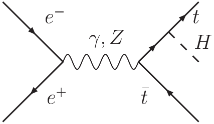
# The Top Yukawa Coupling at 500 GeV

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# Outline



- Physical motivation
- Measurement at the ILC at 500 GeV
- Nonrelativistic QCD
- NLL QCD effects
- WIP: NLL electroweak effects

# Measurement
## Motivation



Open questions:

- Generation of mass?
- Electroweak symmetry breaking?

In the SM: Higgs mechanism

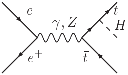
| | predicted? | LHC | $\frac{\Delta x}{x}$ |
|---|---|---|---|
| Mass | – | $\surd$ | 1% |
| Gauge couplings | $e m_w$ | $\surd$ | 10-20% |
| Self-couplings | $(\frac{m_h}{m_w})^{(2)}$ | – | – |
| Yukawa couplings | $\frac{m_f}{v}$ | top, $\tau$ | 30-40% |

- Couplings discriminate between models
- Need ILC to measure them

Here:

Top Yukawa coupling at ILC

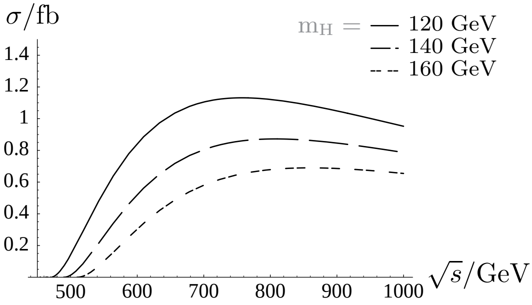
- How?
  - Measurement of $\sigma(e^+e^- \to t\bar{t}H)$




  - Known: Born CS [Gaemers, Gounaris, Djouadi, Kalinowski ]
    One-loop CS [Dawson, Reina, Belanger, Dittmaier, Denner, Roth, Weber . . . ]

    New: NLL CS in vNRQCD

Total Born CS for $m_t = 180$ GeV



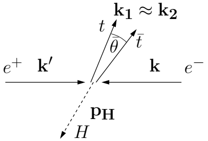

Dedicated Studies:

- At 800 GeV: $\delta Y_t / Y_t \approx 5 - 10\%$ [Juste,Gay]
- At 500 GeV: $\approx$ 25% [Juste]

$2m_t + m_H \geq 475$ GeV $\Rightarrow$ Phase space is small

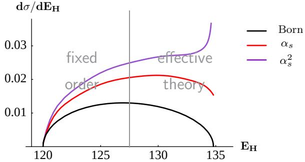
# ILC at 500 GeV
## Theoretical Challenges

Difficulties for $v \ll 1$:




$$v = \sqrt{1 - \frac{4m_t^2}{(k_1+k_2)^2}}$$

- Coulomb singularities $\sim (\alpha_s / v)^n$
- $(\alpha_s \log v)^n$ singularities: $\log \frac{E}{p}$, $\log \frac{p}{m}$
- Fixed-order theory breaks down




At 500 GeV: v is always small
Phase space is non-relativistic

$\Rightarrow$ vNRQCD

$$\frac{d\sigma}{dE_H} \sim v \sum \left(\frac{\alpha_s}{v}\right)^n (\alpha_s \ln v)^n \left(1\,(LL) + \#\,\alpha_s\,(NLL)\right)$$

# NLL QCD Effects



Cross section $\sigma(e^+e^- \to t\bar{t}H)$

$$\frac{d\sigma}{dE_H}(E_H \approx E_H^{\max}) \sim \Big[f_0^2\, c_0^2(\mu, \sqrt{s}, m_t, m_H) + f_1^2\, c_1^2(\mu, \sqrt{s}, m_t, m_H)\Big] \operatorname{Im} G^{\mathrm{NLL}}_{\mathrm{Coulomb}}$$

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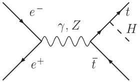
$f_{0,1}^2$: electroweak information in the endpoint

$$\sim \left(\frac{d\sigma_{0,1}}{dE_H}\right)_{Born} \text{ for } E_H \to E_{H,max}$$



# NLL QCD Effects



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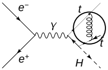
$c_{0,1}^2$: hard QCD corrections in the endpoint




[Denner, Dittmaier, Roth Weber]

$f_{0,1}^2$: electroweak information in the endpoint

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# NLL QCD Effects



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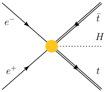
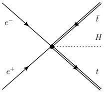
Currents

$$\mathbf{O_p} = f_0\, c_0(\mu, \sqrt{s}, m_H) \left( \psi^\dagger_{\vec{p}} \; \tilde{\chi}_{-\vec{p}} \right) + f_1\, c_1(\mu, \sqrt{s}, m_H) \left( \psi^\dagger_{\vec{p}} \; \vec{\sigma} \; \tilde{\chi}_{-\vec{p}} \right)$$




$= f_i\, c_i$



# NLL QCD Effects



Cross section $\sigma(e^+e^- \to t\bar{t}H)$

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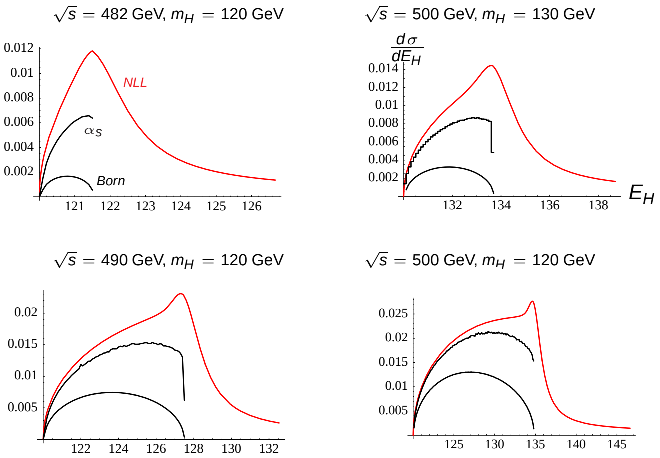
Known:
- Renormalization group running of $c_{0,1}$
- $G^{\mathrm{NLL}}_{\mathrm{Coulomb}}$

New:
- Matching conditions $f_{0,1},\ c_{0,1}(\mu = m_t)$
- Inclusion of $e^+e^-$ polarization
- Formula for $\sigma_{\mathrm{tot}}$

Ongoing:
- Top-decay effects

# NLL QCD Effects
## Differential Cross Section






$\Rightarrow$ Behavior far from threshold is well reproduced

$\Rightarrow$ Increase of total cross section

# $e^+e^-$ Polarization



Cross section depends on the helicity of $e^+$ and $e^-$:


Total cross section:

$$\sigma_{pol} = \frac{1}{4}(1+P_+)(1-P_-)\,\sigma^+ + \frac{1}{4}(1-P_-)(1+P_+)\,\sigma^-$$
$$= \sigma_{\text{unpol}}\left[1 - P_- P_+ - A_{LR}(P_+ - P_-)\right]$$

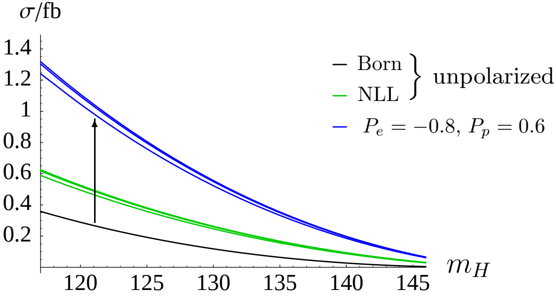
$P_\pm$: degree of $e^\pm$ polarization

left-right asymmetry: $A_{LR} = \dfrac{\sigma^- - \sigma^+}{\sigma^- + \sigma^+}$

$\Rightarrow$ Increase of $\sigma_{tot}$ by polarization possible

# $e^+e^-$ Polarization

Total Cross Section for $\sqrt{s} = 500$ GeV, $m_t = 175$ GeV




$\sim$ 400% increase vs. unpolarized Born cross section

$\Rightarrow$ Decrease of statistical uncertainty by $\sim$ 50%

Power Counting:

$$g \sim g' \sim v \sim \alpha_s$$

Top decay:

At LL: $E \to E + \mathrm{i}\Gamma_t$

At NNLL: Imaginary Wilson coefficients

Tops decay into $b - W$-pairs

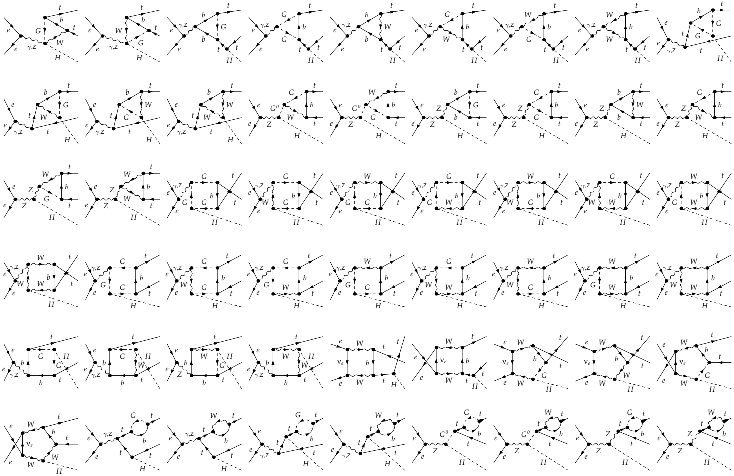
- Cutkosky: Identify cuts corresponding to $bW$-cuts
- At $t\bar{t}$ threshold: Gram determinant often vanishes
  $\Rightarrow$ Use appropriate reduction method [Denner,Dittmaier]

# Electroweak Effects

## The $t\bar{t}H$ Case



### The relevant diagrams for $t\,\bar{t}H$



# Electroweak Effects
## The $t\bar{t}H$ Case



- Sensitivity to Re[G]

  Optical Theorem: $\sigma_{\text{tot}} \sim \text{Im}[c_W\, G]$

$$G^{\text{LL}}_{\text{Coulomb}}(0,0,E) = \frac{m_t^2}{4\,\pi}\left\{ iv - c_F\,\alpha_s \left[ \frac{1}{4\epsilon} + \ln\left(\frac{-im_t v}{\mu}\right) + \psi\left(1 - \frac{i\,c_f\,\alpha_s}{2v}\right) \right] \right\}$$

# Electroweak Effects

## The $t\bar{t}H$ Case

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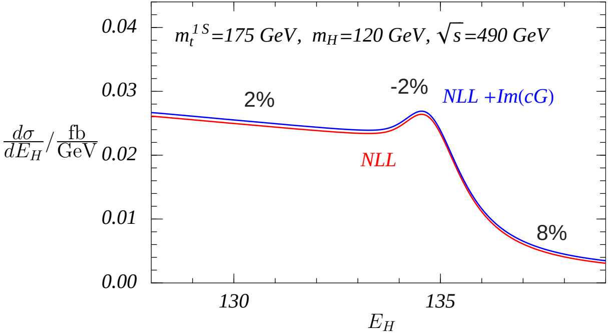
$\Rightarrow$ Line-shape is changed



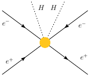
# Electroweak Effects
## The $t\bar{t}H$ Case

- Sensitivity to Re[G]

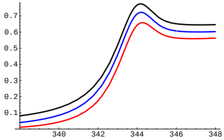
  Optical Theorem: $\sigma_{\rm tot} \sim {\rm Im}[c_W\, G]$

$$G^{\rm LL}_{\rm Coulomb}(0,0,E) = \frac{m_t^2}{4\,\pi}\left\{iv - c_F\,\alpha_s\left[\frac{1}{4\epsilon} + \ln\left(\frac{-im_t v}{\mu}\right) + \psi\left(1 - \frac{i\,c_f\,\alpha_s}{2v}\right)\right]\right\}$$

$\Rightarrow$ RGE running of new operator




- Phase Space Matching: Cut on $Q^2$ of the tops
  - Result for $t\bar{t}$:




[Hoang, Reisser]

# Conclusion

- Top Yukawa coupling for test of EWSB
- ILC at 500 GeV: Phase space is non-relativistic
  $\Rightarrow$ vNRQCD
- Completed:
  - Strong and electroweak matching conditions at $\mathcal{O}(NLL)$
  - Effects of $e^+e^-$ polarization
  - Formula for the total cross section
- Increase of total cross section of up to 400%
  - $\delta Y_t / Y_t \approx 10 - 15\%$ might be possible
- Work in progress:
  - Electroweak NLL decay effects
  - Phase space matching