SO(10) SUSY GUTs with family symmetries: the test of FCNCs

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Outline

偏差
The DR Model: an SO(10) SUSY GUT with $D_3$ family symmetry

- Top-down approach to the MSSM+$\nu$
- Successful fit to quark & lepton masses, CKM & PMNS matrices

偏差
Including FCNCs: the only way to test the pattern of SUSY particles’ masses & mixings predicted by the model

偏差
Details on the analysis: global fit to low-energy observables, FCNCs directly in the $\chi^2$ function
The model by Dermíšek & Raby (DR) is

- an SO(10) SUSY GUT
- with an additional $D_3 \times [U(1) \times Z_2 \times Z_3]$ family symmetry
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Why SUSY GUTs

Tantalizing unification of LEP-measured SM couplings

🤔 after MSSM running to high energies.
Maybe just a coincidence. Maybe not.
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Why SO(10)

- Complete quark-lepton unification: single 16 representation for each family
- Natural inclusion of $\nu_R$ in each 16. See-saw mechanism easily incorporated.
- Can explain the pattern of quark/lepton masses and mixings, through family symmetries or (few) extra fermion multiplets

D. Guadagnoli, Euroflavour07, November 14 – 16, 2007
Global symmetries: are believed *not* to arise in string theory

Local (i.e. gauge) symmetries: typically enhance FCNCs

Discrete symmetries can arise e.g.

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- directly from compactifications in string theory
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Family symmetries: “isospin” example

1. We know that in SO(10) the $16_i$ contains the fermions of the $i$-th generation
   Let $\{16_1, 16_2\}$ transform as an isospin doublet and $16_3$ as a singlet

2. Then let us introduce:
   - “flavon” fields $\phi$: transform under the “family isospin”, are SO(10) singlets
   - Froggatt-Nielsen fields $\chi$: transform under the “family isospin”, are 16’s of SO(10)
Now one can build up the following interactions:

\[
\begin{align*}
&16_{1,2} \cdot [\text{Higgs}] \cdot \chi, \\
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Yukawa unification only for 3\textsuperscript{rd} generation fermions
Now one can build up the following interactions:

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One then **breaks spontaneously** the family symmetry through \( \langle \phi \rangle = v_{\text{ev}} \) and **integrates out** the \( \chi \), with \( M_\chi \approx \text{GUT scale} \).

Mass terms for, say, quarks are generated through diagrams like:

\[
\bar{q} M_q q \propto \bar{q} \frac{\langle \phi \rangle}{M_\chi} \langle \text{Higgs} \rangle q
\]

- FN states implement a sort of “see-saw” mechanism to make quark masses \( \ll \) GUT scale
- Quark mass hierarchies are understood in terms of the *sequential* breaking of the family symmetry through \( \langle \phi \rangle = v_{\text{ev}} \)
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\langle \phi \rangle \quad \langle \text{Higgs} \rangle
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\[
16_{i} \quad \bar{\chi} \quad \chi \quad 16_{i}
\]

\( \bar{q} M_{q} q \propto \bar{q} \frac{\langle \phi \rangle}{M_{\chi}} \langle \text{Higgs} \rangle q \)

The DR model realizes the above mechanism with the smallest (non-Abelian) **discrete analogue** of “isospin”, i.e. the \( D_{3} \) group.

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- Quark mass hierarchies are understood in terms of the **sequential** breaking of the family symmetry through \( \langle \phi \rangle = v_{\text{ev}} \)

With 11 family-symmetry (real) parameters, it successfully describes quark & lepton masses and mixings.

With a total of 24 parameters (less than in the SM+\( \nu \)) the whole MSSM+\( \nu \) parameter space is fixed.

*D.Guadagnoli, Euroflavour07, November 14 – 16, 2007*
It is worthwhile to perform a more detailed analysis of the DR model.

The aim is to test the SUSY mass spectrum and mixings predicted by the model.

The SUSY spectrum will affect flavor-changing neutral current (FCNC) processes.

Strategy: perform a global fit to the model parameters, including FCNCs among the observables in the fit.

Albrecht, Altmannshofer, Buras, D.G., Straub, JHEP ’07
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- coupling and unified scale: $\alpha_G, M_G$
- soft SUSY-breaking params at $M_G$
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- right-handed neutrino masses $M_{Ri}$
- initial conditions
- define “see-saw” scale
- integrate out RH $\nu$
- Y’s, soft SUSY pars, $\alpha_i, M_{Ri}$
- MSSM+RH $\nu$ RGEs

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- right-handed neutrino masses $M_{Ri}$
- $\mu$-term and $\tan\beta$ at the EW scale

\[ Y's, \text{ soft SUSY pars, } \alpha_i, M_{Ri} \]

\[ \text{MSSM+RH } \nu \text{ RGEs} \]

\[ \text{integrate out RH } \nu \]

\[ \text{MSSM RGEs} \]

\[ \text{integrate out SUSY} \]

\[ \text{SM RGEs} \]

\[ \text{enter EWSB} \]

\[ \text{define "see-saw" scale} \]

\[ \text{initial conditions} \]

\[ \text{define "see-saw" scale} \]

\[ \text{EW scale} \]

\[ \text{Compute observables} \]

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We test the model, through the following observables $O_i$:

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**lepton masses**
- $M_\tau$
- $M_\mu$
- $M_e$
- $\Delta m_{31}^2$
- $\Delta m_{21}^2$

**CKM & PMNS**
- $|V_{us}|$
- $|V_{ub}|$
- $|V_{cb}|$
- $\sin 2\beta$
- $\sin^2 \theta_{12}$
- $\sin^2 \theta_{23}$
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**FCNCs**
- $\epsilon_K$
- $\text{BR}[B_s \rightarrow \mu^+ \mu^-]$
- $\text{BR}[B \rightarrow X_s \gamma]$
- $\text{BR}[\bar{B} \rightarrow X_s l^+ l^-]$
- $\text{BR}[B_u \rightarrow \tau \nu]$
- $\Delta M_s/\Delta M_d$
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  - $\Delta M_s / \Delta M_d$

These observables $O_i$ enter a $\chi^2$ function, defined as

$$\chi^2[\text{model pars}] \equiv \sum_{i=1}^{N_{\text{obs}}} \frac{[f_i[\text{model pars}]-O_i]^2}{\left(\frac{\sigma_i^2}{2}\right)_{\text{exp}} + \left(\frac{\sigma_i^2}{2}\right)_{\text{theo}}}$$

where $f_i$ is the model prediction for $O_i$.

The $\chi^2$ function is then minimized upon variation of the model parameters.
Detailed chart of the fitting procedure

Set initial conditions:
\{Y's, soft SUSY pars, \alpha_i, M_{Ri}\}

Integrate out \nu_{Ri}

Get all MSSM pars:
\{Y's, soft SUSY pars, \alpha_i, M_y\}

Compute:
- SUSY spectrum
- EWSB conditions
  [including one-loop corr. to masses and mixings]

Compute:
- FCNC observables

Variation of:
- textures
- soft SUSY pars
  [all but m_{Hu}, m_{Hd}]
- \(M_G, \alpha_G, \epsilon_3\)

Evaluate \chi^2 function

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The DR model is characterized by $\tan\beta \approx 50$ because of SO(10). Hence all the FCNC observables need be computed in the MSSM at large $\tan\beta$. 

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FCNCs considered in the analysis:
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**BR [$B_s \rightarrow \mu^+ \mu^-$]**

For large $\tan\beta$ (and sizable $A_t$), dominated by double penguins with neutral Higgses

Enhancement going as:

$$\text{BR} [B_s \rightarrow \mu^+ \mu^-] \propto A_t^2 \frac{\tan^6 \beta}{M_A^4}$$

(Old) upper bound from CDF

$$\text{BR} [B_s \rightarrow \mu^+ \mu^-]_{\exp} < 1.0 \times 10^{-7}$$

$M_A > 450$ GeV

Generic bound valid for all the heavy Higgs masses

**$\Delta M_s$**

Again, double penguin dominance

Suppression going as:

$$\Delta M_s \propto -\frac{m_b m_s}{M_W^2} A_t^2 \frac{\tan^4 \beta}{M_A^2}$$

Within the DR model, typical corrections to $\Delta M_s$ do not exceed $-5\%$

D. Guadagnoli, Euroflavour07, November 14 – 16, 2007
\[ \text{BR}[\bar{B} \to X_s \gamma] = (3.55 \pm 0.26) \times 10^{-4} \quad \{ \text{HFAG average} \} \]

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The theory prediction for \( b \to s \gamma \) must be “SM-like”
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Very rough \( b \rightarrow s \gamma \) formula

\[ \Gamma \left[ B \rightarrow X_s \gamma \right] \approx \frac{G_F^2 \alpha_{\text{e.m.}}}{32 \pi^4} \left| V_{ts}^* V_{tb} \right|^2 m_b^5 \left| C_7^\text{eff} (\mu_b) \right|^2 + \ldots \]

with \( C_7^\text{eff} (\mu_b) = C_7^\text{eff} (\mu_b) + C_7,\text{DR} (\mu_b) \)

Subleading corr’s & contrib’s negligible in the DR model

In order to end up with a “SM-like” prediction, one must have either

\[ C_{7,\text{DR}} (\mu_b) \ll C_{7,\text{SM}}^\text{eff} (\mu_b) \]

or

\[ C_{7,\text{DR}} (\mu_b) \approx -2 C_{7,\text{SM}}^\text{eff} (\mu_b) \]

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**Notes**

- This solution is highly conspired
- SUSY is here *not* a “correction” to the SM result
- The theoretical control on the SUSY part should then be at least as good as in the SM
- In absence of it, we find e.g. a *strong* sensitivity to the SUSY matching scale

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Within the DR model, dominant NP contributions are from charginos and Higgses. Gluinos play a minor role.

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Main features:

- Higgs contrib’s *add up* to the SM ones. However, Higgs contrib’s are made small by the lower bound on \( M_A \) placed by \( B_s \to \mu^+\mu^- \).
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In the DR model, chargino contrib's can be very large. As a matter of fact:

\[ \mu \cdot A_t < 0 \quad \text{"prefers" the fine-tuned case:} \]

---

D. Guadagnoli, Euroflavour07, November 14 – 16, 2007
At variance with $b \to s\gamma$, the decay $b \to s l^+ l^-$ is sensitive to the sign of $C_7^\text{eff}$

$$
\frac{d\Gamma[\bar{B} \to X_s l^+ l^-]}{d\hat{s}} \propto (1 + 2\hat{s}) \left( |\tilde{C}_9^\text{eff}(\hat{s})|^2 + |\tilde{C}_{10}^\text{eff}(\hat{s})|^2 \right) + 4 \left( 1 + \frac{2}{\hat{s}} \right) |C_7^\text{eff}|^2 + 12 C_7^\text{eff} \text{Re} \left( \tilde{C}_9^\text{eff}(\hat{s}) \right)
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\[ \hat{s} = (p_\mu + p_\nu)^2 / m_b^2 \]
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\[ \hat{s} = (p_{\mu} + p_{\mu})^2 / m_b^2 \]

BaBar & Belle average

\[ 10^6 \times \text{BR}[\bar{B} \to X_s l^+ l^-]_{\text{exp}} = 1.60 \pm 0.51 \]

SM

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The sign of $C_7^{\text{eff}}$ is the same as in the SM, unless $C_9$ and $C_{10}$ are significantly affected by NP.

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\[\hat{s} = (p_\mu^0 + p_\mu^0)^2/m_b^2\]

\[
10^6 \times \text{BR}[\bar{B} \to X_s l^+ l^-]_{\text{exp}}^{\text{low} - \hat{s}} = 1.60 \pm 0.51 \quad \text{SM} \quad C_7^{\text{eff}} \to -C_7^{\text{eff}}
\]

BaBar & Belle average

Maximal ranges within the MFV MSSM (to which the low-energy DR model belongs)

Gambino, Haisch, Misiak, PRL '05

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Ali, Lunghi, Greub, Hiller, PRD '02

The sign of $C_7^{\text{eff}}$ is the same as in the SM, unless $C_9$ and $C_{10}$ are significantly affected by NP

D. Guadagnoli, Euroflavour07, November 14 – 16, 2007
Some “reference” fits

univ. scalar term \( m_{16} = 4 \text{ TeV} \),
univ. trilinear \( A_0 \approx -2 m_{16} \),
\( \mu > 0 \)

The solution \( A_0 \approx -2 m_{16} \) helps
3rd generation Yukawa unification

To fit \( m_b \), a cancellation between these terms is required: \( |A_t| > m_{\tilde{g}} \)

Helped by \( A_0 \approx -2 m_{16} \)

\[
m_b(M_Z) = m_b^{(0)} \left[ 1 + \mu \tan \beta \left( \frac{2\alpha_s m_{\tilde{g}}}{3\pi m_b^2} + \frac{\lambda_i^2}{16\pi^2 m_t^3} A_t \right) + \Delta m_b^{\log} \right]
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Blazek, Dermisek, Raby, PRL & PRD (‘02)

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$\Rightarrow A_0 \approx -2 m_{16}$

$m_{\tilde{t}} \ll m_{\tilde{b}}$

Blazek, Dermisek, Raby, PRL & PRD (‘02)

This solution prefers $C_7 = -2 C_7^{SM}$.

Imposing $C_7 > 0$, one has:

$$B_s \rightarrow \mu^+ \mu^- \text{ implies } M_A \geq 450 \text{ GeV}$$

$b \rightarrow s \gamma$ is chargino dominated

D.Guadagnoli, Euroflavour07, November 14 – 16, 2007
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$m_t \ll m_b$

Blazek, Dermisek, Raby, PRL & PRD (‘02)

Only way-out: decoupling

$m_t \geq 2$ TeV

D. Guadagnoli, Euroflavour 07, November 14 – 16, 2007
Some “reference” fits: continued

\[
\begin{aligned}
m_{16} &= 4 \text{ TeV}, \quad \mu \geq 0 \\
|A_0| &\neq 2 m_{16}
\end{aligned}
\]

In this case one has NO inverted mass hierarchy:

One typically finds solutions with both \( m_t, \ m_b \) heavy, and \( A_t \) small

\[
m_b(M_Z) = m_b^{(0)} \left[ 1 + \mu \tan \beta \left( \frac{2 \alpha_s}{3 \pi} \frac{m_\tilde{b}}{m_b^2} + \frac{\lambda_t^2}{16 \pi^2} \frac{A_t}{m_t^2} \right) + \Delta m_b^{\log} \right]
\]

D. Guadagnoli, Euroflavour07, November 14 – 16, 2007
Some “reference” fits: continued

\[
\begin{aligned}
  m_{16} &= 4 \text{ TeV, } \mu \gtrless 0 \\
  |A_0| &\neq 2 m_{16}
\end{aligned}
\]

Easy to fit \( m_b \), since in this case these two terms are both small

\[
m_b(M_Z) = m_b^{(0)} \left[ 1 + \mu \tan \beta \left( \frac{2 \alpha_s m_b}{3 \pi m_b^2} + \frac{\lambda_i^2 A_t}{16 \pi^2 m_i^2} \right) + \Delta m_b^{\log} \right]
\]

In this case one has NO inverted mass hierarchy:

One typically finds solutions with both \( m_t, m_b \) heavy, and \( A_t \) small

Now \( b \to s \gamma \) is OK

\[
\begin{aligned}
  m_t \gtrsim 2.6 \text{ TeV} \\
  M_A \gtrsim 1.5 \text{ TeV}
\end{aligned}
\]

But typical masses are

In this case EWSB finds generically very large masses for heavy Higgses

D. Guadagnoli, Euroflavour07, November 14 – 16, 2007
**The Big Picture**

**Fit strategy:**
choose $m_{16} \in [4, 10] \text{ TeV}$, then let the fit determine the other param’s.

**Dictionary**
$m_{16} =$ univ. soft scalar term
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## Benchmark Cases

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$A_0 \approx -2 m_{16}$ | Inverted mass hierarchy (IMH)  
$C_7 \simeq -C_7^{SM}$ | $\geq 400$ | OK | 3σ too large [Gambino et al.]

*D.Guadagnoli, Euroflavour07, November 14 – 16, 2007*
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| 2. $m_{16} = 4 \text{ TeV}, \mu > 0$
$A_0 \approx -2 m_{16}$ | Like 1, but forcing $C_7 > 0$
$C_7 \simeq 0+$ | $\geq 600$ | $5\sigma$ too low | OK |
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| $m_{16} = 6$ TeV, $\mu > 0$  
$A_0 \approx -2 \, m_{16}$ | Like 2, but try increase $m_{16}$  
$C_7 > 0$ | $\geq 1200$ | $2.3\sigma$ too low | OK |
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Fit strategy: choose $m_{16} \in [4,10] \text{ TeV}$, then let the fit determine the other param’s.

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<td>No IMH: $</td>
<td>A_0</td>
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*D.Guadagnoli, Euroflavour07, November 14 – 16, 2007*
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| **3** $m_{16} = 6$ TeV, $\mu > 0$  
$A_0 \approx -2m_{16}$ | Like **2**, but try increase $m_{16}$  
$C_7 > 0$ | $\geq 1200$ | 2.3$\sigma$ too low | OK |
| **4** $m_{16} = 4$ TeV, $\mu < 0$  
$A_0 \neq -2m_{16}$ | No IMH: $|A_0| < 2m_{16}$  
$\Rightarrow$ small $A_t$  
$C_7 > 0$ | $\geq 2600$ | OK | OK |
| **5** $m_{16} \geq 6$ TeV, $\mu > 0$  
$A_0 \neq -2m_{16}$ | Like **4**, but $\mu > 0$: requires larger $m_{16}$  
$C_7 > 0$ | $\geq 2300$ | OK | OK |

*D.Guadagnoli, Euroflavour07, November 14 – 16, 2007*
BR $[B_u \rightarrow \tau \nu]$: a possible additional problem

The DR model fits successfully quark & lepton masses and mixings. However, among the predictions one gets:

$|V_{ub}^{\text{DR}}| \approx 3.26 \times 10^{-3}$ which is somehow "too small"

The $B_u \rightarrow \tau \nu$ mode is a way to display the problem
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\begin{align*}
\text{In the SM} \\
\text{The } B_u \to \tau \nu \text{ decay is affected by a large error due to the poor knowledge of } \quad F_B & \approx F_{B_s} \\
\text{One considers one of the "Ikado" ratios} \quad & \quad \frac{\text{BR} \left[ B_u \to \tau \nu \right]_{\text{SM}}}{\frac{\Delta M_{d,s}^{\text{SM}}}{\tau_B}} \quad \text{with reduced hadronic uncertainties [mostly } \hat{B}_{B_s} \text{]} 
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\]

Correspondingly, one predicts

\[
10^4 \times \text{BR} \left( B_u \to \tau \nu \right)_{\text{SM}} = \begin{cases} 
0.87 \pm 0.11 , & \text{with} |V_{ub}|_{\text{UTfit}} = 3.66(15) \times 10^{-3} \\
1.31 \pm 0.23 , & \text{with} |V_{ub}|_{\text{incl}} = 4.49(33) \times 10^{-3} 
\end{cases}
\]

versus

\[
10^4 \times \text{BR} \left( B_u \to \tau \nu \right)_{\text{exp}} = 1.31 \pm 0.48 \quad \text{Belle & BaBar average}
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In the DR model

\[ \frac{\text{BR} \left( B_u \rightarrow \tau \nu \right)_{\text{DR}}}{\text{BR} \left( B_u \rightarrow \tau \nu \right)_{\text{SM}}} = \left[ 1 - \frac{m^2_{B^+}}{m^2_{H^+}} \frac{\tan^2 \beta}{1 + \epsilon_0 \tan \beta} \right]^2 \frac{|V_{ub}^{\text{DR}}|^2}{|V_{ub}^{\text{SM}}|^2} \]

\[ \text{suppression:} \]

Hou; Akeroyd+Recksiegel; Isidori+Paradisi

Typical prediction:

\[ 10^4 \times \text{BR} \left( B_u \rightarrow \tau \nu \right)_{\text{DR}} \leq 0.6 \]
Conclusions

The DR model, an SO(10) SUSY GUT with a $D_3$ family symmetry, provides detailed, testable predictions for the low-energy MSSM. It successfully describes quark and lepton masses and mixings.

The DR model is however challenged when probed against the simultaneous description of quark FCNC data. This test is accomplished through a global fit, with FCNCs appearing directly in the $\chi^2$ function.

FCNCs offer a unique probe to the SUSY mass spectrum predicted by the model. They turn out into a discriminating test for the model itself.
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D.Guadagnoli, Euroflavour07, November 14 – 16, 2007
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Work in progress in this direction. See David Straub’s talk.