\mu + h.c\ ,} \quad U = \begin{pmatrix} u \\ c \\ t \end{pmatrix},\ D = \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

  - $\mathbf{1}_\mu$: standard V-A leptonic current
  - $\mathcal{V}_{\text{eff}}$, $\mathcal{A}_{\text{eff}}$: 3x3 complex matrices of effective couplings.

2.1 SM case: $\mathcal{V}_{eff} = -\mathcal{A}_{eff} = V_{CKM}$

- Unitarity of V_{CKM}: $|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} = 1$
- V_{ud} from $0^+ \rightarrow 0^+$ superallowed β decays: $\mathcal{V}_{ud} = 0.97418(26)$

  [Towner&Hardy'07]
- V_{us} from unitarity $|V_{us}|^2 = 1 - |V_{ud}|^2$

⇨ F_K/F_π, $f_+(0)$ are known from experiments.

- From $\Gamma(\pi \to \mu\nu(\gamma)) \sim \left| F_\pi \; \mathcal{A}_{eff}^{ud} \right|$ ⇨ $\hat{F}_\pi = (92.1 \pm 0.2)$ MeV
- From $\dfrac{\Gamma(K \to \mu\nu(\gamma))}{\Gamma(\pi \to \mu\nu(\gamma))} \sim \left| \dfrac{F_K \mathcal{A}_{eff}^{us}}{F_\pi \mathcal{A}_{eff}^{ud}} \right|$ ⇨ $\dfrac{\widehat{F}_K}{\widehat{F}_\pi} = 1.192 \pm 0.007$
- From $\Gamma(K^0 \to \pi^+ e^- \nu) \sim \left| f_+(0) \; \mathcal{V}_{eff}^{us} \right|$ ⇨ $\widehat{f}_+(0) = 0.9574 \pm 0.0052$

2.3 Physics beyond the SM

- If Physics Beyond the Standard Model, then F_π, F_K/F_π, $f_+(0)$ unknown

- Ex: $V_{eff} \neq A_{eff}$: Low Energy Effective Theory (LEET) **[Hirn&Stern'05]** **(C.f Talk M.Oertel)**

  3 parameters at NLO:
  - modification of the left-handed couplings: δ
  - Modification of the right-handed couplings of quarks to W (RHCs):
    - In the non-strange sector: ε_{NS}
    - In the strange sector: ε_S

- Order of magnitude: $\varepsilon_{NS} < 1\%$, ε_S~few % if the hierarchy of the right-handed mixing matrix is inverted.

- Expression of the effective couplings in terms of the Cabibbo angle and the 3 parameters neglecting V_{ub}:

$$\left|\mathcal{V}_{eff}^{ud}\right|^2 = \cos^2\hat{\theta} \qquad \text{and} \qquad \left|\mathcal{V}_{eff}^{us}\right|^2 = \sin^2\hat{\theta}\left(1+2\frac{\delta+\varepsilon_{NS}}{\sin^2\hat{\theta}}\right)\left(1+2\varepsilon_S-2\varepsilon_{NS}\right)$$

$$\left|\mathcal{A}_{eff}^{ud}\right|^2 = \cos^2\hat{\theta}\left(1-4\varepsilon_{NS}\right) \qquad \left|\mathcal{A}_{eff}^{us}\right|^2 = \sin^2\hat{\theta}\left(1+2\frac{\delta+\varepsilon_{NS}}{\sin^2\hat{\theta}}\right)\left(1-2\varepsilon_S-2\varepsilon_{NS}\right)$$

- Unitarity of V_{eff}: $\left|\mathcal{V}_{eff}^{ud}\right|^2+\left|\mathcal{V}_{eff}^{us}\right|^2 = 1+\Delta_{\text{unitarity}} = 1+2(\delta+\varepsilon_{NS})+2(\varepsilon_S-\varepsilon_{NS})\sin^2\hat{\theta}$

- We obtain $F_\pi = \widehat{F}_\pi\left(1+2\varepsilon_{NS}\right)$

$$\left(\frac{F_K}{F_\pi}\right)^2 = \left(\frac{\widehat{F}_K}{\widehat{F}_\pi}\right)^2 \frac{1+2\left(\varepsilon_S-\varepsilon_{NS}\right)}{1+\frac{2}{\sin^2\hat{\theta}}\left(\delta+\varepsilon_{NS}\right)} \qquad \text{and} \qquad \left[f_+^{K^0\pi^-}(0)\right]^2 = \left[\widehat{f}_+^{K^0\pi^-}(0)\right]^2 \frac{1-2\left(\varepsilon_S-\varepsilon_{NS}\right)}{1+\frac{2}{\sin^2\hat{\theta}}\left(\delta+\varepsilon_{NS}\right)}$$

$$\Delta_{\text{unitarity}} = \sin^2\hat{\theta}\left(\left|\frac{\widehat{f}_+^{K^0\pi^-}(0)}{f_+^{K^0\pi^-}(0)}\right|^2-1\right)$$

⇒ $\Delta_{\text{unitarity}}$ small

- Order of magnitude: F_π not that much affected.

- In a more general framework: parametrization of our ignorance of F_K/F_π, $f_+(0)$.

$$\left(\frac{F_K}{F_\pi}\right)^2 = \alpha \left(\frac{\widehat{F}_K}{\widehat{F}_\pi}\right)^2 \qquad \text{and} \qquad \left[f_+^{K^0\pi^-}(0)\right]^2 = \beta \left[\widehat{f}_+^{K^0\pi^-}(0)\right]^2$$

Physics beyond the SM if α and $\beta \neq 1$

- Try to determine some of these quantities from K_{l3} experiments

3. ChPT at 2 loops in the isospin limit

[Bijnens&Talavera'03]

- Consider the quantity:

$$\tilde{f}_0(t) = f_+(t) + \frac{t}{(m_K^2 - m_\pi^2)}\left(f_-(t) + 1 - \frac{F_K}{F_\pi} \right) = f_0(t) + \frac{t}{(m_K^2 - m_\pi^2)}\left(1 - \frac{F_K}{F_\pi} \right)$$

No dependence on the L_i at p^4, only via p^6 contribution:

$$\tilde{f}_0(t) = 1 - \frac{8}{F_\pi^4}\left(C_{12}^r + C_{34}^r\right)\left(m_K^2 - m_\pi^2\right)^2 + 8\frac{t}{F_\pi^4}\left(2C_{12}^r + C_{34}^r\right)\left(m_K^2 + m_\pi^2\right)^2$$

$$- \frac{8}{F_\pi^4} t^2 C_{12}^r + \overline{\Delta}(t) + \Delta(0)$$

⟹ Only 2 LECs C12 and C34 !
Can be determined by the slope
and curvature.

- Δ(t) and Δ(0) : contributions from loops: → F_π, the LECs L_i (L_5 ⟺ F_K/F_π) can be calculated at $\mathcal{O}(p^6)$ with the knowledge of the L_i at $\mathcal{O}(p^4)$ in the physical region

$$\overline{\Delta}(t) = -0.25763t + 0.833045t^2 + 1.25252t^3 \quad \left[K_{l3}^0\right]$$

$$\Delta(0) = -0.0080 \pm 0.0057\left[loops\right] \pm 0.0028\left[L_i^r\right]$$ (new fit ? (WG on K))

- Consider the quantity:

$$\tilde{f}_0(t) = f_+(t) + \frac{t}{(m_K^2 - m_\pi^2)}\left(f_-(t) + 1 - \frac{F_K}{F_\pi} \right) = f_0(t) + \frac{t}{(m_K^2 - m_\pi^2)}\left(1 - \frac{F_K}{F_\pi} \right)$$

No dependence on the L_i at p^4, only via p^6 contribution:

$$\tilde{f}_0(t) = 1 - \frac{8}{F_\pi^4}\left(C_{12}^r + C_{34}^r\right)\left(m_K^2 - m_\pi^2\right)^2 + 8\frac{t}{F_\pi^4}\left(2C_{12}^r + C_{34}^r\right)\left(m_K^2 + m_\pi^2\right)^2$$

$$- \frac{8}{F_\pi^4} t^2 C_{12}^r + \overline{\Delta}(t) + \Delta(0)$$

⇨ Only 2 LECs C12 and C34 ! Can be determined by the slope and curvature.

- The expression of $f_+(0)$ at 2 loops in the isospin limit:

$$f_+(0) = 1 + \Delta(0) - \frac{8}{F_\pi^4}\left(C_{12}^r + C_{34}^r\right)\Delta_{K\pi}^2$$

4. Dispersive Representation of the Kπ scalar form factor

[Bernard, Oertel, P. and Stern'06]

- Dispersive parametrization twice subtracted at t=0 and t=$\Delta_{K\pi}$

$$\bar{f}_0(t) \equiv \frac{f_0^{K^0\pi^-}(t)}{f_+^{K^0\pi^-}(0)} = \exp\left[\frac{t}{\Delta_{K\pi}}\left(\ln C - G(t)\right)\right]$$

Phase of the ff

$$\text{with} \qquad G(t) = \frac{\Delta_{K\pi}(\Delta_{K\pi} - t)}{\pi} \int_{t_{\pi K}}^{\infty} \frac{ds}{s} \frac{\phi(s)}{(s-\Delta_{K\pi})(s-t)}$$

- Allow to measure C:

  Remember:

$$\boxed{C = \bar{f}_0(\Delta_{K\pi}) = \frac{F_K}{F_\pi}\frac{1}{f_+(0)} + \Delta_{CT}} \quad \Leftrightarrow \quad \ln C = 0.2188(35) + \underbrace{2\left(\varepsilon_S - \varepsilon_{NS}\right) + \Delta_{CT}/B_{\exp}}_{\Delta\varepsilon}$$

Depend only one 1 RHCs combination : 2 (ε_S-ε_{NS})

- Slope and curvature of the form factor: $\overline{f}_0(t) = 1 + \lambda_0 \frac{t}{m_\pi^2} + \frac{1}{2}\lambda_0' \left(\frac{t}{m_\pi^2}\right)^2 + \dots$

- ϕ(s): $s < \Lambda : \ \phi(s) = \delta^{s,\frac{1}{2}}_{\pi,K}(t)$ [Watson theorem]

  $s > \Lambda : \ \phi(s) = \pi \pm \pi$

End of elastic region

$$\lambda_0 = \frac{m_\pi^2}{\Delta_{K\pi}}\left(\ln C - G(0)\right)$$

and

$$\lambda_0' = \lambda_0^2 - 2\frac{m_\pi^4}{\Delta_{K\pi}} G'(0)$$
$$= \lambda_0^2 + \left(4.16 \pm 0.50\right) \text{x } 10^{-4}$$

$$G(0) = 0.0398(40)$$

5. Matching

$$\widetilde{f}_0(t) = 1 - \frac{8}{F_\pi^4}\left(C_{12}^r + C_{34}^r\right)\left(m_K^2 - m_\pi^2\right)^2 + 8\frac{t}{F_\pi^4}\left(2C_{12}^r + C_{34}^r\right)\left(m_K^2 + m_\pi^2\right)^2$$

$$- \frac{8}{F_\pi^4} t^2 C_{12}^r + \overline{\Delta}(t) + \Delta(0)$$

- Taking the derivative:

$$\Longrightarrow \quad \lambda_0 f_+(0) = \frac{m_\pi^2}{\Delta_{K\pi}}\left(\frac{F_K}{F_\pi} - 1\right) + \frac{8 m_\pi^2 \Sigma_{K\pi}}{F_\pi^4}\left(2C_{12}^r + C_{34}^r\right) + m_\pi^2 \overline{\Delta}'(0)$$

- And derivate 2 times:

$$\Longrightarrow \quad \lambda_0' f_+(0) = -\frac{16 m_\pi^4}{F_\pi^4} C_{12}^r + m_\pi^4 \overline{\Delta}''(0) \qquad (1)$$

- Combine with the two loop result for f$_+$(0)

$$\Longrightarrow \quad f_+(0) = 1 + \Delta(0) - \frac{8}{F_\pi^4}\left(C_{12}^r + C_{34}^r\right)\Delta_{K\pi}^2 \qquad (2)$$

- From (1)+(2) $\Longrightarrow$ $2C_{12}^r + C_{34}^r$ $\Longrightarrow$ $\lambda_0 = f\left(\frac{F_K}{F_\pi}, f_+(0), \lambda_0'\right)$

- Use dispersive representation

$$\lambda_0^{'} = \lambda_0^2 \;-\; 2\frac{m_\pi^4}{\Delta_{K\pi}} G^{'}(0)$$

$$= \lambda_0^2 + \left(4.16 \pm 0.50\right) \text{x } 10^{-4}$$

+ Rewrite $f_+(0) \rightarrow \widehat{f}_+(0)$ and $\frac{F_K}{F_\pi} \rightarrow \frac{\widehat{F}_K}{\widehat{F}_\pi}$

$$\Rightarrow \quad \lambda_0 = -\frac{m_\pi^2}{\Sigma_{K\pi}}\left(1 - \sqrt{1 - 2\frac{\Sigma_{K\pi}^2}{\Delta_{K\pi}}\left(\frac{Y}{\Delta_{K\pi}} - G\,'(0)\right)}\right)$$

with

$$Y = 1 - \frac{\Delta_{K\pi}}{\Sigma_{K\pi}}\frac{\widehat{F}_K}{\widehat{F}_\pi}\frac{1}{\widehat{f}_+(0)}\left(1 + 2(\varepsilon_S - \varepsilon_{NS})\right)$$

$$- \frac{1}{\widehat{f}_+(0)}\left(1 + \Delta(0) + \frac{\Delta_{K\pi}^2}{2}\overline{\Delta}\,''(0) - \frac{\Delta_{K\pi}}{\Sigma_{K\pi}}\left(1 - \Delta_{K\pi}\overline{\Delta}\,'(0)\right)\right)\left(1 + \varepsilon_S - \varepsilon_{NS}\right)\sqrt{1 + \frac{2(\delta + \varepsilon_{NS})}{\sin^2\hat{\theta}}}$$

Depend on 2 combinations of NP parameters contrary to InC.

- What do we have ? « Input Parameters »

  ε_S-ε_{NS} and δ+ ε_{NS} ⟺ α and β which parametrize our ignorance of F_K/F_π, and $f_+(0)$.

- What do we obtain ?
  - F_K/F_π
  - $f_+(0)$
  - λ_0

- And then

$$C_{12}^r = \frac{F_\pi^4}{16}\left(-\frac{\lambda_0' f_+(0)}{m_\pi^4} + \bar{\Delta}''(0)\right)$$

and

$$C_{34}^r = \frac{F_\pi^4}{8\Delta_{K\pi}^2}\left(1+\Delta(0)-f_+(0)\right)-C_{12}^r$$

$$\Delta_{CT} = \mathrm{B}_{\exp}\left(\frac{\Delta_{K\pi}}{m_\pi^2}\lambda_0 + G(0) - \ln B_{\exp} - 2\left(\varepsilon_S - \varepsilon_{NS}\right)\right)$$

with $B_{\exp} = 1.2446(41)$

5. Results and Conclusion

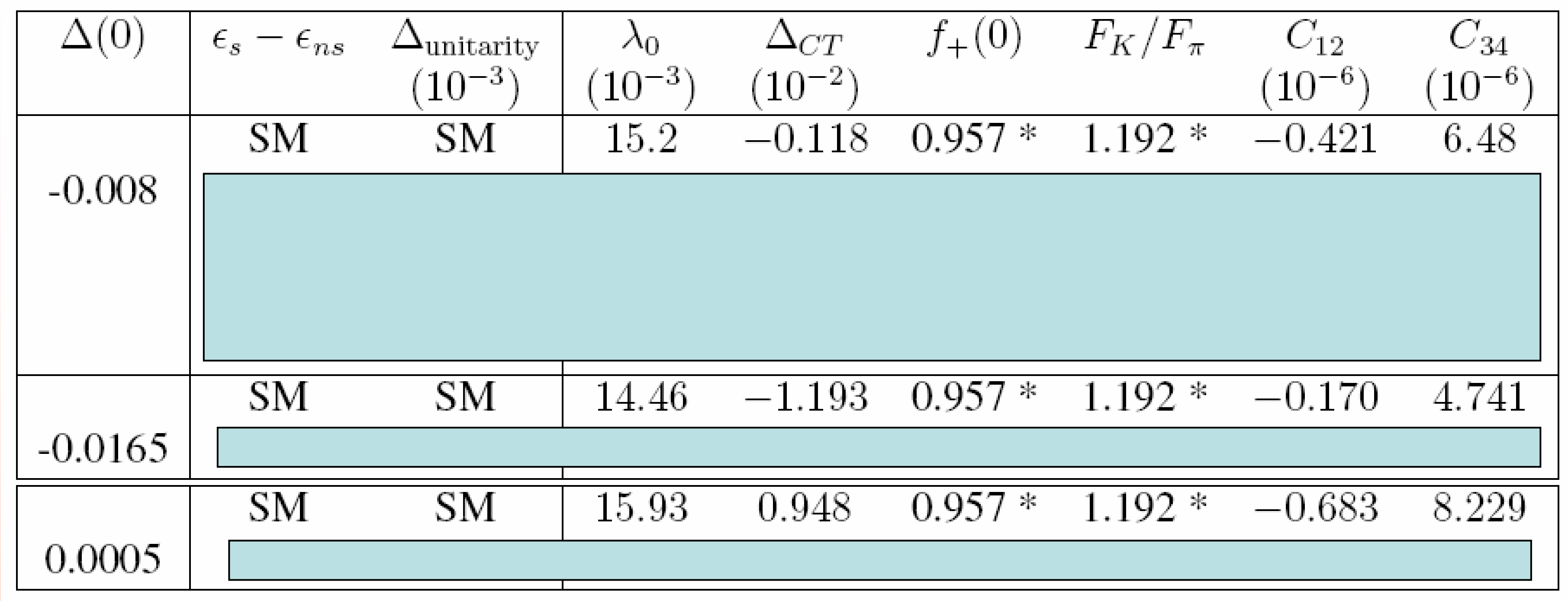
5.1 Results

- We will present trends and not exact results: use of $\Delta(0)$ and $\bar{\Delta}(t)$ from **[Bijnens&Talavera] determined with** F_K/F_π **=1.22 and** F_π **=92.4 MeV**

$\Rightarrow$ **Redo the fit varying** F_K/F_π **and** F_π**.**

- **We vary** $\Delta(0)$ in its error bars, give the largest uncertainty.

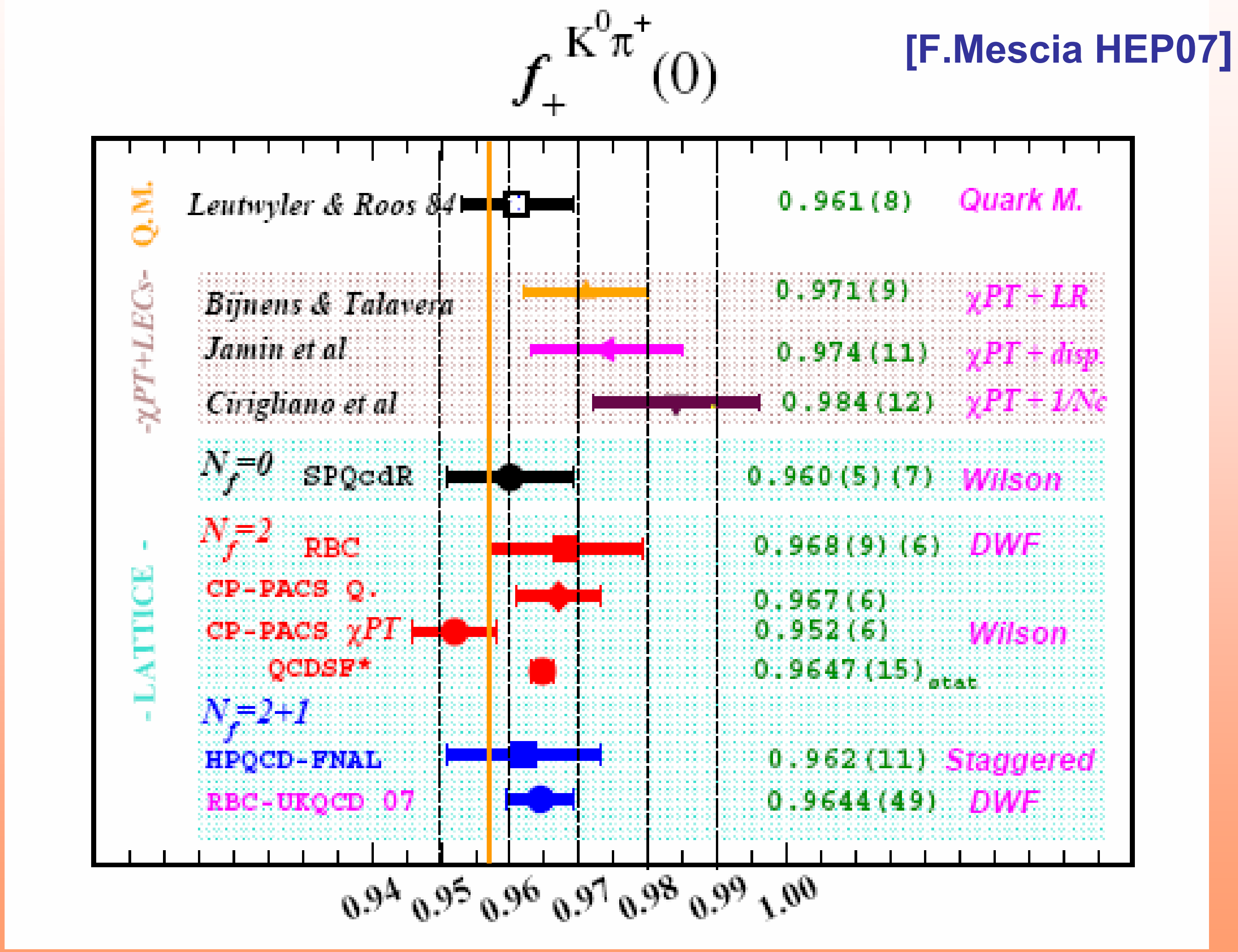
1) SM case



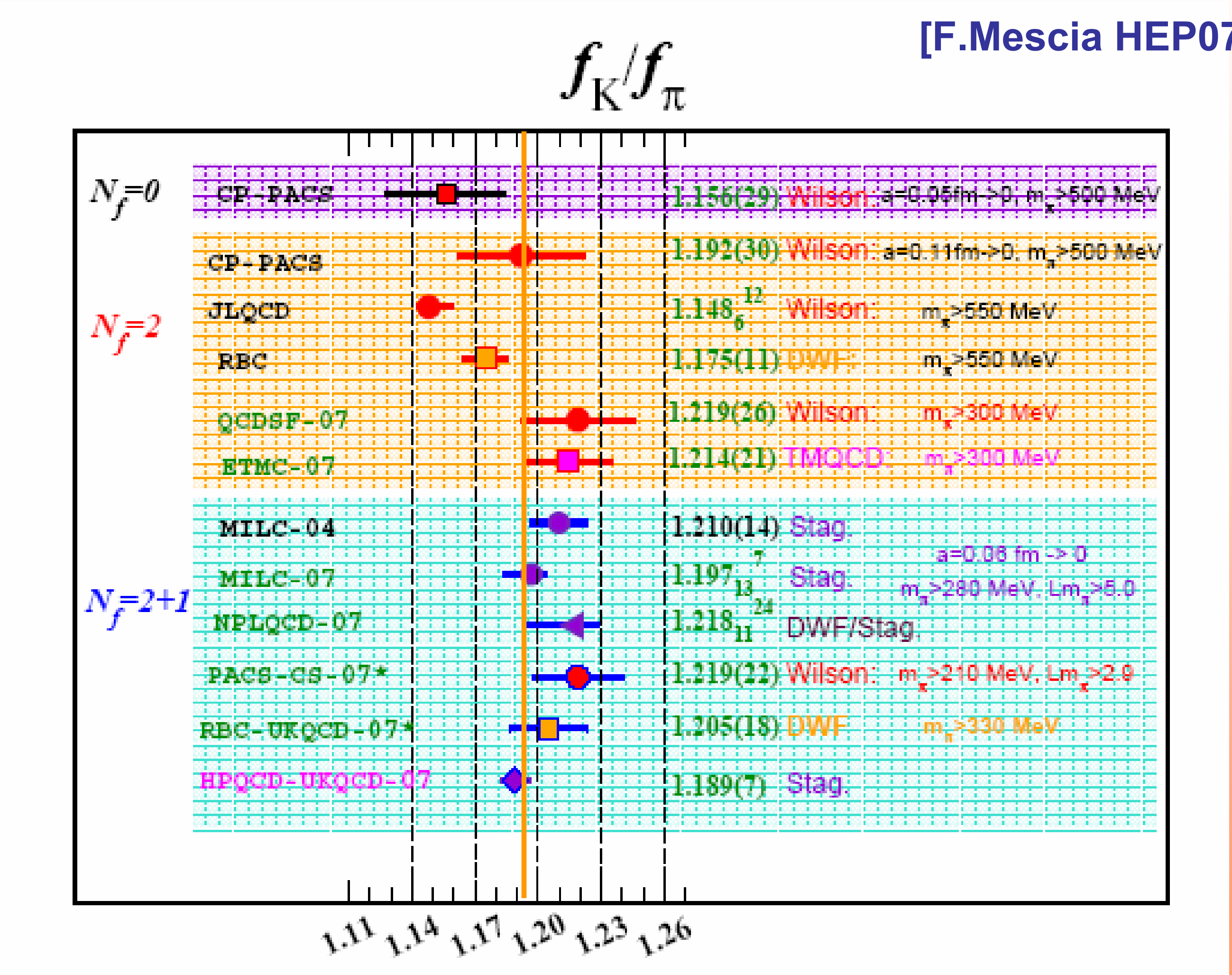
$\Delta(0)$	$\epsilon_s - \epsilon_{ns}$	$\Delta_{\text{unitarity}}$ (10^{-3})	λ_0 (10^{-3})	Δ_{CT} (10^{-2})	$f_+(0)$	F_K/F_π	C_{12} (10^{-6})	C_{34} (10^{-6})
-0.008	SM	SM	15.2	-0.118	0.957 *	1.192 *	-0.421	6.48
-0.0165	SM	SM	14.46	-1.193	0.957 *	1.192 *	-0.170	4.741
0.0005	SM	SM	15.93	0.948	0.957 *	1.192 *	-0.683	8.229

- F_K/F_π and $f_+(0)$ in agreement with the lattice results: on the lower side of the lattice results for F_K/F_π.
- $f_+(0) = 0.961(8)$ **[Leutwyler&Roos]**
- $f_+(0) = 0.984(12)$ **[Cirigliano et al]**

Lattice Results



Lattice Results



[F.Mescia HEP07]

$\Delta(0)$	$\epsilon_s - \epsilon_{ns}$	$\Delta_{\text{unitarity}}$ (10^{-3})	λ_0 (10^{-3})	Δ_{CT} (10^{-2})	$f_+(0)$	F_K/F_π	C_{12} (10^{-6})	C_{34} (10^{-6})
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- RChPT models: $C_{12} \sim -1\text{x}10^{-5}$ for M_s~980MeV (a_0)
  For $1GeV \leq M_s \leq 1.5GeV$ ⇨ $-9\ 10^{-6} \leq C_{12} \leq -1.8\ 10^{-6}$
  Evolving to the ρ scale ⇨ $-7.8\ 10^{-6} \leq C_{12} \leq 4\ 10^{-6}$

- $C_{12}=(0.3\pm5.4)\text{x}10^{-7}$, $\lambda_0=0.0157(1)$ with c.v. $f_+(0)=0.976$ **[Jamin, Oller&Pich]**
  $\lambda_0=0.0147(4)$ with $f_+(0)=0.972(12)$ and $F_K/F_\pi=1.203(16)$

- λ_0 on the large side of experimental results

$\Delta(0)$	$\epsilon_s - \epsilon_{ns}$	$\Delta_{\text{unitarity}}$ (10^{-3})	λ_0 (10^{-3})	Δ_{CT} (10^{-2})	$f_+(0)$	F_K/F_π	C_{12} (10^{-6})	C_{34} (10^{-6})
-0.008	SM	SM	15.2	-0.118	0.957 *	1.192 *	-0.421	6.48
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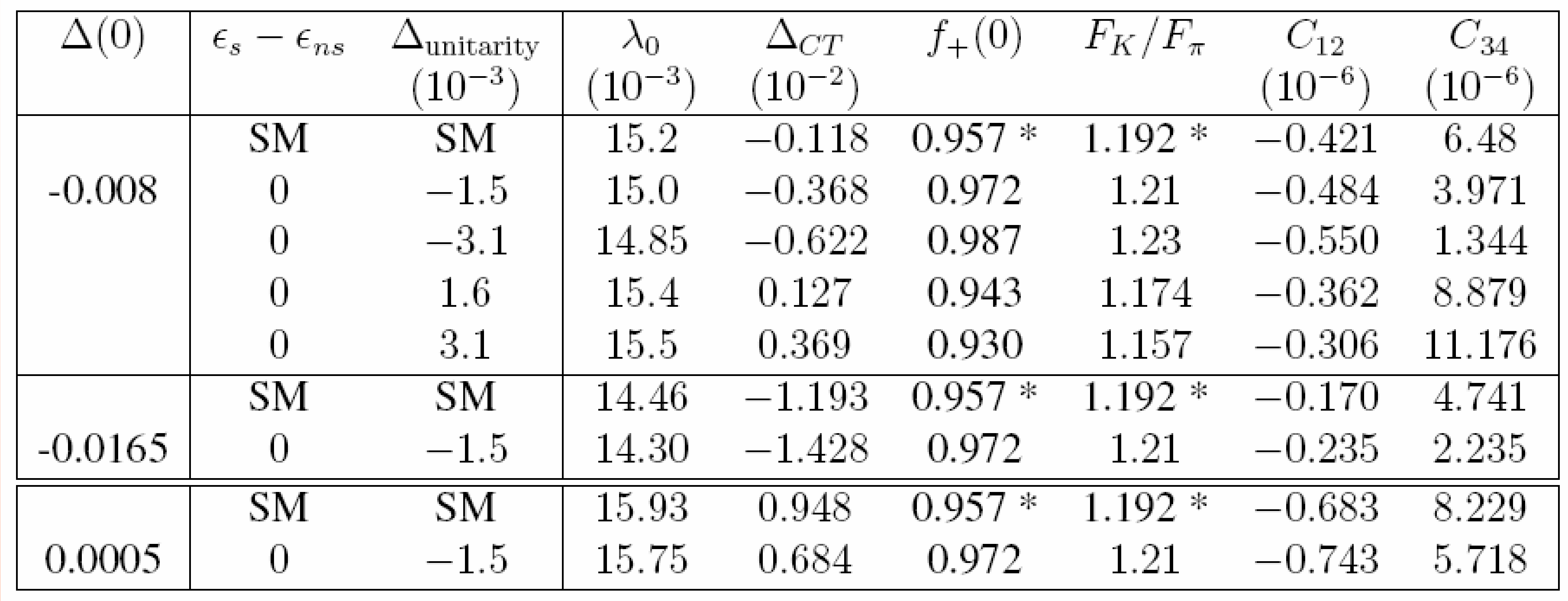
0.0005	SM	SM	15.93	0.948	0.957 *	1.192 *	-0.683	8.229

- Small value of Δ_{CT} in agreement with the NLO result $\Delta_{CT}^{NLO} = (-3.5 \pm 8)\,\mathrm{x}10^{-3}$
  One recovers $\Delta_{CT}^{loops} = -6.2\mathrm{x}10^{-3}$ **[Bijnens&Ghorbani'07]**

- Rather large variations of C_i Δ_{CT} and λ_0 with $\Delta(0)$
  ⇨ large uncertainties

2) Breaking of $\Delta_{\text{unitarity}}$ and ε_S-ε_{NS}=0



$\Delta(0)$	$\epsilon_s - \epsilon_{ns}$	$\Delta_{\text{unitarity}}$ (10^{-3})	λ_0 (10^{-3})	Δ_{CT} (10^{-2})	$f_+(0)$	F_K/F_π	C_{12} (10^{-6})	C_{34} (10^{-6})
-0.008	SM	SM	15.2	−0.118	0.957 *	1.192 *	−0.421	6.48
	0	−1.5	15.0	−0.368	0.972	1.21	−0.484	3.971
	0	−3.1	14.85	−0.622	0.987	1.23	−0.550	1.344
	0	1.6	15.4	0.127	0.943	1.174	−0.362	8.879
	0	3.1	15.5	0.369	0.930	1.157	−0.306	11.176
-0.0165	SM	SM	14.46	−1.193	0.957 *	1.192 *	−0.170	4.741
	0	−1.5	14.30	−1.428	0.972	1.21	−0.235	2.235
0.0005	SM	SM	15.93	0.948	0.957 *	1.192 *	−0.683	8.229
	0	−1.5	15.75	0.684	0.972	1.21	−0.743	5.718

- When $\Delta_{\text{unitarity}}$ increases, λ_0 , Δ_{CT} , C_{12} and F_K/F_π decrease whereas C_{34} , and $f_+(0)$ increase.

$$\Delta_{\text{unitarity}} = \sin^2\hat{\theta}\left(\left|\frac{\widehat{f}_+^{K^0\pi^-}(0)}{f_+^{K^0\pi^-}(0)}\right|^2 - 1\right)$$

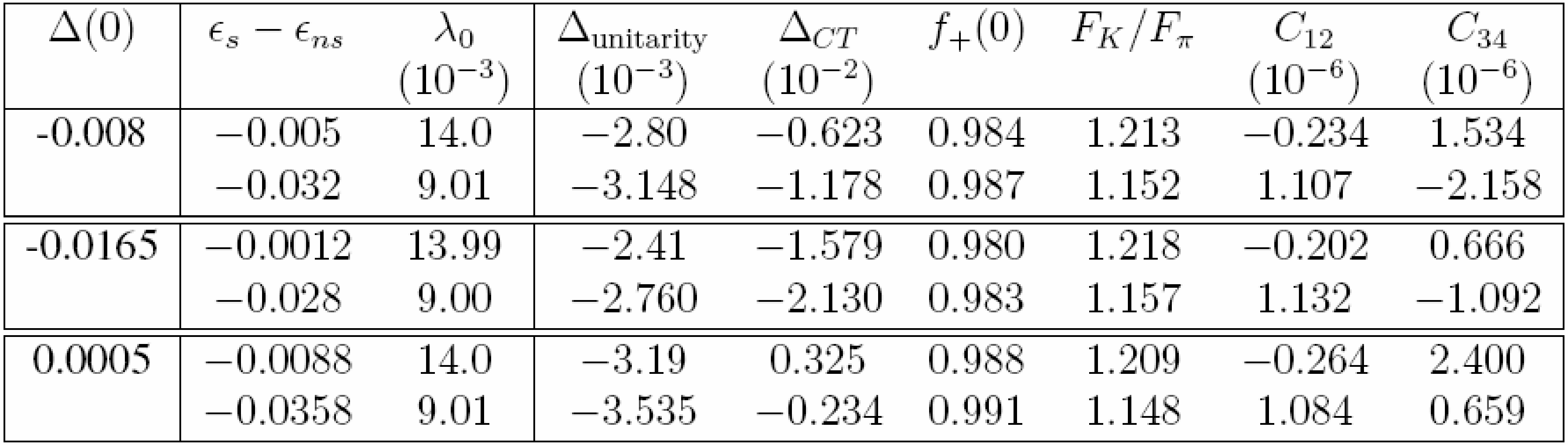
Lattice variation for $\Delta_{\text{unitarity}}$

$\Rightarrow$ $\boxed{0.0148 \le \lambda_0 \le 0.0154}$

3) Allow for physics beyond the SM: use experimental knowledge of λ_0 and $\Delta\varepsilon$ obtained with dispersive fit.

$$\ln C = 0.2188(35) + \underbrace{2\left(\varepsilon_S - \varepsilon_{NS}\right) + \Delta_{CT} / B_{\exp}}_{\Delta\,\varepsilon}$$

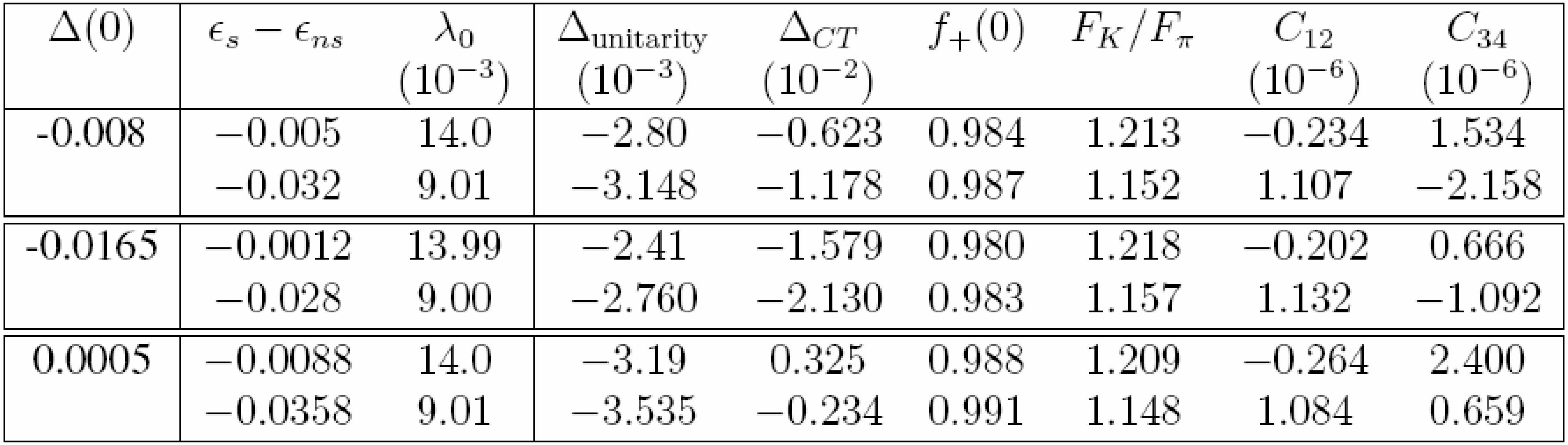
- NA48 $\lambda_0 = (8.88 \pm 1.24)\text{x}10^{-3}$ and $\Delta\varepsilon = -0.0075(14)$ **[Phys.Letter. B.647]**
- KLOE $\lambda_0 = (14.0 \pm 2.10)\text{x}10^{-3}$ and $\Delta\varepsilon = -0.0015(25)$ **[hep-ex/0710.4470]**



$\Delta(0)$	$\epsilon_s - \epsilon_{ns}$	λ_0 (10^{-3})	$\Delta_{\text{unitarity}}$ (10^{-3})	Δ_{CT} (10^{-2})	$f_+(0)$	F_K/F_π	C_{12} (10^{-6})	C_{34} (10^{-6})
-0.008	-0.005	14.0	-2.80	-0.623	0.984	1.213	-0.234	1.534
	-0.032	9.01	-3.148	-1.178	0.987	1.152	1.107	-2.158
-0.0165	-0.0012	13.99	-2.41	-1.579	0.980	1.218	-0.202	0.666
	-0.028	9.00	-2.760	-2.130	0.983	1.157	1.132	-1.092
0.0005	-0.0088	14.0	-3.19	0.325	0.988	1.209	-0.264	2.400
	-0.0358	9.01	-3.535	-0.234	0.991	1.148	1.084	0.659

3) Allow for physics beyond the SM: use experimental knowledge of λ_0 and $\Delta\varepsilon$

- NA48 $\lambda_0 = (8.88 \pm 1.24)\mathrm{x}10^{-3}$ and $\Delta\varepsilon = -0.0075(14)$ [Phys.Letter. B.647]



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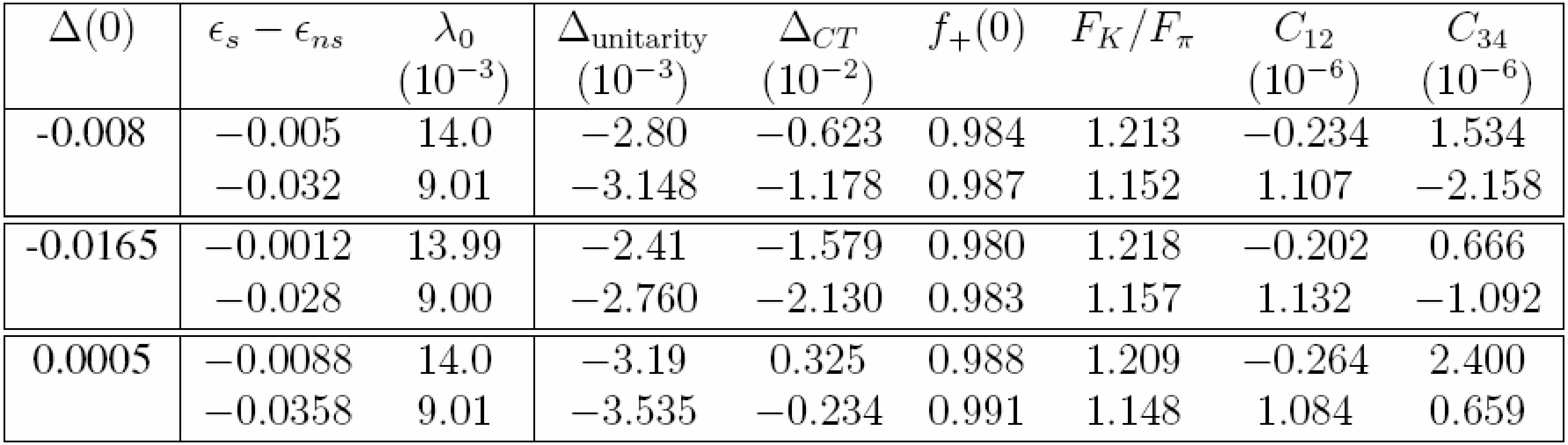
- F_K/F_π rather small
- Large contribution from RHCs
- Δ_{CT} on the large side of the NLO result,
- C_{34} becomes negative, C_{12} is positive.

3) Allow for physics beyond the SM: use experimental knowledge of λ_0 and $\Delta\varepsilon$

- KLOE $\lambda_0 = (14.0 \pm 2.10)\text{x}10^{-3}$ and $\Delta\varepsilon = -0.0015$ **[hep-ex/0710.4470]**



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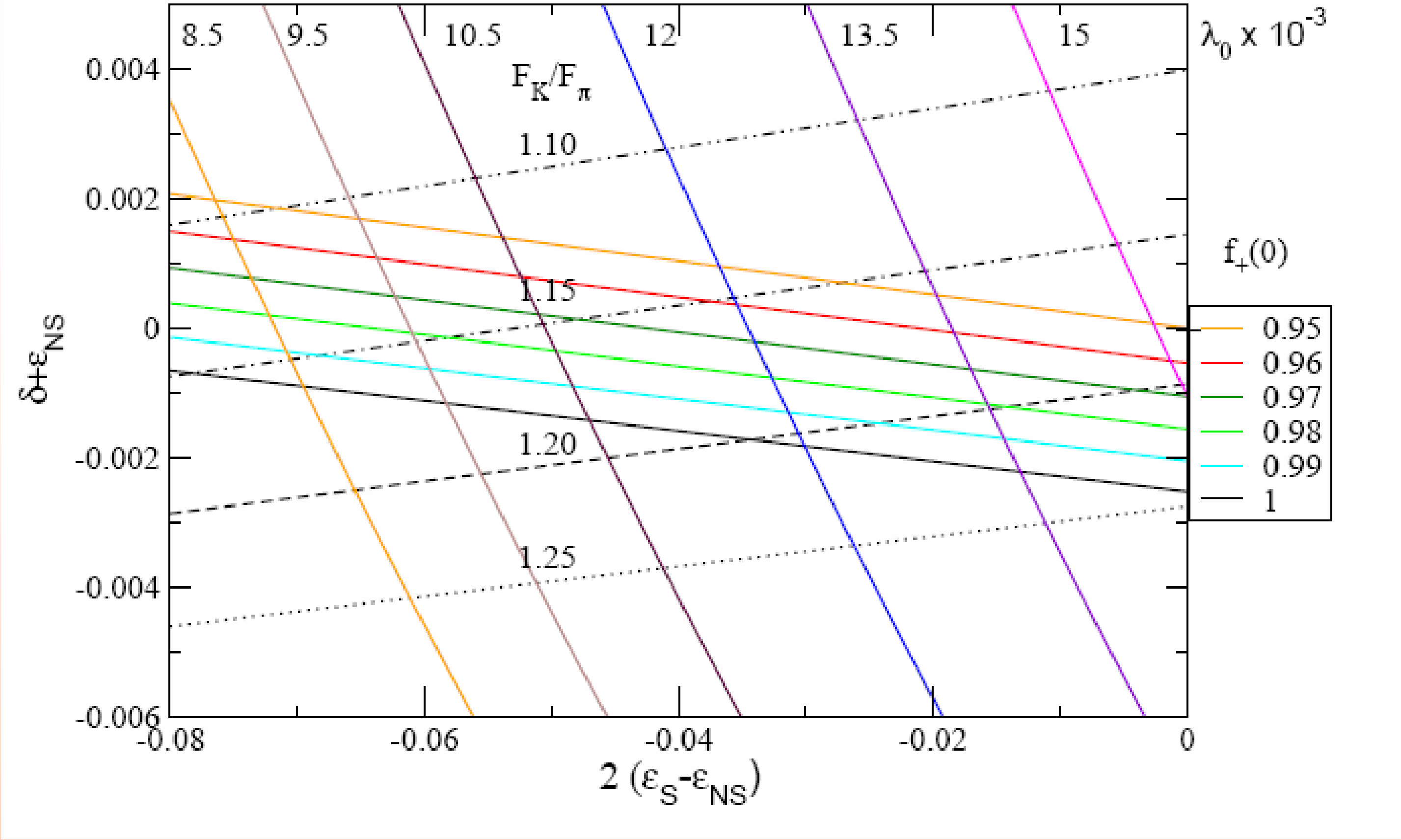
	-0.0358	9.01	-3.535	-0.234	0.991	1.148	1.084	0.659

- Results for F_K/F_π and $f_+(0)$ compatible with lattice results, small RHCs for the central value.



Attention: Small uncertainties on F_K/F_π and $f_+(0)$ purely experimental but large uncertainties on λ_0 from $\Delta(0)$! Here, $\Delta(0) = -0.008$.

5.2 Conclusion and outlook

- Matching the K_{l3} two loop computation + experimental results using dispersive representation offer the opportunity to determine $f_+(0)$, C_{12}, C_{34}, F_K/F_π , Δ_{CT}.

- Uncertainties too large at the moment to extract these quantities, need of
  - more precise and consistent fits
  - more precise lattice determinations
  - more precise scalar form factor measurements

- KLOE result consistent with lattice computations of $f_+(0)$ and $F_K/F_{\pi.}$
- NA48 result needs smaller value of F_K/F_π than thought.

- At present the experimental results but also the lattice results do not exclude Physics beyond the SM in K_{l3} decays.

- Outlook: -systematic study of uncertainties
  -consistent matching.
  - include IB