Light two-particle matrix elements of the S=$-$1 vector current

B.Moussallam (I.P.N Orsay)

Introduction

∗ Remark: Analyticity properties in QCD useful in data analysis (see talks Emilie Passemar, Micaela Oertel)

∗ Discussed here: application to $f_+^{K\pi}(t)$ [to be compared with results on τ decays from Belle, Babar]

∗ Previous work: application to scalar form factors

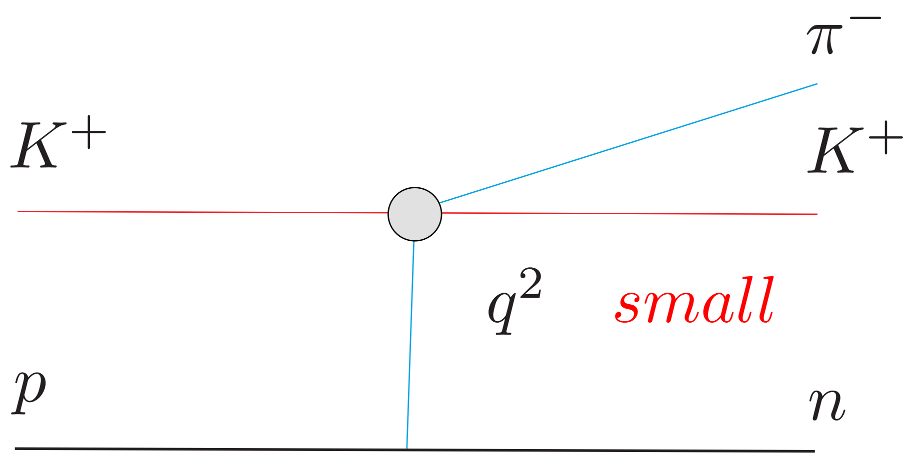
- Donoghue, Gasser, Leutwyler (1990) ($\pi\pi$)
- Jamin, Oller, Pich (2001) (πK)
- Used in ChPT: not tested ?

∗ More specifically:

- Dispersion relations link different experimental results
- Start from πK scattering in P-wave (LASS experiments)
- construct $f_+^{K\pi}(t)$ [Modulo a few additional inputs].

Experimental inputs on πK scattering

* **High statistics** production experiments Estabrooks (1979), Aston (LASS) (1988)



* Complete determination of P-wave $\pi K \to \pi K$ T-matrix element in energy range: $0.9 \leq E \leq 2.5$ GeV

* **Future:** $D \to \pi K l \nu_l$ (FOCUS (2005))

* **Details on inelasticity:** (LASS): $\pi K \to \pi\pi K$, $\pi K \to \pi\pi\pi K$
  - Crucial observation: quasi two-body channels dominate
  - Mainly: $K^*\pi$, ρK
  - Rates in resonance regions:

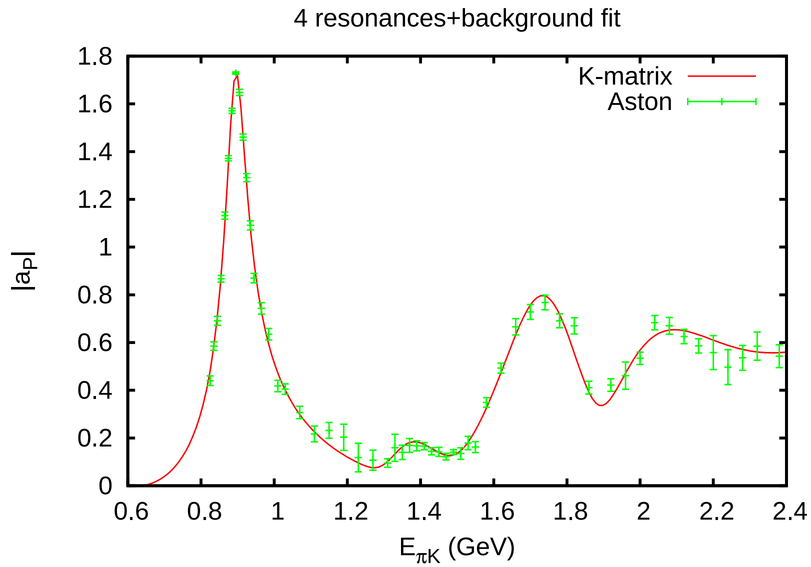
	$K\pi$	$K^*\pi$	$K\rho$
$K^*(892)$	100	0.0	0.0
$K^*(1410)$	6.6 ± 1	> 40	< 7
$K^*(1680)$	38.7 ± 2.5	$29.9^{+2.2}_{-4.7}$	$31.4^{+4.7}_{-2.1}$

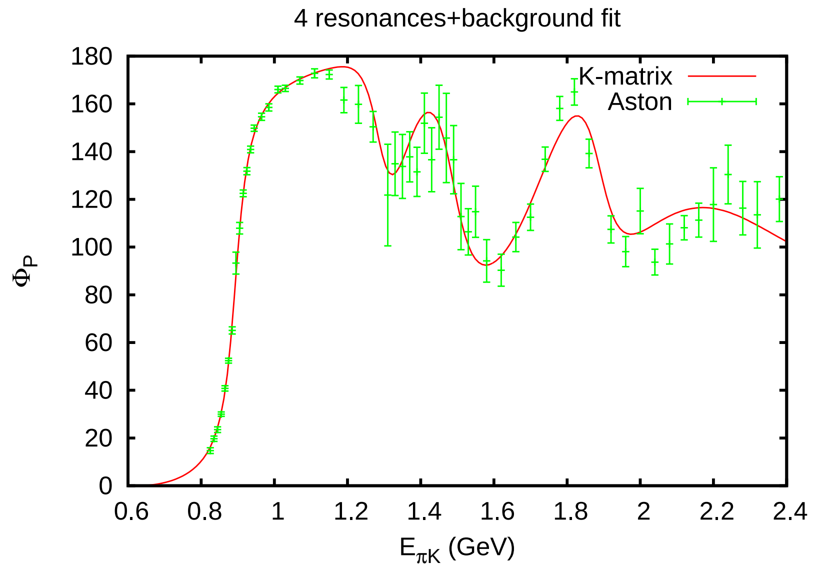
  - (Motivated) approximation: ignore other inelastic channels

* **Fits:** include elastic as well as inelastic constraints

Unitary fits to πK scattering

* T matrix with 3×3 unitarity (K matrix method)
* 16 parameters. Energy range: $0.8 \leq E \leq 2.5$ GeV
* Illustration of results ($\pi^+ K^- \to \pi^+ K^-$):





∗ Resonance contributions from resonance chiral Lagrangian (good chiral behaviour + identification of parameters $M_V(i)$, $g_V(i)$, $\sigma_V(i)$)

∗ $\mathcal{L} = \sum_{n=1}^{n=4} \left[\mathcal{L}_K^{(n)} + \mathcal{L}_g^{(n)} + \mathcal{L}_\sigma^{(n)}\right]$ with:

$$\mathcal{L}_g^{(n)} - = \frac{-i}{2} g_V(n) \, \mathrm{tr}\,(V_{\mu\nu}^{(n)}[u_\mu, u_\nu])$$

$$\mathcal{L}_\sigma^{(n)} = \frac{1}{2} \sigma_V(n) \, \epsilon^{\mu\nu\rho\sigma} \mathrm{tr}\,(V_{\mu\nu}^{(n)}\{u_\rho, V_\sigma\})$$

∗

	Fit	Prades(1993)
$g_V(1)$	0.073	0.083
$\sigma_V(1)$	0.26	0.25

∗ Non-resonance contributions: phenomenological parametrization

From T-matrix to form factors

- $S = -1$ vector current matrix elements:

$$\langle K^+(p_K)|\bar{u}\gamma^\mu s|\pi^0(p_\pi)\rangle = \frac{1}{\sqrt{2}}\left[f_+^{K\pi}(t)\,(p_K+p_\pi)^\mu + f_-^{K\pi}(t)\,(p_K-p_\pi)^\mu\right]$$

$$\langle K^{*+}(p_V,\lambda)|\bar{u}\gamma_\mu s|\pi^0(p_\pi)\rangle = \epsilon_{\mu\nu\alpha\beta}\, e^{*\nu}(\lambda) p_V^\alpha p_\pi^\beta\, H_2(t)$$

$$\langle \rho^0(p_V,\lambda)|\bar{u}\gamma_\mu s|K^-(p_K)\rangle = -\epsilon_{\mu\nu\alpha\beta}\, e^{*\nu}(\lambda) p_V^\alpha p_K^\beta\, H_3(t)$$

- Analyticity + asymptotic QCD

$$H_i(t) = \frac{1}{\pi}\int_{(m_\pi+m_K)^2}^{\infty} \frac{dt'}{t'-t}\,\mathrm{Im}\, H_i(t') \qquad (1)$$

- [with $H_1(t) = f_+^{K\pi}(t)$]

* Unitarity relations (time reversal invariance)

$$\mathrm{Im}\, H_i(t) = \sum_{j=1}^{3} T_{ij}^*(t) H_j(t) \frac{2(q_j(t))^3 \tau_j(t)}{\sqrt{t}\,\tau_i(t)} \theta(t - t_j) \qquad (2)$$

* (1) + (2) Closed set of eqs. [Muskhelishvili-Omnès (MO) type]
* Simple (one-variable form factors) because two-body channels
* Well known equations, rare attempts at solving them

* Remark: Watson's theorem (when $t \leq t_{inel}$).

Treatment of MO equations

∗ Input to eq. (2) known up to $t_0 \simeq 6$ GeV^2. In region $[t_0, \infty]$ smooth cutoff generated from S-matrix,

$$S(t) = \exp[2iD(t)] \qquad (D \text{ hermitian})$$

∗ Index of MO operator:

$$D_{ij}(\infty) = 0 \; (i \neq j) \qquad \sum D_{ii}(\infty) = N\pi$$

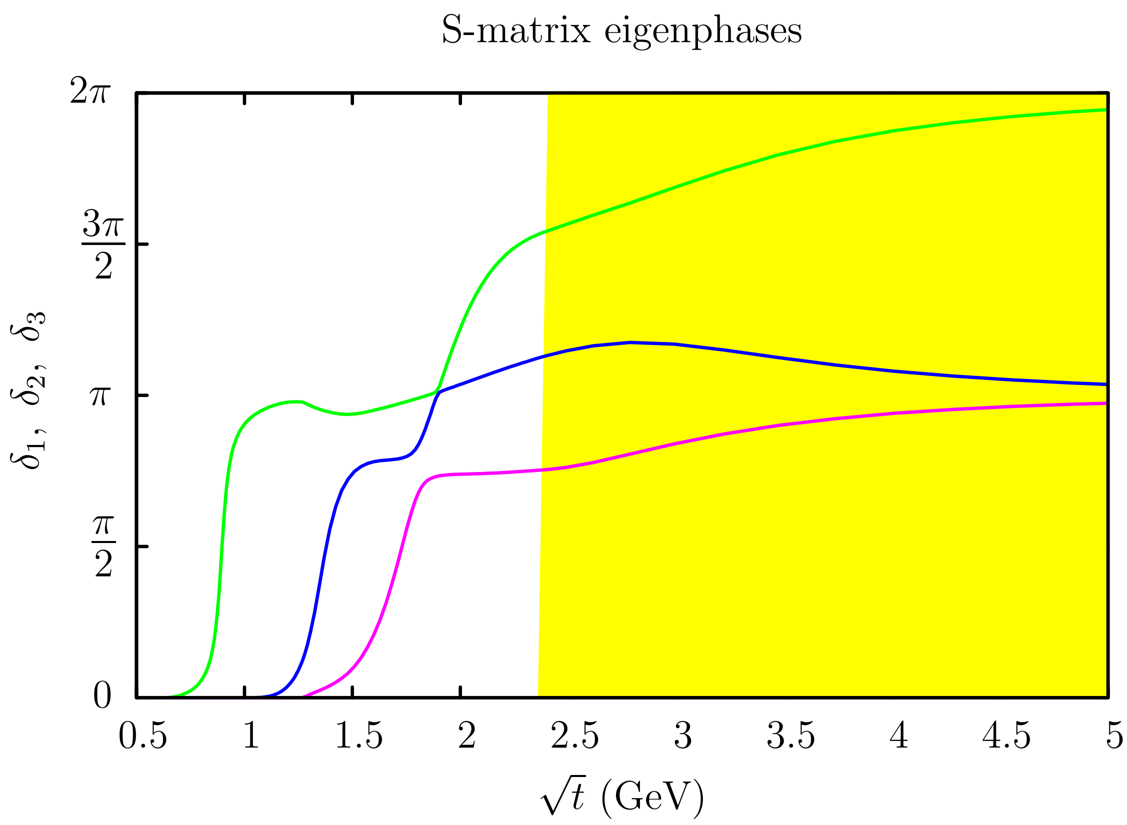
∗ Theorem : N = number of independent conditions to be imposed on $H_i(t)$

∗ At $t = t_0$

$$\sum D_{ii}(t_0) \simeq 3.5\pi$$

∗ Choose $N = 4$

∗ Illustration of asymptotic interpolation [eigenvalues of D]



∗ impose four conditions: $H_1(0)$, $H_2(0)$, $H_3(0)$ and $H_1(\infty)$

Conditions at $t = 0$ and $t = \infty$

∗ $t = \infty$, Brodsky-Lepage +flavour symmetry (used with average $\alpha_s = 0.2$)

$$f_+^{K\pi}(-t)\Big|_{t\to\infty} = \frac{16\pi\sqrt{2}\,\alpha_s(t)F_\pi^2}{t}$$

∗ $t = 0$

– $f_+^{K\pi}(0) = 0.977 + O(m_s - \hat{m})^3$ [Gasser, Leutwyler (1985), +talk by J. Bijnens]

– Flavour symmetry limit (+large N_c) from $\Gamma(\rho^+ \to \pi^+\gamma)$.

$$H_2(0) = -H_3(0) = (1.54 \pm 0.08)\ \mathrm{GeV}^{-1}$$

– First order symmetry breaking:

$$\begin{aligned} H_2(0) &= \quad (1.41 \pm 0.09 - 65.4\,a)\ \mathrm{GeV}^{-1} \\ H_3(0) &= (-1.34 \pm 0.07 - 65.4\,a)\ \mathrm{GeV}^{-1} \end{aligned}$$

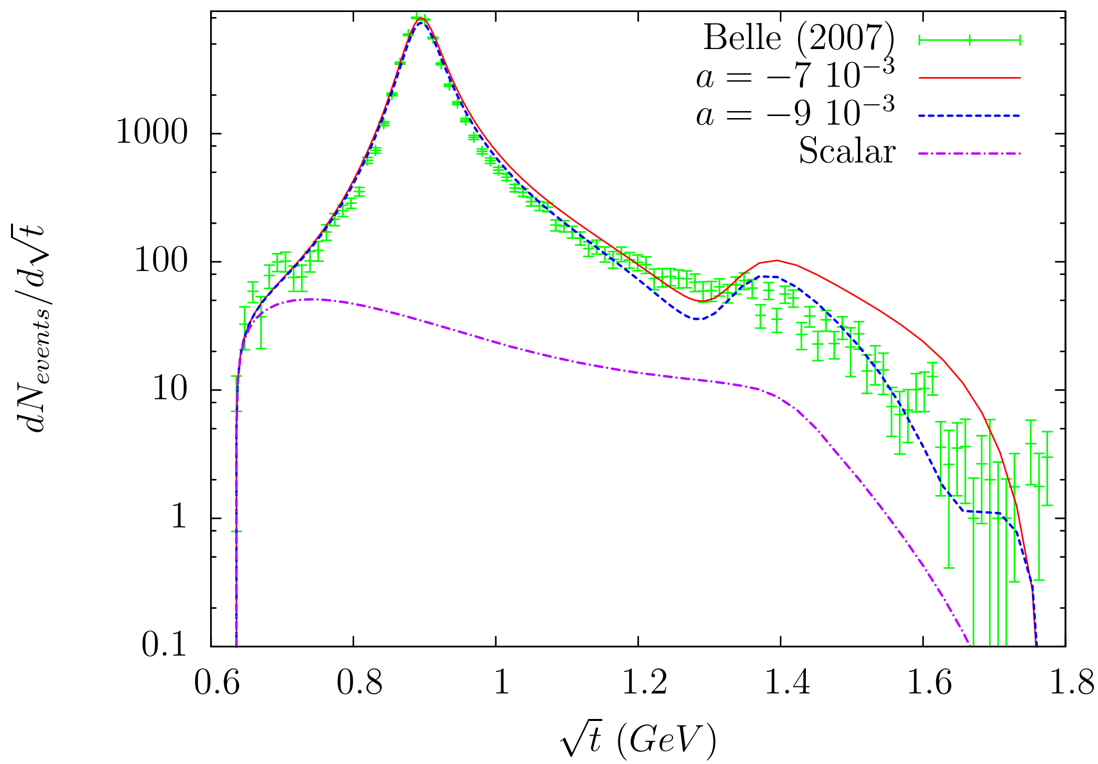
– One undetermined parameter left : order of magnitude $O(10^{-3})$

Comparison with τ decay data

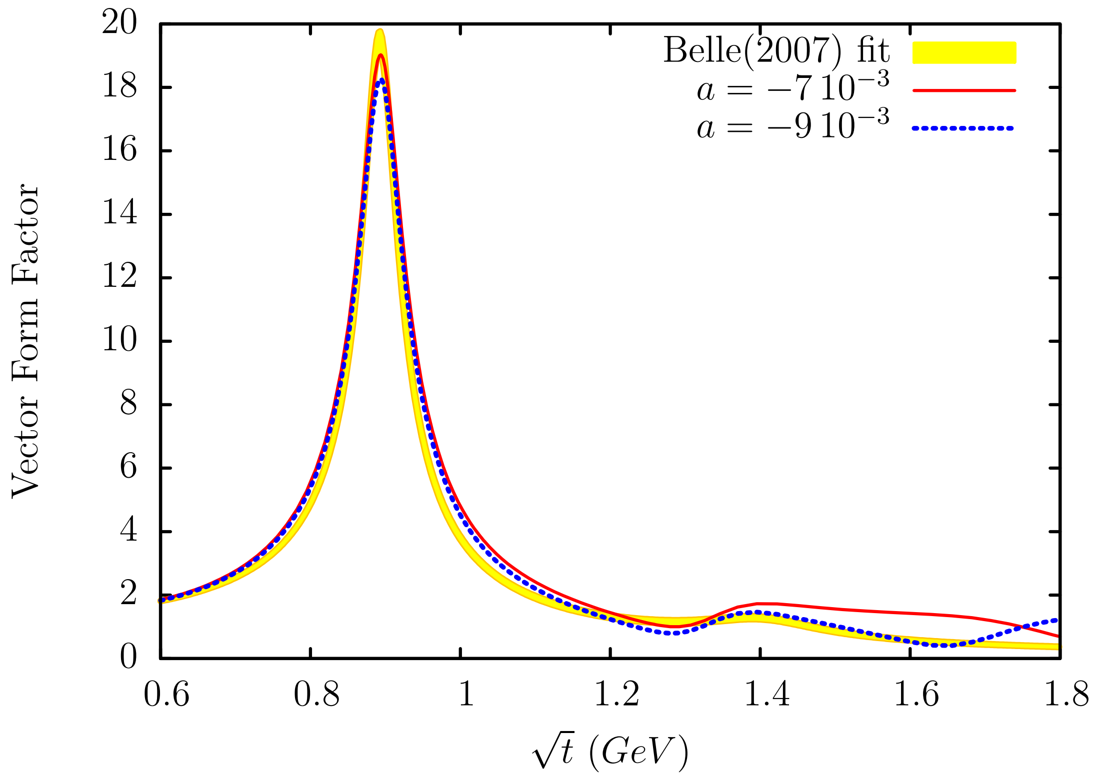
$\ast$ Recently Belle (2007): $\tau \to K_S^0 \pi^- \nu_\tau$ and Babar (2007): $\tau \to K^- \pi^0 \nu_\tau$ [statistics: $\times 10^3$ compared to Aleph (1999)]

$\ast$ Scalar form factor $f_0^{K\pi}(t)$ also needed [borrowed from Jamin, Oller, Pich (2001)]

$\ast$ Parameter a: adjusted to branching ratio:
$a \simeq -7\,10^{-3}$ (PDG),
$a \simeq -9\,10^{-3}$ (Babar/Belle)



* Comparison of form factor $f_+^{K\pi}$ with Belle's determination.



* Resonance $K^*(1680)$ suppressed

* Isospin breaking for $K^*(892)$ Unlike PDG !

$$M_{K^{*0}} \simeq M_{K^{*+}} \qquad \Gamma_{K^{*0}} > \Gamma_{K^{*+}}$$

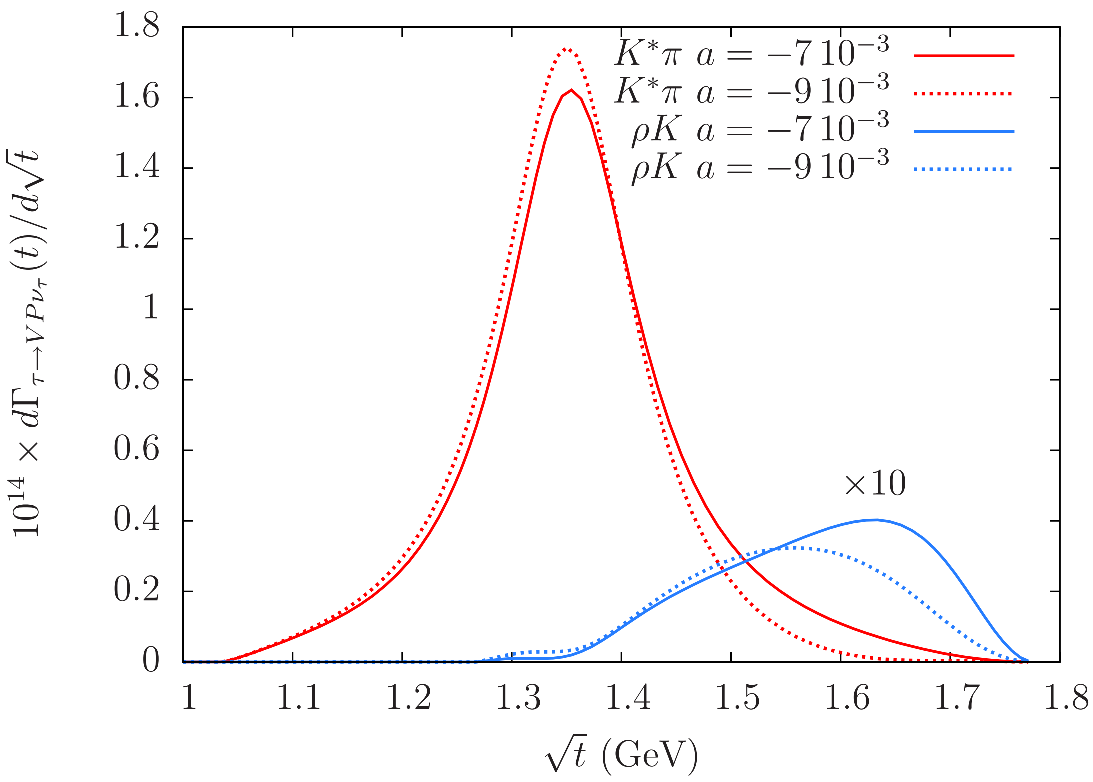
* Predictions for

$$\tau \longrightarrow K^*\pi\nu_\tau \qquad\qquad \tau \longrightarrow \rho K\nu_\tau$$

* Branching ratio:

$$R(\tau \longrightarrow K^*(1410) \longrightarrow K^*\pi) \simeq 1.35\,10^{-3}$$

(agrees with `Aleph (1999)`)



* Low energy expansion:

$$f_+^{K\pi}(t) = f_+^{K\pi}(0)\,[1 + \lambda'_+\, t/m^2_{\pi^+} + 1/2\lambda''_+\, t^2/m^4_{\pi^+} + \ldots]$$

	λ'_+	λ''_+
$a = -7\,10^{-3}$	$26.3\,10^{-3}$	$1.30\,10^{-3}$
$a = -9\,10^{-3}$	$25.5,\,10^{-3}$	$1.25\,10^{-3}$
Jamin et al (2006)	$26.2\,10^{-3}$	$1.37\,10^{-3}$
Kloe(2006,2007)	$(25.6 \pm 1.5 \pm 0.9)\,10^{-3}$	$(1.4 \pm 0.7 \pm 0.4)\,10^{-3}$

* More optimal use of τ data possible

Conclusions

* Starting from T-matrix, including realistic inelasticity $\longrightarrow$ construct vector form factors (analyticity...)

* One parameter left, agreement with Belle (2007) reasonable (but not perfect). Isospin breaking ?

* This supports previous similar calculations of scalar $\pi\pi$ or πK form factors.

* Experimental separation of $f_+^{\pi K}$ and $f_0^{\pi K}$ would be very useful

* Combined fit LASS and Belle could improve low energy predictions

* Other application of form factors B decays into 3-body (factorization approximation)