Ultrasoft Renormalization of the potentials in NRQCD

Maximilian Stahlhofen

in collaboration with André Hoang

Max-Planck-Institut für Physik, München

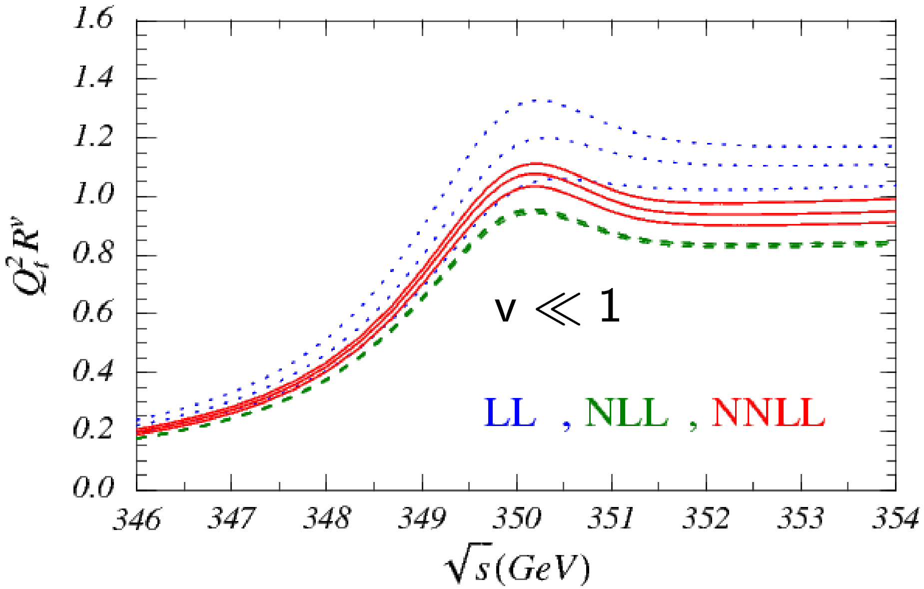
Outline

- Top Quark Threshold Physics
- vNRQCD
- Renormalization of the Potentials
  - $\frac{1}{m^2}$ - Potentials
  - $\frac{1}{mk}$ - Potentials
- Status of Calculations / Summary

Top Quark Threshold Physics

Top Physics at the ILC:

Focus: $t\bar{t}$ - production at threshold $(e^+ e^- \to t\bar{t})$



$\Gamma_t \approx 1.5\,\text{GeV} \gg \Lambda_{QCD} \Rightarrow$

- nonpert. effects suppressed

  [Fadin, Khoze]

- no sharp resonance peak

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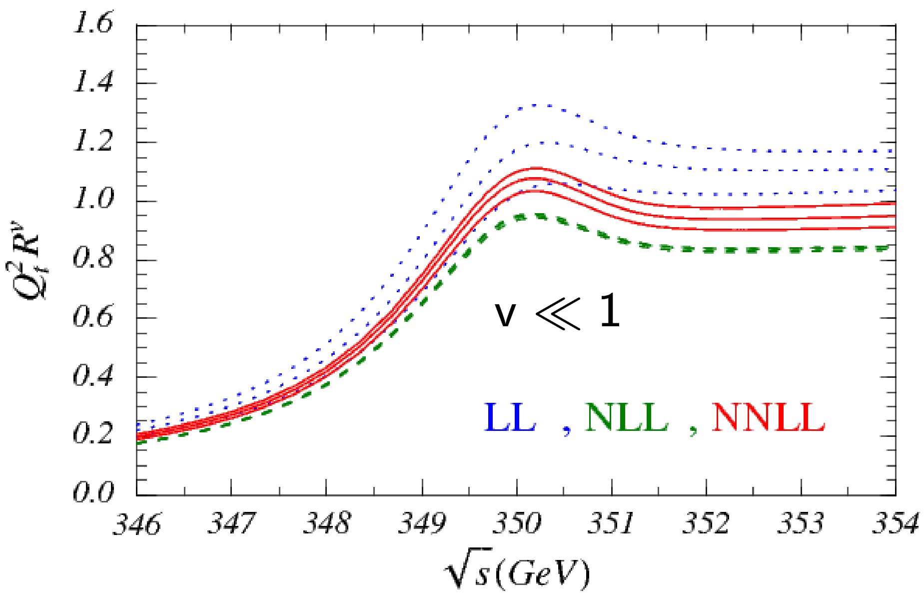

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Aim: precise determination of

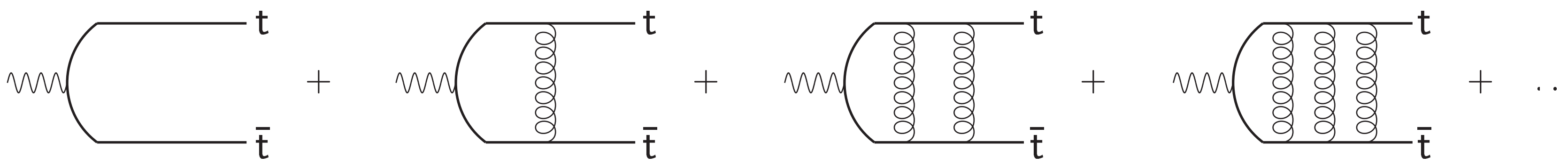
m_t	status: $\delta m_t \sim 100\,\mathrm{MeV}$ ✓
$y_t,\ \alpha_s,\ \Gamma_t$	status: $\delta\sigma_{tot}^{theo}/\sigma_{tot} \sim 6\,\%$ (NNLL incomplete)

needed: $< 3\,\%$

Top Quark Threshold Physics

Problem of Coulomb singularities:

Production threshold $\Rightarrow$ $\mathsf{v} \sim \alpha_s \sim 0.1$ $\Rightarrow$ breakdown of perturbation theory



~ 1 $\qquad$ $\sim \alpha_s/\mathsf{v}$ $\qquad$ $\sim (\alpha_s/\mathsf{v})^2$ $\qquad$ $\sim (\alpha_s/\mathsf{v})^3$

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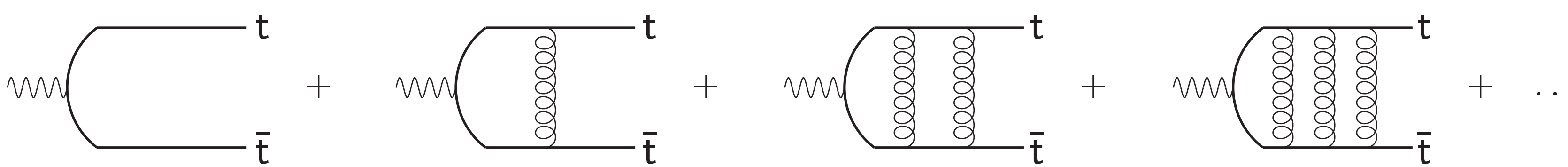

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Solution:

nonrelativistic effective field theory $\boxed{\text{NRQCD}}$

$\rightarrow$ summation of $\left(\frac{\alpha_s}{\mathsf{v}}\right)^n$- terms using Schrödinger formalism (LO)

vNRQCD

Problem of large logarithms:

$$\boxed{\text{3 scales:} \quad m_t \gg \vec{p} \sim m_t v \gg E_{kin} \sim m_t v^2 \quad (\sim \Gamma_t \gg \Lambda_{QCD})}$$

(soft) (ultrasoft)

$\Rightarrow$ log's at the dyn. scales:

$$\ln\left(\frac{m^2}{E^2}\right), \ \ln\left(\frac{m^2}{\mathbf{p}^2}\right), \ \ln\left(\frac{\mathbf{p}^2}{E^2}\right) \qquad \text{e.g.:} \quad \alpha_s \ln\left(\frac{m^2}{E^2}\right) \sim 1$$

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Solution:

two renormalization scales: $\boxed{\mu_s \sim mv, \ \mu_u \sim mv^2}$

correlation: $E \sim \frac{\mathbf{p}^2}{m} \to \mu_u \sim \frac{\mu_s^2}{m} \quad \longrightarrow \quad \mu_s = m\nu, \ \mu_u = m\nu^2 \quad \Rightarrow$ $\boxed{\text{'v'NRQCD}}$

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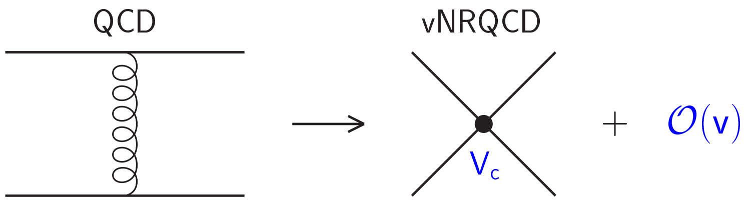
$\Rightarrow$ RGE's resum $\underbrace{[\alpha_s \ln v]^n}_{\text{LL}},\ \underbrace{\alpha_s [\alpha_s \ln v]^n}_{\text{NLL}},\ \underbrace{\alpha_s^2 [\alpha_s \ln v]^n}_{\text{NNLL}},\ \ldots$ terms

vNRQCD

- Nonresonant dof’s integrated out, e.g.:



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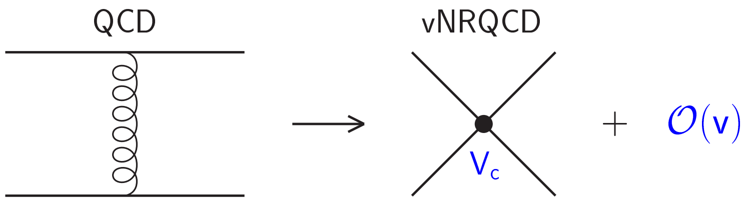
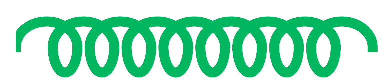


- Resonant dof's $\rightarrow$ fields in the vNRQCD Lagrangian:

nonrel. quark:	$(E, \mathbf{p}) \sim (mv^2, mv)$	$\psi_{\mathbf{p}}(x)$	
soft gluon:	$(q_0, \mathbf{q}) \sim (mv, mv)$	$A_{\mathbf{q}}(x)$	
ultrasoft gluon:	$(q_0, \mathbf{q}) \sim (mv^2, mv^2)$	$A(x)$	

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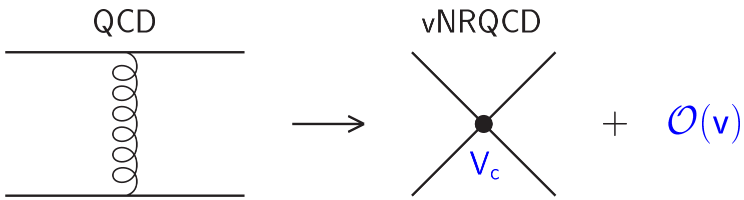

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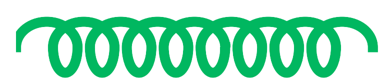
ultrasoft gluon:	$(q_0, \mathbf{q}) \sim (mv^2, mv^2)$	$A(x)$	

- Systematic expansion in v $\Rightarrow$ consistent power counting in $v \sim \alpha_s$

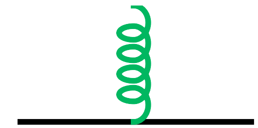
vNRQCD

[Luke, Manohar, Rothstein]

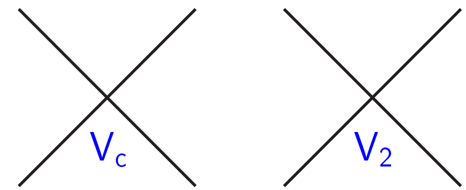
$$\boxed{\mathcal{L}_{\mathrm{vNRQCD}} = \mathcal{L}_{\mathrm{usoft}} + \mathcal{L}_{\mathrm{pot}} + \mathcal{L}_{\mathrm{soft}}}$$

$$D^{\mu} = \partial^{\mu} + igA^{\mu}(x)$$

$$\mathcal{L}_{\mathrm{usoft}} : \quad \psi^{\dagger}_{\mathbf{p}}(x)\left[iD^0 - \frac{(\mathbf{p}-i\mathbf{D})^2}{2m} + \ldots \right]\psi_{\mathbf{p}}(x) + \ldots$$



$$\mathcal{L}_{\mathrm{pot}} : \quad - V \; \psi^{\dagger}_{\mathbf{p}'}\psi_{\mathbf{p}}\chi^{\dagger}_{-\mathbf{p}'}\chi_{-\mathbf{p}} + \ldots$$



$$V \sim \frac{\mathcal{V}_c}{\mathbf{k}^2} + \frac{\mathcal{V}_k \pi^2}{m\mathbf{k}} + \frac{\mathcal{V}_r(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2}{m^2} + \frac{\mathcal{V}_s}{m^2}\mathbf{S}^2 + \ldots$$

$(\mathbf{k} = \mathbf{p}' - \mathbf{p})$

vNRQCD

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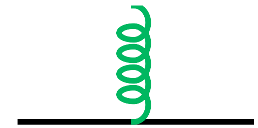

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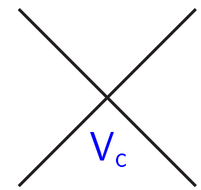
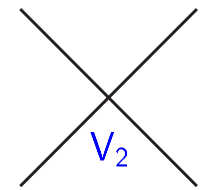
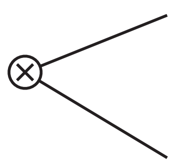




$$V \sim \frac{\mathcal{V}_c}{\mathbf{k}^2} + \frac{\mathcal{V}_k \pi^2}{m\mathbf{k}} + \frac{\mathcal{V}_r(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2}{m^2} + \frac{\mathcal{V}_s}{m^2}\mathbf{S}^2 + \ldots \qquad (\mathbf{k} = \mathbf{p}' - \mathbf{p})$$

external production/annihilation current (CMS):

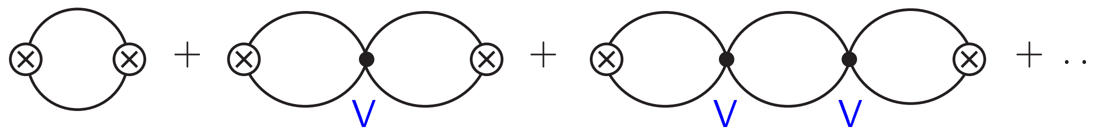


$$\sim \quad c_1(\nu) \cdot \underbrace{\vec{j}_1^{\,\text{eff}}(x)}_{\psi^\dagger_{\mathbf{p}}\vec{\sigma}(i\sigma_2)\chi^*_{-\mathbf{p}}} + \ldots$$

Renormalization of the Potentials

Anomalous dimension of V contributes to $\sigma(e^+ e^- \to t\,\bar{t}\,)$:

$\sigma_{\mathrm{tot}} \sim \mathrm{Im}\Big[$



$+ \cdots \Big]$

$$\sim \quad |c_1(\nu)|^2 \cdot \mathrm{Im}\left[-i \int d^4x\ e^{i\hat{q}x} \langle 0|\, T\, \vec{j}_1^{\,\mathrm{eff}\,*}(x)\, \vec{j}_1^{\,\mathrm{eff}}(0)\, |0\rangle \right]$$

$$\sim \quad |c_1(\nu)|^2 \cdot \mathrm{Im}\left[G(0, 0, E, \nu) \right]$$

G^{NNLL} known ✓

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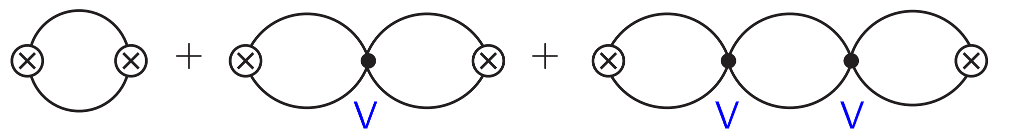

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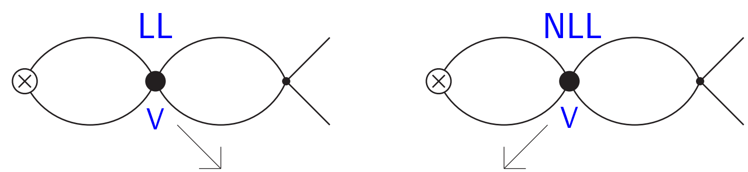
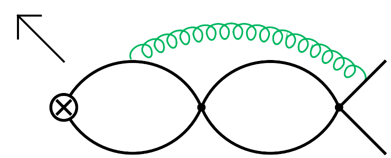
current renormalization



$$\ln\left[\frac{c_1(\nu)}{c_1(1)}\right] = \underbrace{\xi^{\,\rm LL}}_{0} + \xi^{\,\rm NLL} + \xi^{\rm NNLL}_{\rm mix} + \xi^{\rm NNLL}_{\rm non-mix}$$

[Luke, Manohar, Rothstein, Pineda, Hoang, Stewart]



[Hoang]

Renormalization of the Potentials

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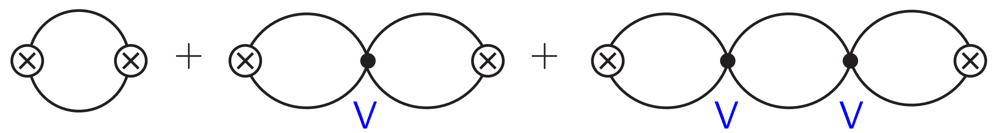

$$+ \cdots \Big]$$

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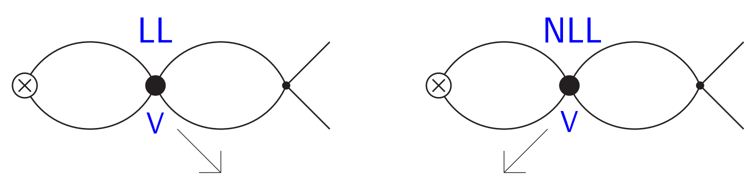

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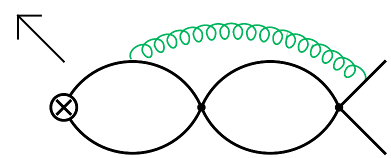

[Hoang]

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missing $\Rightarrow$ $\boxed{V^{\rm NLL}(\nu)}$ needed!

Renormalization of the Potentials

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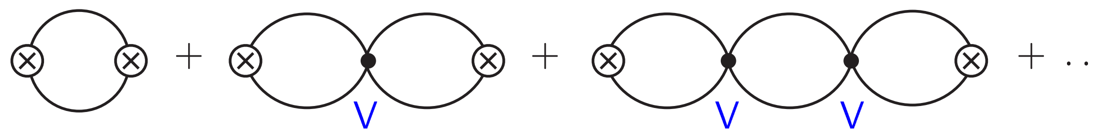
$\sigma_{\rm tot} \sim \mathrm{Im}\Big[$  $\Big]$

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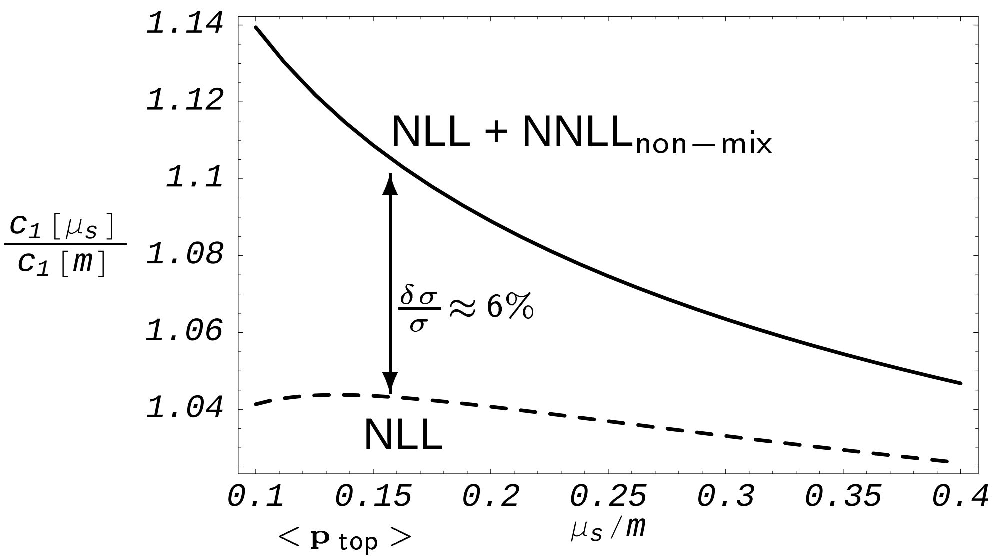


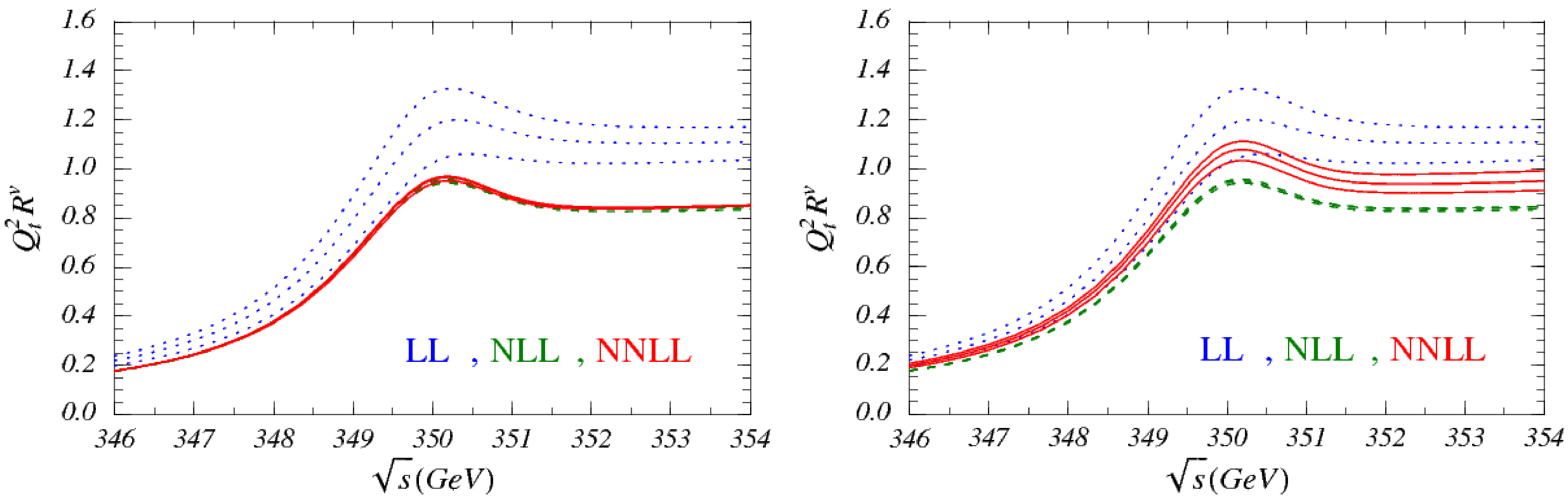
missing NNLL$_{\rm mix}$ contribution
may reduce theortical error of $\sigma_{\rm tot}$

$\Rightarrow$ $\boxed{V^{\rm NLL}(\nu)}$ needed!

Renormalization of the Potentials

NNLL$_{\rm nonmix}$ contribution to c_1 has large effect on $\sigma_{\rm tot}$:

[Hoang, Teubner]



NLL running of c_1 is used for NNLL

$\Rightarrow$ small scale (ν) dependence

NNLL$_{\rm nonmix}$ of c_1 is included in NNLL

$\Rightarrow$ large scale (ν) dependence

$\Rightarrow$ large theoretical uncertainty ($\sim 6\,\%$) of $\sigma_{\rm tot}$

Renormalization of the Potentials

$$\text{NLL}: \quad \nu\frac{\partial}{\partial\nu}\ln[\mathsf{c}_1(\nu)] \;=\; -\frac{\mathcal{V}_{\mathsf{c}}(\nu)}{16\pi^2}\left[\frac{\mathcal{V}_{\mathsf{c}}(\nu)}{4}+\mathcal{V}_2(\nu)+\mathcal{V}_{\mathsf{r}}(\nu)+\mathbf{S}^2\,\mathcal{V}_{\mathsf{s}}(\nu)\right]+\frac{1}{2}\mathcal{V}_{\mathsf{k}}(\nu)$$

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NLL (two loop) anomalous dimension of V:

- soft NLL running of V_s known: tiny effect [Pineda, Steinhauser, Penin, Smirnov]

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- Relevant potentials for $c_1(\nu)$ at NNLL_{mix} affected by NLL ultrasoft renormalization:

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$$\frac{\mathcal{V}_{\mathsf{k}}\pi^2}{\mathsf{m}\mathbf{k}}, \quad \frac{\mathcal{V}_{\mathsf{r}}(\mathbf{p}^2 + \mathbf{p}'^2)}{2\mathsf{m}^2\mathbf{k}^2}, \quad \frac{\mathcal{V}_2}{\mathsf{m}^2}$$

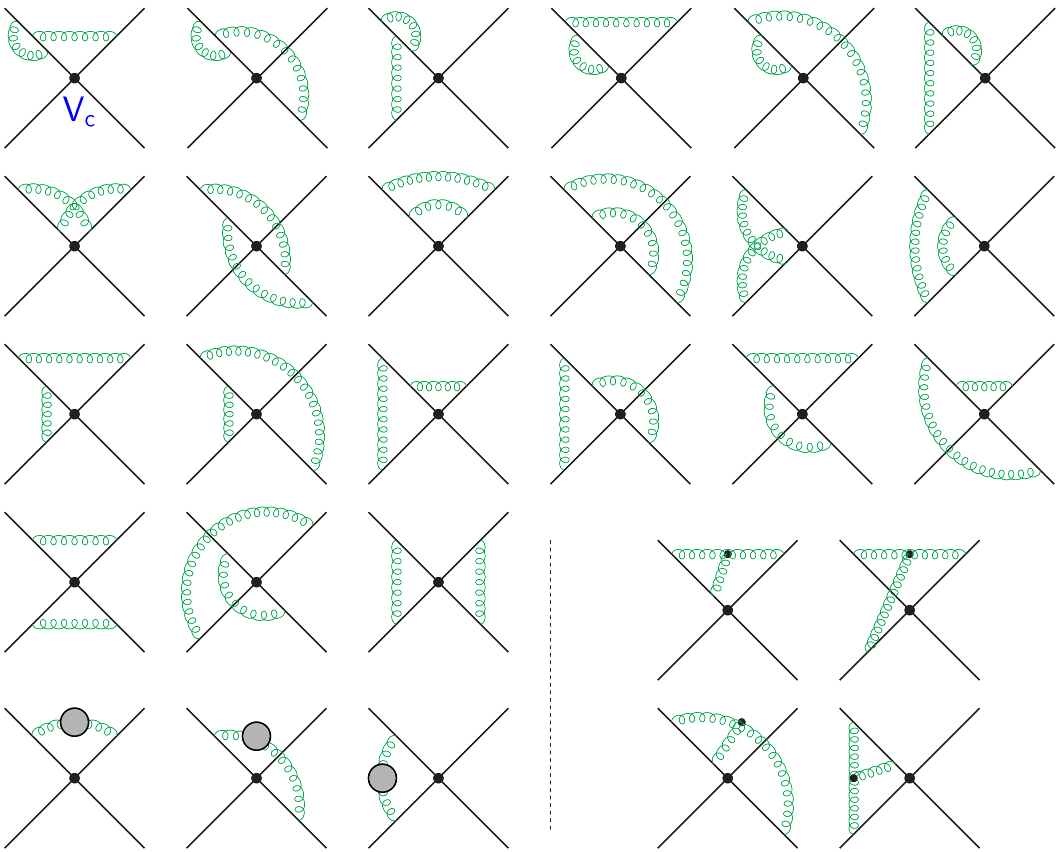
- $\mathcal{V}_{\mathsf{k}}^{\mathsf{LL}}$ is dominant contribution to $\mathsf{c}_1(\nu)^{\mathsf{NLL}}$. Same for $\mathcal{V}_{\mathsf{k}}^{\mathsf{NLL}}$ at NNLL_{mix}?

Renormalization of the $1/m^2$ - Potentials

$$\text{NLL}: \quad \nu\frac{\partial}{\partial\nu}\ln[c_1(\nu)] = -\frac{\mathcal{V}_c(\nu)}{16\pi^2}\left[\frac{\mathcal{V}_c(\nu)}{4} + \underbrace{\mathcal{V}_2(\nu)+\mathcal{V}_r(\nu)} + \mathbf{S}^2\,\mathcal{V}_s(\nu)\right] + \frac{1}{2}\mathcal{V}_k(\nu)$$

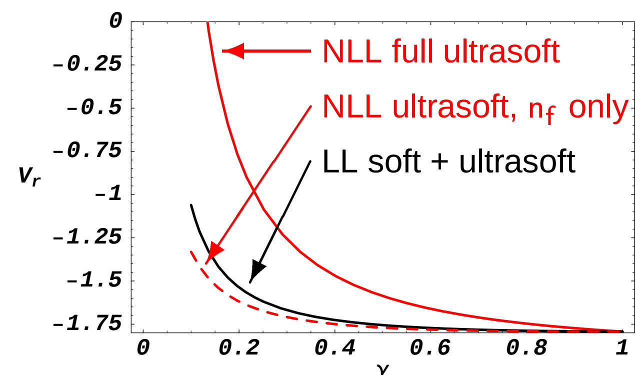
renormalize directly

$\mathcal{O}(10^3)$ diagrams



e.g.:

$$\Rightarrow \delta V_r^{2\,\text{loop}} \xrightarrow{\text{RGE}} V_r^{\text{NLL}}(\nu)$$



$\boxed{V_2^{\text{NLL}} \text{ and } V_r^{\text{NLL}} \text{ complete } \checkmark}$

[M.S., Hoang]

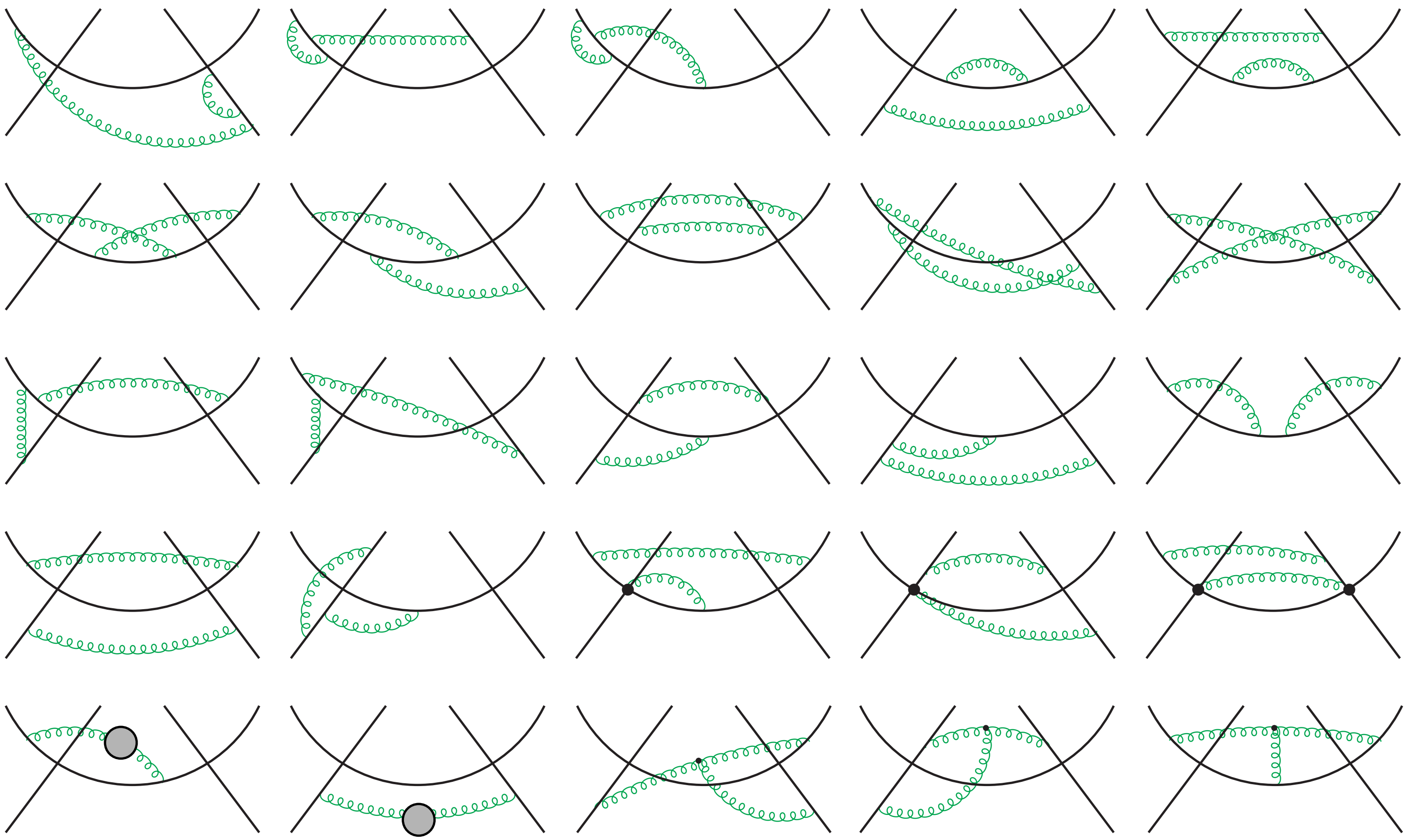
$\boxed{\text{usoft corrections} \sim v^2: \quad V_c \times (\text{usoft corr.}) \rightarrow \delta V_{2/r}}$ (general statement)

Renormalization of the $1/\mathrm{mk}$ - Potentials

$$\mathrm{NLL:}\quad \nu\frac{\partial}{\partial\nu}\ln[c_1(\nu)] \;=\; -\frac{\mathcal{V}_c(\nu)}{16\pi^2}\left[\frac{\mathcal{V}_c(\nu)}{4}+\mathcal{V}_2(\nu)+\mathcal{V}_r(\nu)+\mathbf{S}^2\,\mathcal{V}_s(\nu)\right]+\frac{1}{2}\underbrace{\mathcal{V}_k(\nu)}$$

renormalize indirectly

6 external HQ legs, $\mathcal{O}(10^4)$ diagrams

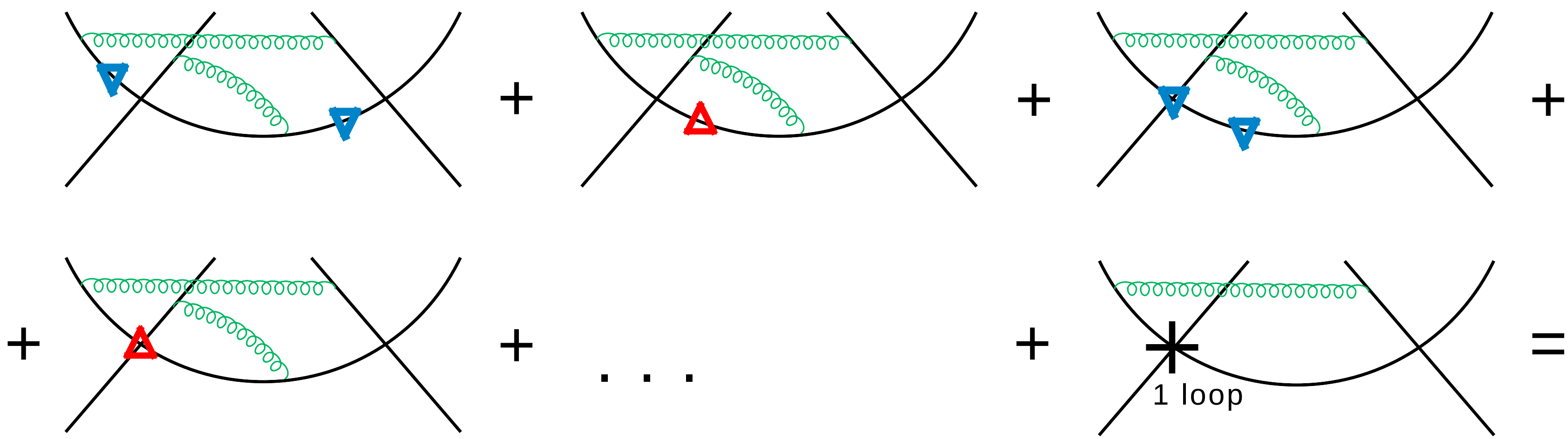


Work in progress !

usoft corrections $\sim v^2$

Renormalization of the $1/\mathrm{mk}$ - Potentials

Example for one topology: ($\overline{\mathrm{MS}}$, Dim.Reg.)

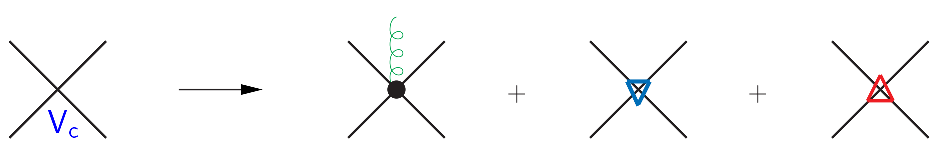


$$= \quad \frac{\alpha_U^2}{\epsilon^2} A + \frac{\alpha_U^2}{\epsilon} B \qquad [\alpha_U = \alpha_s(mv^2)]$$

Terms in $\mathcal{L}_{\mathrm{vNRQCD}}$:

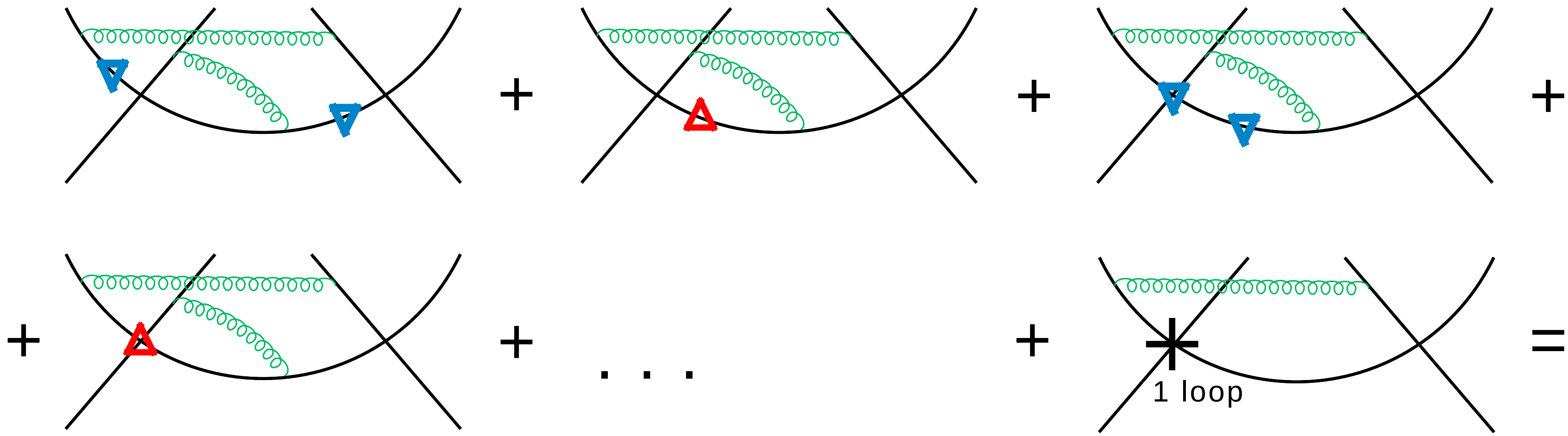
$$\psi_{\mathbf{p}}^\dagger \Big[iD^0 - \frac{(\mathbf{p}-i\mathbf{D})^2}{2m} + \ldots \Big] \psi_{\mathbf{p}} = \psi_{\mathbf{p}}^\dagger \Big[\underbrace{\Big(i\partial^0 - \frac{\mathbf{p}^2}{2m}\Big)}_{\text{HQ propagator}} - gA^0 + \frac{i\mathbf{p}\nabla}{m} + g\frac{\mathbf{p}\mathbf{A}}{m} + \frac{\nabla^2}{2m} + \ldots \Big] \psi_{\mathbf{p}}$$

similar for $\frac{\mathcal{V}_c}{\mathbf{k}^2}$:



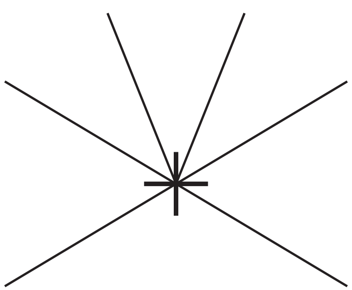
Renormalization of the $1/mk$ - Potentials

Example for one topology: ($\overline{\mathrm{MS}}$, Dim.Reg.)



$$= \quad \frac{\alpha_U^2}{\epsilon^2} A + \frac{\alpha_U^2}{\epsilon} B \qquad [\alpha_U = \alpha_s(mv^2)]$$

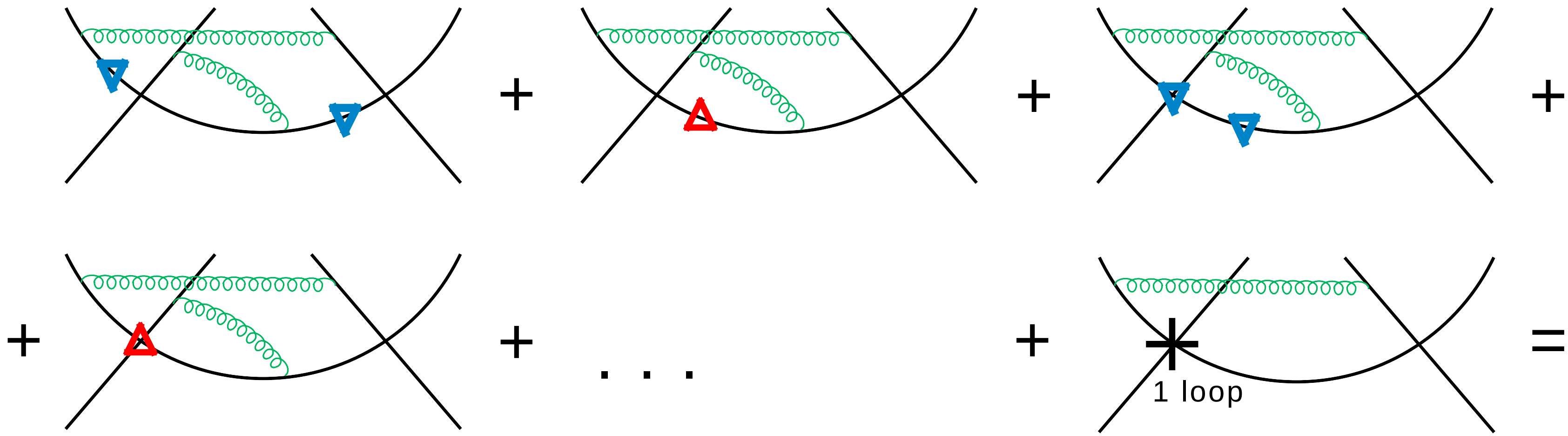
Absorb divergence by new 6Q - Operator:



$$= -\left(\frac{\alpha_U^2}{\epsilon^2} A + \frac{\alpha_U^2}{\epsilon} B \right)$$

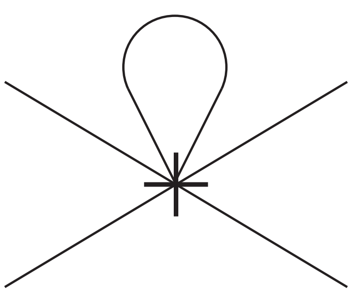
Renormalization of the $1/mk$ - Potentials

Example for one topology: ($\overline{\mathrm{MS}}$, Dim.Reg.)



$$= \quad \frac{\alpha_{\mathrm{U}}^2}{\epsilon^2}\mathrm{A} + \frac{\alpha_{\mathrm{U}}^2}{\epsilon}\mathrm{B} \qquad [\alpha_{\mathrm{U}} = \alpha_{\mathrm{s}}(\mathrm{m}v^2)]$$

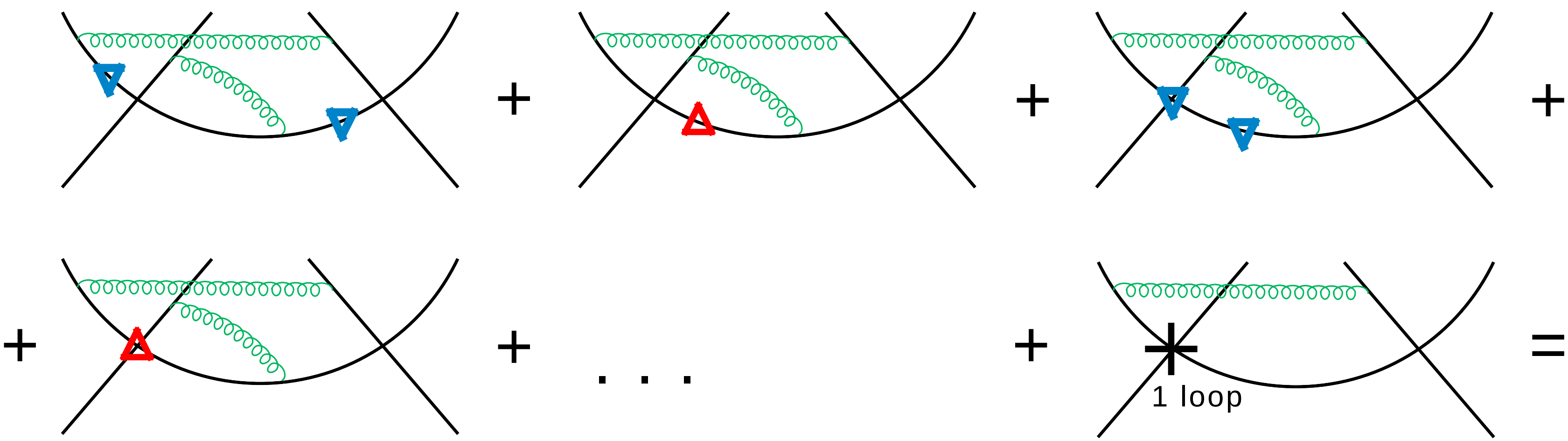
Close finite HQ - Loop:



$$= \boxed{\frac{\alpha_{\mathrm{U}}^2}{\epsilon^2}\mathrm{A}' + \frac{\alpha_{\mathrm{U}}^2}{\epsilon}\mathrm{B}'}$$

Renormalization of the $1/\mathrm{mk}$ - Potentials

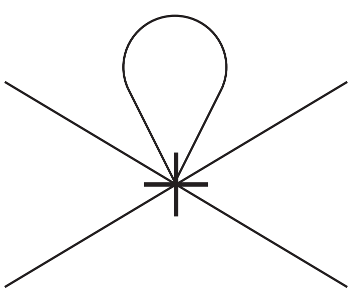
Example for one topology: ($\overline{\mathrm{MS}}$, Dim.Reg.)



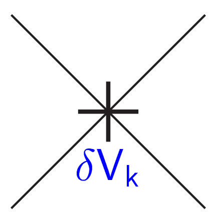
$$= \quad \frac{\alpha_U^2}{\epsilon^2}\mathsf{A} + \frac{\alpha_U^2}{\epsilon}\mathsf{B}$$

$[\alpha_U = \alpha_s(mv^2)]$

Close finite HQ - Loop:



$$= \boxed{\frac{\alpha_U^2}{\epsilon^2}\mathsf{A}' + \frac{\alpha_U^2}{\epsilon}\mathsf{B}'} \longrightarrow$$



$$\boxed{\delta V_k^{2\text{loop}} \xrightarrow{\text{RGE}} \mathcal{V}_k^{\text{NLL}}(\nu)}$$

Status of Calculations / Summary

- Ultrasoft NLL running of the potentials V_k, V_r, V_2 is essential for a precise prediction of $\sigma_{\mathrm{tot}}(e^+ e^- \to t\,\bar{t})$ at threshold.

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  $\delta c_1 = (-1.9\%, -0.5\%)$ for $\nu = 0.1, 0.2$

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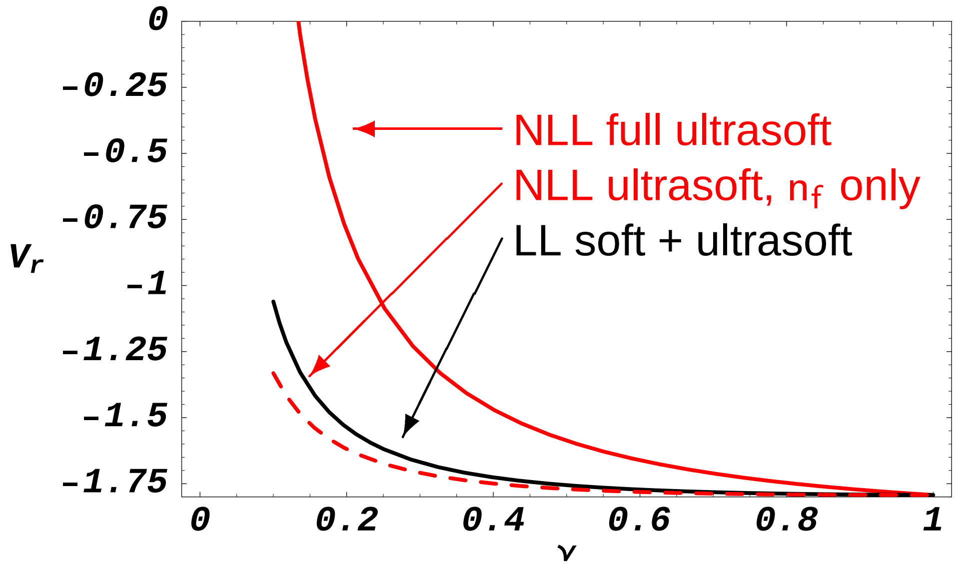
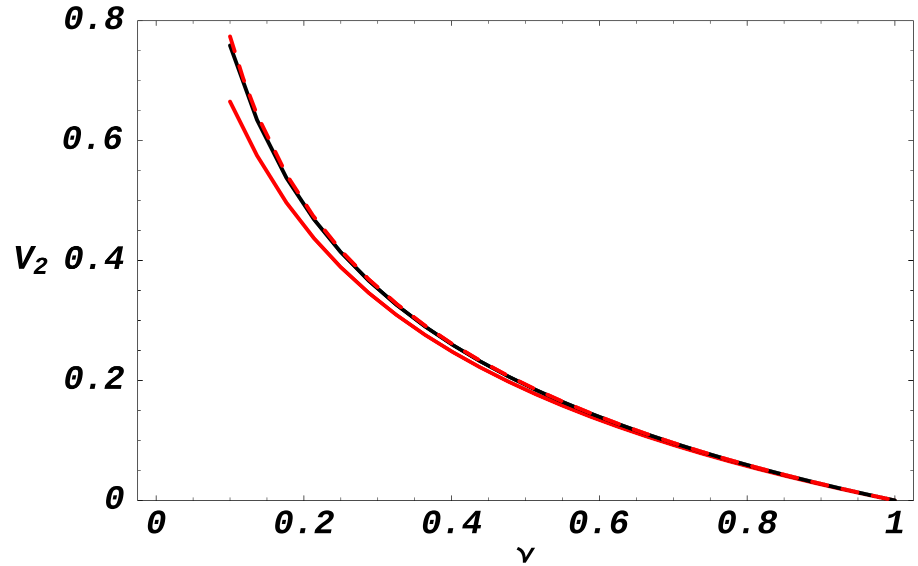
- What about $\mathcal{V}_k$ (dominant at NLL)? $\longrightarrow$ w.i.p.
- Current status of the calculation: $[\alpha_S = \alpha_s(mv),\, \alpha_U = \alpha_s(mv^2)]$

Contribution	order $/\alpha_S$	V_k	V_r	V_2	V_s
soft + usoft LL	$(\alpha_S \ln v)^n$, $(\alpha_U \ln v)^n$	✓	✓	✓	✓
usoft NLL n_f	$n_f\, \alpha_U (\alpha_U \ln v)^n$	✓	✓	✓	0
full usoft NLL	$\alpha_U (\alpha_U \ln v)^n$	w.i.p.	✓	✓	0
soft NLL	$\alpha_S (\alpha_S \ln v)^n$	–	–	–	✓

Back Up: Status of Calculations

Results for $\frac{1}{m^2}$ potentials:





Back Up: Status of Calculations

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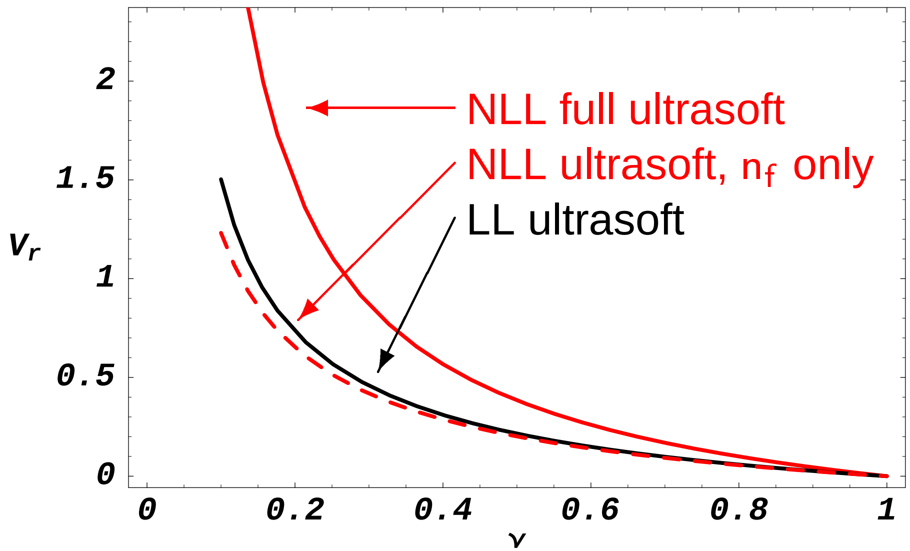

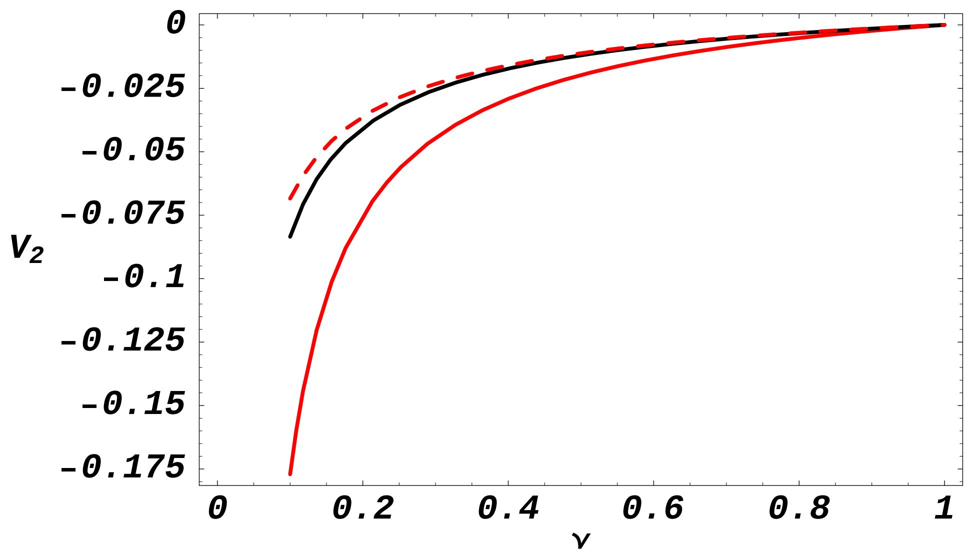

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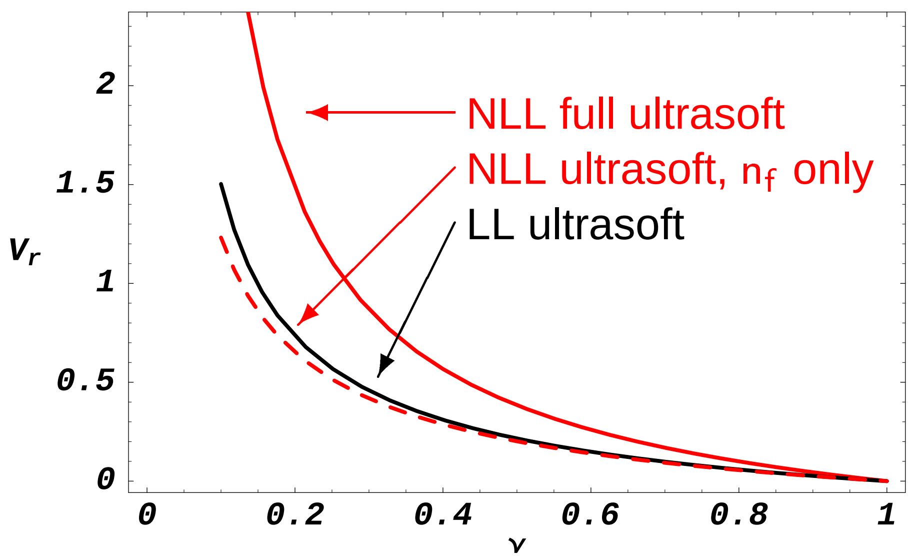
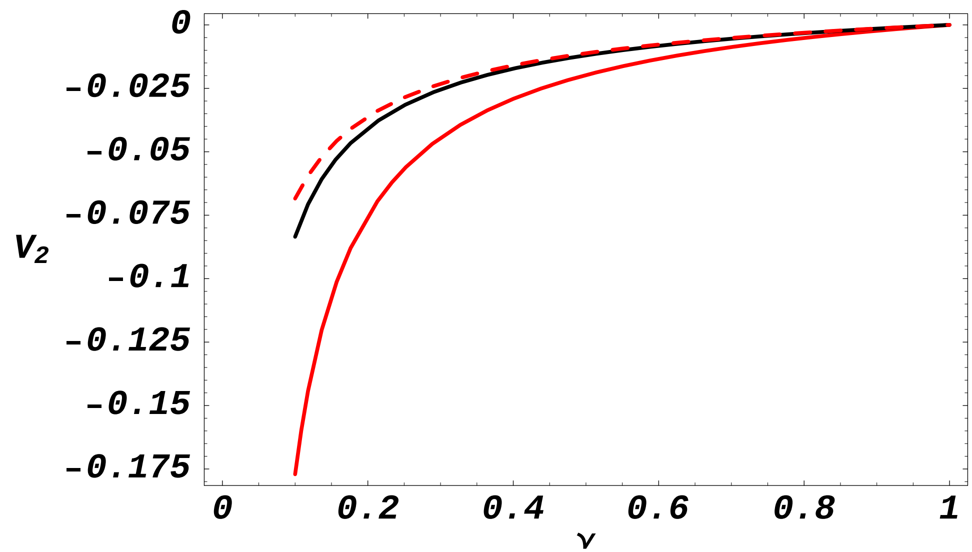
Results for $\frac{1}{m^2}$ potentials:

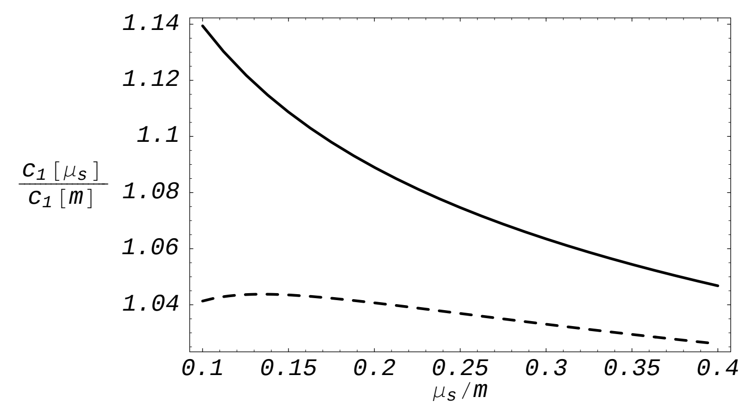




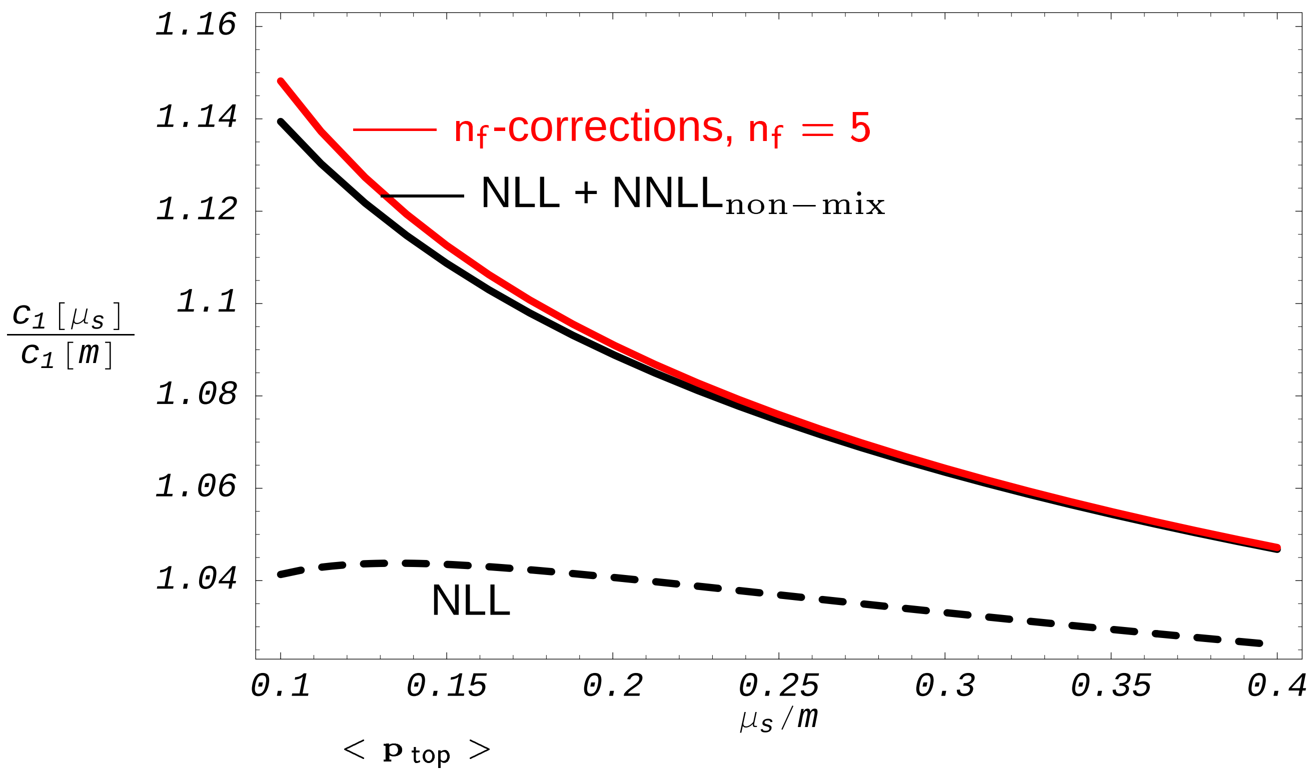
- Analysis shows that $\boxed{\text{usoft LL} \sim \text{usoft NLL}}$

$\Rightarrow$ Big NNLL_{mix} contributions to c_1 expected

$\Rightarrow$ may compensate $\text{NNLL}_{\text{nonmix}}$ and reduce ν dependence of c_1!



Back Up: Old n_f Result



Back Up: Extra Formulae

$$
\begin{aligned}
\nu \frac{\partial}{\partial \nu} \ln[c_1(\nu)] \quad = \quad & -\frac{\mathcal{V}_c^{(0)}(\nu)}{16\pi^2}\left[\frac{\mathcal{V}_c^{(0)}(\nu)}{4} + \mathcal{V}_2^{(2)}(\nu) + \mathcal{V}_r^{(2)}(\nu) + \mathbf{S}^2\, \mathcal{V}_s^{(2)}(\nu)\right] \\
& + \frac{1}{2}\mathcal{V}_k^{(1)}(\nu) + \alpha_s^2(m\nu)\left[3\mathcal{V}_{k1}^{(1)}(\nu) + 2\mathcal{V}_{k2}^{(1)}(\nu)\right]
\end{aligned}
$$

$$
v \cong \alpha_s(mv) = \frac{4\pi}{\beta_0 \ln(m^2 v^2/\Lambda_{QCD}^2)} \quad \Rightarrow \quad v \cong \alpha_s \cong 0.14
$$

$$
v \equiv \sqrt{\frac{\sqrt{s}-2m_t}{m_t}} \to \sqrt{\frac{\sqrt{s}-2m_t+i\Gamma_t}{m_t}}
$$

[Fadin, Khoze]