# Finite lifetime effects in nonrelativistic top pair production

Christoph Reißer

Max-Planck-Institut für Physik, Munich

in collaboration with
André H. Hoang and Pedro Ruiz-Femenía

# Outline

- Motivation
- Top instability, electroweak effects
- Phase space matching
- Numerical analysis
- Outlook, Summary

# Nonrelativistic top pairs

$e^+e^-$ collisions: c.m. energy $\sqrt{s} \approx 340 - 360$ GeV

- Top quarks are nonrelativistic $\boxed{v = \sqrt{1 - \frac{4m_t^2}{s}} \ll 1}$

$\Rightarrow$ Perturbation theory in $\alpha_s$ breaks down $\left(\frac{\alpha_s}{v}\right)^n \sim 1$

$\Rightarrow$ Nonrelativistic QCD $\simeq$ Schrödinger theory at LO

# Nonrelativistic top pairs

$e^+e^-$ collisions: c.m. energy $\sqrt{s} \approx 340 - 360$ GeV

- Top quarks are nonrelativistic $v = \sqrt{1 - \frac{4m_t^2}{s}} \ll 1$

  $\Rightarrow$ Perturbation theory in $\alpha_s$ breaks down $\left(\frac{\alpha_s}{v}\right)^n \sim 1$

  $\Rightarrow$ Nonrelativistic QCD $\simeq$ Schrödinger theory at LO

- Top quarks decay fast: $t \to Wb$ $\Gamma_t \approx 1.5\ \text{GeV} \gg \Lambda_{\text{QCD}}$

  $\Rightarrow$ No bound states

  $\Rightarrow$ Non-perturbative effects suppressed Fadin, Khoze (JETP Lett. 46, 1987)

# Nonrelativistic top pairs

$e^+e^-$ collisions: c.m. energy $\sqrt{s} \approx 340 - 360$ GeV

- Top quarks are nonrelativistic $v = \sqrt{1 - \frac{4m_t^2}{s}} \ll 1$

  $\Rightarrow$ Perturbation theory in $\alpha_s$ breaks down $\left(\frac{\alpha_s}{v}\right)^n \sim 1$

  $\Rightarrow$ Nonrelativistic QCD $\simeq$ Schrödinger theory at LO

- Top quarks decay fast: $t \to Wb$ $\Gamma_t \approx 1.5\ \mathrm{GeV} \gg \Lambda_{QCD}$

  $\Rightarrow$ No bound states

  $\Rightarrow$ Non-perturbative effects suppressed Fadin, Khoze (JETP Lett. 46, 1987)

  $\longrightarrow$ Instead of process $e^+e^- \to t\bar{t}$ consider $e^+e^- \to bW^+\bar{b}W^-$ or even include W decay products

  $\longrightarrow$ Interferences of double and single resonant diagrams

  $\longrightarrow$ New theoretical concepts for treatment beyond LO

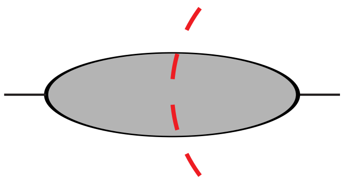
# vNRQCD (stable quarks)

Total cross section from $e^+e^- \to e^+e^-$ using the Optical Theorem

Strassler, Peskin (Phys. Rev. D 43, 1991)

$$\sigma_{\rm tot} \quad \propto \quad \mathrm{Im}\left[ i \sum_{\mathbf{p},\mathbf{p}'} \int d^4x\, e^{-i\hat{q}\cdot x} \left\langle 0 \middle| T\left( C(\mu)\mathbf{O}^\dagger_{\mathbf{p}}(0)\right)\left( C(\mu)\mathbf{O}_{\mathbf{p}'}(x)\right) \middle| 0 \right\rangle \right]$$

$$\propto \quad \mathrm{Im}\left[ C(\mu)^2\, G(0,0,E) \right]$$



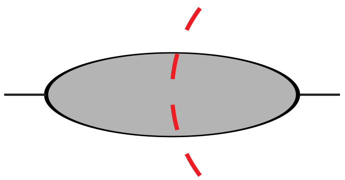
# vNRQCD (stable quarks)

Total cross section from $e^+e^- \to e^+e^-$ using the Optical Theorem

Strassler, Peskin (Phys. Rev. D 43, 1991)

$$\sigma_{\rm tot} \quad \propto \quad {\rm Im}\left[ i \sum_{\mathbf{p},\mathbf{p}'} \int d^4x\, e^{-i\hat{q}\cdot x} \left\langle 0 \left| T \left( C(\mu) \mathbf{O}^\dagger_{\mathbf{p}}(0) \right) \left( C(\mu) \mathbf{O}_{\mathbf{p}'}(x) \right) \right| 0 \right\rangle \right]$$

$$\propto \quad {\rm Im}\left[ C(\mu)^2\, G(0,0,E) \right]$$



$$\left( -\frac{\nabla^2}{m_t} - \frac{\nabla^4}{4m_t^3} + V(\mathbf{r}) - E \right) G(\mathbf{r},\mathbf{r}',E) = \delta^{(3)}(\mathbf{r}-\mathbf{r}')$$

# Theory status

## QCD effects

$\left(\frac{\alpha_s}{v}\right)^n \sim 1$ $\quad$ $(\alpha_s \ln v)^m \sim 1$

- LL ✓
- NLL ✓
- NNLL (almost) $\quad \rightarrow \delta\sigma_{\rm tot}/\sigma_{\rm tot} \sim \pm 6\,\%$ $\quad \rightarrow$ talk by Maximilian Stahlhofen

# Theory status

## QCD effects

$\left(\frac{\alpha_s}{v}\right)^n \sim 1$ $\qquad$ $(\alpha_s \ln v)^m \sim 1$

- LL ✓
- NLL ✓
- NNLL (almost) $\quad \rightarrow \delta\sigma_{\rm tot}/\sigma_{\rm tot} \sim \pm 6\,\%$ $\quad \rightarrow$ talk by Maximilian Stahlhofen

## Electroweak effects

$\Gamma_t \sim m_t \alpha \approx E_{\rm kin} \sim m_t \alpha_s^2$

- LL: $\quad E \rightarrow E + i\Gamma_t$ ✓
- NLL: – NNLO phase space divergencies $\Rightarrow$ NLL RG effects
  – phase space matching $\rightarrow$ w.i.p.
- NNLL: – real matrixelement corrections
  – imaginary matrixelement corrections $\rightarrow$ interference diagrams
  – NNLL running from phase space divergencies $\rightarrow$ not yet started

# Unstable quarks in vNRQCD

## Effective theory for unstable particles

- Replacement rule $E \rightarrow E + i\Gamma_t$ at LL    Fadin, Khoze (JETP Lett. 46, 1987)
- Complex matching conditions
  $\longrightarrow$ at NNLL contain interferences
  $\longrightarrow$ UV phase space divergencies
  $\longrightarrow$ Phase space matching necessary
- Effective Lagrangian non-hermitian
- Total cross section through the optical theorem using unitarity of the underlying theory

# Unstable quarks in vNRQCD

## Effective theory for unstable particles

- Replacement rule $E \to E + i\Gamma_t$ at LL   Fadin, Khoze (JETP Lett. 46, 1987)
- Complex matching conditions
  $\longrightarrow$ at NNLL contain interferences
  $\longrightarrow$ UV phase space divergencies
  $\longrightarrow$ Phase space matching necessary
- Effective Lagrangian non-hermitian
- Total cross section through the optical theorem using unitarity of the underlying theory

$\Rightarrow$ Contributions from real $Wb$ final states included in EFT matching conditions

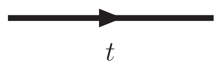
$\Rightarrow$ EFT does not describe details of decay mechanism
$\longrightarrow$ inclusive treatment

$\gg$ In analogy to absorptive processes in the optical theory

# Unstable quarks in vNRQCD

Power counting: $D^0 \sim m_t v^2, \quad \mathbf{p}^2 \sim m_t^2 v^2, \quad \Gamma_t \sim m_t \alpha \sim m_t \alpha_s^2$

Full theory:




$\bar{\psi}(\not{p} - m_t)\psi$

↓

Effective theory:

$$\mathcal{L}_{\rm bil} = \psi_{\mathbf{p}}^\dagger(x)\left[iD^0 - \frac{\mathbf{p}^2}{2m_t} + \frac{\mathbf{p}^4}{8m_t^3}\right]\psi_{\mathbf{p}}(x)$$

$\sim m_t v^2$

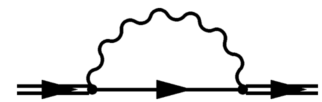
stable propagator: $\dfrac{i}{p^0 - \frac{\mathbf{p}^2}{2m_t}}$

# Unstable quarks in vNRQCD

Power counting: $D^0 \sim m_t v^2, \quad \mathbf{p}^2 \sim m_t^2 v^2, \quad \Gamma_t \sim m_t \alpha \sim m_t \alpha_s^2$

Full theory:


$\bar{\psi}(\not{p} - m_t)\psi$

$i\,\Sigma_t =$ 


$\mathrm{Im}\,\Sigma_t = \frac{\Gamma_t}{2}$

Effective theory:

$$\mathcal{L}_{\rm bil} = \psi_{\mathbf{p}}^\dagger(x)\left[iD^0 - \frac{\mathbf{p}^2}{2m_t} + \frac{\mathbf{p}^4}{8m_t^3}\right]\psi_{\mathbf{p}}(x), \;\; \delta\mathcal{L}_{\rm bil} = \psi_{\mathbf{p}}^\dagger\left[\delta m_t + i\frac{\Gamma_t}{2} - i\frac{\Gamma_t}{2}\frac{\mathbf{p}^2}{2m_t^2}\right]\psi_{\mathbf{p}}$$

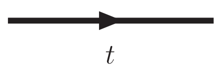
$\sim m_t v^2$

stable propagator: $\dfrac{i}{p^0 - \frac{\mathbf{p}^2}{2m_t}}$

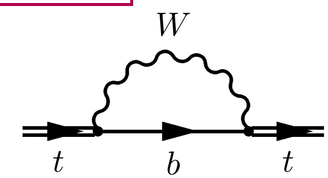
# Unstable quarks in vNRQCD

Power counting: $D^0 \sim m_t v^2, \quad \mathbf{p}^2 \sim m_t^2 v^2, \quad \Gamma_t \sim m_t \alpha \sim m_t \alpha_s^2$

Full theory:




$\bar{\psi}(\not{p} - m_t)\psi$

$i\,\Sigma_t =$ 


$\mathrm{Im}\,\Sigma_t = \frac{\Gamma_t}{2}$

Effective theory:

$$\mathcal{L}_{\mathrm{bil}} = \psi_{\mathbf{p}}^\dagger(x)\left[iD^0 - \frac{\mathbf{p}^2}{2m_t} + \frac{\mathbf{p}^4}{8m_t^3}\right]\psi_{\mathbf{p}}(x), \quad \delta\mathcal{L}_{\mathrm{bil}} = \psi_{\mathbf{p}}^\dagger\left[\delta m_t + i\frac{\Gamma_t}{2} - i\frac{\Gamma_t}{2}\frac{\mathbf{p}^2}{2m_t^2}\right]\psi_{\mathbf{p}}$$

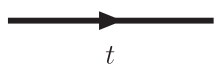
$\sim m_t v^2$ $\quad$ $v \sim \alpha_s$ $\quad$ $\sim m_t \alpha_s^2$ $\quad$ $\sim m_t \alpha_s^4$

stable propagator: $\dfrac{i}{p^0 - \frac{\mathbf{p}^2}{2m_t}}$

# Unstable quarks in vNRQCD

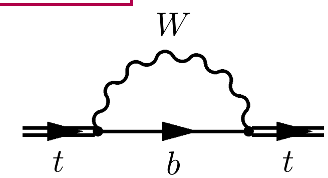
Power counting: $D^0 \sim m_t v^2, \quad \mathbf{p}^2 \sim m_t^2 v^2, \quad \Gamma_t \sim m_t \alpha \sim m_t \alpha_s^2$

Full theory:




$\bar{\psi}(\not{p} - m_t)\psi$

$i\,\Sigma_t =$




$\mathrm{Im}\,\Sigma_t = \frac{\Gamma_t}{2}$

Effective theory:

$$\mathcal{L}_{\rm bil} = \psi_{\mathbf{p}}^\dagger(x)\left[iD^0 - \frac{\mathbf{p}^2}{2m_t} + \frac{\mathbf{p}^4}{8m_t^3}\right]\psi_{\mathbf{p}}(x), \quad \delta\mathcal{L}_{\rm bil} = \psi_{\mathbf{p}}^\dagger\left[\delta m_t + i\frac{\Gamma_t}{2} - i\frac{\Gamma_t}{2}\frac{\mathbf{p}^2}{2m_t^2}\right]\psi_{\mathbf{p}}$$

$\sim m_t v^2$ $\qquad$ $v \sim \alpha_s$ $\qquad$ $\sim m_t \alpha_s^2$ $\qquad$ $\sim m_t \alpha_s^4$

stable propagator: $\dfrac{i}{p^0 - \frac{\mathbf{p}^2}{2m_t}}$

unstable propagator:

$$\frac{i}{p^0 - \frac{\mathbf{p}^2}{2m_t} + i\frac{\Gamma_t}{2}}$$

NNLL time dilatation correction



# Instability beyond LL (inclusive)

Quark bilinear operators:

- Dilatation of lifetime at NNLL
- $\mathcal{O}(\alpha_s)$, $\mathcal{O}(\alpha_s^2)$, $\mathcal{O}(\alpha)$ corrections to $\Gamma_t$ at NLL and NNLL

  Jeżabek, Kühn; Blokland, Czarnecki, Ślusarczyk, Tkachov

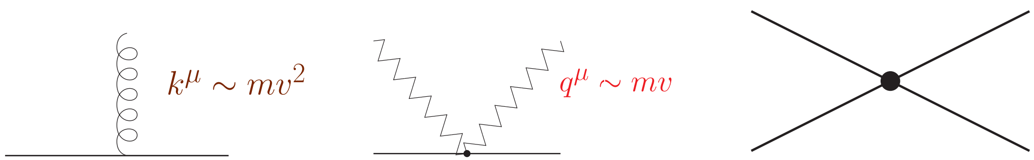
# Instability beyond LL (inclusive)

## Quark bilinear operators:

- Dilatation of lifetime at NNLL
- $\mathcal{O}(\alpha_s)$, $\mathcal{O}(\alpha_s^2)$, $\mathcal{O}(\alpha)$ corrections to $\Gamma_t$ at NLL and NNLL

  Jeżabek, Kühn; Blokland, Czarnecki, Ślusarczyk, Tkachov

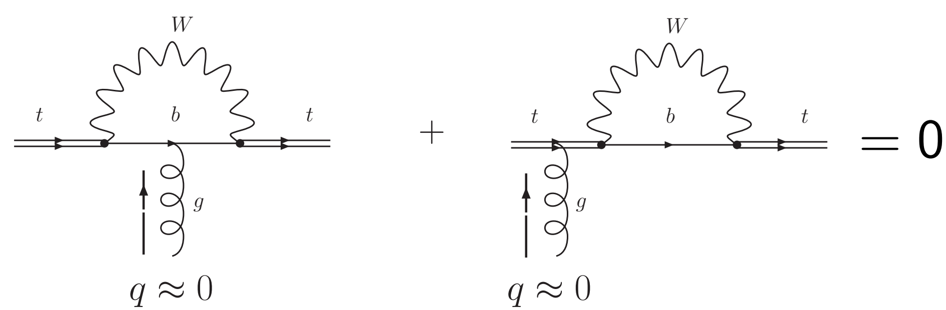
## Gluon interactions and potentials:




- electroweak corrections either beyond NNLL or contributions to $\sigma_{\rm tot}$ cancel due to gauge invariance

  Melnikov, Yakovlev (Phys. Lett. B324, 1994)

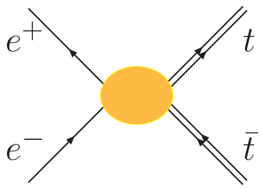
  Fadin, Khoze, Martin, Stirling (1995)



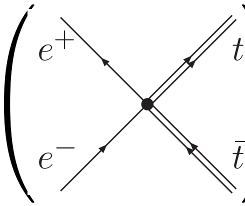
# Electroweak matching beyond LL

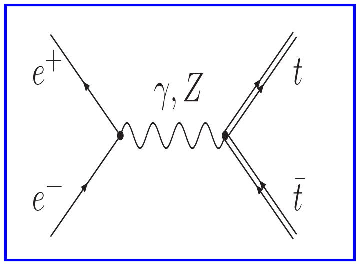
Currents:

$m_t\,\alpha \sim m_t\,\alpha_s^2$




$$= \left[ C_{LL}^{born} + C_{NLL}^{QCD} + C_{NNLL}^{QCD} + C_{NNLL}^{ew} + i\,C_{NNLL}^{bW,abs} + \ldots \right] \cdot$$

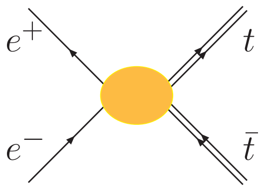


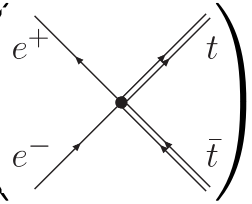
# Electroweak matching beyond LL

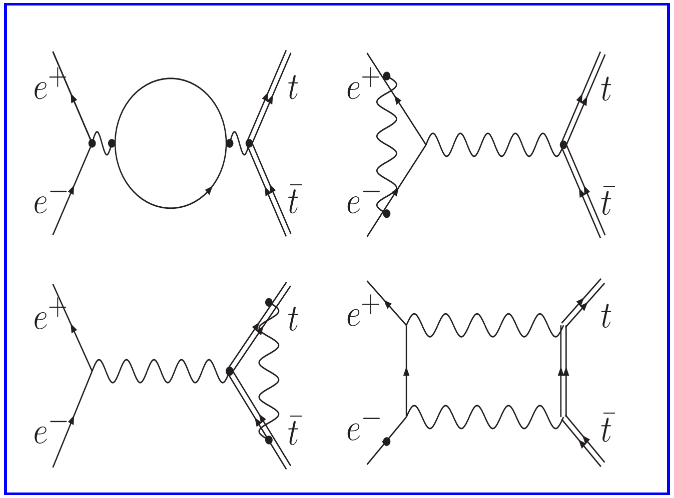
Currents:

$m_t\,\alpha \sim m_t\,\alpha_s^2$




$$= \Big[ C_{\rm LL}^{\rm born} + C_{\rm NLL}^{\rm QCD} + C_{\rm NNLL}^{\rm QCD} + C_{\rm NNLL}^{\rm ew} + i\, C_{\rm NNLL}^{\rm bW,abs} + \dots \Big] \cdot \Big( \quad \Big)$$



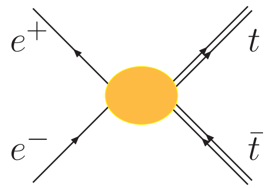

Real parts of ew. 1-loop diagrams $\mathcal{O}(\alpha)$
(pure QED diagrams not included)

$\Rightarrow$ NNLL hard usual
electroweak effects

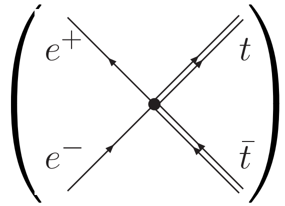
# Electroweak matching beyond LL

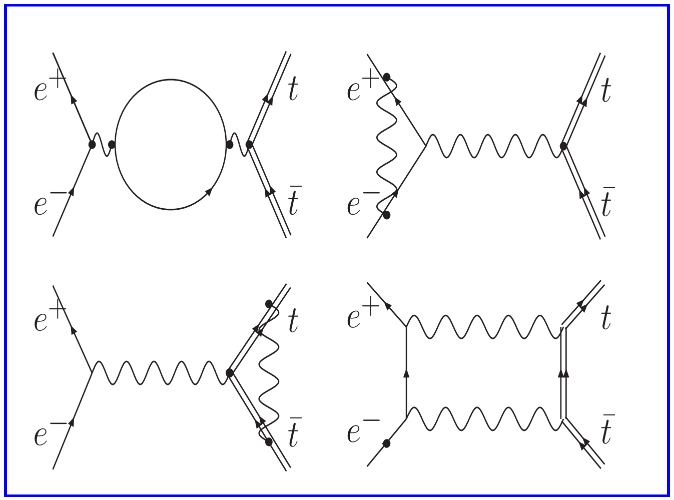
## Currents:

$m_t\,\alpha \sim m_t\,\alpha_s^2$




$$= \Big[ C_{LL}^{born} + C_{NLL}^{QCD} + C_{NNLL}^{QCD} + C_{NNLL}^{ew} + i\,C_{NNLL}^{bW,abs} + \dots \Big] \cdot \Big( \quad \Big)$$

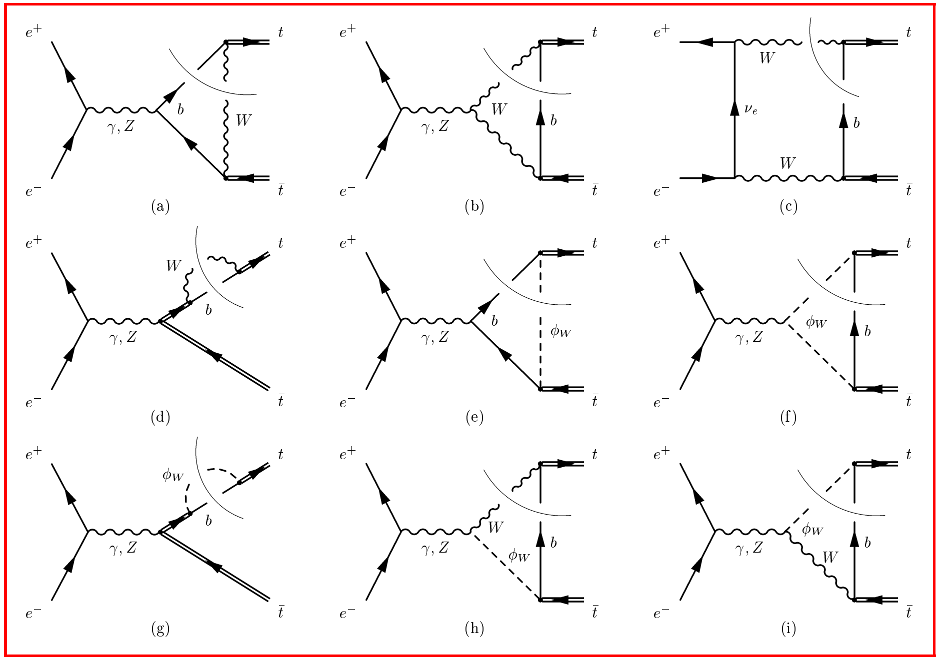



Real parts of ew. 1-loop diagrams $\mathcal{O}(\alpha)$

(pure QED diagrams not included)

$\Rightarrow$ NNLL hard usual electroweak effects



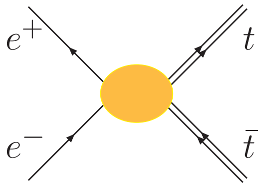

bW-cuts of electroweak 1-loop diagrams $\mathcal{O}(\alpha)$

bW-cuts are gauge invariant

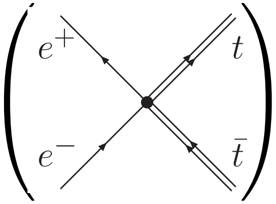
bW treated as stable particles

$\Rightarrow$ NNLL interference effects

# Phase space divergence




$$= \Big[ C_{LL}^{\mathrm{born}} + C_{NLL}^{\mathrm{QCD}} + C_{NNLL}^{\mathrm{QCD}} + C_{NNLL}^{\mathrm{ew}} + i\, C_{NNLL}^{\mathrm{bW,abs}} + \ldots \Big] \cdot$$

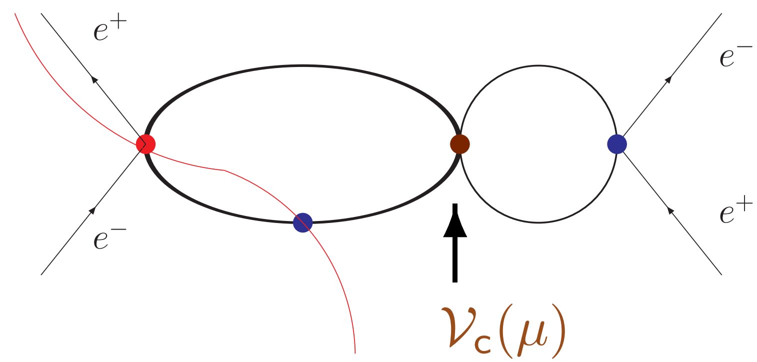



- NNLL interference correction

$$\Delta^{\Gamma,1}\sigma_{\mathrm{tot}} = 2\, N_c \Big\{ 2\, C_{LL}^{\mathrm{born}}\, C_{NNLL}^{\mathrm{abs,bW}}\, \mathrm{Re}[G_{LL}] + \ldots \Big\}$$

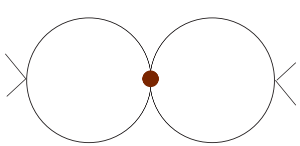
contains logarithmic UV phase space divergence

$$\boxed{C_{NNLL}^{\mathrm{abs,bW}}\, \mathcal{V}_c(\mu)\, \frac{1}{\epsilon}} \quad \sim$$



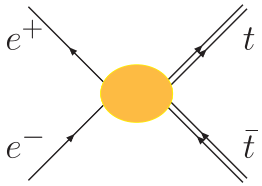

$$\mathcal{V}_c(\mu) = -4\pi C_F \alpha_s(\mu)$$

from $\mathcal{O}(\alpha_s)$ term in Green function

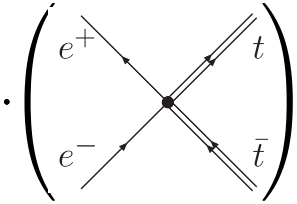


$$\mathrm{i}_{LL}^{\mathcal{O}(\alpha_s)} = \alpha_s(\mu)\, C_F \frac{m_t^2}{4\pi} \left[ \frac{1}{4\epsilon} - \ln\left( \frac{-i m_t v}{\mu} \right) + \frac{1}{2} - \ln 2 \right]$$

# Phase space divergence




$$= \Big[ \mathsf{C}_{\mathrm{LL}}^{\mathrm{born}} + \mathsf{C}_{\mathsf{NLL}}^{\mathrm{QCD}} + \mathsf{C}_{\mathsf{NNLL}}^{\mathrm{QCD}} + \mathsf{C}_{\mathsf{NNLL}}^{\mathrm{ew}} + \mathsf{i}\,\mathsf{C}_{\mathsf{NNLL}}^{\mathrm{bW,abs}} + \ldots \Big] \cdot$$

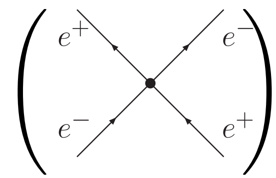



- NNLL interference correction

$$\Delta^{\Gamma,1}\sigma_{\mathrm{tot}} = 2\,\mathsf{N_c} \Big\{ 2\,\mathsf{C}_{\mathrm{LL}}^{\mathrm{born}}\,\mathsf{C}_{\mathrm{NNLL}}^{\mathsf{abs,bW}}\,\mathrm{Re}[\mathsf{G}_{\mathrm{LL}}] + \ldots \Big\}$$

contains logarithmic UV phase space divergence

- NLL mixing effect: Anomalous dimension: $\mathsf{i}\,\mathsf{C}(\mu) \cdot$




≫ Running correction $\Delta^{\Gamma,2}\sigma_{\mathrm{tot}}$

– $\sqrt{\mathsf{s}}$-independent

– scale-dependent

≫ Matching coefficient

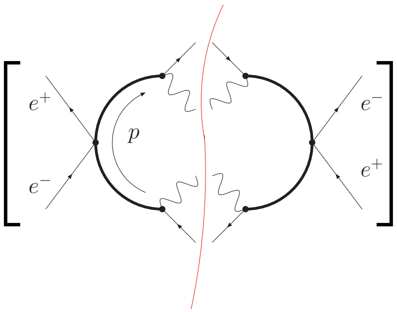
$\mathsf{C}(\mu = \mathsf{m_t}, \Lambda)$

determination by

phase space matching

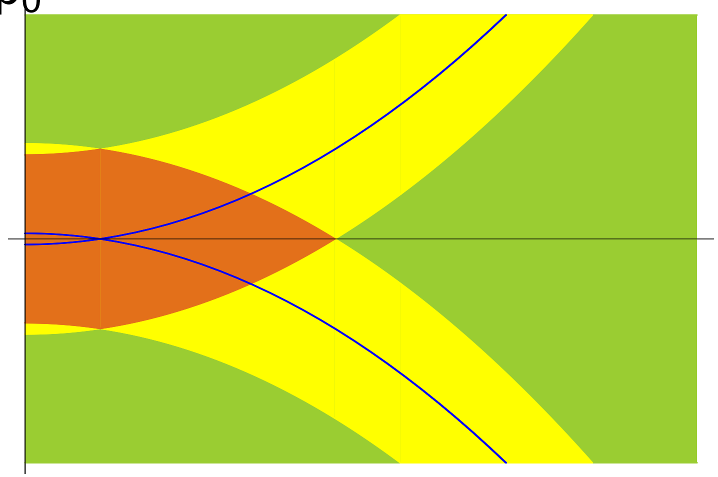
# Phase space matching

Hoang, Ruiz-Femenía, CJR (w.i.p.)

$$\sigma_{\rm tot} \sim \Bigg[$$  $$\Bigg]$$

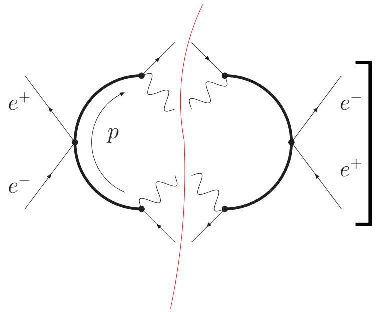
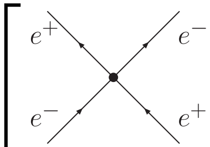
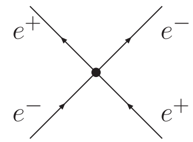


$$\sim \int_{-\infty}^{+\infty} \mathrm{d}p_0 \int_{0}^{+\infty} \mathrm{d}|\mathbf{p}|\, |\mathbf{p}|^2 \frac{\Gamma_t^2}{|\underbrace{\frac{E}{2} + p_0 - \frac{\mathbf{p}^2}{2m_t}}_{(q^2 - m_t^2)/2m_t} + i\frac{\Gamma_t}{2}|^2 \, |\frac{E}{2} - p_0 - \frac{\mathbf{p}^2}{2m_t} + i\frac{\Gamma_t}{2}|^2}$$



# Phase space matching

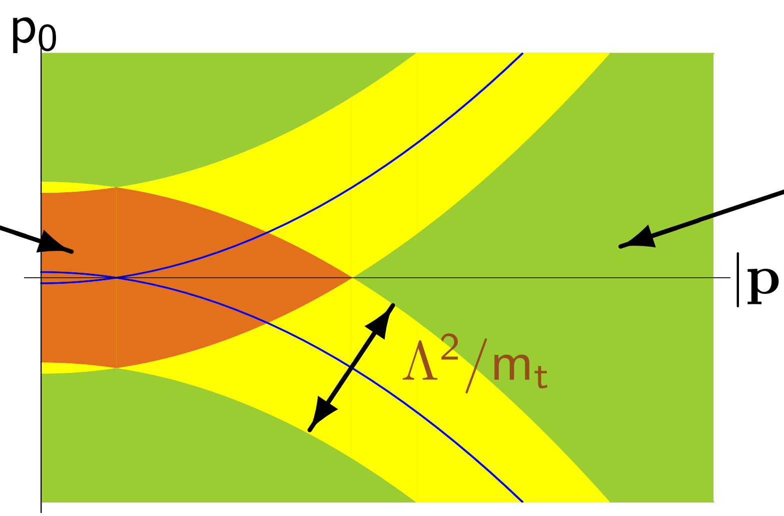
Hoang, Ruiz-Femenía, CJR (w.i.p.)

$\sigma_{\mathrm{tot},\Lambda} \sim \Big[$  $\Big] \quad + \mathrm{Im}\Big[$ $iC(\mu,\Lambda)$  $+$ $i\tilde{C}(\mu,\Lambda)\frac{\hat{E}}{m_t}$  $+ \ldots \Big]$



$$\sim \int dp_0 \int d|\mathbf{p}|\,|\mathbf{p}|^2 \frac{\Gamma_t^2}{|\underbrace{\frac{E}{2}+p_0-\frac{\mathbf{p}^2}{2m_t}}_{(q^2-m_t^2)/2m_t}+i\frac{\Gamma_t}{2}|^2\,|\frac{E}{2}-p_0-\frac{\mathbf{p}^2}{2m_t}+i\frac{\Gamma_t}{2}|^2}$$

$(q^2 - m_t^2) \ll \Lambda^2 \lesssim m_t^2$
double resonant
nonrel. expansion valid
$\Lambda$: cut on top and anti-top invariant masses




unphysical region of EFT
single/non-resonant
subtracted in local exp.
by matching conditions
$C(m_t,\Lambda), \tilde{C}(m_t,\Lambda), \ldots$

alternative approach see Beneke, Falgari, Schwinn, Signer, Zanderighi (2007)

# Phase space cutoff

## Physical cutoff $\Lambda$

- Cutoff corresponds to maximal invariant mass of an experimentally measured $\mathrm{Wb}$ pair that is assigned to a top decay event

  Cross section is differential in experimental parameter $\Lambda$: $\sigma(\Lambda)$

# Phase space cutoff

Physical cutoff $\Lambda$

- Cutoff corresponds to maximal invariant mass of an experimentally measured $\mathrm{Wb}$ pair that is assigned to a top decay event

  Cross section is differential in experimental parameter $\Lambda$: $\sigma(\Lambda)$

Formal treatment: OPE, imaginary coefficients

# Phase space cutoff

Physical cutoff $\Lambda$

- Cutoff corresponds to maximal invariant mass of an experimentally measured $\mathrm{Wb}$ pair that is assigned to a top decay event
  Cross section is differential in experimental parameter $\Lambda$: $\sigma(\Lambda)$

Formal treatment: OPE, imaginary coefficients

Cutoff scaling: $\boxed{\Lambda^2 \lesssim m_t^2}$

Captures resonance region, excludes unphysical parts of phase space

- Power counting breaking: natural scaling $\Lambda^2 \sim 2\, m_t E \sim m_t^2 v^2$
  - – Higher dimension operators will not be suppressed
  - + But: $\frac{\Lambda}{m_t} < 1$ yields sufficient suppression (choose e.g. $\Lambda \approx 0.6\, m_t$)
  - $\rightarrow$ mild power counting breaking

# Phase space cutoff

Physical cutoff $\Lambda$

- Cutoff corresponds to maximal invariant mass of an experimentally measured $\mathrm{Wb}$ pair that is assigned to a top decay event
  Cross section is differential in experimental parameter $\Lambda$: $\sigma(\Lambda)$

Formal treatment: OPE, imaginary coefficients

Cutoff scaling: $\boxed{\Lambda^2 \lesssim m_t^2}$

Captures resonance region, excludes unphysical parts of phase space

- Power counting breaking: natural scaling $\Lambda^2 \sim 2\, m_t E \sim m_t^2 v^2$
  – Higher dimension operators will not be suppressed
  + But: $\frac{\Lambda}{m_t} < 1$ yields sufficient suppression (choose e.g. $\Lambda \approx 0.6\, m_t$)
  $\rightarrow$ mild power counting breaking
- Good convergence of the $\left(\frac{E}{\Lambda}\right)^n \left(\frac{\Gamma_t}{\Lambda}\right)^m$ expansion

# Phase space cutoff

Physical cutoff $\Lambda$

- Cutoff corresponds to maximal invariant mass of an experimentally measured $Wb$ pair that is assigned to a top decay event
  Cross section is differential in experimental parameter $\Lambda$: $\sigma(\Lambda)$

Formal treatment: OPE, imaginary coefficients

Cutoff scaling: $\boxed{\Lambda^2 \lesssim m_t^2}$

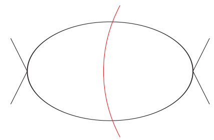
Captures resonance region, excludes unphysical parts of phase space

- Power counting breaking: natural scaling $\Lambda^2 \sim 2\, m_t E \sim m_t^2 v^2$
  – Higher dimension operators will not be suppressed
  + But: $\frac{\Lambda}{m_t} < 1$ yields sufficient suppression (choose e.g. $\Lambda \approx 0.6\, m_t$)
  $\rightarrow$ mild power counting breaking
- Good convergence of the $\left(\frac{E}{\Lambda}\right)^n \left(\frac{\Gamma_t}{\Lambda}\right)^m$ expansion
- Expansion for $\alpha_s$: $\left(\frac{\alpha_s}{v}\right)^n \rightarrow \left(\alpha_s \frac{m_t}{\Lambda}\right)^n$
  $\Rightarrow$ Phase space loop corrections parametrically suppressed by $\alpha_s^n$

# Phase space corrections

Formal counting $\boxed{\Lambda^2 \lesssim m_t^2}$ Numerically $\Lambda = \sqrt{2\,m_t \times 30\,\text{GeV}} \approx 100$ GeV

NLL Leading $^3S_1$ current



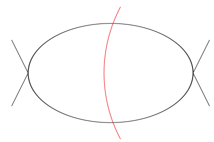
$$\sim i \left[ \# \frac{\Gamma_t}{\Lambda} + \frac{\# m_t \hat{E} \Gamma_t + \# m_t \Gamma_t^2}{\Lambda^3} + \ldots \right]$$

$$\sim i \, [-0.014 - 0.00007 + \ldots]$$

# Phase space corrections

Formal counting $\Lambda^2 \lesssim m_t^2$  Numerically $\Lambda = \sqrt{2\, m_t \times 30\,\text{GeV}} \approx 100$ GeV

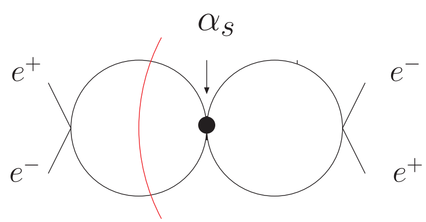
NLL  Leading $^3S_1$ current



$$\sim i \left[ \# \frac{\Gamma_t}{\Lambda} + \frac{\# m_t \hat{E} \Gamma_t + \# m_t \Gamma_t^2}{\Lambda^3} + \ldots \right]$$

$$\sim i\, [-0.014 - 0.00007 + \ldots]$$

NNLL  Coulomb insertion



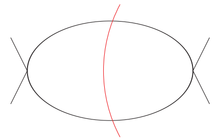

$$\sim i \left[ \# \, \alpha_s \, \frac{m_t \Gamma_t}{\Lambda^2} + \ldots \right]$$

$$\sim i\, [-0.007 + \ldots]$$

# Phase space corrections

Formal counting $\boxed{\Lambda^2 \lesssim m_t^2}$ Numerically $\Lambda = \sqrt{2\, m_t \times 30\, \mathrm{GeV}} \approx 100$ GeV

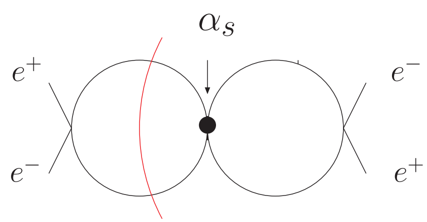
NLL Leading $^3S_1$ current



$$\sim i \left[ \# \frac{\Gamma_t}{\Lambda} + \frac{\# m_t \hat{E} \Gamma_t + \# m_t \Gamma_t^2}{\Lambda^3} + \ldots \right]$$

$$\sim i \, [-0.014 - 0.00007 + \ldots]$$

NNLL Coulomb insertion




$$\sim i \left[ \# \, \alpha_s \, \frac{m_t \Gamma_t}{\Lambda^2} + \ldots \right]$$

$$\sim i \, [-0.007 + \ldots]$$

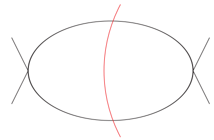
Power counting breaking terms:

Insertions of higher dimension operators, e.g. kinetic energy correction

# Phase space corrections

Formal counting $\Lambda^2 \lesssim m_t^2$ Numerically $\Lambda = \sqrt{2\, m_t \times 30\,\text{GeV}} \approx 100$ GeV

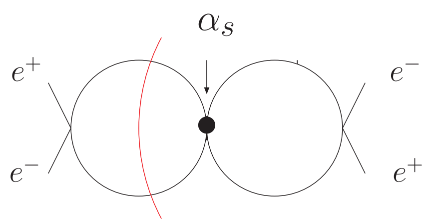
NLL Leading $^3S_1$ current



$$\sim i \left[\# \frac{\Gamma_t}{\Lambda} + \frac{\# m_t \hat{E} \Gamma_t + \# m_t \Gamma_t^2}{\Lambda^3} + \dots\right]$$

$$\sim i\,[-0.014 - 0.00007 + \dots]$$

NNLL Coulomb insertion



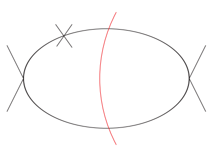

$$\sim i \left[\# \alpha_s \frac{m_t \Gamma_t}{\Lambda^2} + \dots\right]$$

$$\sim i\,[-0.007 + \dots]$$

Power counting breaking terms:

Insertions of higher dimension operators, e.g. kinetic energy correction

NLL $\frac{\mathbf{p}^4}{8 m_t^3}$



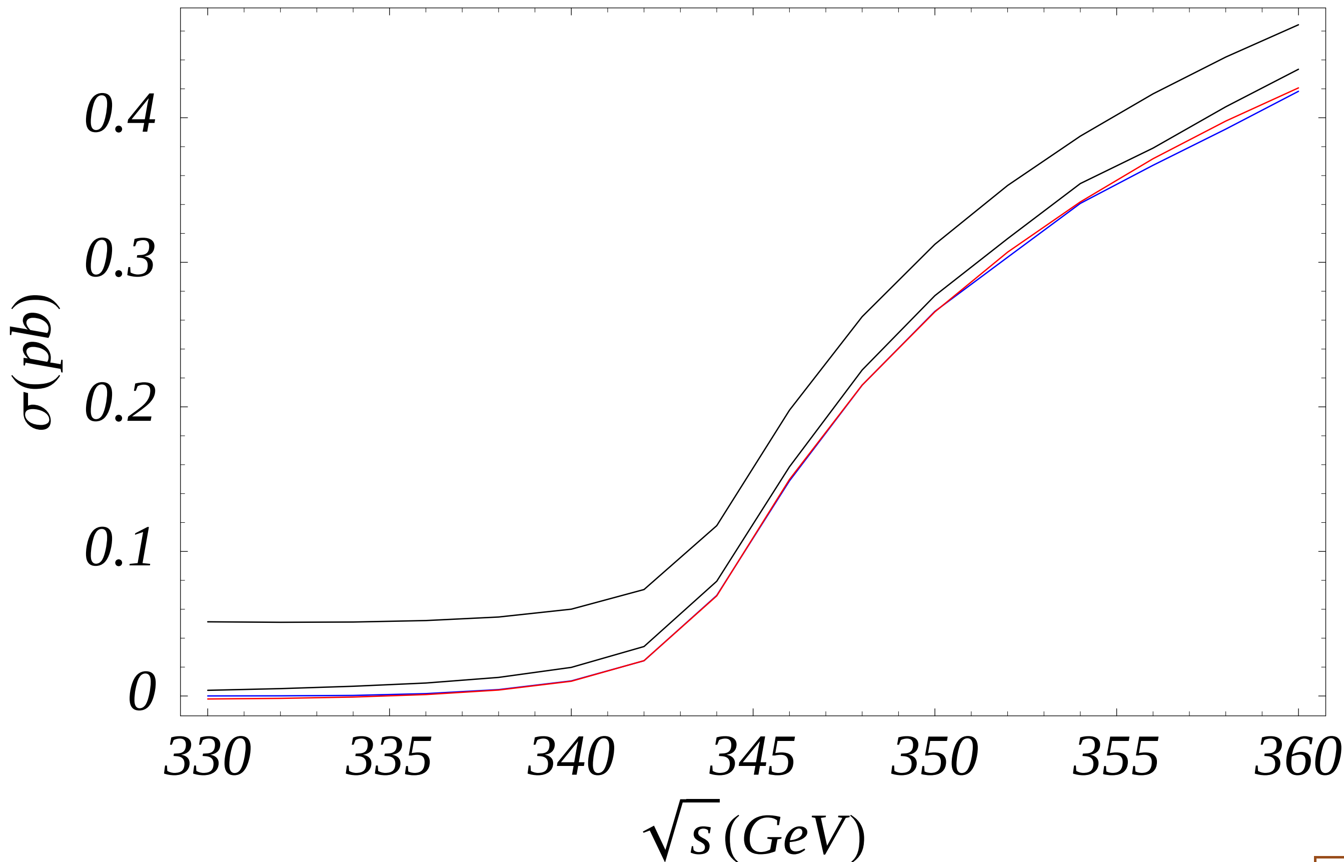
$$\sim i \left[\# \frac{\Gamma_t \Lambda}{m_t^2} + \dots\right]$$

$$\sim i\,[0.001 + \dots]$$

$\Rightarrow$ Suppressed through “mild” power counting breaking

# Phase space corrections

Numerical analysis: $\mathcal{O}(\alpha_s^0)$ cross section




— NRQCD prediction without phase space cut

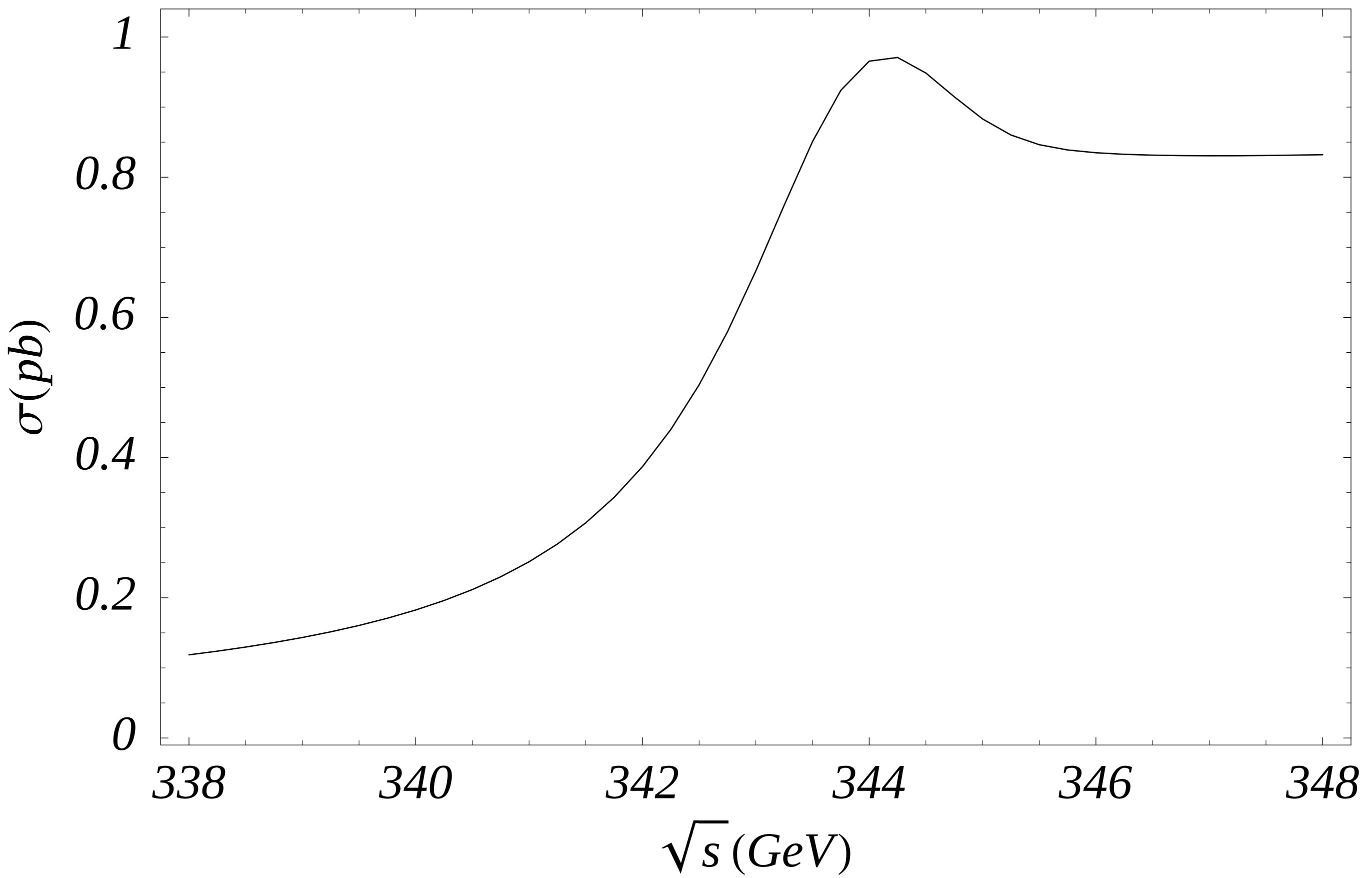
— MadGraph without invariant mass cut

— NRQCD with phase space corrections, cut $\Lambda^2 = 2\,m_t \times 10\,\text{GeV}$

— MadGraph with invariant mass cut $\Lambda^2 = 2\,m_t \times 10\,\text{GeV}$

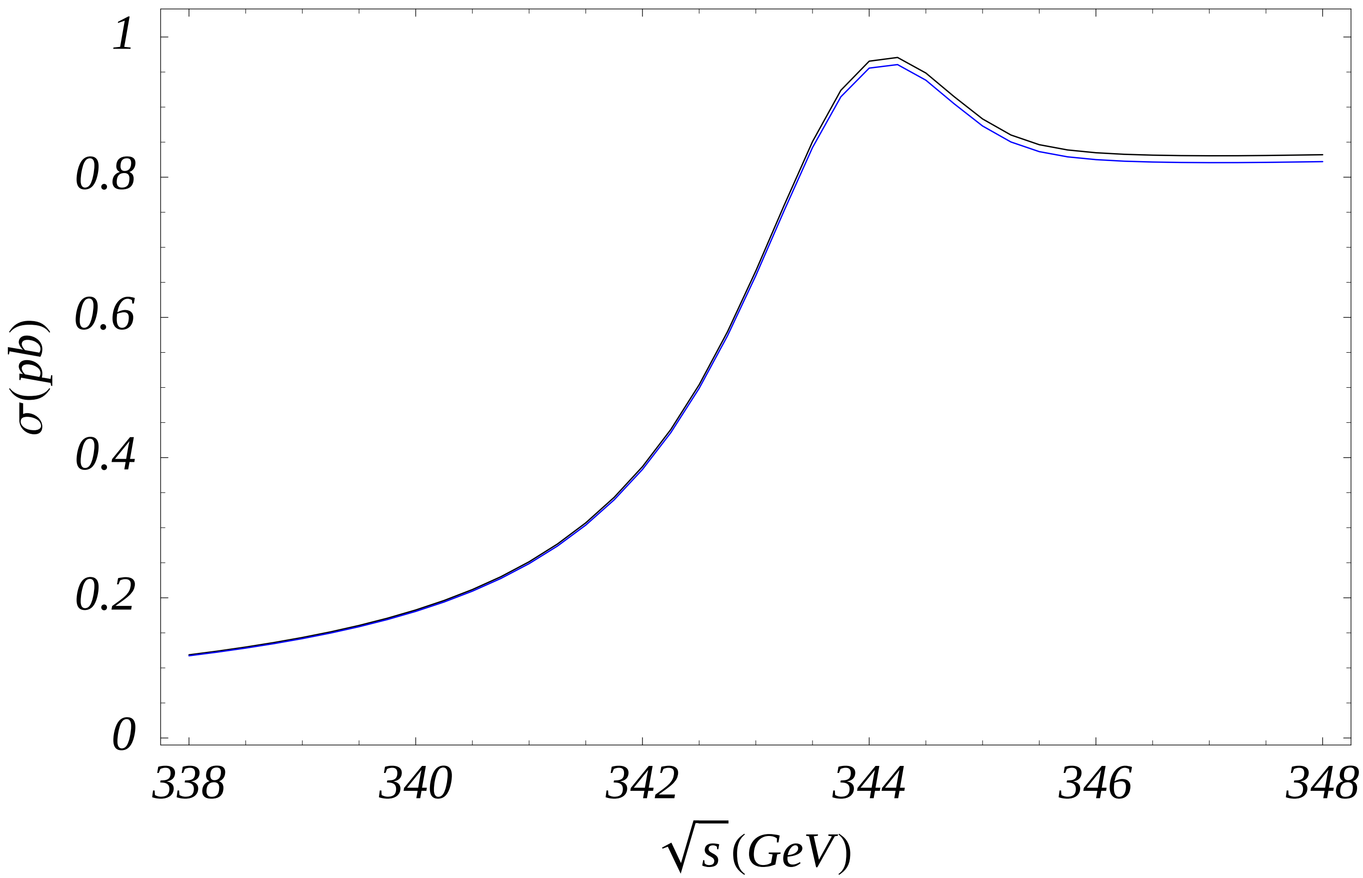
Phase space corrections are numerically at NLO

# Numerical analysis




— Default NRQCD cross section (NNLL Coulomb Green function)

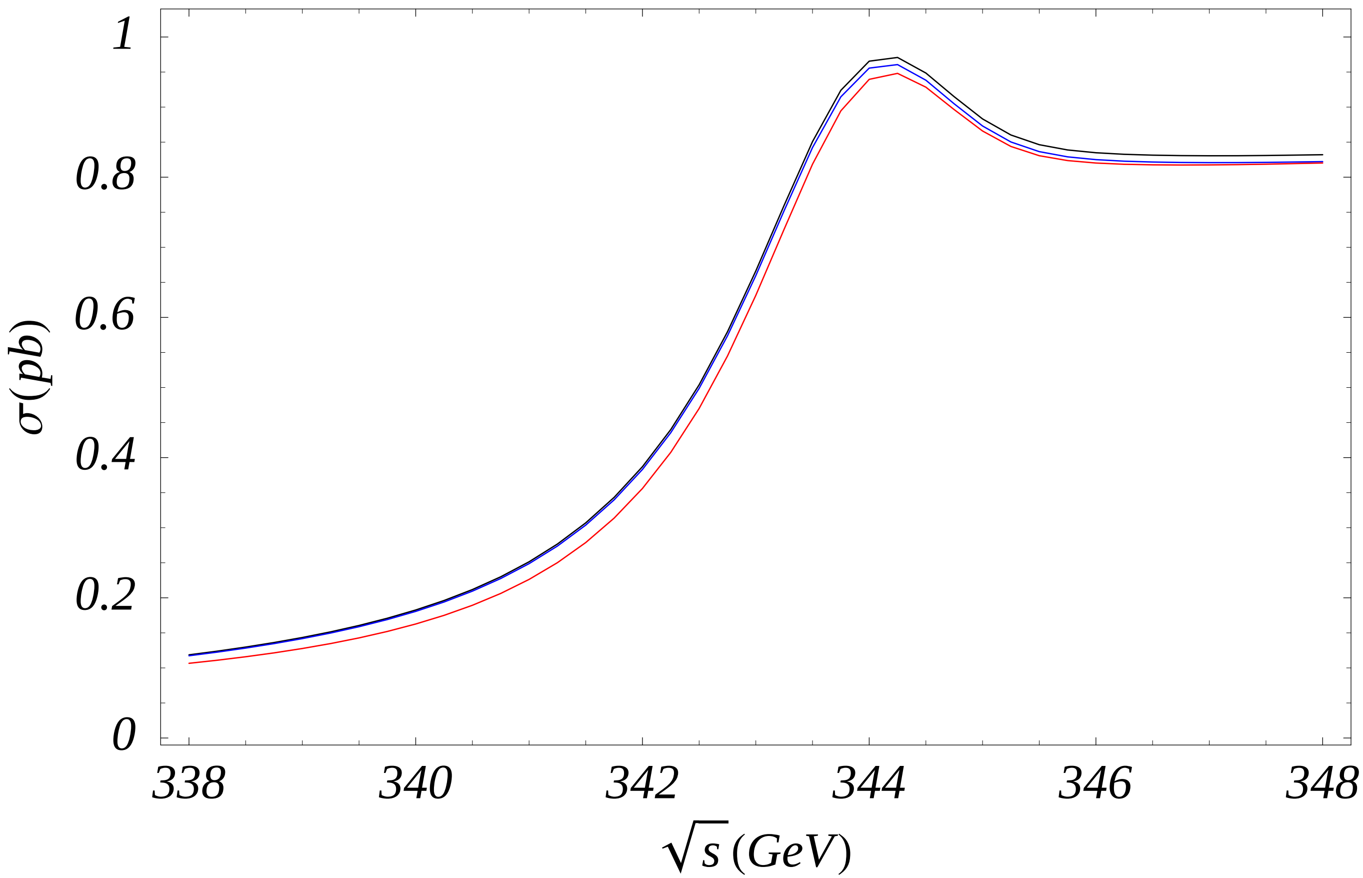
# Numerical analysis




— Default NRQCD cross section (NNLL Coulomb Green function)

— Default plus NNLL usual electroweak corrections (no pure QED)

# Numerical analysis

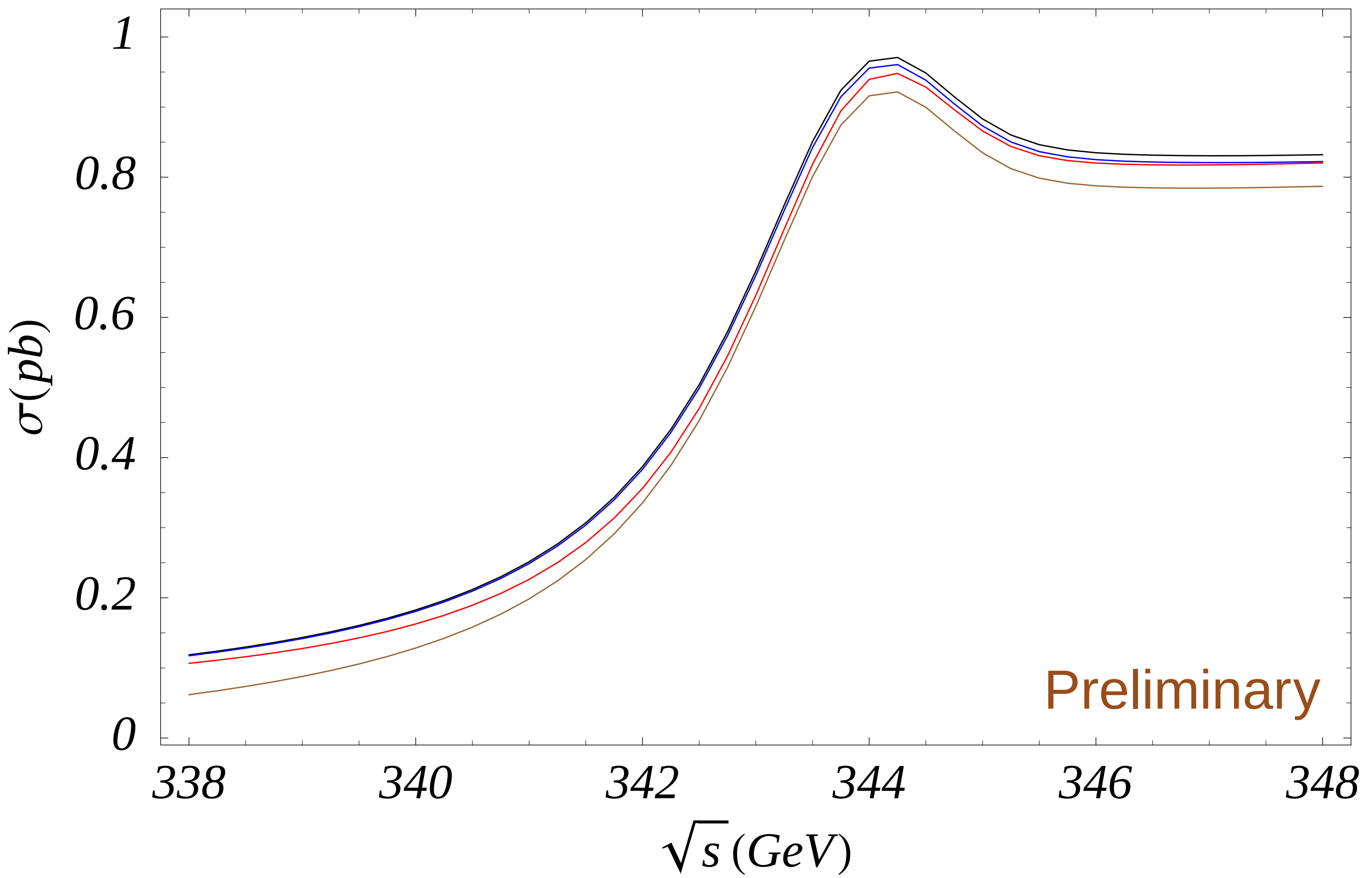



— Default NRQCD cross section (NNLL Coulomb Green function)

— Default plus NNLL usual electroweak corrections (no pure QED)

— Default plus NLL + NNLL absorptive electroweak corrections

# Numerical analysis




— Default NRQCD cross section (NNLL Coulomb Green function)

— Default plus NNLL usual electroweak corrections (no pure QED)

— Default plus NLL + NNLL absorptive electroweak corrections

— Default plus NLO + NNLO phase space corrections

# Outlook

- Completion of phase space matching

# Outlook

- Completion of phase space matching
- Investigate effects of ultrasoft gluons in phase space matching

# Outlook

- Completion of phase space matching
- Investigate effects of ultrasoft gluons in phase space matching
- $\mathcal{O}(\alpha_s)$ corrections to imaginary current matching conditions $\rightarrow$ NNLL running of $(e^+e^-)(e^+e^-)$ operators

# Outlook

- Completion of phase space matching
- Investigate effects of ultrasoft gluons in phase space matching
- $\mathcal{O}(\alpha_{\mathsf{s}})$ corrections to imaginary current matching conditions $\rightarrow$ NNLL running of $(\mathrm{e}^{+}\mathrm{e}^{-})(\mathrm{e}^{+}\mathrm{e}^{-})$ operators
- QED contributions: ISR, Coulomb singularities

# Summary

- Threshold scan allows for precise $m_t, y_t, \Gamma_t, \alpha_s$ determination
- Effective theory approach crucial to sum up threshold contributions

# Summary

- Threshold scan allows for precise $m_t, y_t, \Gamma_t, \alpha_s$ determination
- Effective theory approach crucial to sum up threshold contributions

Unstable top leads to

- Complex matching conditions
- UV phase space divergencies
- Matching conditions for the $t\bar{t}$ phase space that depend on definition of “threshold top pair event”
- Cutoff involves mild power counting breaking
- Phase space corrections are large (NLO)