
Ultrasoft Renormalization of the potentials in NRQCD

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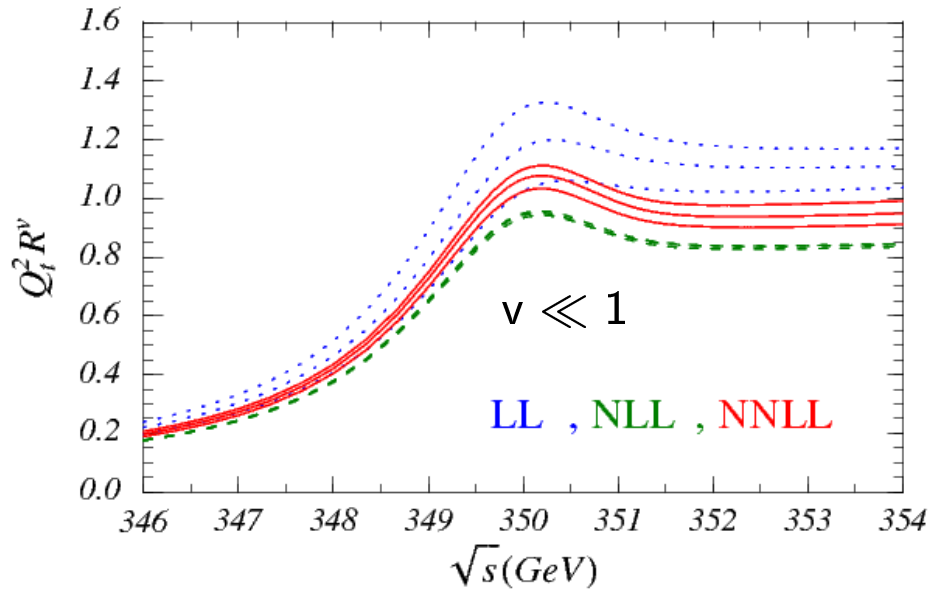
Outline

- Top Quark Threshold Physics
- vNRQCD
- Renormalization of the Potentials
 - ▷ $\frac{1}{m^2}$ - Potentials
 - ▷ $\frac{1}{mk}$ - Potentials
- Status of Calculations / Summary

Top Quark Threshold Physics

Top Physics at the ILC:

Focus: $t\bar{t}$ - production at threshold ($e^+e^- \rightarrow t\bar{t}$)



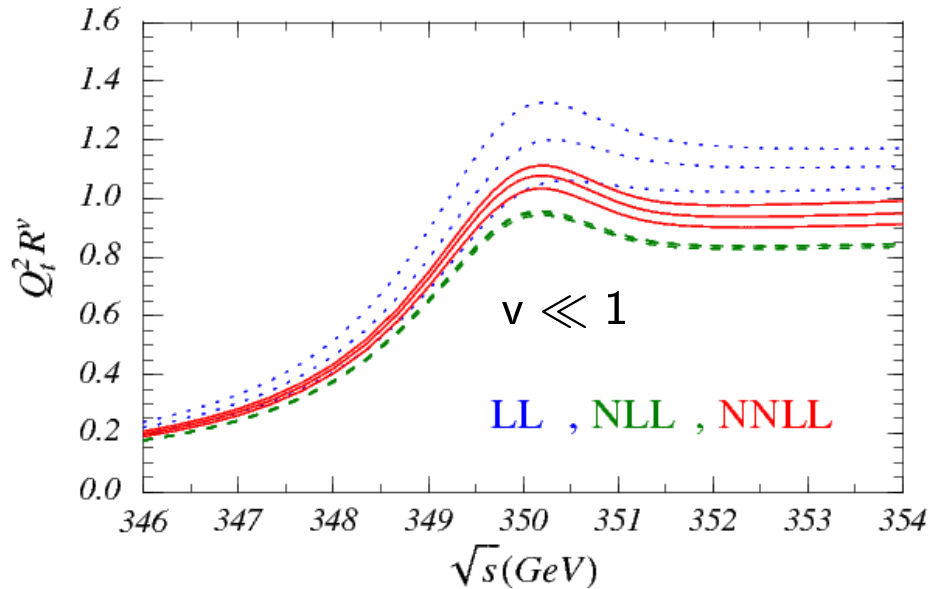
$$\Gamma_t \approx 1.5 \text{ GeV} \gg \Lambda_{\text{QCD}} \Rightarrow$$

- nonpert. effects suppressed [Fadin, Khoze]
- no sharp resonance peak

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Aim: precise determination of

m_t

status: $\delta m_t \sim 100 \text{ MeV} \checkmark$

y_t, α_s, Γ_t

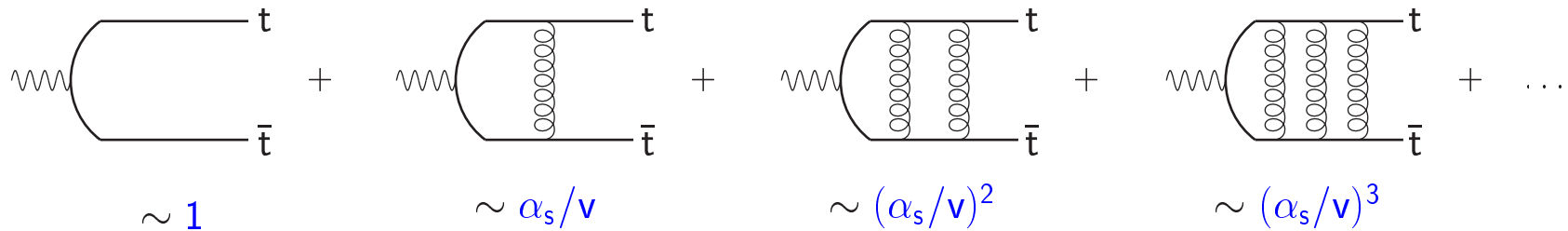
status: $\delta \sigma_{\text{tot}}^{\text{theo}} / \sigma_{\text{tot}} \sim 6\%$ (NNLL incomplete)

needed: $< 3\%$

Top Quark Threshold Physics

Problem of Coulomb singularities:

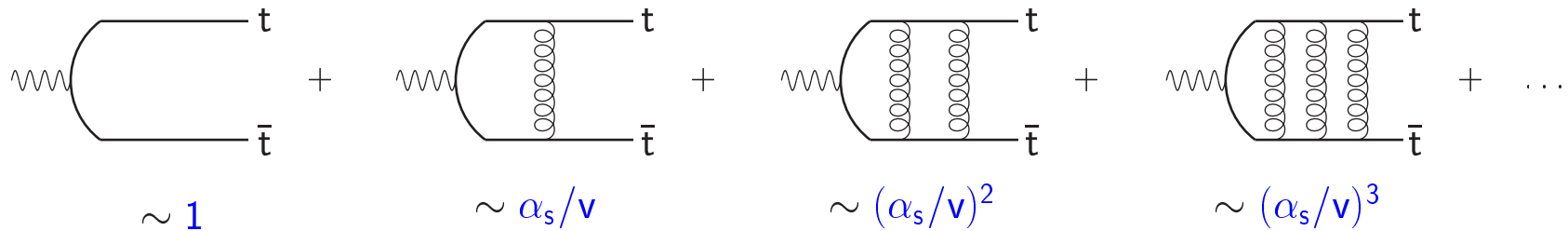
Production threshold $\Rightarrow v \sim \alpha_s \sim 0.1 \Rightarrow$ breakdown of perturbation theory



Top Quark Threshold Physics

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Solution:

nonrelativistic effective field theory NRQCD

\rightarrow summation of $\left(\frac{\alpha_s}{v}\right)^n$ - terms using Schrödinger formalism (LO)

Problem of large logarithms:

$$\boxed{\text{3 scales: } m_t \gg \vec{p} \sim m_t v \gg E_{\text{kin}} \sim m_t v^2 \quad (\sim \Gamma_t \gg \Lambda_{\text{QCD}})}$$

(soft) (ultrasoft)

⇒ log's at the dyn. scales:

$$\ln\left(\frac{m^2}{E^2}\right), \ln\left(\frac{m^2}{p^2}\right), \ln\left(\frac{p^2}{E^2}\right) \quad \text{e.g.: } \alpha_s \ln\left(\frac{m^2}{E^2}\right) \sim 1$$

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Solution:

two renormalization scales:

$$\boxed{\mu_s \sim m v, \mu_u \sim m v^2}$$

correlation: $E \sim \frac{p^2}{m} \rightarrow \mu_u \sim \frac{\mu_s^2}{m} \longrightarrow \mu_s = m v, \mu_u = m v^2 \Rightarrow$ 'v'NRQCD

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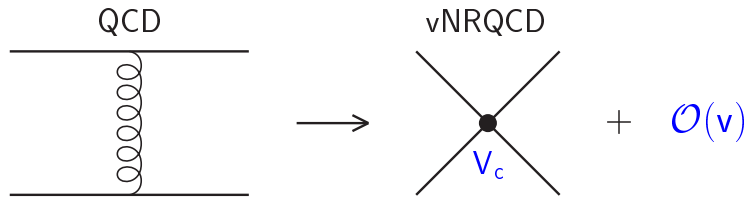
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⇒ RGE's resum $[\alpha_s \ln v]^n$, $\alpha_s [\alpha_s \ln v]^n$, $\alpha_s^2 [\alpha_s \ln v]^n$, ... terms

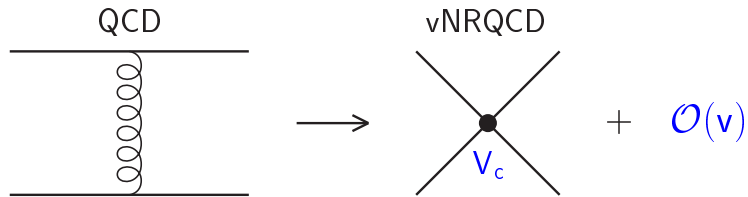
LL NLL NNLL

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vNRQCD

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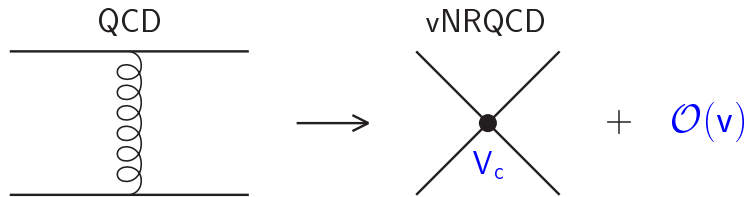


- Resonant dof's \rightarrow fields in the vNRQCD Lagrangian:

nonrel. quark:	$(E, \mathbf{p}) \sim (mv^2, mv)$	$\psi_{\mathbf{p}}(x)$	
soft gluon:	$(q_0, \mathbf{q}) \sim (mv, mv)$	$A_{\mathbf{q}}(x)$	
ultrasoft gluon:	$(q_0, \mathbf{q}) \sim (mv^2, mv^2)$	$A(x)$	

vNRQCD

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- Systematic expansion in $v \Rightarrow$ consistent power counting in $v \sim \alpha_s$

vNRQCD

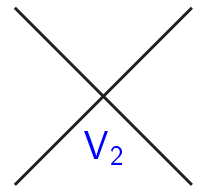
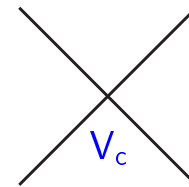
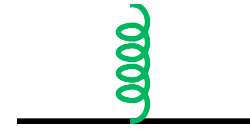
[Luke, Manohar, Rothstein]

$$\mathcal{L}_{\text{vNRQCD}} = \mathcal{L}_{\text{usoft}} + \mathcal{L}_{\text{pot}} + \mathcal{L}_{\text{soft}}$$

$$\mathcal{L}_{\text{usoft}} : \psi_{\mathbf{p}}^\dagger(x) \left[i\mathbf{D}^0 - \frac{(\mathbf{p} - i\mathbf{D})^2}{2m} + \dots \right] \psi_{\mathbf{p}}(x) + \dots$$

$$\mathcal{L}_{\text{pot}} : -V \psi_{\mathbf{p}'}^\dagger \psi_{\mathbf{p}} \chi_{-\mathbf{p}'}^\dagger \chi_{-\mathbf{p}} + \dots$$

$$V \sim \frac{\mathcal{V}_c}{\mathbf{k}^2} + \frac{\mathcal{V}_k \pi^2}{m\mathbf{k}} + \frac{\mathcal{V}_r (\mathbf{p}^2 + \mathbf{p}'^2)}{2m^2 \mathbf{k}^2} + \frac{\mathcal{V}_2}{m^2} + \frac{\mathcal{V}_s}{m^2} \mathbf{S}^2 + \dots$$



($\mathbf{k} = \mathbf{p}' - \mathbf{p}$)

vNRQCD

[Luke, Manohar, Rothstein]

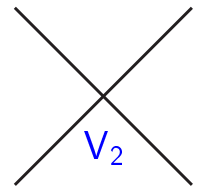
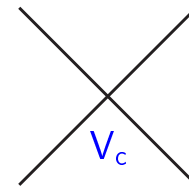
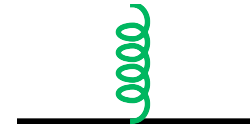
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$$(\mathbf{k} = \mathbf{p}' - \mathbf{p})$$



external production/annihilation current (CMS):

$$\otimes \sim c_1(\nu) \cdot \underbrace{\vec{J}_1^{\text{eff}}(x)}_{\psi_{\mathbf{p}}^\dagger \vec{\sigma}(i\sigma_2) \chi_{-\mathbf{p}}^*} + \dots$$

Renormalization of the Potentials

Anomalous dimension of V contributes to $\sigma(e^+ e^- \rightarrow t \bar{t})$:

$$\sigma_{\text{tot}} \sim \text{Im} \left[\text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \dots \right]$$

The diagrams show a series of vacuum polarization corrections to the photon propagator. Diagram 1 is a simple photon loop. Diagram 2 is a photon loop with a top quark loop, where the top quark loop is labeled with a blue V . Diagram 3 is a photon loop with two top quark loops, each labeled with a blue V .

$$\sim |c_1(\nu)|^2 \cdot \text{Im} \left[-i \int d^4x e^{i\hat{q}x} \langle 0 | T \vec{j}_1^{\text{eff}*}(x) \vec{j}_1^{\text{eff}}(0) | 0 \rangle \right]$$

$$\sim |c_1(\nu)|^2 \cdot \text{Im} [G(0, 0, E, \nu)]$$

G^{NNLL} known ✓

Renormalization of the Potentials

Anomalous dimension of V contributes to $\sigma(e^+ e^- \rightarrow t \bar{t})$:

$$\sigma_{\text{tot}} \sim \text{Im} \left[\text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \dots \right]$$

Diagram 1: A circle with two external lines, each marked with a cross (⊗).
 Diagram 2: Two circles connected at a central vertex, with a blue 'V' label below the vertex. Each external line has a cross (⊗).
 Diagram 3: Three circles connected in a chain at two central vertices, with blue 'V' labels below each vertex. Each external line has a cross (⊗).

$$\sim |c_1(\nu)|^2 \cdot \text{Im} \left[-i \int d^4x e^{i\hat{q}x} \langle 0 | T \vec{j}_1^{\text{eff}*}(x) \vec{j}_1^{\text{eff}}(0) | 0 \rangle \right]$$

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current renormalization

$$\ln \left[\frac{c_1(\nu)}{c_1(1)} \right] = \underbrace{\xi^{\text{LL}}}_0 + \xi^{\text{NLL}} + \xi_{\text{mix}}^{\text{NNLL}} + \xi_{\text{non-mix}}^{\text{NNLL}}$$

Diagram 1 (LL): Two circles connected at a central vertex, with a blue 'V' label below the vertex. Each external line has a cross (⊗).
 Diagram 2 (NLL): Three circles connected in a chain at two central vertices, with a blue 'V' label below the middle vertex. Each external line has a cross (⊗).
 Diagram 3 (NNLL): Three circles connected in a chain at two central vertices, with a green wavy line (representing a gluon) connecting the two outer vertices. Each external line has a cross (⊗).
 [Hoang]

[Luke, Manohar, Rothstein, Pineda, Hoang, Stewart]

Renormalization of the Potentials

Anomalous dimension of V contributes to $\sigma(e^+ e^- \rightarrow t \bar{t})$:

$$\sigma_{\text{tot}} \sim \text{Im} \left[\text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \dots \right]$$

Diagram 1: A circle with two external lines (crosses) and a vertex \otimes .
 Diagram 2: Two circles sharing a vertex \bullet , with external lines \otimes and a vertex V at the bottom.
 Diagram 3: Three circles in a chain, with external lines \otimes and vertices V at the bottom.

$$\sim |c_1(\nu)|^2 \cdot \text{Im} \left[-i \int d^4x e^{i\hat{q}x} \langle 0 | T \vec{j}_1^{\text{eff}*}(x) \vec{j}_1^{\text{eff}}(0) | 0 \rangle \right]$$

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 Diagram 2 (NLL): Two circles sharing a vertex \bullet , with external lines \otimes and a vertex V at the bottom.
 Diagram 3 (NNLL): Two circles sharing a vertex \bullet , with external lines \otimes and a vertex V at the bottom, and a green wavy line on top. [Hoang]

[Luke, Manohar, Rothstein, Pineda, Hoang, Stewart]

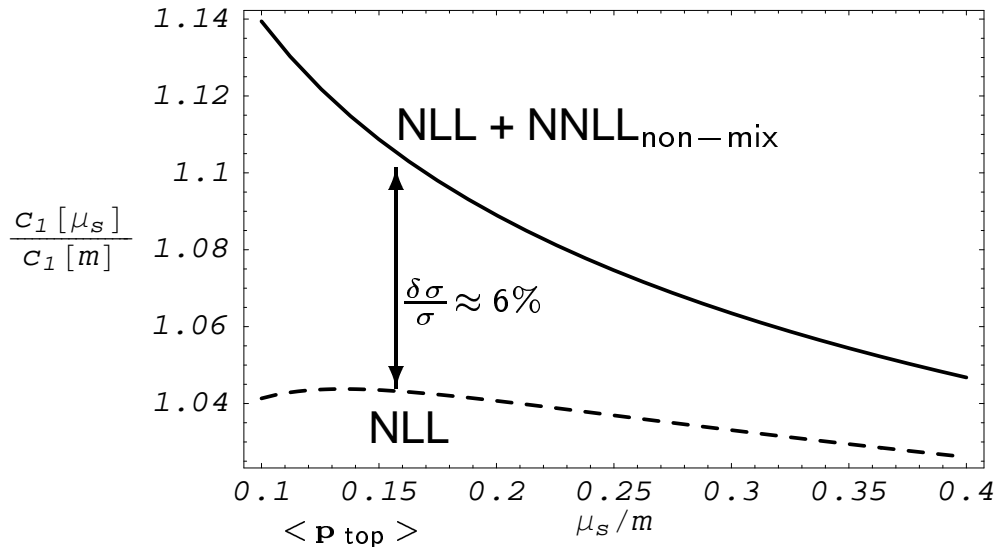
missing \Rightarrow $V^{\text{NLL}}(\nu)$ needed!

Renormalization of the Potentials

Anomalous dimension of V contributes to $\sigma(e^+ e^- \rightarrow t \bar{t})$:

$$\begin{aligned} \sigma_{\text{tot}} &\sim \text{Im} \left[\text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \dots \right] \\ &\sim |c_1(\nu)|^2 \cdot \text{Im} \left[-i \int d^4x e^{i\hat{q}x} \langle 0 | T \vec{j}_1^{\text{eff}*}(x) \vec{j}_1^{\text{eff}}(0) | 0 \rangle \right] \\ &\sim |c_1(\nu)|^2 \cdot \text{Im} [G(0, 0, E, \nu)] \end{aligned}$$

G^{NNLL} known ✓

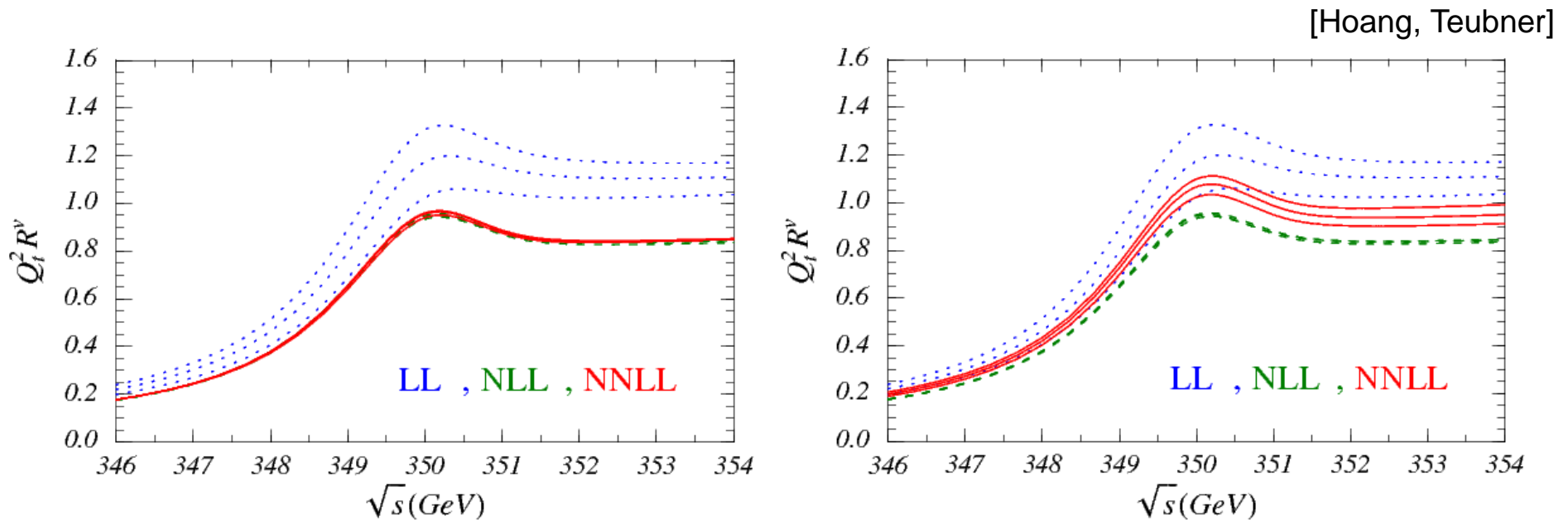


missing NNLL_{mix} contribution
may reduce theoretical error of σ_{tot}

\Rightarrow $V^{\text{NLL}}(\nu)$ needed!

Renormalization of the Potentials

NNLL_{nonmix} contribution to c_1 has large effect on σ_{tot} :



NLL running of c_1 is used for **NNLL**

⇒ small scale (ν) dependence

NNLL_{nonmix} of c_1 is included in **NNLL**

⇒ large scale (ν) dependence

⇒ large theoretical uncertainty ($\sim 6\%$) of σ_{tot}

Renormalization of the Potentials

$$\text{NLL : } \nu \frac{\partial}{\partial \nu} \ln[\mathbf{c}_1(\nu)] = -\frac{\mathcal{V}_c(\nu)}{16\pi^2} \left[\frac{\mathcal{V}_c(\nu)}{4} + \mathcal{V}_2(\nu) + \mathcal{V}_r(\nu) + \mathbf{S}^2 \mathcal{V}_s(\nu) \right] + \frac{1}{2} \mathcal{V}_k(\nu)$$

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NLL (two loop) anomalous dimension of V :

- soft NLL running of V_s known: tiny effect

[Pineda, Steinhauser, Penin, Smirnov]

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$$\alpha_s(m\nu^2) \simeq 0.27 > \alpha_s(m\nu) \simeq 0.15 \quad (\nu \simeq 0.1)$$

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- Relevant potentials for $\mathbf{c}_1(\nu)$ at NNLL_{mix} affected by NLL **ultrasoft** renormalization:

$$\frac{\mathcal{V}_k \pi^2}{m\mathbf{k}}, \quad \frac{\mathcal{V}_r(\mathbf{p}^2 + \mathbf{p}'^2)}{2m^2\mathbf{k}^2}, \quad \frac{\mathcal{V}_2}{m^2}$$

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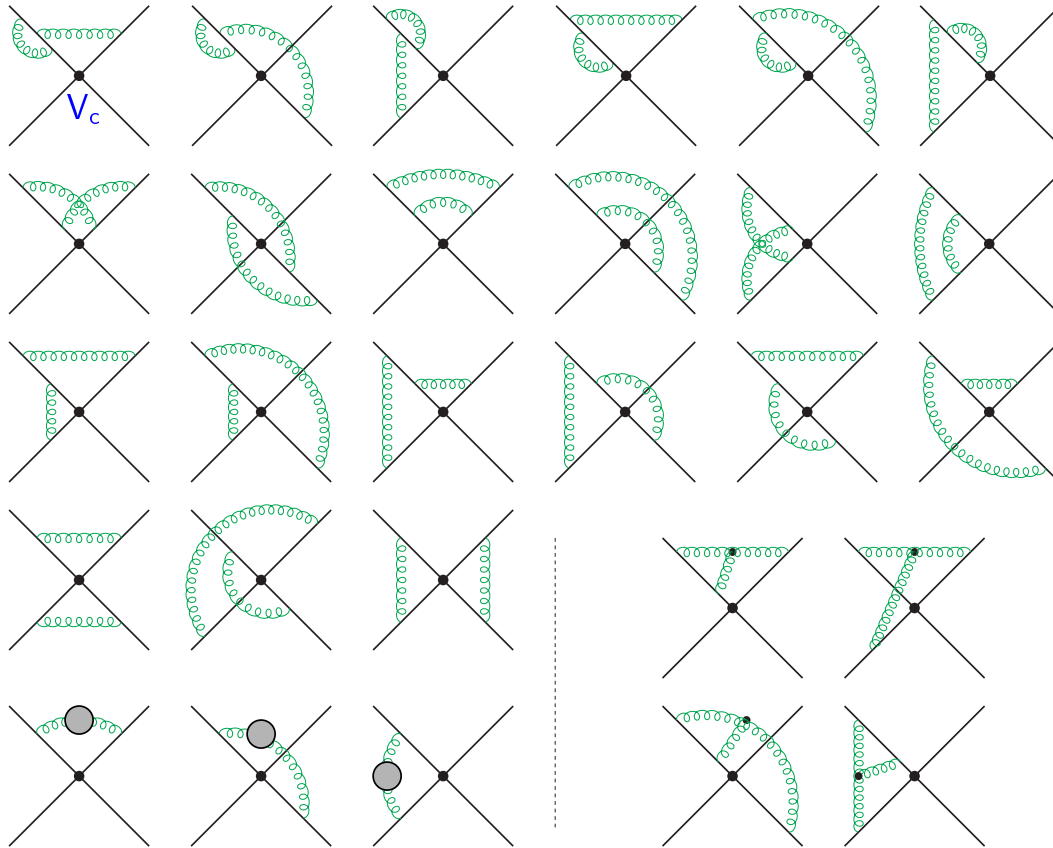
$$\frac{\mathcal{V}_k \pi^2}{m\mathbf{k}}, \quad \frac{\mathcal{V}_r(\mathbf{p}^2 + \mathbf{p}'^2)}{2m^2\mathbf{k}^2}, \quad \frac{\mathcal{V}_2}{m^2}$$

- $\mathcal{V}_k^{\text{LL}}$ is dominant contribution to $\mathbf{c}_1(\nu)^{\text{NLL}}$. Same for $\mathcal{V}_k^{\text{NLL}}$ at NNLL_{mix} ?

Renormalization of the $1/m^2$ - Potentials

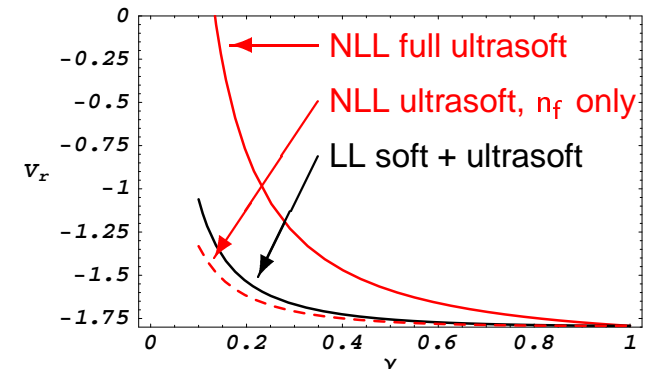
$$\text{NLL : } \nu \frac{\partial}{\partial \nu} \ln[c_1(\nu)] = -\frac{\mathcal{V}_c(\nu)}{16\pi^2} \left[\frac{\mathcal{V}_c(\nu)}{4} + \underbrace{\mathcal{V}_2(\nu) + \mathcal{V}_r(\nu)}_{\text{renormalize directly}} + \mathbf{S}^2 \mathcal{V}_s(\nu) \right] + \frac{1}{2} \mathcal{V}_k(\nu)$$

$\mathcal{O}(10^3)$ diagrams



e.g.:

$$\Rightarrow \delta V_r^{2\text{loop}} \xrightarrow{\text{RGE}} V_r^{\text{NLL}}(\nu)$$



V_2^{NLL} and V_r^{NLL} complete ✓

[M.S., Hoang]

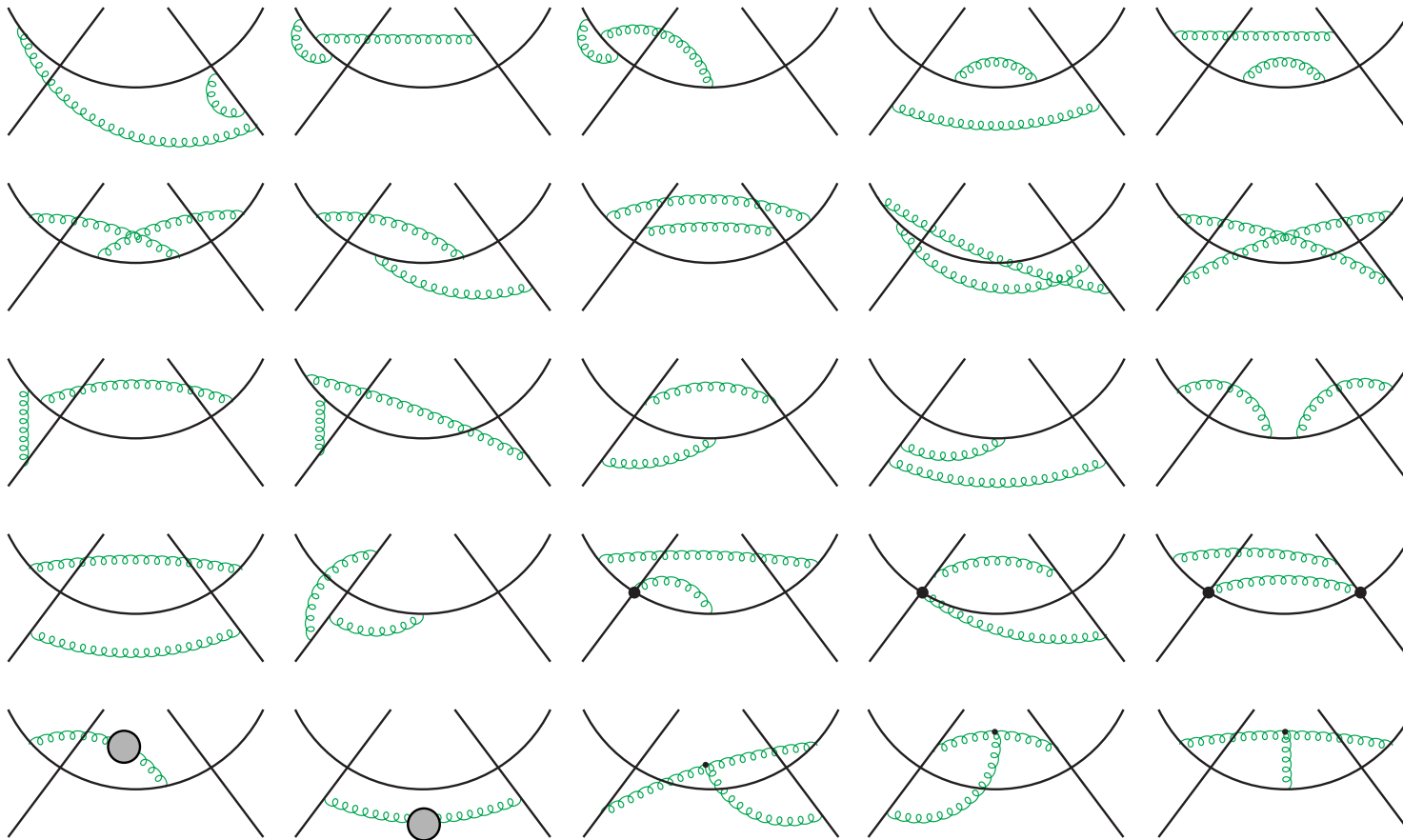
usoft corrections $\sim \nu^2$: $V_c \times (\text{usoft corr.}) \rightarrow \delta V_{2/r}$ (general statement)

Renormalization of the $1/mk$ - Potentials

$$\text{NLL : } \nu \frac{\partial}{\partial \nu} \ln[\mathcal{C}_1(\nu)] = -\frac{\mathcal{V}_c(\nu)}{16\pi^2} \left[\frac{\mathcal{V}_c(\nu)}{4} + \mathcal{V}_2(\nu) + \mathcal{V}_r(\nu) + \mathbf{S}^2 \mathcal{V}_s(\nu) \right] + \frac{1}{2} \underbrace{\mathcal{V}_k(\nu)}$$

6 external HQ legs, $\mathcal{O}(10^4)$ diagrams

renormalize indirectly



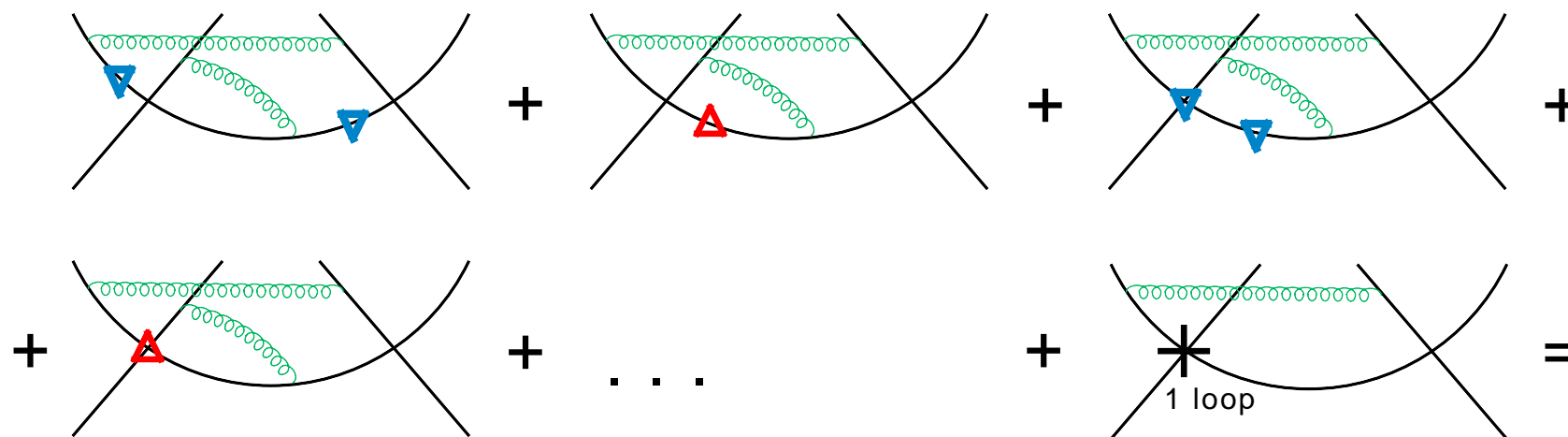
Work in progress !

soft corrections $\sim v^2$

Renormalization of the $1/mk$ - Potentials

Example for one topology:

(\overline{MS} , Dim.Reg.)



$$= \frac{\alpha_U^2}{\epsilon^2} A + \frac{\alpha_U^2}{\epsilon} B$$

$$[\alpha_U = \alpha_s(mv^2)]$$

Terms in $\mathcal{L}_{\text{vNRQCD}}$:

$$\psi_{\mathbf{p}}^\dagger \left[iD^0 - \frac{(\mathbf{p} - i\mathbf{D})^2}{2m} + \dots \right] \psi_{\mathbf{p}} = \psi_{\mathbf{p}}^\dagger \left[\underbrace{\left(i\partial^0 - \frac{\mathbf{p}^2}{2m} \right)}_{\text{HQ propagator}} - gA^0 + \frac{i\mathbf{p}\nabla}{m} + g\frac{\mathbf{p}\mathbf{A}}{m} + \frac{\nabla^2}{2m} + \dots \right] \psi_{\mathbf{p}}$$

similar for $\frac{V_c}{k^2}$:



Renormalization of the $1/mk$ - Potentials

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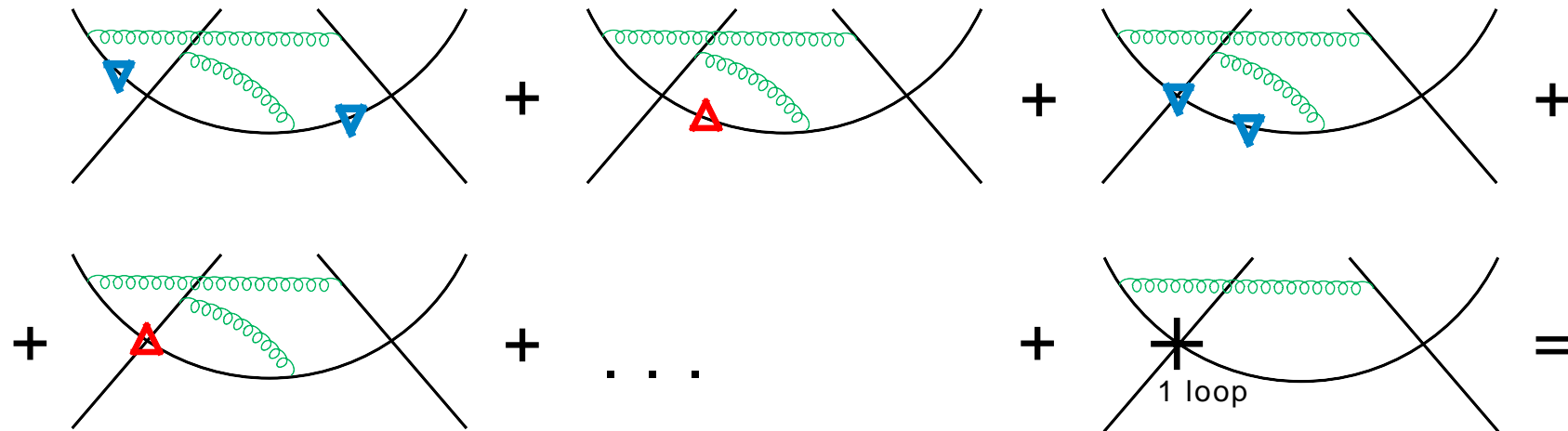
Absorb divergence by new 6Q - Operator:

$$= - \left(\frac{\alpha_U^2}{\epsilon^2} A + \frac{\alpha_U^2}{\epsilon} B \right)$$

Renormalization of the $1/mk$ - Potentials

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(\overline{MS} , Dim.Reg.)



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$$[\alpha_U = \alpha_s(mv^2)]$$

Close **finite** HQ - Loop:

$$= \boxed{\frac{\alpha_U^2}{\epsilon^2} A' + \frac{\alpha_U^2}{\epsilon} B'}$$

Renormalization of the $1/mk$ - Potentials

Example for one topology:

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Close **finite** HQ - Loop:

$$= \frac{\alpha_U^2}{\epsilon^2} A' + \frac{\alpha_U^2}{\epsilon} B' \rightarrow \delta V_k \quad \delta V_k^{2loop} \xrightarrow{RGE} \mathcal{V}_k^{NLL}(\nu)$$

Status of Calculations / Summary

- Ultrasoft NLL running of the potentials V_k , V_r , V_2 is essential for a precise prediction of $\sigma_{\text{tot}}(e^+ e^- \rightarrow t \bar{t})$ at threshold.

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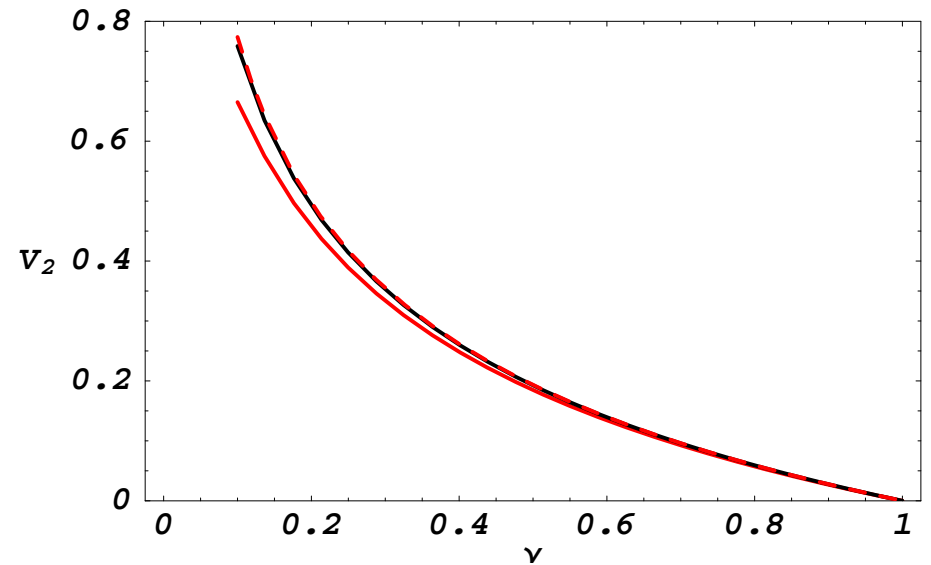
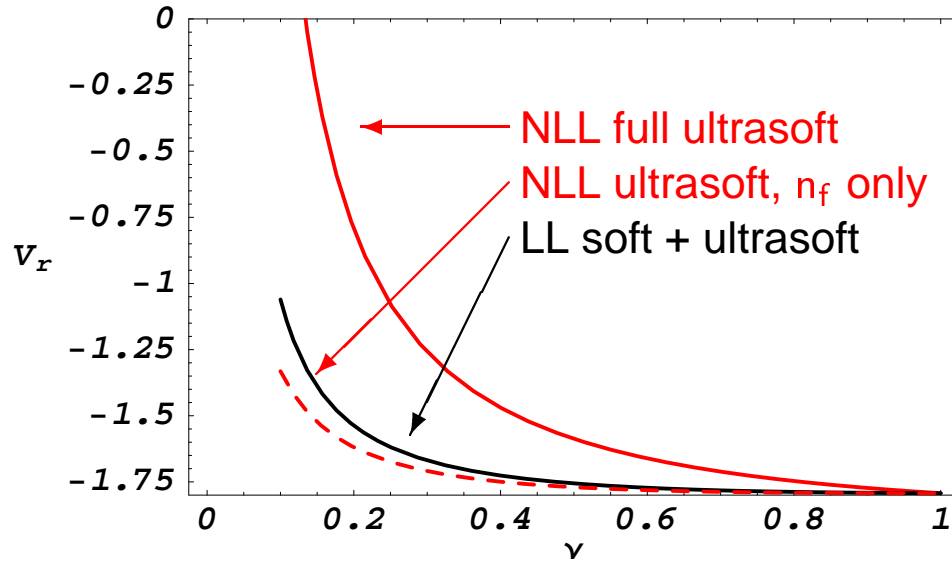
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Contribution	order / α_S	V_k	V_r	V_2	V_s
soft + usoft LL	$(\alpha_S \ln v)^n, (\alpha_U \ln v)^n$	✓	✓	✓	✓
usoft NLL n_f	$n_f \alpha_U (\alpha_U \ln v)^n$	✓	✓	✓	0
full usoft NLL	$\alpha_U (\alpha_U \ln v)^n$	w.i.p.	✓	✓	0
soft NLL	$\alpha_S (\alpha_S \ln v)^n$	—	—	—	✓

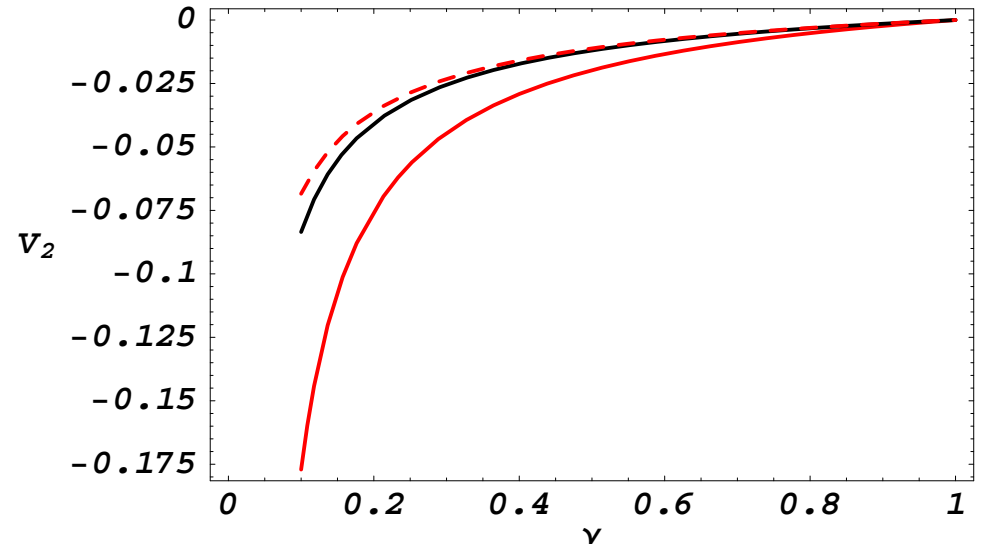
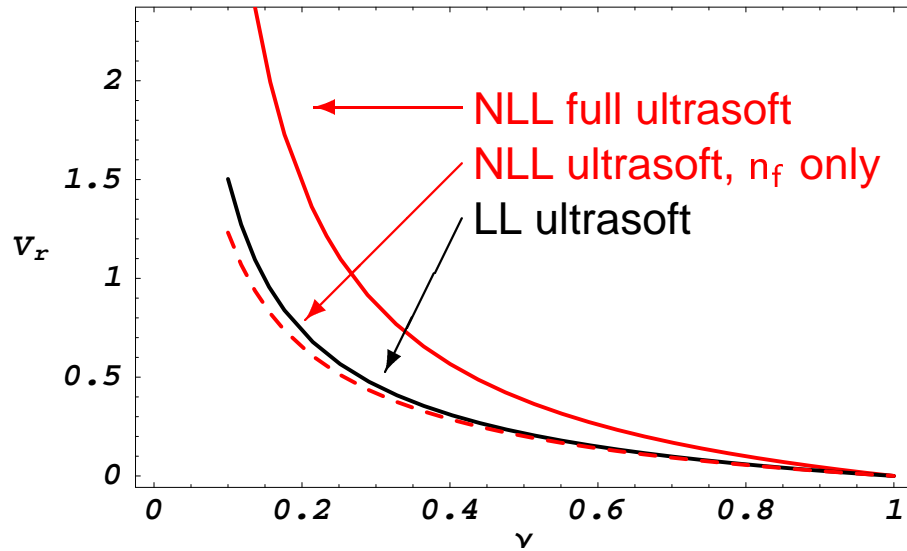
Back Up: Status of Calculations

Results for $\frac{1}{m^2}$ potentials:



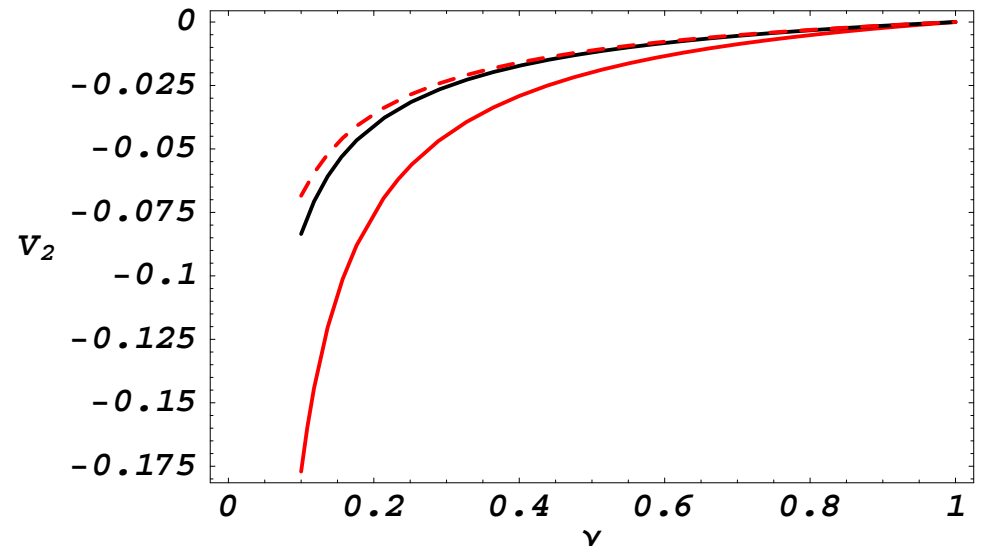
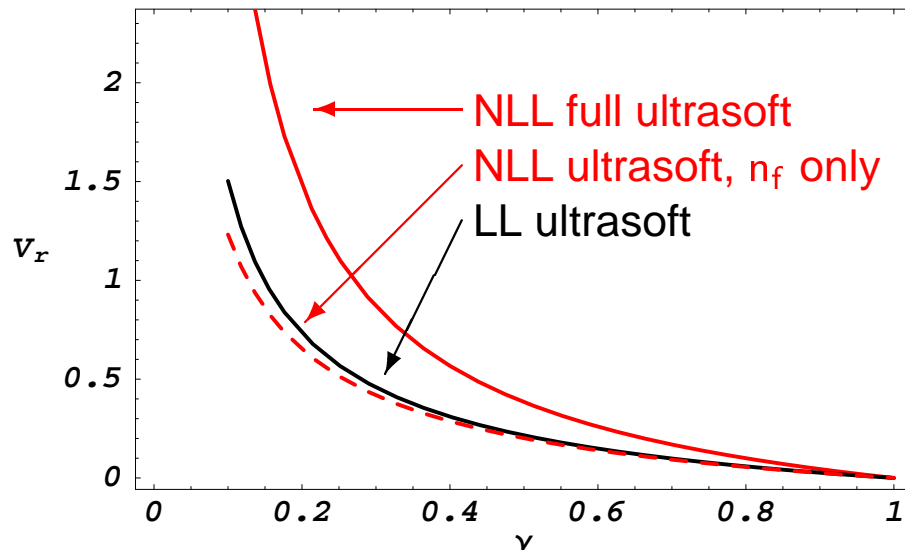
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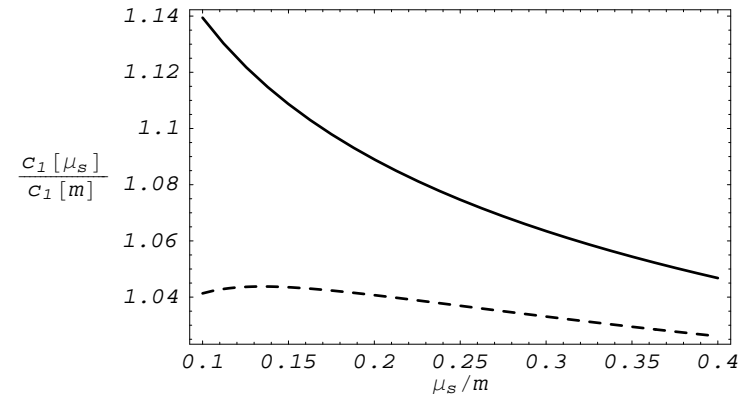
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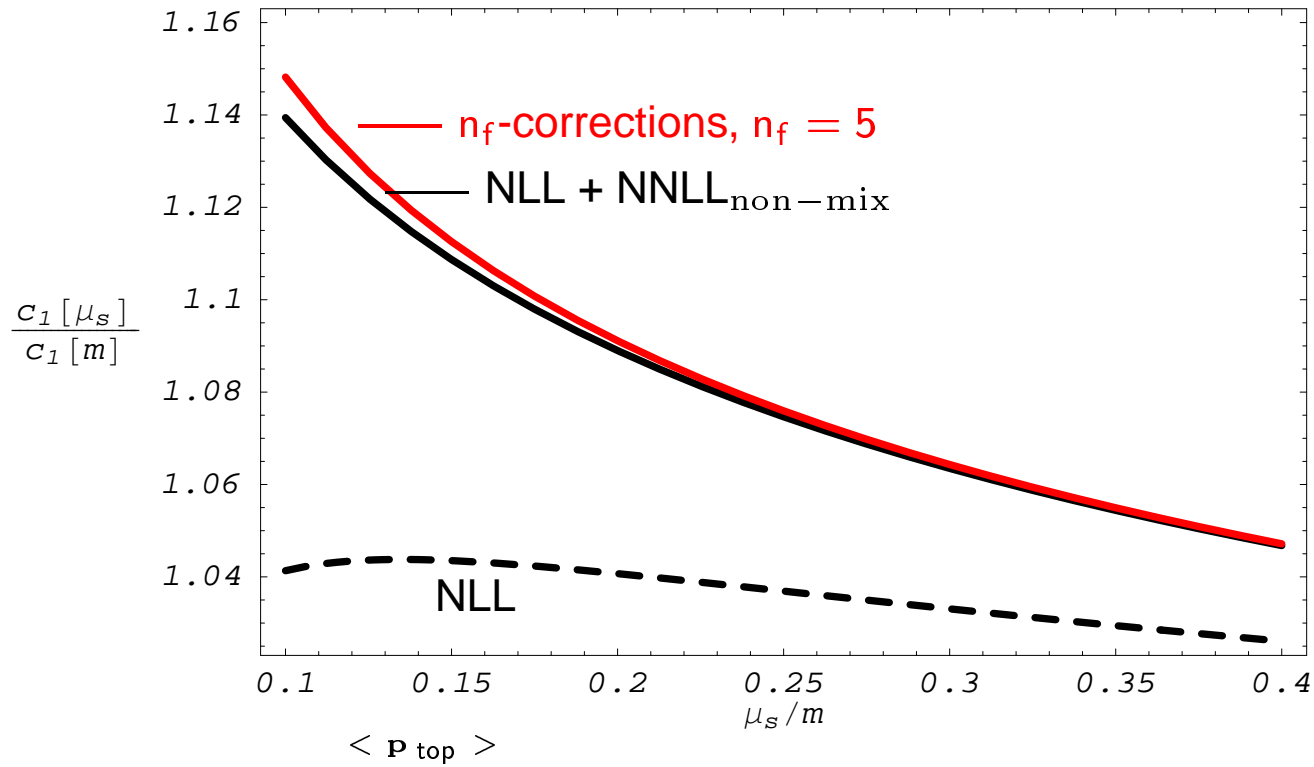
- Analysis shows that usoft LL \sim usoft NLL

\Rightarrow Big NNLL_{mix} contributions to c_1 expected

\Rightarrow may compensate $\text{NNLL}_{\text{nonmix}}$ and reduce ν dependence of c_1 !



Back Up: Old n_f Result



Back Up: Extra Formulae

$$\nu \frac{\partial}{\partial \nu} \ln[c_1(\nu)] = -\frac{\mathcal{V}_c^{(0)}(\nu)}{16\pi^2} \left[\frac{\mathcal{V}_c^{(0)}(\nu)}{4} + \mathcal{V}_2^{(2)}(\nu) + \mathcal{V}_r^{(2)}(\nu) + \mathbf{S}^2 \mathcal{V}_s^{(2)}(\nu) \right] + \frac{1}{2} \mathcal{V}_k^{(1)}(\nu) + \alpha_s^2(m\nu) [3\mathcal{V}_{k1}^{(1)}(\nu) + 2\mathcal{V}_{k2}^{(1)}(\nu)]$$

$$v \cong \alpha_s(mv) = \frac{4\pi}{\beta_0 \ln(m^2 v^2 / \Lambda_{\text{QCD}}^2)} \Rightarrow v \cong \alpha_s \cong 0.14$$

$$v \equiv \sqrt{\frac{\sqrt{s} - 2m_t}{m_t}} \rightarrow \sqrt{\frac{\sqrt{s} - 2m_t + i\Gamma_t}{m_t}} \quad [\text{Fadin, Khoze}]$$