FIFTY YEARS THAT CHANGED OUR PHYSICS

LAL, 21 Nov. 2016

Jean Iliopoulos

ENS, Paris

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- Each one involved new physical concepts, new mathematical tools and new champions

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- They were conservative : Things changed just enough so that they could remain the same
- Yet, they influenced profoundly our way of looking at the fundamental laws of Nature
- They were mostly rejected by the champions of the previous revolutions

A bit of history

- **The rules of counting states in a statistical ensemble**

Boltzmann, Gibbs, Planck, Natanson, Ehrenfest, Fowler, ...

- **The Bose-Einstein rule**

Bose (1924), Einstein (1924)

- **The Pauli exclusion principle**

Pauli (1925)

- **The Fermi-Dirac rule**

Fermi (1926), Dirac (1926)

- **Applications** *(mostly incorrect)* **to various physical systems**

Einstein, Heisenberg, Dirac, Pauli, Hund, Dennison, Wigner, ...

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  Rule : What comes out must be in

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- Bohr versus Pauli

  Bohr *(et al)* : Conservation laws may be violated in Quantum Mechanics

  Pauli (1930) : $N_1 \rightarrow N_2 + e + \nu$

  $\Rightarrow$ Nuclei are made out of protons electrons *and neutrinos*

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- In 1932 Heisenberg introduces the concept of isospin.
  He puts the proton and the neutron in an $SU(2)$ doublet, but

  In the Bohr-Pauli controversy he sides with Bohr

  He believes that a neutron decays into a proton and an electron, something incompatible with it being a fermion
  "...under suitable circumstances the neutron will break up into a proton and an electron in which case the conservation laws of energy and momentum probably do not apply....The admittedly hypothetical validity of Fermi statistics for neutrons as well as the failure of the energy law in β-decay proves the inapplicability of present quantum mechanics to the structure of the neutron."

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  Tentativo di una teoria della emissione di raggi β.

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- In the Bohr-Pauli controversy Fermi sides with Pauli.

  In the Fermi theory of β-decay the neutrino is a particle like any other.

- But he goes further : he breaks with the prevailing doctrine according to which whatever comes out from a nucleus must be already in. For Fermi a particle, like a photon in a spontaneous emission, is created the moment of the decay.

Fermi's *Tentativo*

- He showed how this could actually happen.

$$
\begin{aligned}
\{a_s(\boldsymbol{p}), a^\dagger_{s'}(\boldsymbol{p}')\} &= \hbar(2\pi)^3 2\omega_p \delta^3(\boldsymbol{p}-\boldsymbol{p}')\,\delta_{ss'} \\
\{a_s(\boldsymbol{p}), a_{s'}(\boldsymbol{p}')\} &= \{a^\dagger_s(\boldsymbol{p}), a^\dagger_{s'}(\boldsymbol{p}')\} = 0
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- It is amazing how fast Fermi's theory was universally accepted. The times were ripe. Quantum Field Theory became the language of particle physics.
- Bohr continued to play with energy non-conserving theories for several years, but he was soon alone.
  A. Pais : "It is clear that Particles and Fields belong to the post-Bohr era."

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- The analytic S-matrix theory
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- Symmetries and Current Algebras, Weak Interactions and CP-violation
  Secondary subjects
- Notice the absence of Quantum Field Theory
  A totally marginal subject

The analytic S-matrix theory

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- A series of (more or less) reasonable axioms formulated directly on the scattering amplitudes.
  - Invariance under Poincaré and internal symmetries
  - Crossing symmetry
  - Unitarity $S = \mathbb{1} + iT \quad SS^\dagger = S^\dagger S = \mathbb{1} \;\Rightarrow 2\,\mathrm{Im}\,T = TT^\dagger$
  - Maximum analyticity
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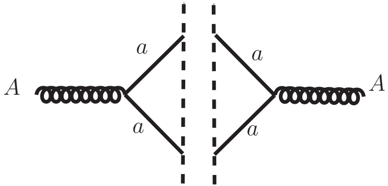
  Not very well defined, fuzzy rules
- An important addition : Analyticity in the complex angular momentum plane (Regge)

Some important by-products

- Cutkosky unitarity relations



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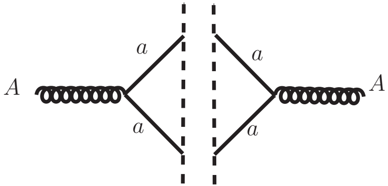

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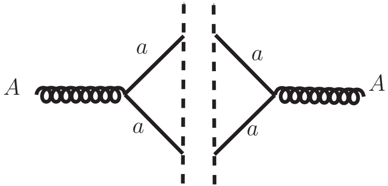
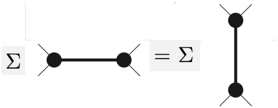


- Bootstrap
- Duality (*Dual Resonance Model*)



The Veneziano amplitude

$$A(s,t) \sim \frac{\Gamma(-1+s/2)\Gamma(-1+t/2)}{\Gamma(-2+(s+t)/2)}$$

This amplitude, appropriately generalised, was the starting point of a concept which turned out to be seminal and important :

The string model

Initially, it was meant to be a theory for hadronic physics and gave rise to interesting phenomenological models

But it was soon realised that it contains a version of quantum gravity
(more about that later)

Symmetries and Current Algebras, Weak Int. and *CPV*

SYMMETRIES

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SYMMETRIES

- The pre-history
  - Space-time symmetries
  - Internal symmetries (Heisenberg 1932, Kemmer 1937, Fermi 1951)
  - Gauge symmetries (Gauss ? ?, Einstein 1914, Fock 1926, Klein 1937, Pauli 1953, Yang and Mills 1954)

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- Early history
  - Higher symmetry (Gell-Mann 1961 (+ Ne'eman)) $SU(3)$
  - Current Algebras (Gell-Mann 1962)

$$[V, V] = V \quad ; \quad [V, A] = A \quad ; \quad [A, A] = V$$

  - Quarks (Gell-Mann 1964 (+Zweig))

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- The importance of anomalies

The Electroweak Standard Model

I. THE WEAK INTERACTIONS. PHENOMENOLOGY

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$$\mathcal{L}_W = \frac{G}{\sqrt{2}} J^{\mu}_{(w)}(x) J^{\dagger}_{(w)\mu}(x)$$

The Electroweak Standard Model

I. THE WEAK INTERACTIONS. PHENOMENOLOGY
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- The Fermi theory of the weak interactions was phenomenologically very successful

$$\mathcal{L}_W = \frac{G}{\sqrt{2}} J^{\mu}_{(w)}(x) J^{\dagger}_{(w)\mu}(x)$$

- But it was a non-renormalisable theory, Fierz 1936

$$d\sigma(\bar{\nu} + p \rightarrow n + e^+) = \frac{G_F^2}{2\pi^2} p_\nu^2 d\Omega$$

$$
\begin{array}{llll}
A \sim & C_0^1(G_F\Lambda^2) & +C_1^1 G_F M^2 & \\
+ & C_0^2(G_F\Lambda^2)^2 & +C_1^2 G_F M^2(G_F\Lambda^2) & +C_2^2(G_F M^2)^2 \\
+ & ..... & & \\
+ & C_0^n(G_F\Lambda^2)^n & +C_1^n G_F M^2(G_F\Lambda^2)^{n-1} & +.... \\
+ & ..... & &
\end{array}
$$

Effective coupling constant : $\lambda = G_F\Lambda^2$

$A \sim \lambda^n + G_F M^2 \lambda^{n-1} + \ldots$

A $\sim$ "leading" + "next-to-leading" + ...

The Theory is valid up to a scale $\sim \Lambda$

$G_F\Lambda^2 \sim 1 \Rightarrow \Lambda \sim 300$ GeV

BUT PRECISION MEASUREMENTS CAN DO BETTER

B.L. Joffe and E.P. Shabalin (1967)

- At leading order

  Limits on Parity and Strangeness violation in strong interactions

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- At next-to-leading order

  Limits on $K^0 \rightarrow \mu^+\mu^-$ and $K^0 - \bar{K}^0$ mass difference

  $G_F \Lambda^2 << 1 \Rightarrow \Lambda \sim 3$ GeV

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- The leading divergences respect all the strong interaction symmetries
  Cl. Bouchiat, J. I., J. Prentki 1968
- Following this line attempts were made to "determine" the properties of the weak interactions, for example to calculate the value of the Cabibbo angle.
  Gatto, Sartori, Tonin ; Cabibbo, Maiani ; Gell-Mann, Goldberger, Kroll, Low

The argument on the leading divergences can, and has been, phrased entirely in terms of currents and symmetries of the strong interactions, although the assumption of an intermediate charged vector boson was always made. The Wilson short distance expansion was not used.

$$A \sim \frac{G}{\sqrt{2}} \int d^4k \ e^{ikx} < a|T(J_{\mu}(x), J_{\nu}(0))|b > \frac{k^{\mu} k^{\nu}/m_W^2}{k^2 - m_W^2}$$

$\Rightarrow$

Only the symmetry properties of the currents are used, not their explicit expression in terms of elementary fields.
The argument can be generalised to all orders in perturbation theory *(J.I.)*

- In principle, the same formalism can be used for the next-to-leading divergences, those which produce FCNC. *(G.I.M.)*

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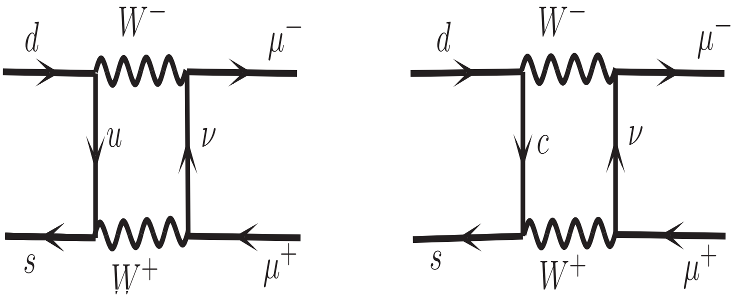
- At this point, however, the paradigm gradually changed from symmetries and currents to the quark model.



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- A model for leptons
  Weinberg 1967 ; Salam 1968
- Both went totally unnoticed

II. THE WEAK INTERACTIONS. FIELD THEORY
Developed in parallel, kind of a sub-culture

Both, the phenomenological approach and the field theory approach, aimed at controlling the divergences of perturbation theory. In the first, you do not know the fields, you do not know the interactions. In the second you start from a given field theory.

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  Lee, Wick 1968
- The electrodynamics of charged vector bosons
  ξ*-limiting formalism* *Lee and Yang ; Lee 1962*

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- Understand why it works. *Becchi, Rouet, Stora ; Tyutin*

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Gauge theories on a space-time lattice

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- The kinetic energy $\bar{\Psi}_n\Psi_{n+\mu} \to \bar{\Psi}_n e^{-\mathrm{i}\Theta_n} e^{\mathrm{i}\Theta_{n+\mu}}\Psi_{n+\mu}$
- Introduce : $U_{n,n+\mu} \quad \to \quad e^{\mathrm{i}\Theta_n} U_{n,n+\mu} e^{-\mathrm{i}\Theta_{n+\mu}}$

Geometry and Dynamics

Gauge theories on a space-time lattice

The dictionary :

- A field $\Psi(x) \quad \Rightarrow \quad \Psi_n$
- A local term such as $\bar{\Psi}(x)\Psi(x) \quad \Rightarrow \quad \bar{\Psi}_n\Psi_n$
- A derivative $\partial_\mu\Psi(x) \quad \Rightarrow \quad (\Psi_n - \Psi_{n+\mu})$
  where $n+\mu$ should be understood as a unit vector joining the point n with its nearest neighbour in the direction μ.
- The kinetic energy term $\bar{\Psi}(x)\partial_\mu\Psi(x) \quad \Rightarrow \quad \bar{\Psi}_n\Psi_n - \bar{\Psi}_n\Psi_{n+\mu}$
- A gauge transformation
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- For a closed path $c = p_{n,n}$ the quantity $\text{Tr}P^{(c)}$ is gauge invariant. $\Rightarrow$ "a curvature"

First conclusion

The 1960's was an extraordinary decade....

although no one at the time had realised that a revolution was taking place !

The renormalisation group and QCD

Contrary to what you may think, the study (rather the re-birth) of the renormalisation group was not initially motivated by the SLAC results on DIS.

A short history

• The RG equation was first written down by Stückelberg and Petermann in 1953

$$[M\frac{\partial}{M} + \beta\frac{\partial}{\partial\lambda} + \gamma_m m\frac{\partial}{\partial m} - n\gamma]\Gamma^{(2n)}(p_1, ..., p_{2n}; m, \lambda; M) = 0$$

It was meant to clarify the meaning of the subtraction in the renormalisation procedure

• Gell-Mann and Low in 1954 observed that it can be used to study the asymptotic behaviour of the theory, but, in the late sixties, the emphasis was to use the equation $\beta = 0$ for QED as an eigenvalue equation to determine α

The renormalisation group and QCD

• In the very late sixties Callan and Symanzik wrote an independent equation, which was *the broken scale invariance Ward identity*

$$\left[m_R \frac{\partial}{\partial m_R} + \beta \frac{\partial}{\partial \lambda_R} + n\gamma\right] \Gamma_R^{(2n)} = m_R^2 \; \delta \; \Gamma_{\phi^2 R}^{(2n)}$$

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• Two physical applications :
(i) Phase transitions and critical phenomena *(Kadanoff, Fischer, Wilson)*
(ii) Scaling properties in DIS $\Rightarrow$ Asymptotic freedom and QCD *(Gross, Politzer, Wilcek)*

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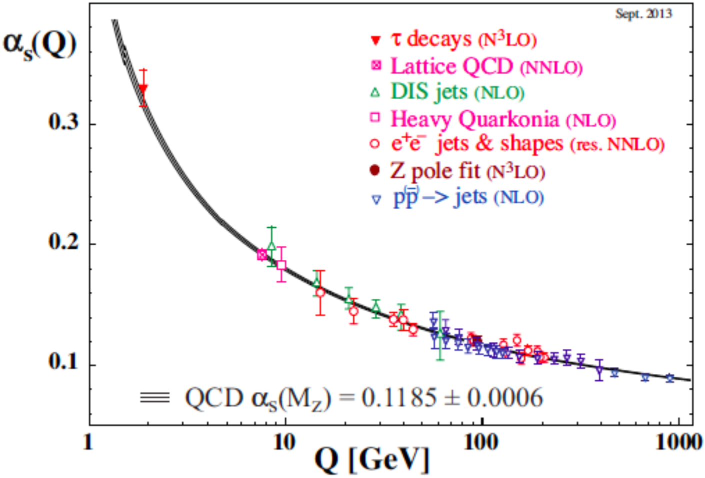
- The synthesis : The DGLAP equations
  The best of two worlds

QCD has been enormously successful

In perturbation



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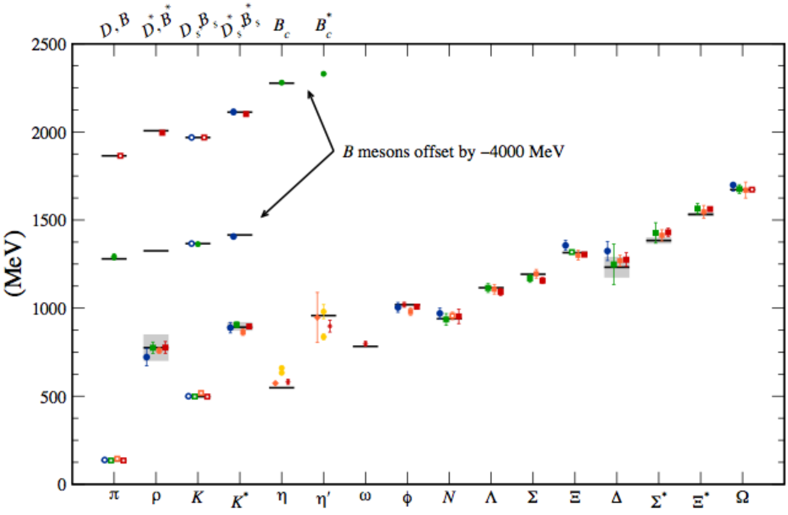
In the non-perturbative region



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- Dynamics=Geometry
  "Let no one ignorant of geometry enter under this roof", Platon

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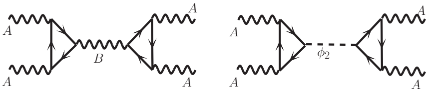
- The weak currents have a vector and an axial part. We know that, in general, we cannot enforce the conservation of both.

$$\partial^\mu j_\mu^{(5)}(x) = \frac{e^2}{8\pi^2}\epsilon_{\nu\rho\sigma\tau}F^{\nu\rho}(x)F^{\sigma\tau}(x)$$



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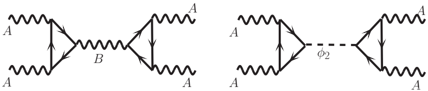

- Anomaly cancellation condition $\mathcal{A} = \sum_i Q_i = 0$

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- Anomalies should be cancelled at all levels
- For the Standard Model, once the τ lepton was found, we could predict the existence of the b and t quarks
- The discovery of a very special anomaly cancellation in string theories, established the super-string theory as the only viable candidate for a quantum gauge theory of all interactions *(Green and Schwarz, 1983)*

The Standard Model and High Energy

Imagine we integrate over all degrees of freedom heavier than a scale M

M does not have to correspond to a physical threshold, although it could !

$\Rightarrow$

We obtain an effective theory in terms of the light, $< M$, degrees of freedom :

$$\mathcal{L}_{\rm eff} = \sum_{i=0}^{\infty} C_i \mathcal{O}_i \qquad (1)$$

By dimensional analysis : $C_i \sim M^{4-d_i}$

$\Rightarrow$

The only dominant operator in the SM is the scalar mass term ϕ^2

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- We are looking forward to the next chapter