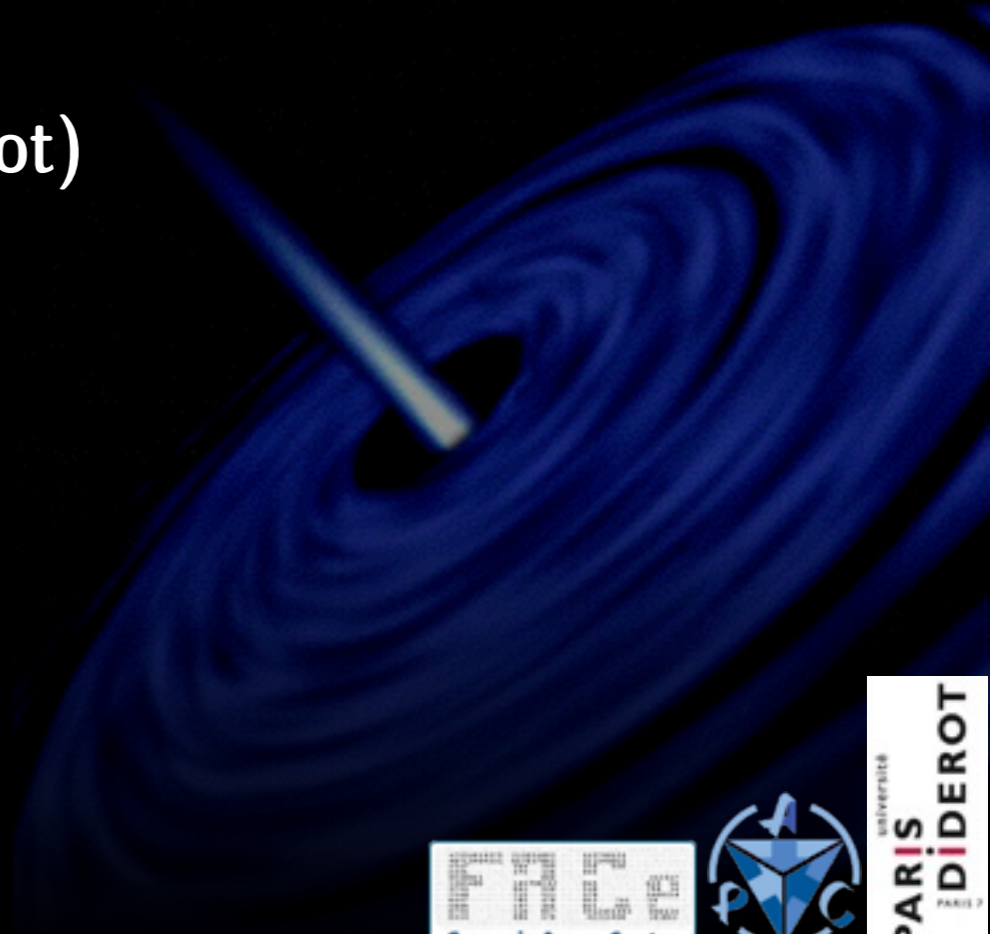




Gravitational waves and space experiments

Antoine Petiteau
(APC – Université Paris-Diderot)

Ecole de Gif
LAL - France
14 September 2017





Overview

- ▶ Introduction
- ▶ Gravitational wave sources in the mHz band
- ▶ LISA: Laser Interferometer Space Antenna
- ▶ Free fall in space: LISAPathfinder
- ▶ Long arm interferometry: Time Delay Interferometry
- ▶ Noise sources
- ▶ Sensitivity
- ▶ Response to GW and orbital motion
- ▶ Data analysis
- ▶ Conclusion



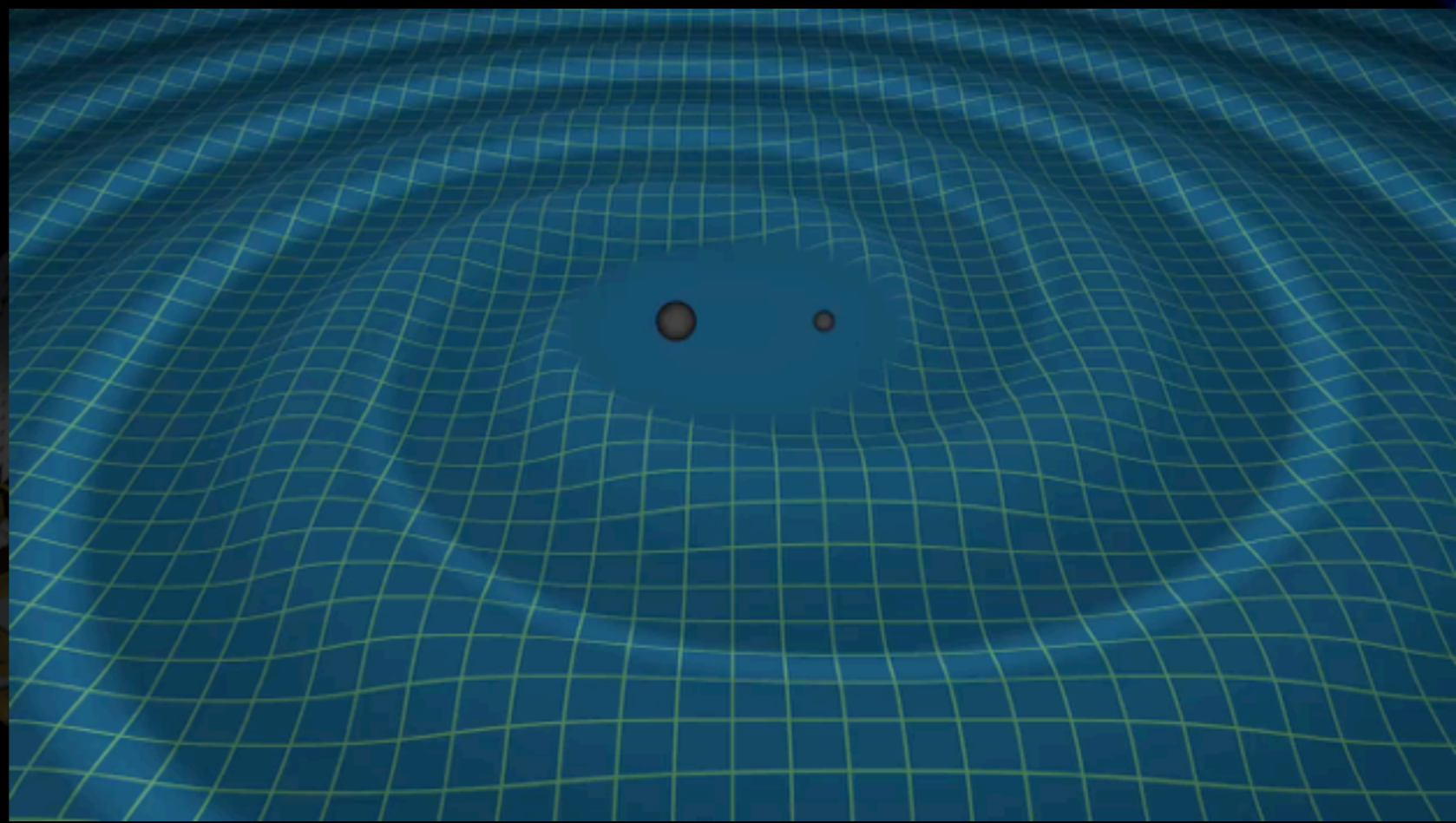
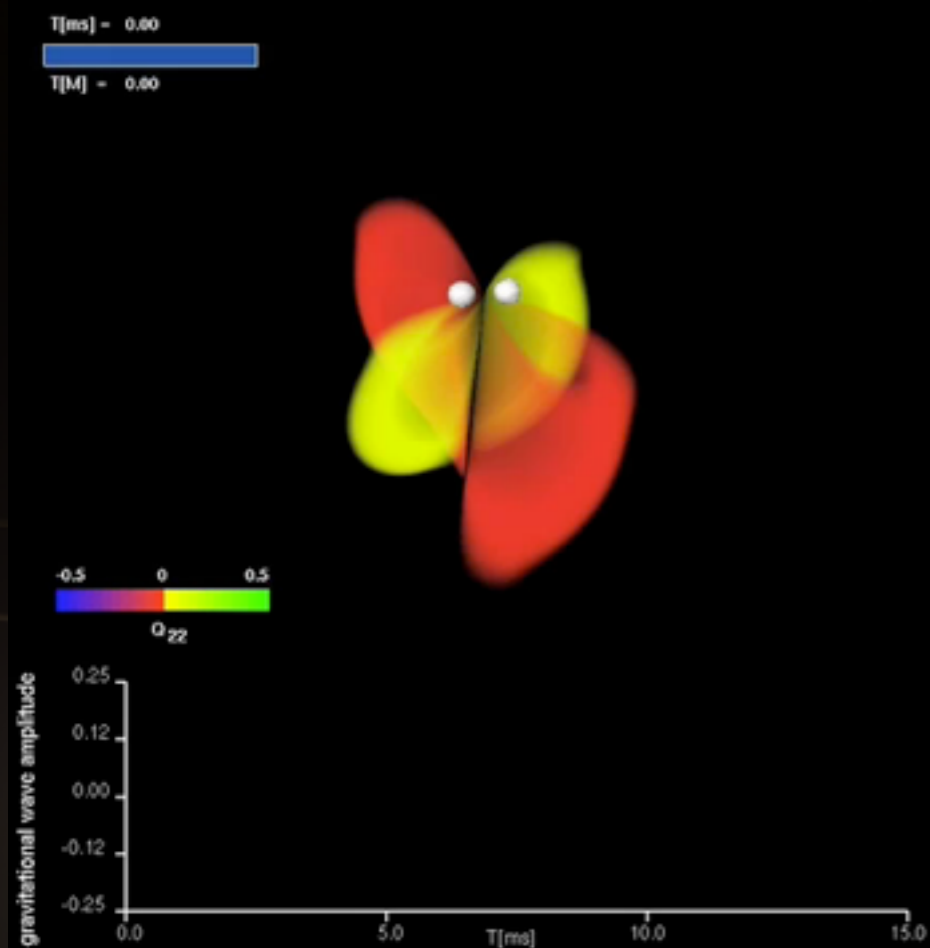
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Emission of GWs

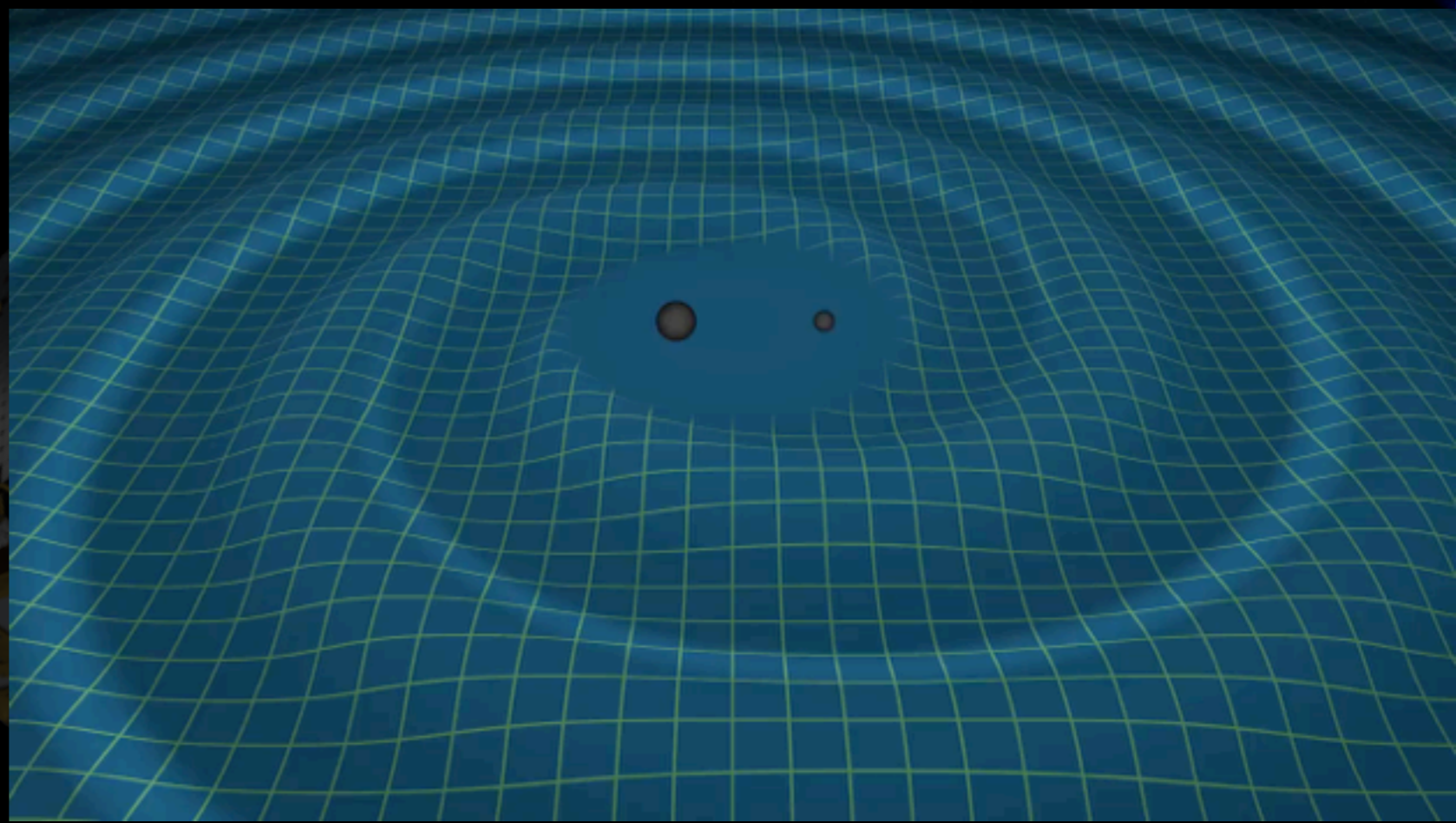
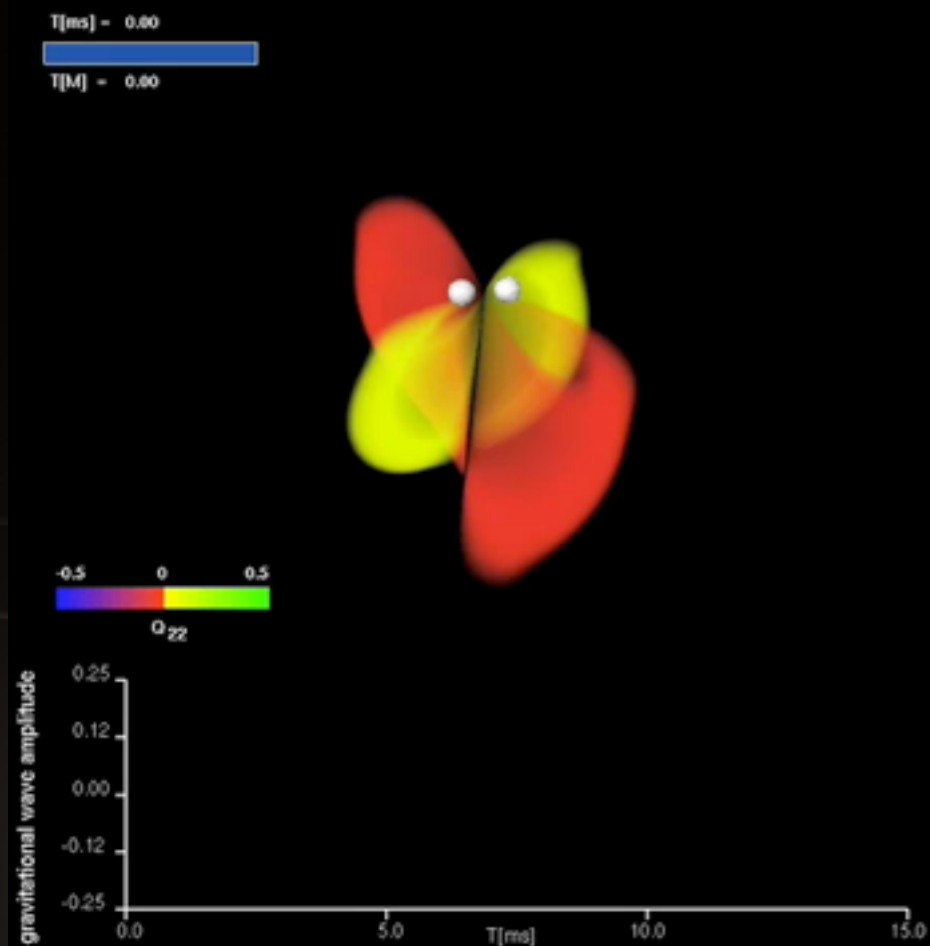
- ▶ A gravitational wave is created during the non-spherical acceleration of one or several massive objects (variation of quadrupolar moment) :
 - **emission**: asymmetric collapse, bodies in orbits or coalescing, ...
 - no emission: isolated, spherical body possibly in rotation





Emission of GWs

- ▶ A gravitational wave is created during the non-spherical acceleration of one or several massive objects (variation of quadrupolar moment) :
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Effects of GWs

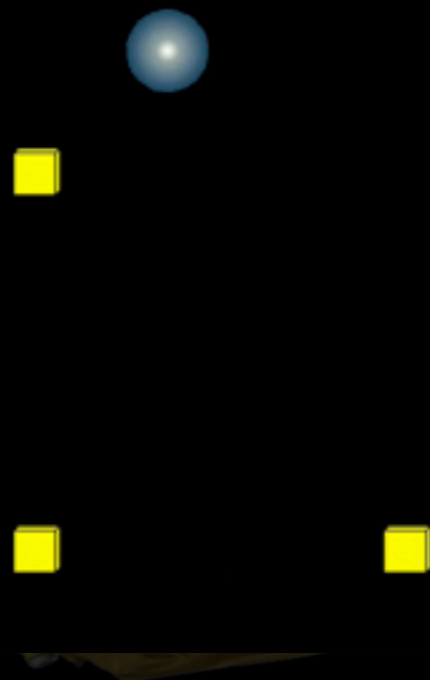
► Modification of distance between 2 objects:

- Elastic deformation **proportional to the distance** between the 2 obj.,
- **Transverse** deformation: perpendicular to the direction of propagation (different from ripples on water !),
- **Two components** of polarisation : h_+ and

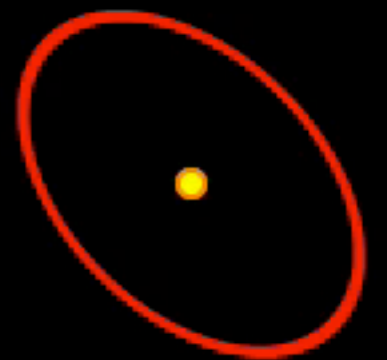
$$\frac{\delta L}{L} = \frac{h}{2}$$

deformation

wave amplitude



+ polarization



x polarization



left polarization



right polarization



Effects of GWs

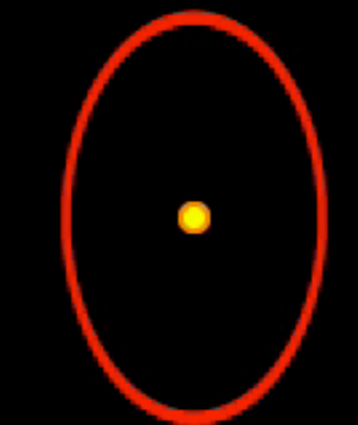
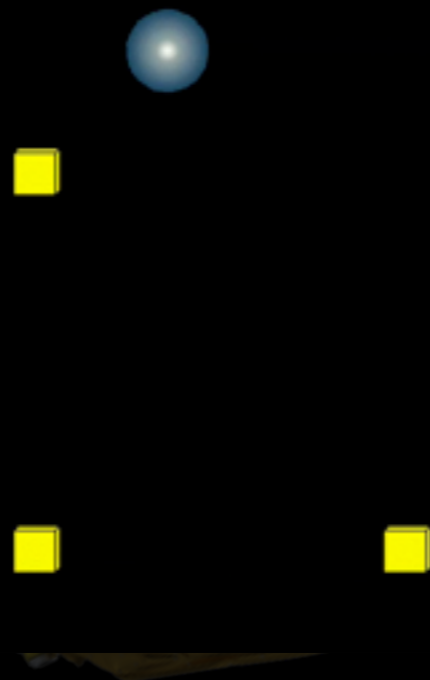
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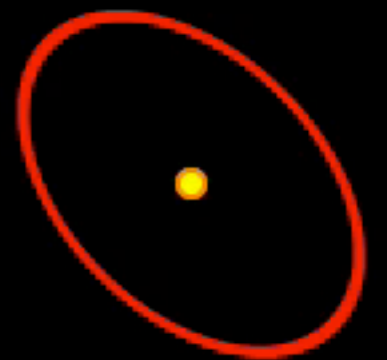
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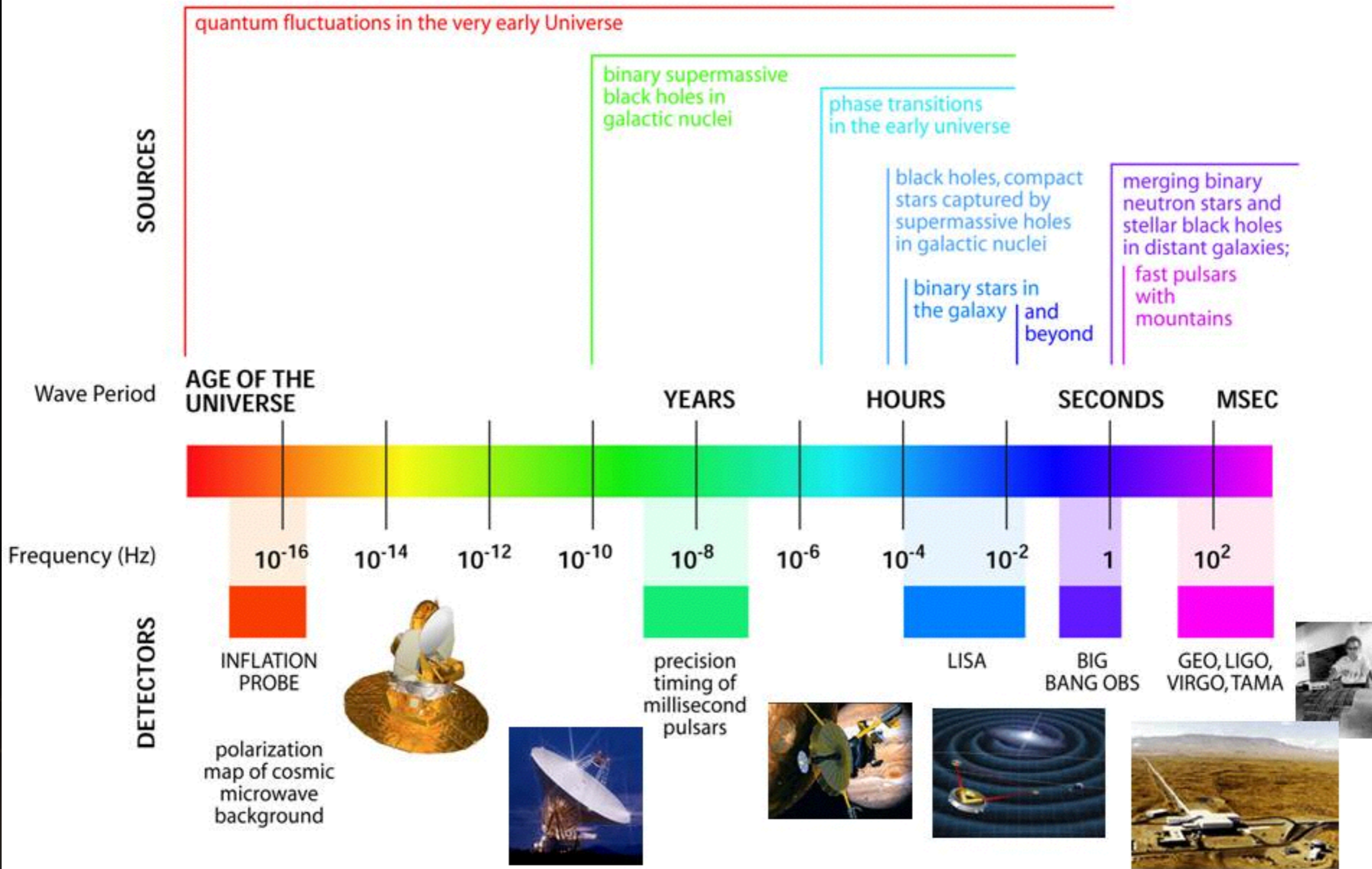


left polarization

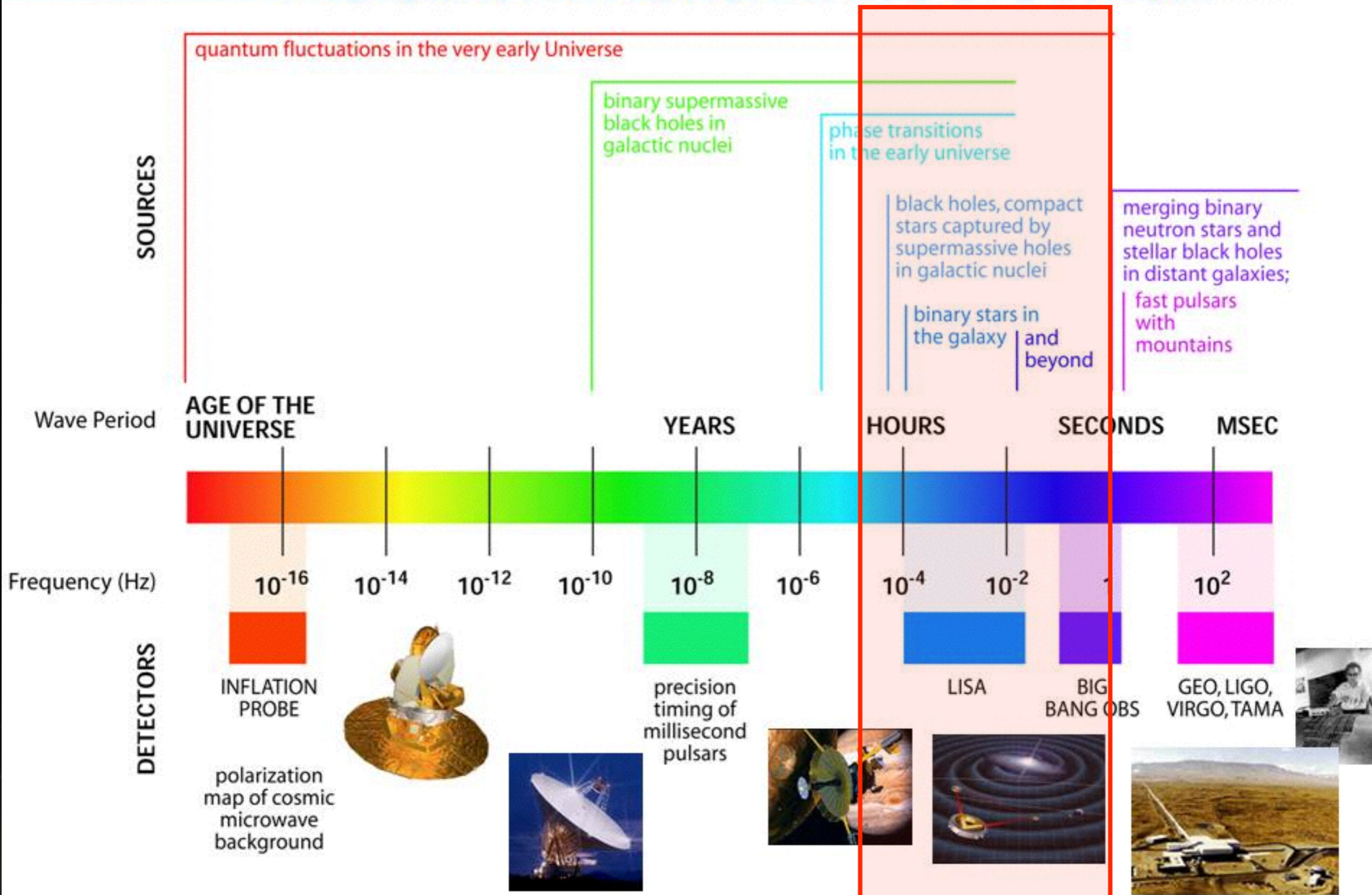


right polarization

THE GRAVITATIONAL WAVE SPECTRUM



THE GRAVITATIONAL WAVE SPECTRUM





Overview

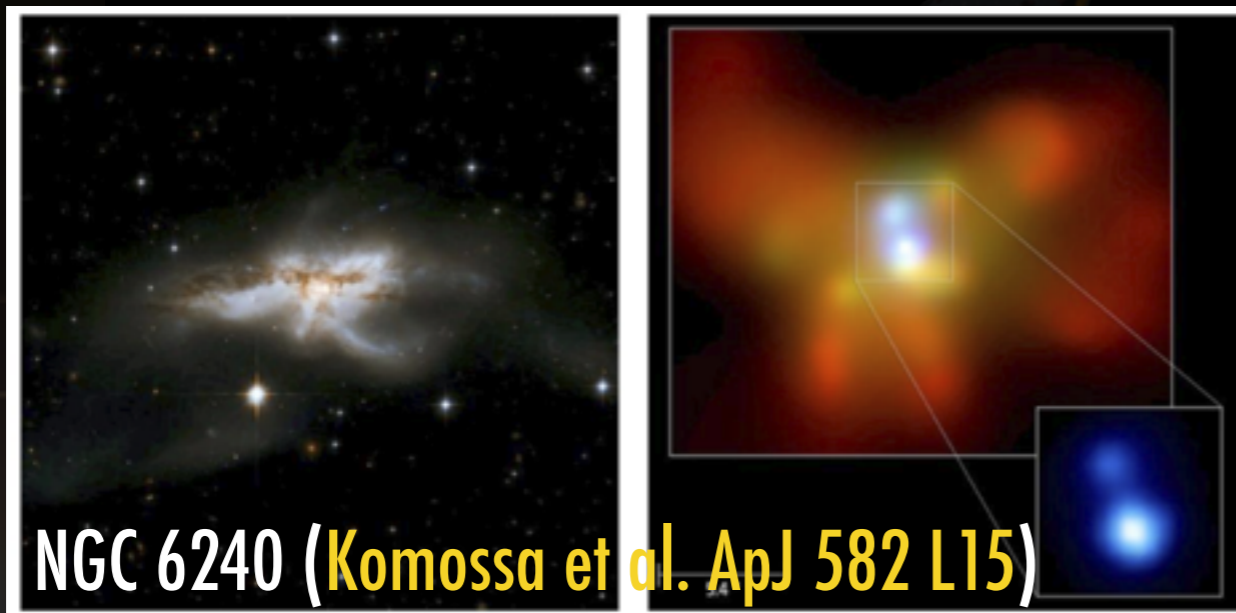
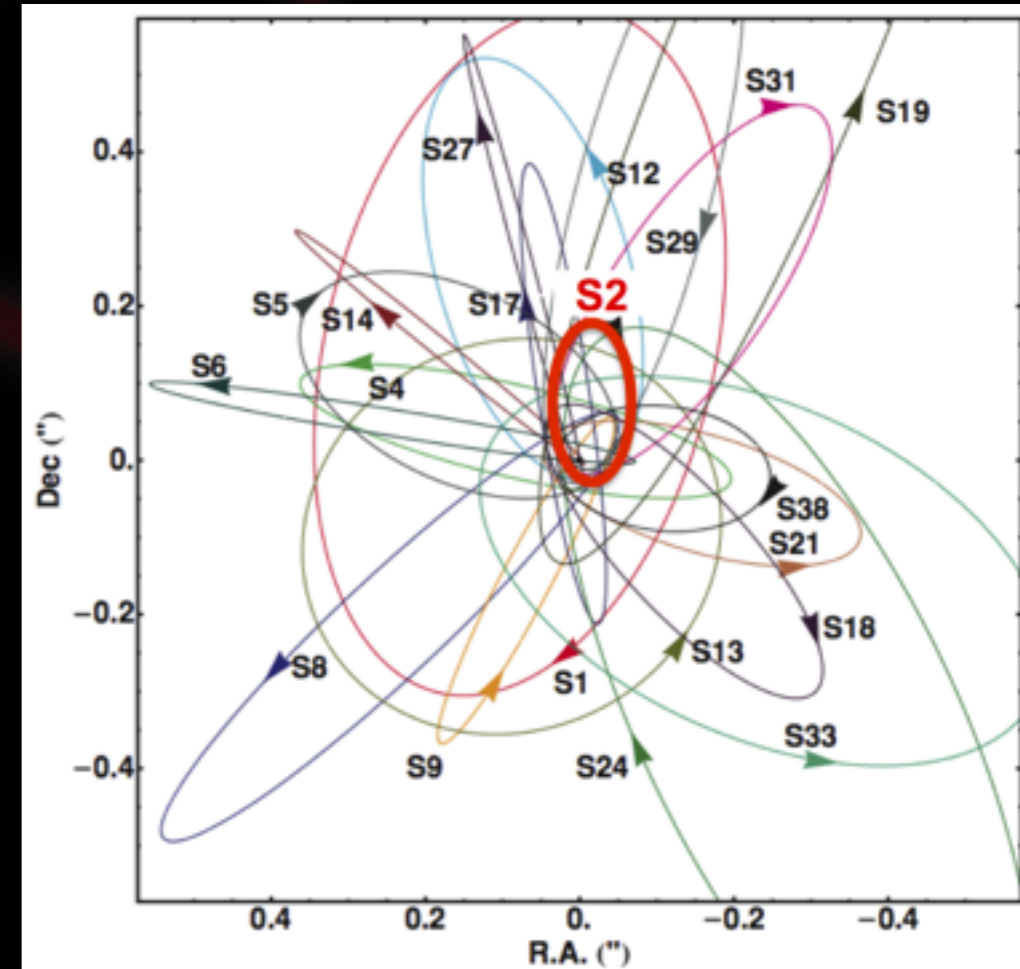
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Supermassive black hole binaries



- ▶ Observations of Sgr A*, a dark massive object of $4.5 \times 10^6 M_{\text{Sun}}$ at the centre of Milky Way.
- ▶ Supermassive Black Hole are indirectly observed in the centre of a large number of galaxies (Active Galactic Nuclei).
- ▶ Observations of galaxies mergers.
 - MBH binaries should exist.
- ▶ Observations of double AGN



NGC 6240 (Komossa et al. ApJ 582 L15)



Antennae galaxies

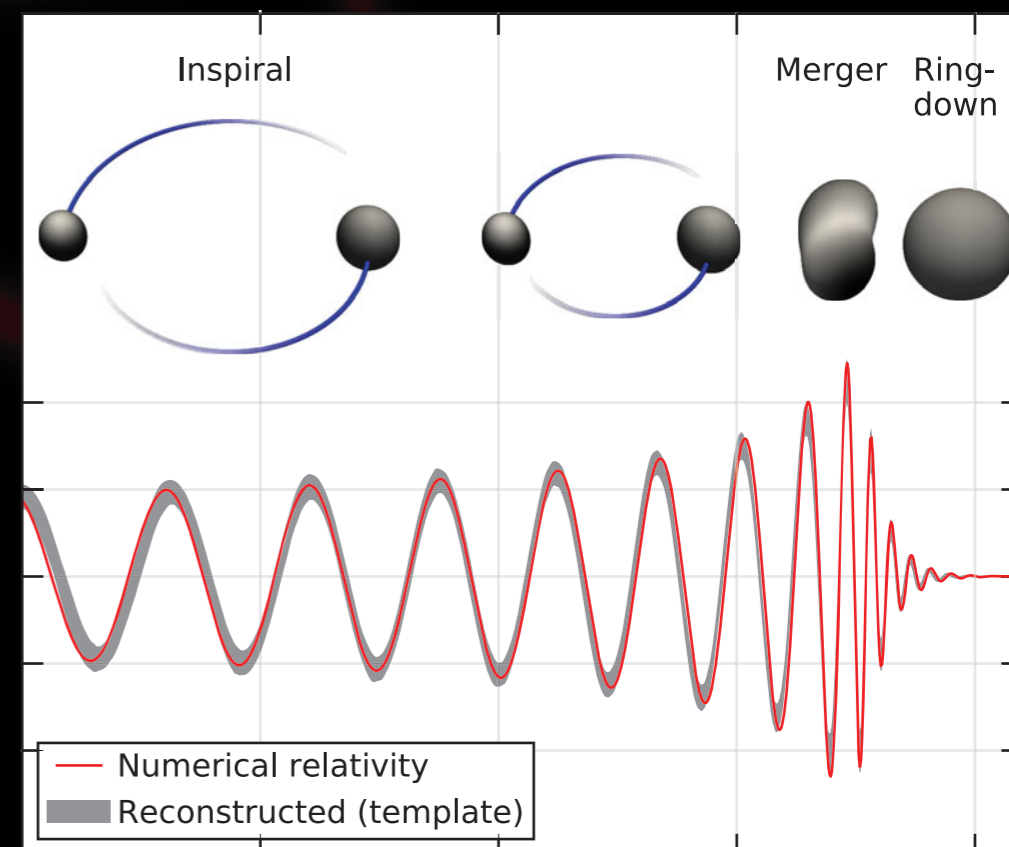


Supermassive black hole binaries



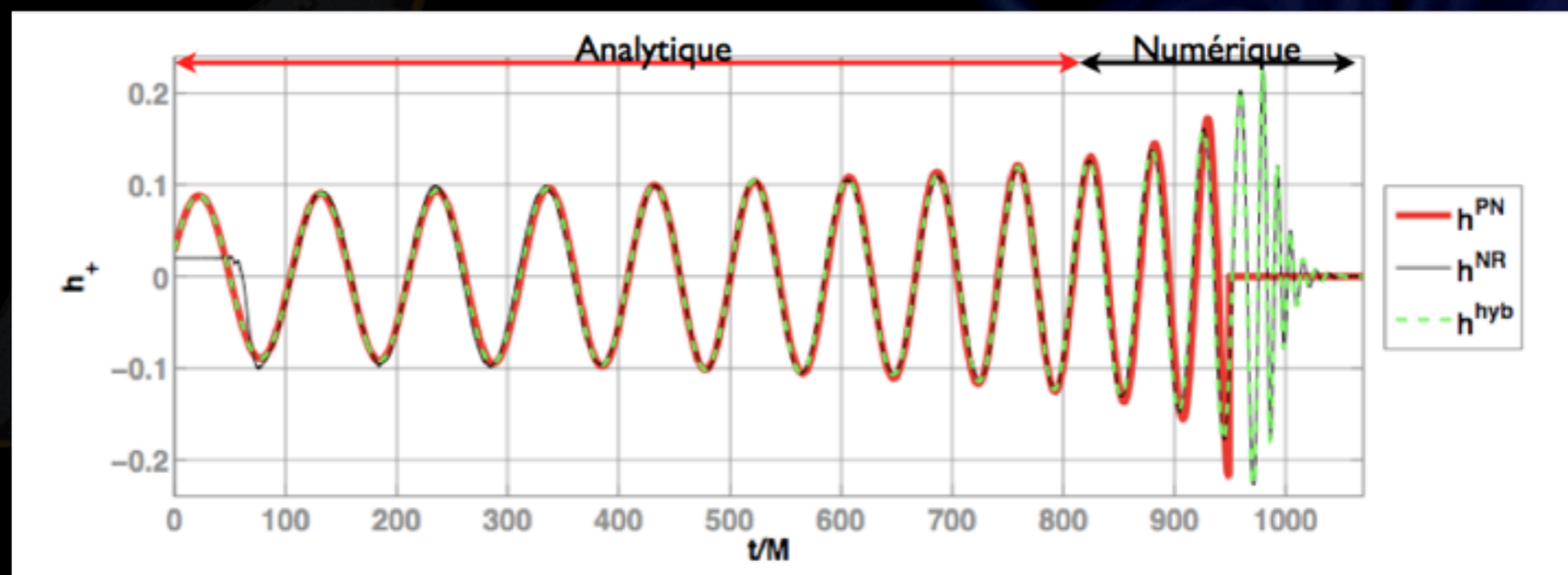
▶ GW emission: 3 phases:

- Inspiral: Post-Newtonian,
- Merger: Numerical relativity,
- Ringdown: Oscillation of the resulting MBH.



▶ No full waveform but several approximations exist :

- Phenomenological waveform,
- Effective One Body,
- ...

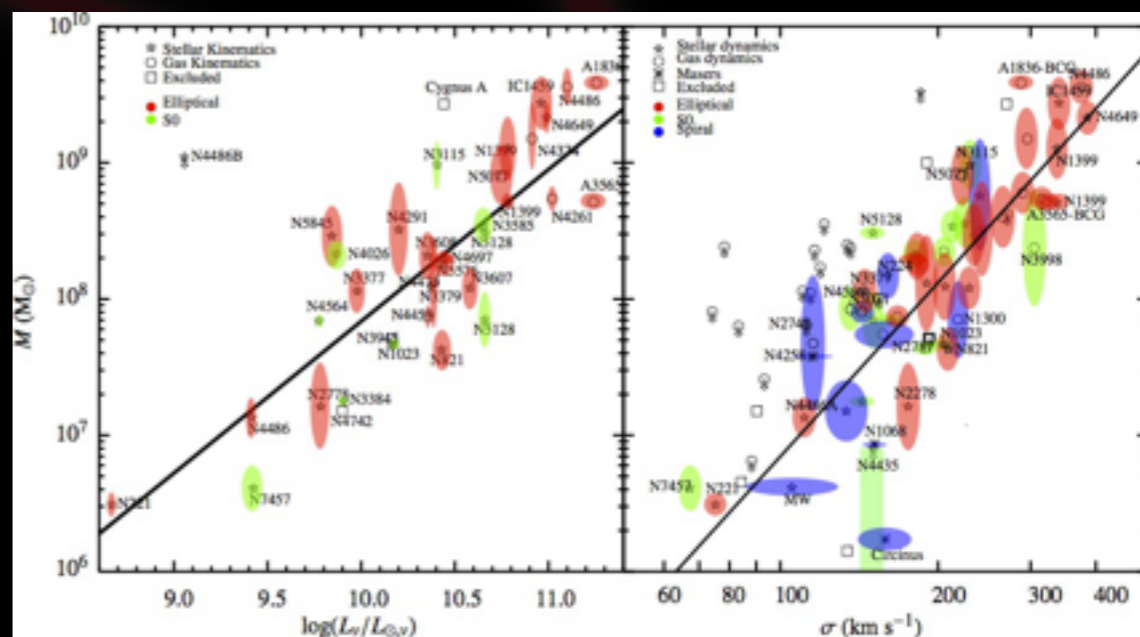
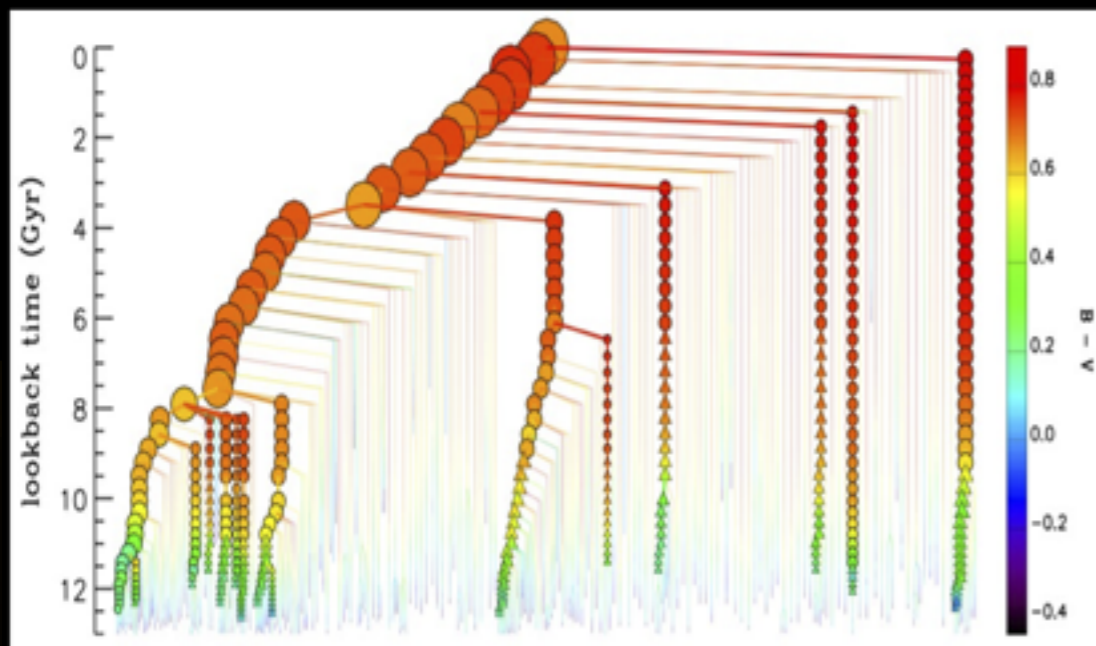




Supermassive black hole binaries

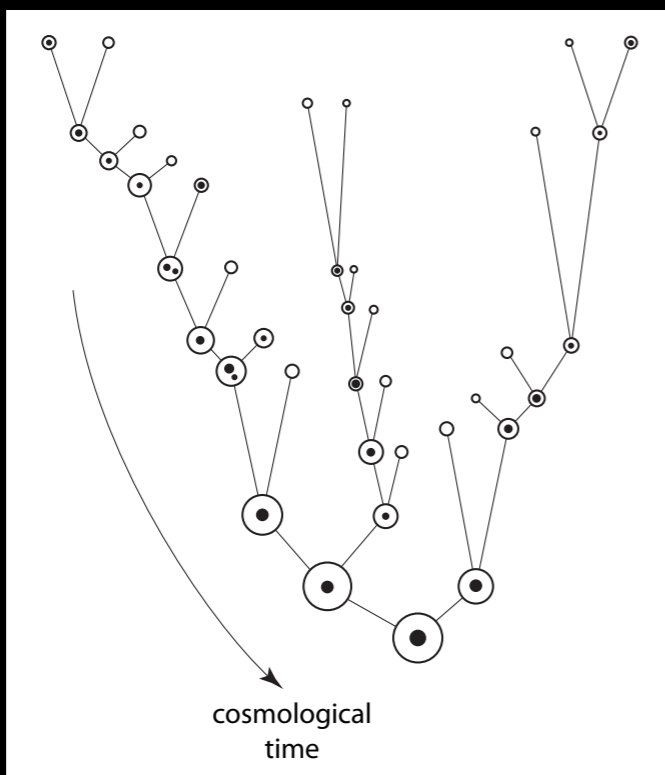
Galaxies merger tree
(cosmological simulation)

"M - σ relation": the speed of stars in bulge is linked to the central MBH mass



From De Lucia et al 2006

Gultekin 2009



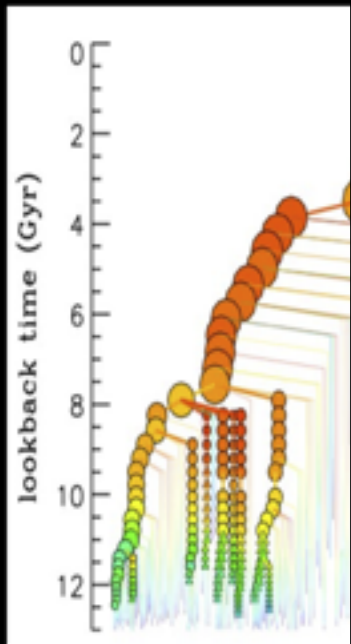


Supermassive black hole binaries

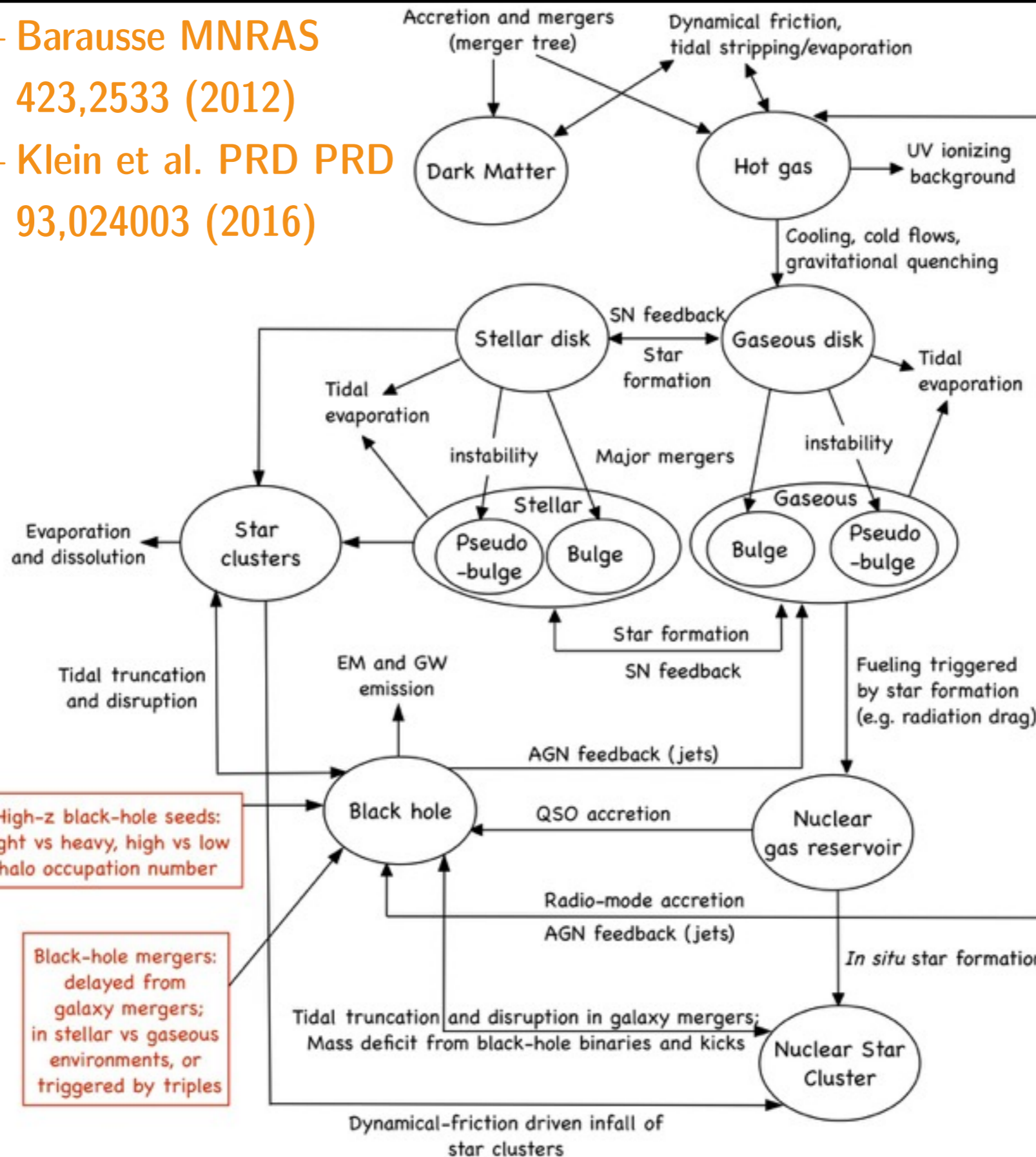
Gal
(cosm

- Barausse MNRAS 423,2533 (2012)
- Klein et al. PRD PRD 93,024003 (2016)

the speed of stars in
the central MBH mass

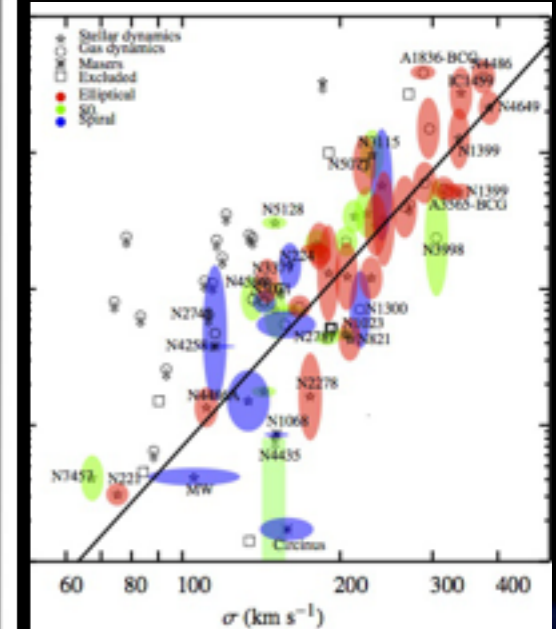


From De Lucia



High-z black-hole seeds:
light vs heavy, high vs low
halo occupation number

Black-hole mergers:
delayed from
galaxy mergers;
in stellar vs gaseous
environments, or
triggered by triples

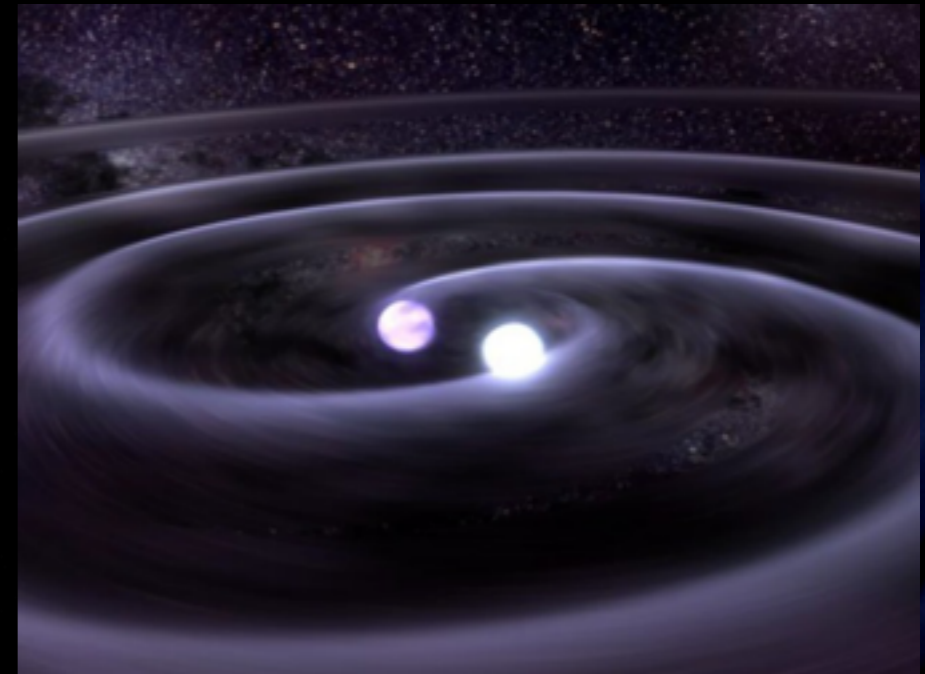




Compact solar mass binaries



- ▶ Large number of stars are in binary system.
- ▶ Evolution in white dwarf (WD) and neutron stars (NS).
=> existence of **WD-WD**, **NS-WD** and **NS-NS binaries**
- ▶ Estimation for the Galaxy: **60 millions**.
- ▶ Gravitational waves:
 - most part in the **slow inspiral** regime (quasi-monochromatic): GW at mHz
 - few are coalescing: GW event of few seconds at $f > 10$ Hz (LIGO/Virgo)
- ▶ Several known system emitting around the mHz

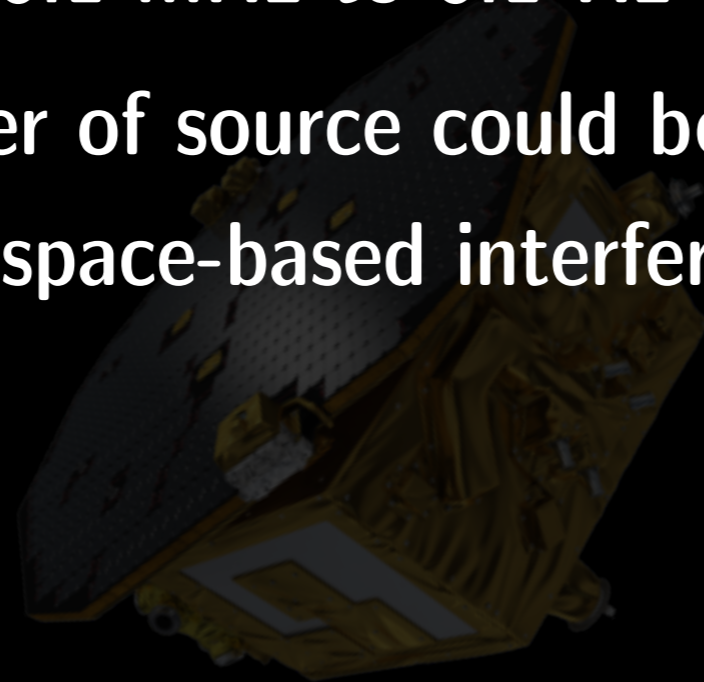
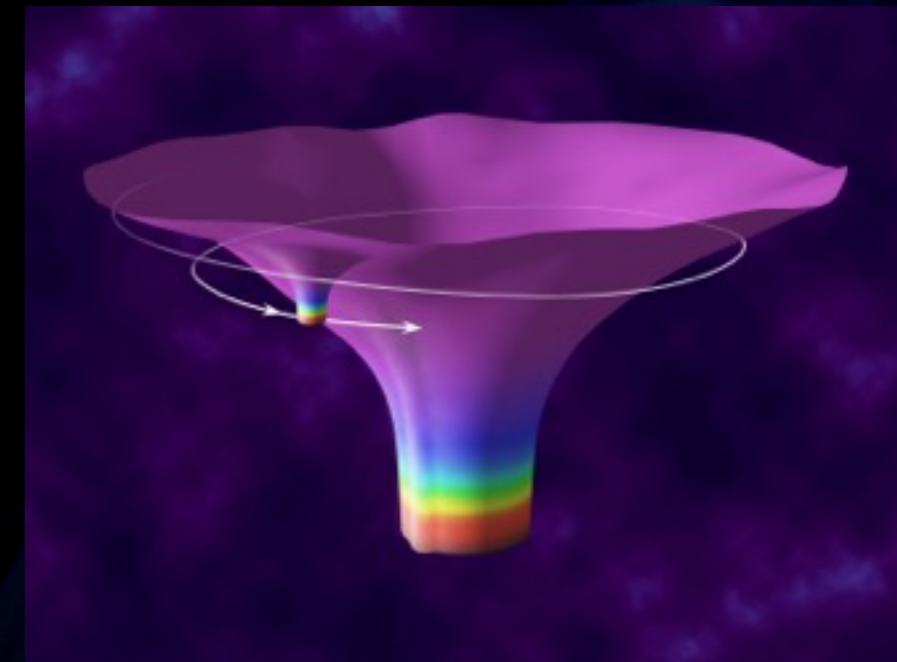
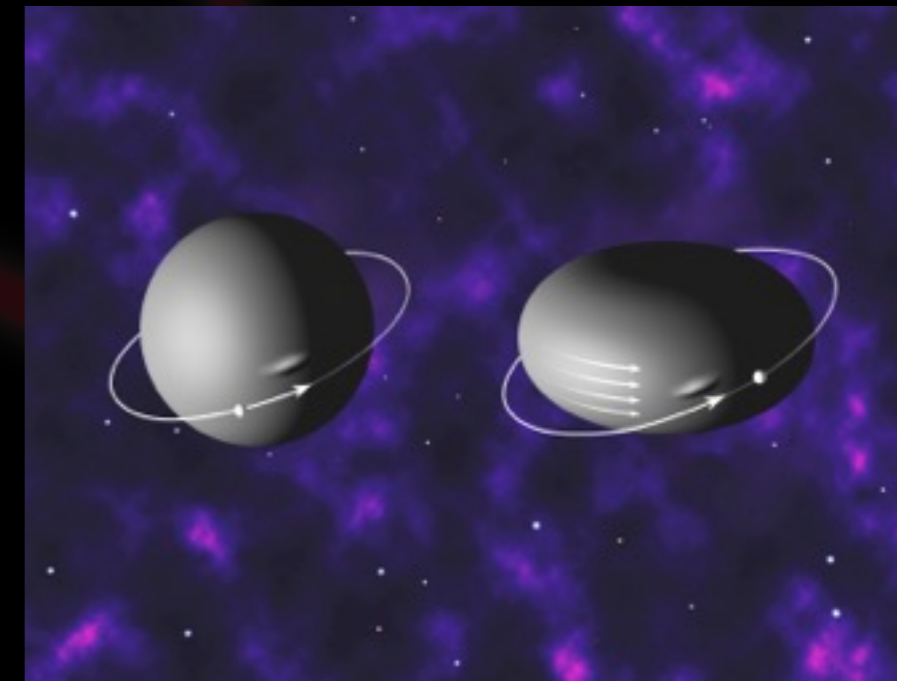




Extreme Mass Ratio Inspirals



- ▶ Capture of a “small” object by massive black hole ($10 - 10^6 M_{\text{Sun}}$)
 - Mass ratio > 200
 - GW gives information on the geometry around the black hole.
 - Test General Relativity in strong field
 - Frequency : 0.1 mHz to 0.1 Hz
 - Large number of source could be observed by space-based interferometer

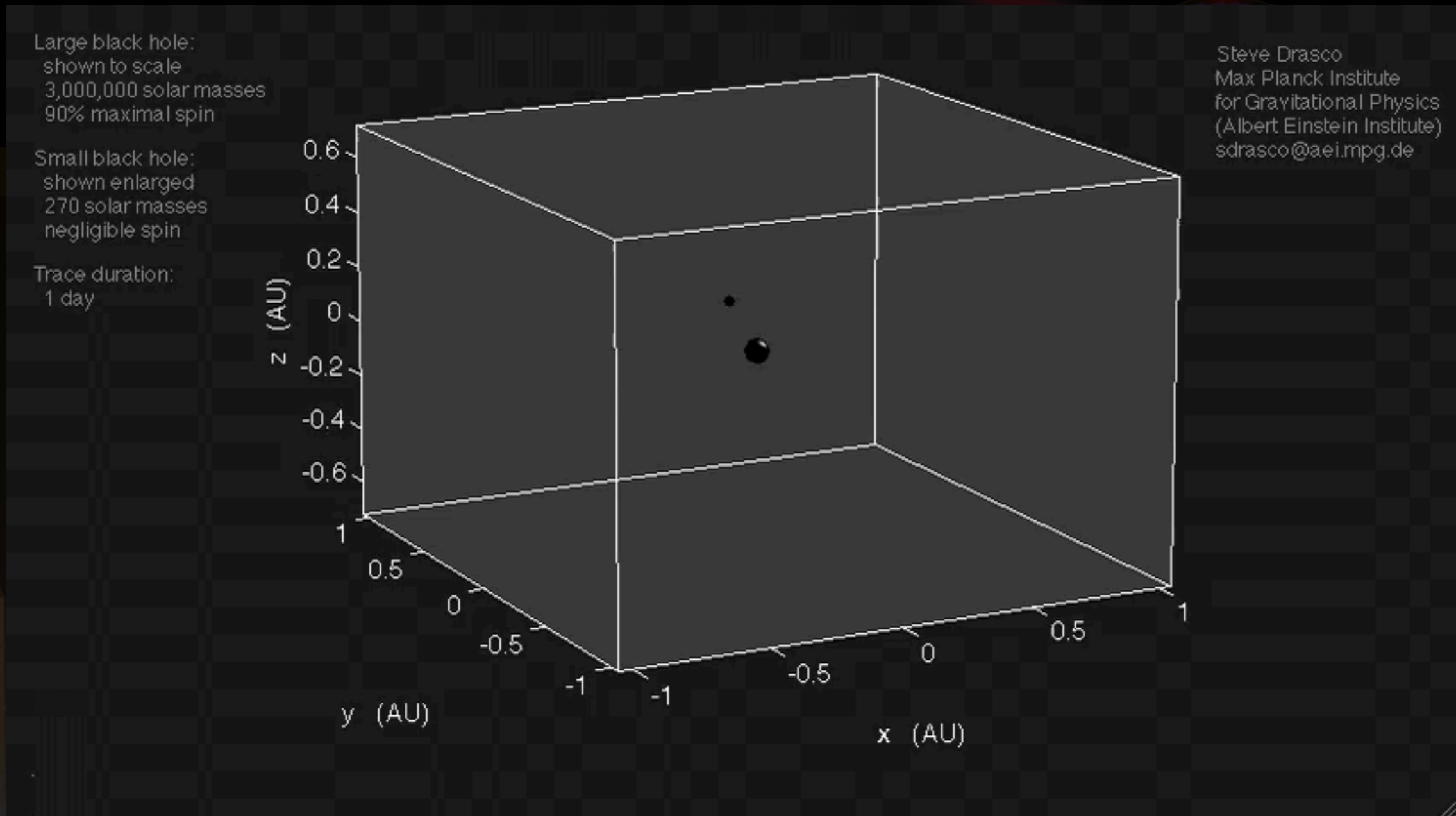




EMRIs



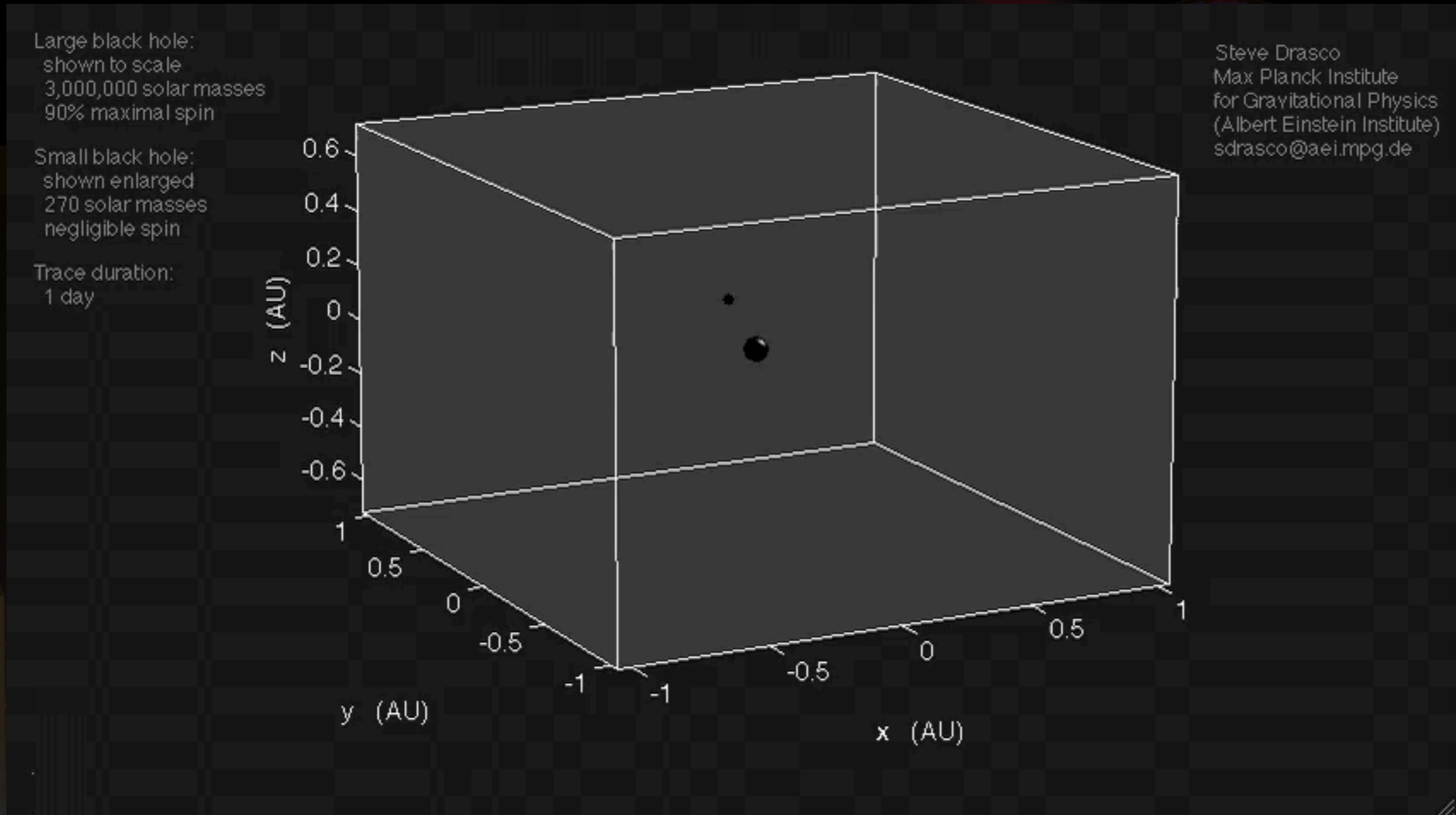
- ▶ **Extreme Mass Ratio Inspiral: small compact objects (10 M_{Sun}) orbiting around a SuperMassive black hole**





EMRIs

- ▶ Extreme Mass Ratio Inspiral: small compact objects (10 M_{Sun}) orbiting around a SuperMassive black hole

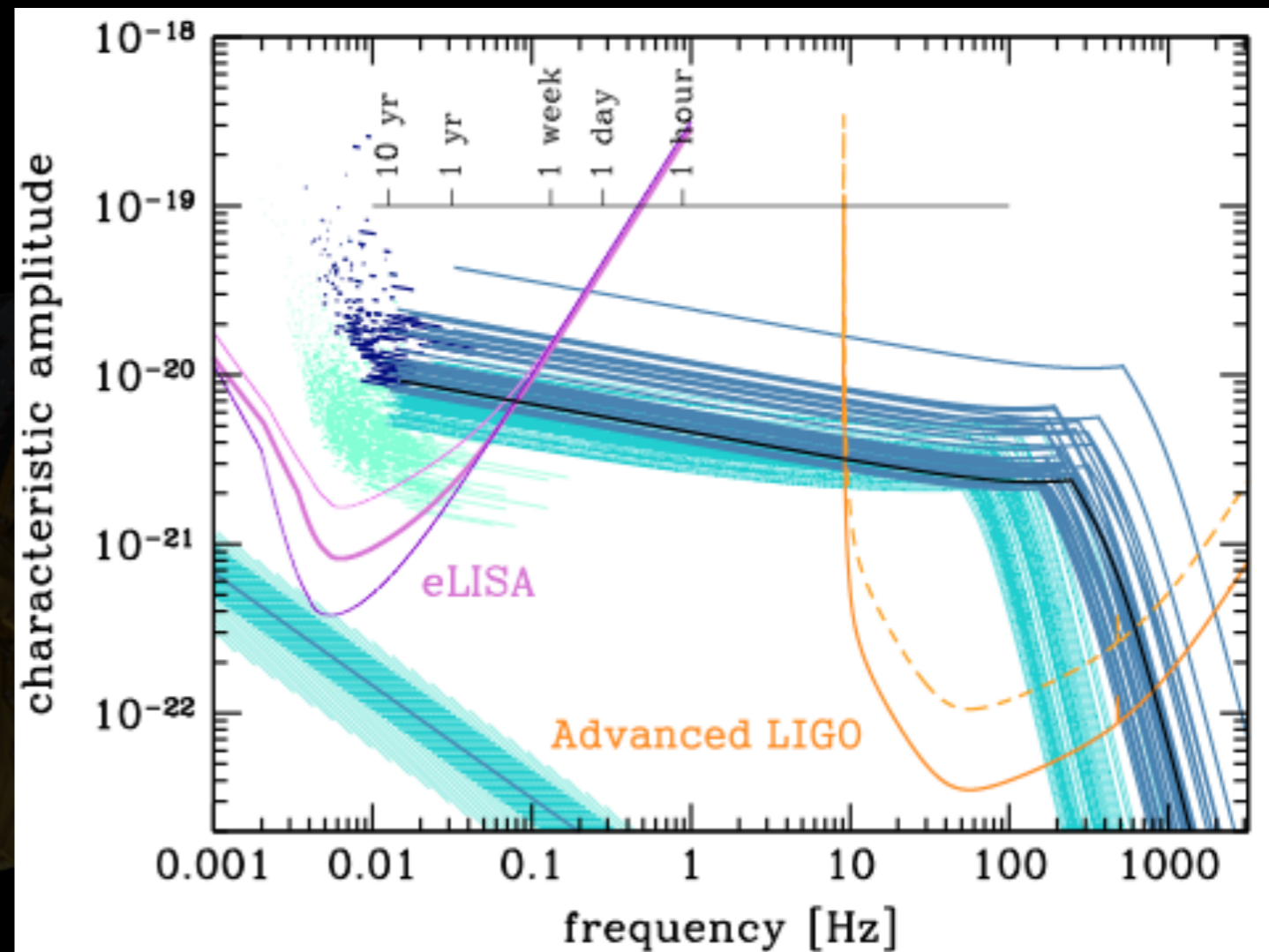




Black Hole Binaries



- ▶ LIGO/Virgo-type sources: binaries with 2 black holes of few tens solar masses.
- ▶ During most part of the inspiral time, emission in the mHz band
=> multi-observatories
GW astronomy

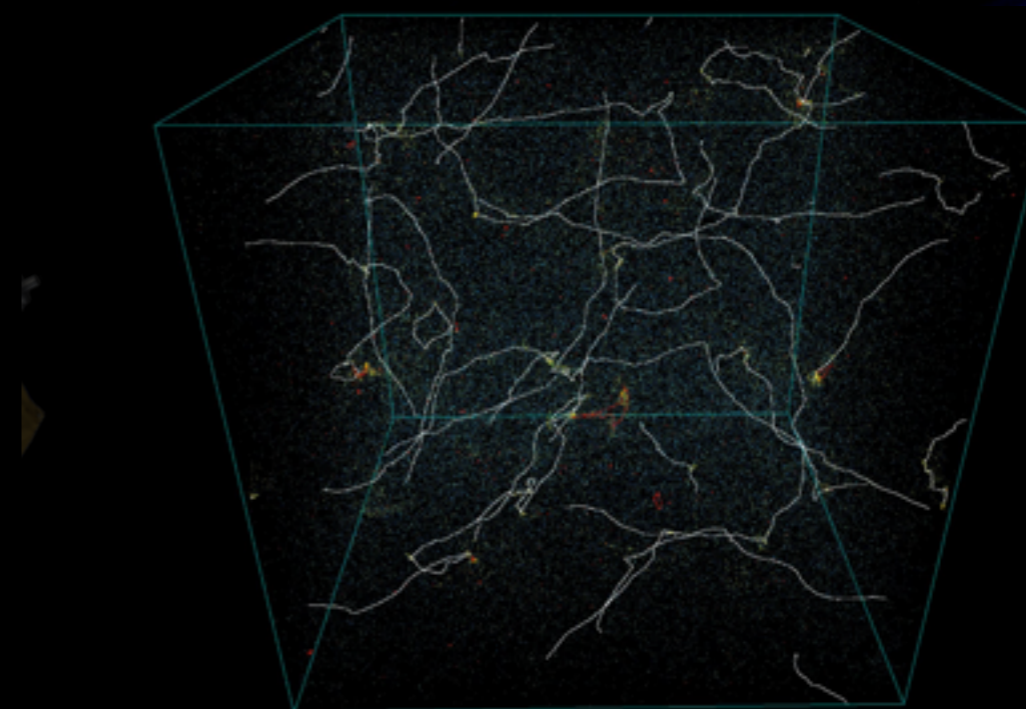
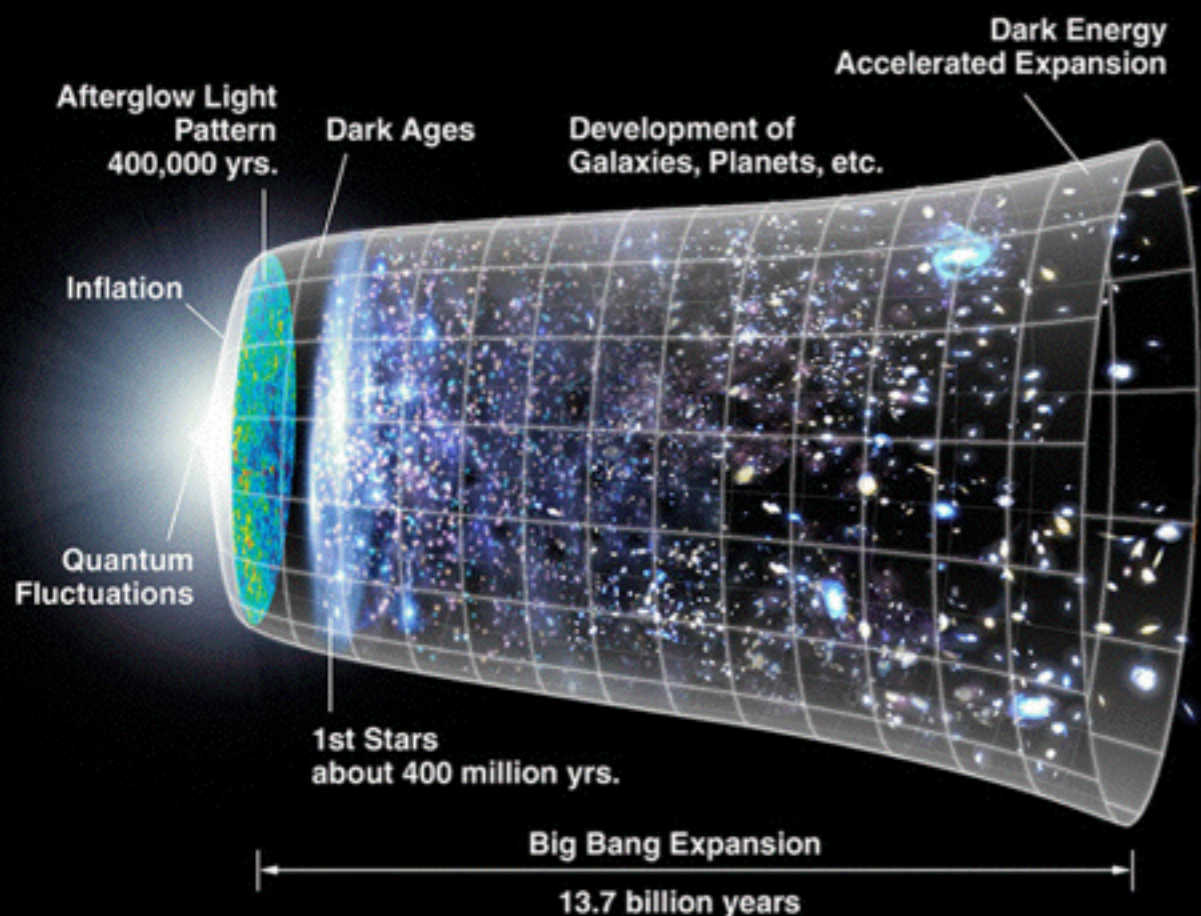
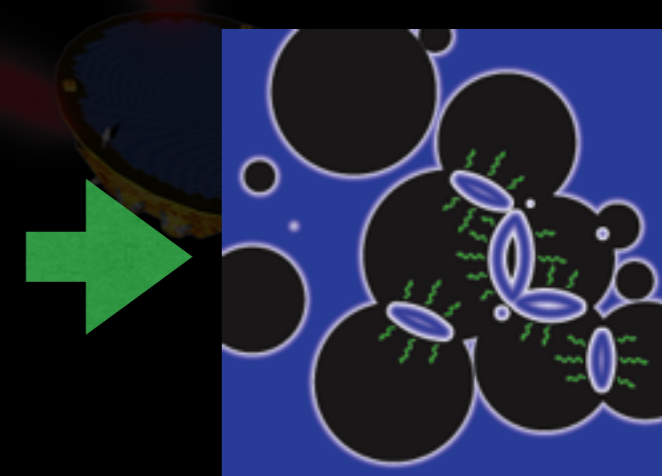
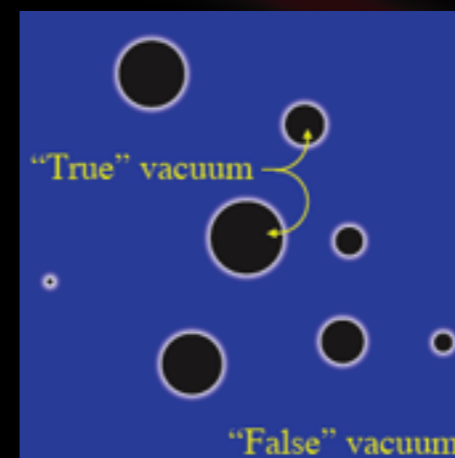


A. Sesana,
PRL 116,
231102 (2016)



Cosmological backgrounds

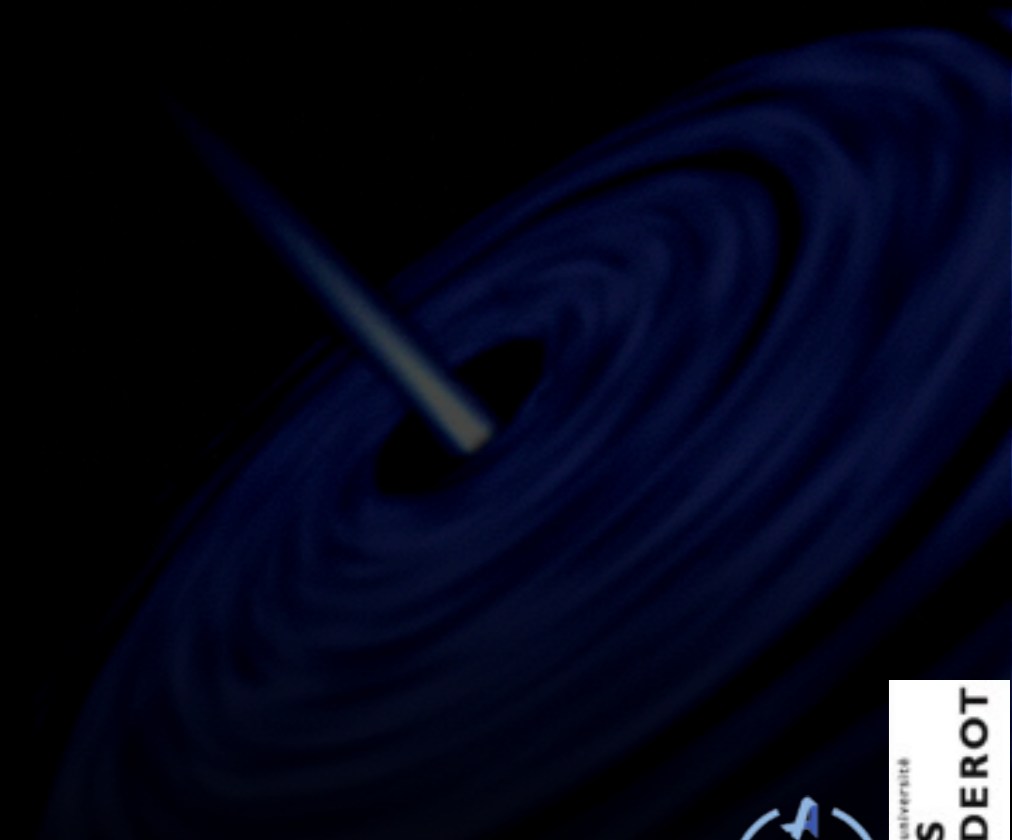
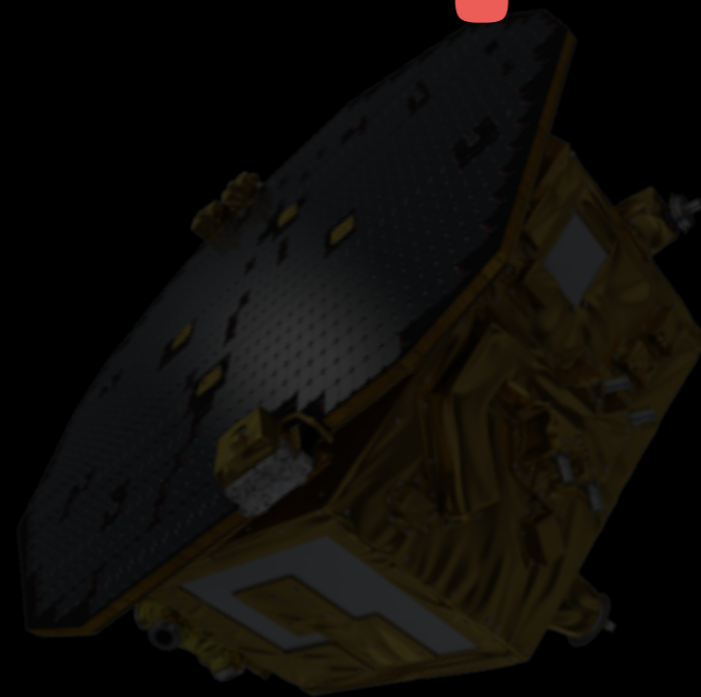
- ▶ Variety of cosmological sources for stochastic background :
 - First order phase transition in the very early Universe
 - Cosmic strings network
 - ...





Unknown sources

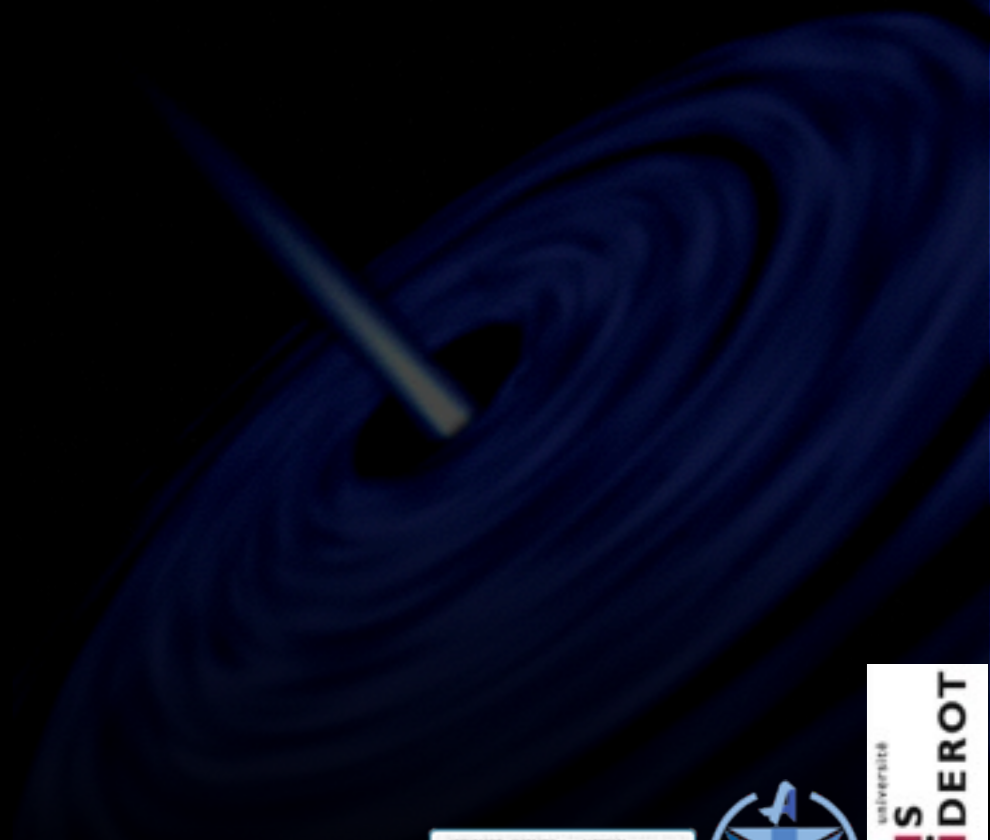
- ▶ High potential of discovery in the mHz GW band ?





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History of LISA

- ▶ 1978: first study based on a rigid structure (NASA)
- ▶ 1980s: studies with 3 free-falling spacecrafts(US)
- ▶ 1993: proposal ESA/NASA: 4 spacecrafts
- ▶ 1996-2000: pre-phase A report
- ▶ 2000-2010: LISA and LISAPathfinder: ESA/NASA mission
- ▶ **2011**: NASA stops => ESA continue: reduce mission
- ▶ 2012: selection of JUICE L1 ESA
- ▶ 2013: selection of ESA L3 : « The gravitational Universe »
- ▶ **2015-2016: success of LISAPathfinder + detection GWs**



LISA at ESA



- ▶ 25/10/2016 : Call for mission
- ▶ 13/01/2017 : submission of «LISA proposal» (LISA consortium)
- ▶ 8/3/2017 : Phase 0 mission (CDF 8/3/17 → 5/5/17)
- ▶ 20/06/2017 : LISA mission approved by SPC
- ▶ 8/3/2017 : Phase 0 payload (CDF June → November 2017)
- ▶ 2018→2020 : competitive phase A : 2 companies compete
- ▶ 2020→2021 : B1: start industrial implementation
- ▶ 2021-2022 : mission adoption
- ▶ During about 8.5 years : construction
- ▶ 2030-2034 : launch Ariane 6.4
- ▶ 1.5 years for transfert
- ▶ 4 years of nominal mission
- ▶ Possible extension to 10 years



GW observations !



« The LISA Proposal »

https://www.elisascience.org/files/publications/LISA_L3_20170120.pdf

LISA
Laser Interferometer Space Antenna

A proposal in response to the ESA call for L3 mission concepts

Lead Proposer
Prof. Dr. Karsten Danzmann

2 Science performance

The science theme of *The Gravitational Universe* is addressed here in terms of Science Objectives (SOs) and Science Investigations (SIs), and the Observational Requirements (ORs) necessary to reach those objectives. The ORs are in turn related to Mission Requirements (MRs) for the noise performance, mission duration, etc. The majority of individual LISA sources will be binary systems covering a wide range of masses, mass ratios, and physical states. From here on, we use M to refer to the total source frame mass of a particular system. The GW strain signal, $h(t)$, called the waveform, together with its frequency domain representation $\hat{h}(f)$, encodes exquisite information about intrinsic parameters of the source (e.g., the mass and spin of the interacting bodies) and extrinsic parameters, such as inclination, luminosity distance and sky location. The assessment of Observational Requirements (ORs) requires a calculation of the Signal-to-Noise-Ratio (SNR) and the parameter measurement accuracy. The SNR is approximately the square root of the frequency integral of the ratio of the signal squared, $\hat{h}(f)^2$, to the sky-averaged sensitivity of the observatory, expressed as power spectral density $S_h(f)$. Shown in Figure 2 is the square root of this quantity, the linear spectral density $\sqrt{S_h(f)}$, for a 2-arm configuration (TDI X). In

the following, any quoted SNRs for the Observational Requirements (ORs) are given in terms of the full 3-arm configuration. The derived Mission Requirements (MRs) are expressed as linear spectral densities of the sensitivity for a 2-arm configuration (TDI X).

The sensitivity curve can be computed from the individual instrument noise contributions, with factors that account for the noise transfer functions and the sky and polarisation averaged response to GWs. Requirements for a minimum SNR level, above which a source is detectable, translate into specific MRs for the observatory. Throughout this section, parameter estimation is done using a Fisher Information Matrix approach, assuming a 4 year mission and 6 active links. For long-lived systems, the calculations are done assuming a very high duty-cycle (> 95%). Requiring the capability to measure key parameters to some minimum accuracy sets MRs that are generally more stringent than those for just detection. Signals are computed according to GR, redshifts using the cosmological model and parameters inferred from the Planck satellite results, and for each class of sources, synthetic models driven by current astrophysical knowledge are used in order to describe their demography. Foregrounds from astrophysical sources, and backgrounds of cosmological origin are also considered.

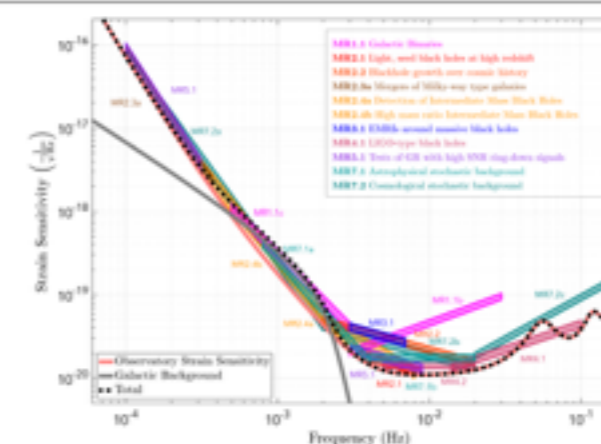


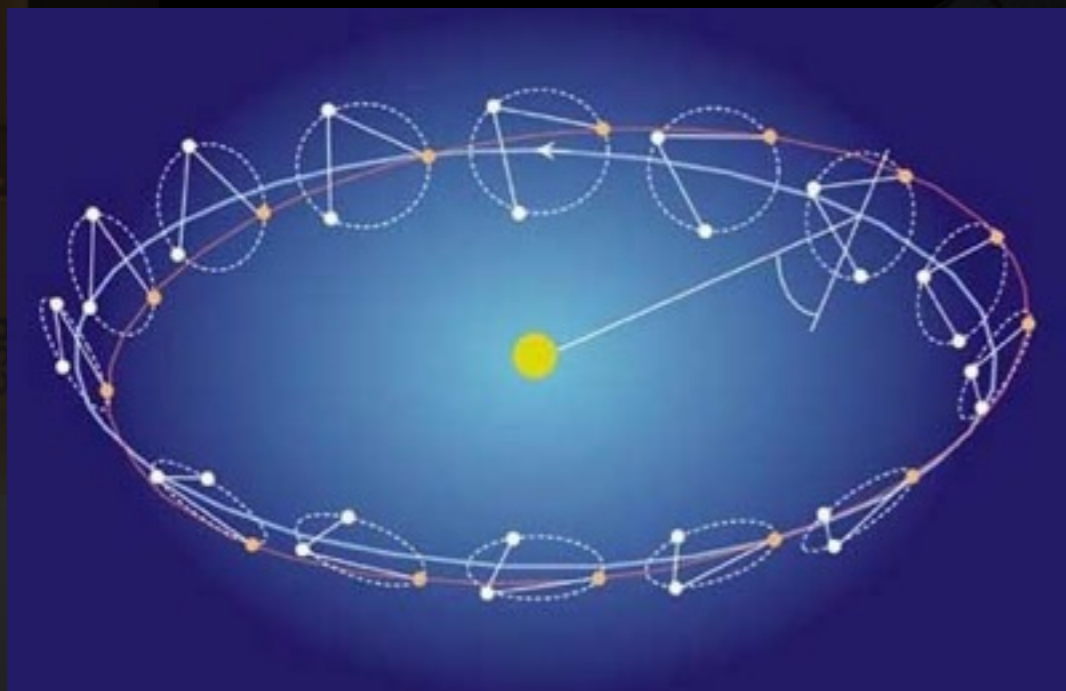
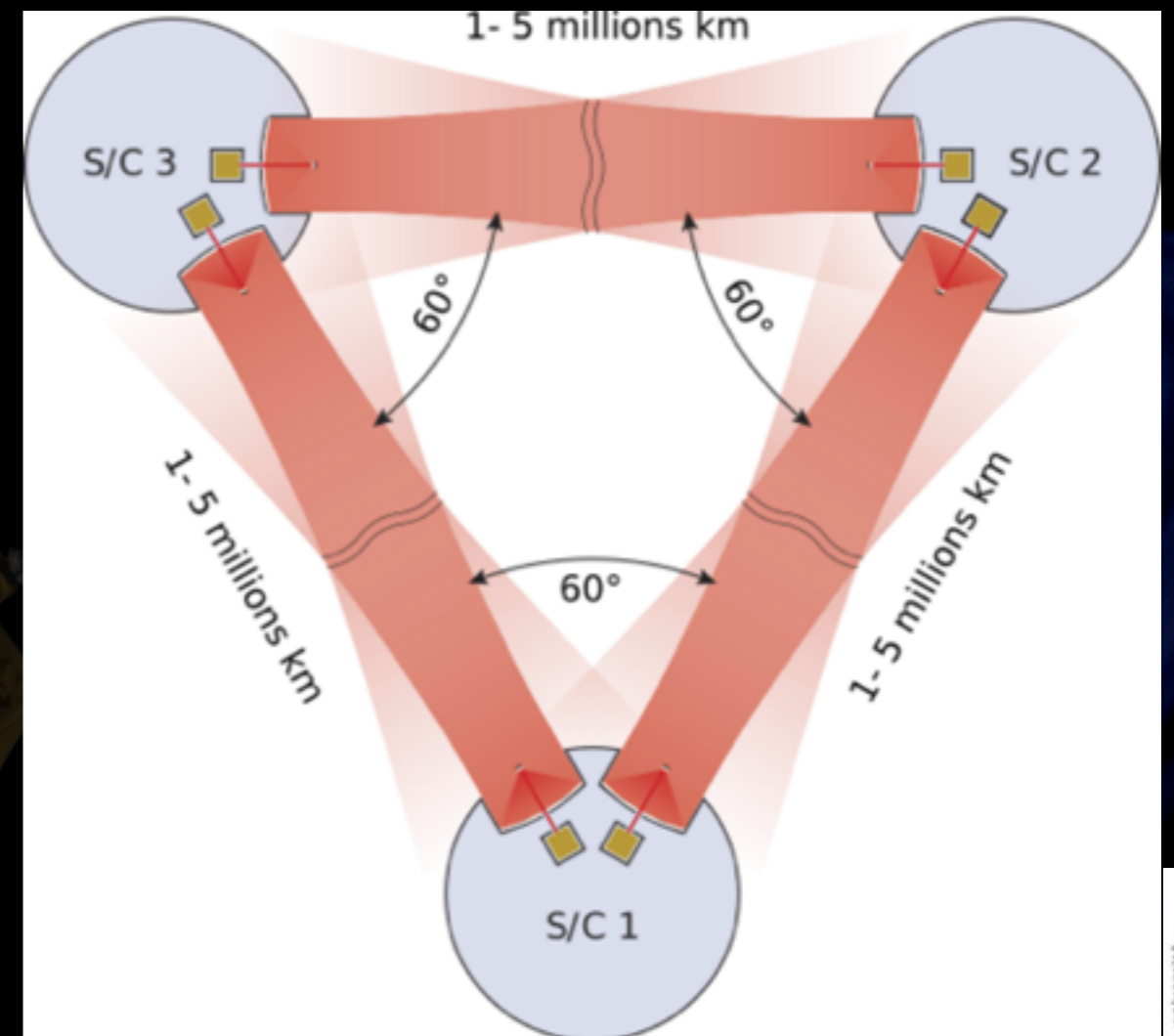
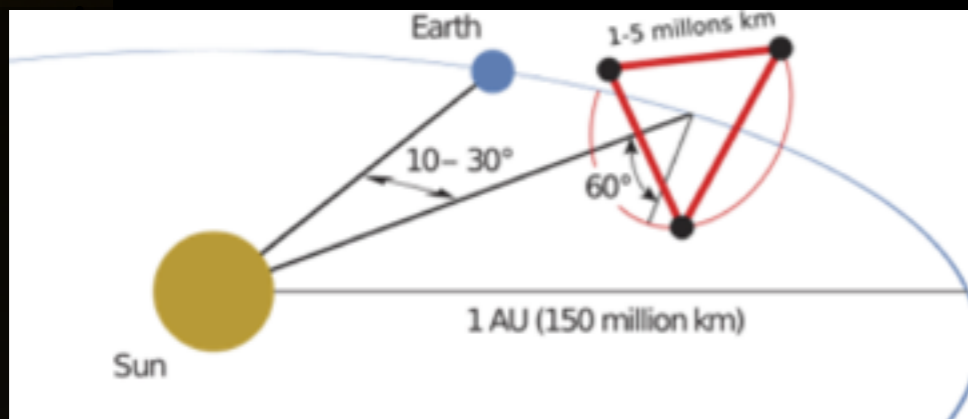
Figure 2: Mission constraints on the sky-averaged strain sensitivity of the observatory for a 2-arm configuration (TDI X), $\sqrt{S_h(f)}$, derived from the threshold systems of each observational requirement.



LISA



- ▶ Laser Interferometer Space Antenna
- ▶ 3 spacecrafts on heliocentric orbits and distant from few millions kilometers (2.5 millions km in the proposal L3)
- ▶ Goal: detect relative distance changes of 10^{-21} : few picometers

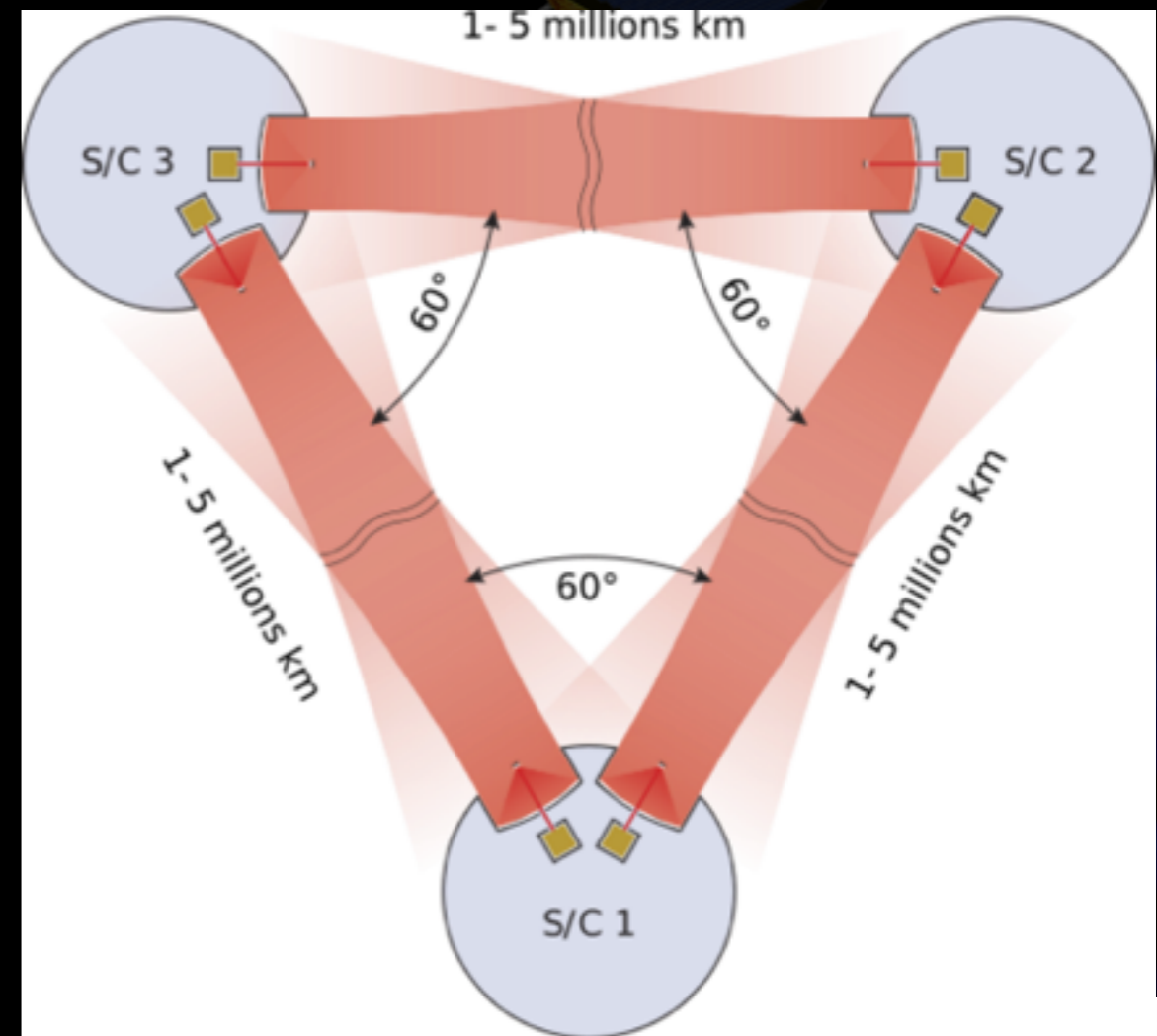
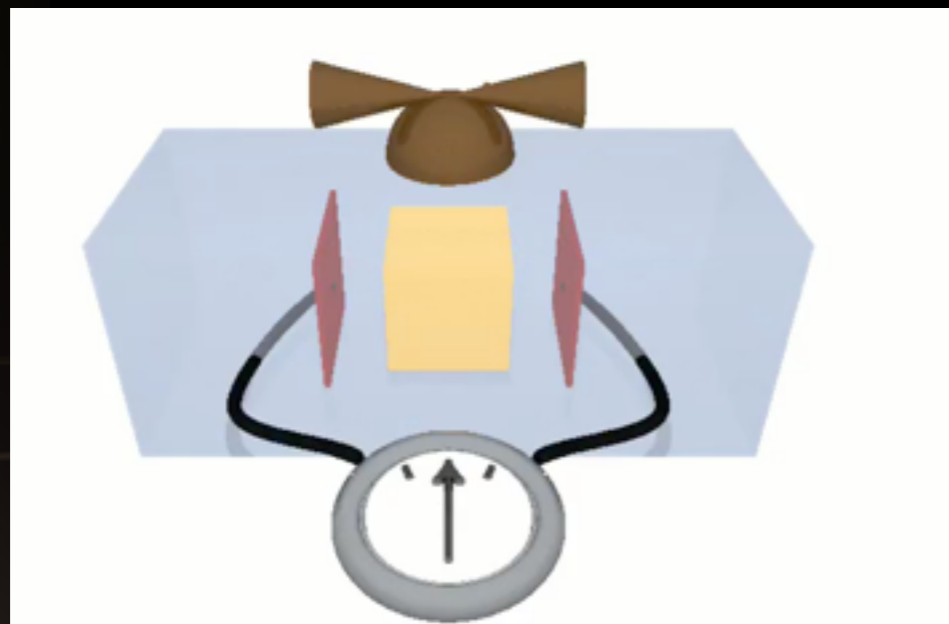




LISA



- ▶ Spacecraft (SC) should only be sensible to gravity:
 - the spacecraft protects test-masses (TMs) from external forces and always adjusts itself on it using micro-thrusters
 - Readout:
 - interferometric (sensitive axis)
 - capacitive sensing

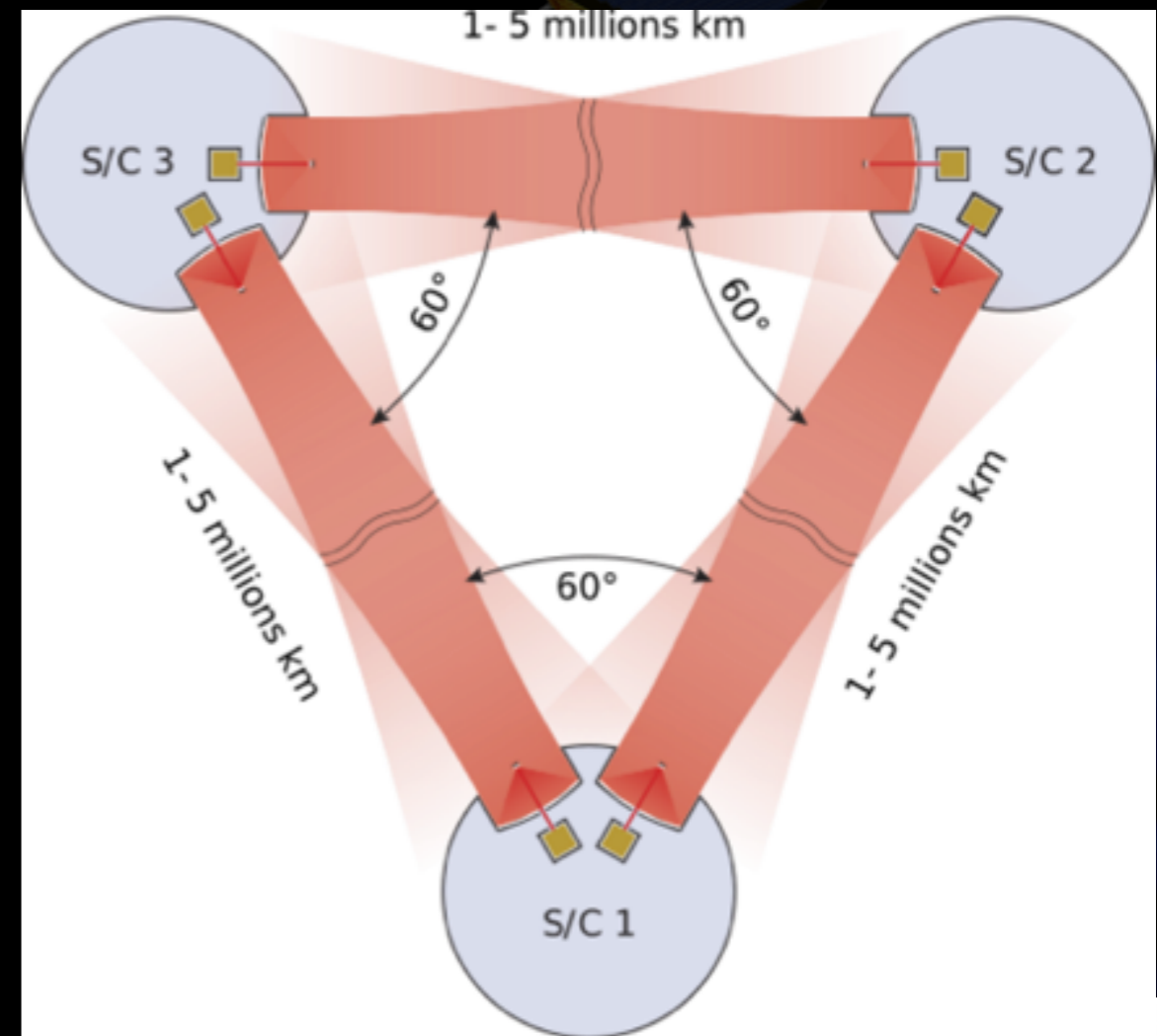
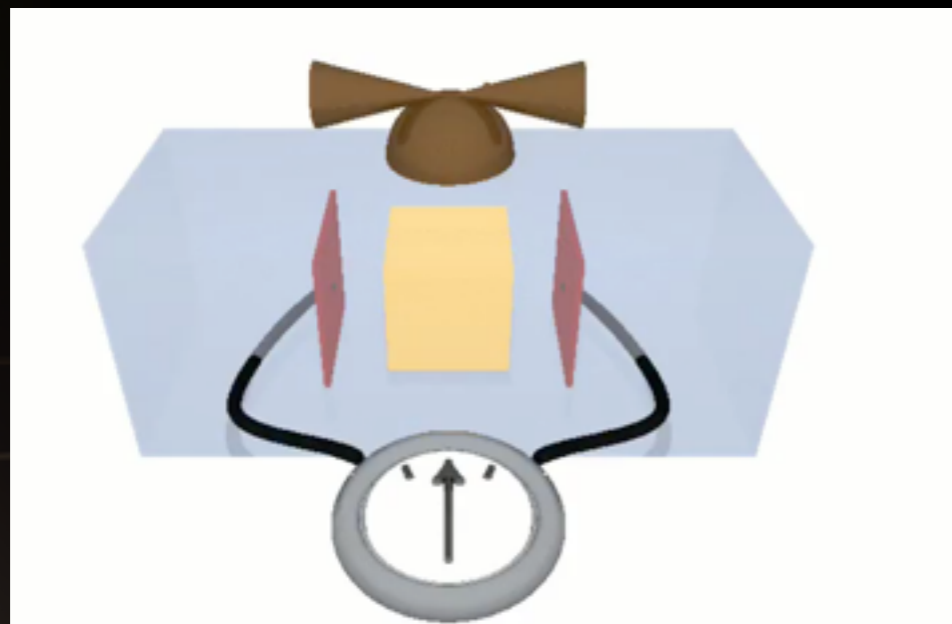




LISA



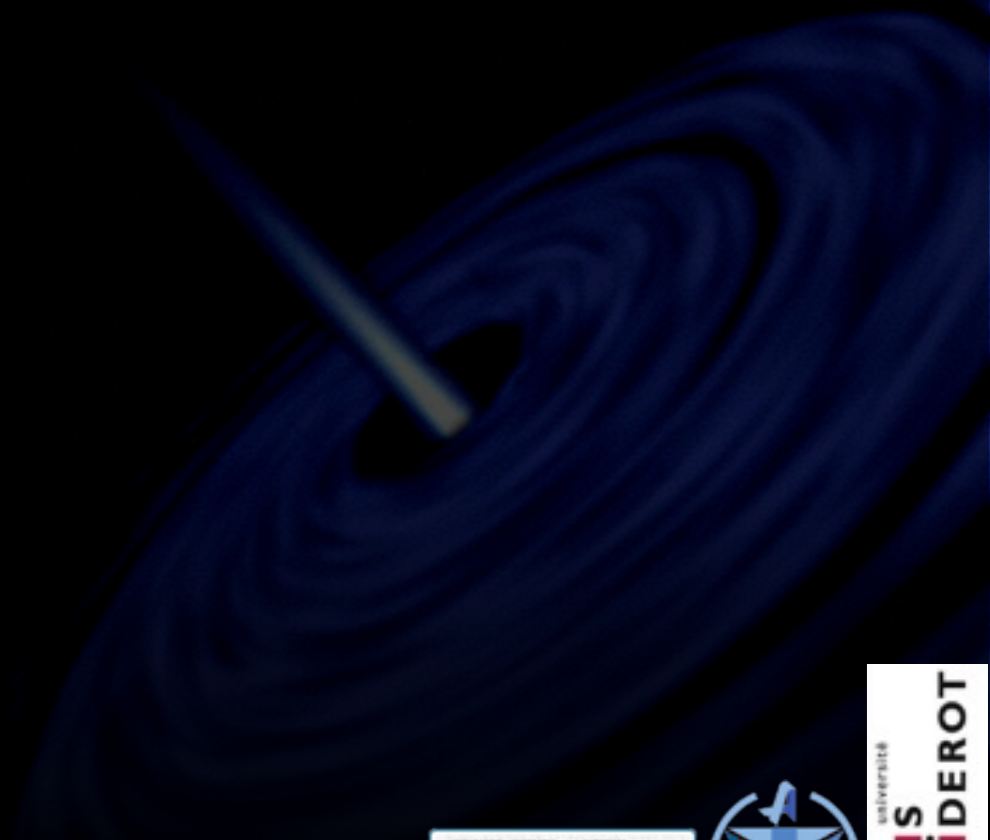
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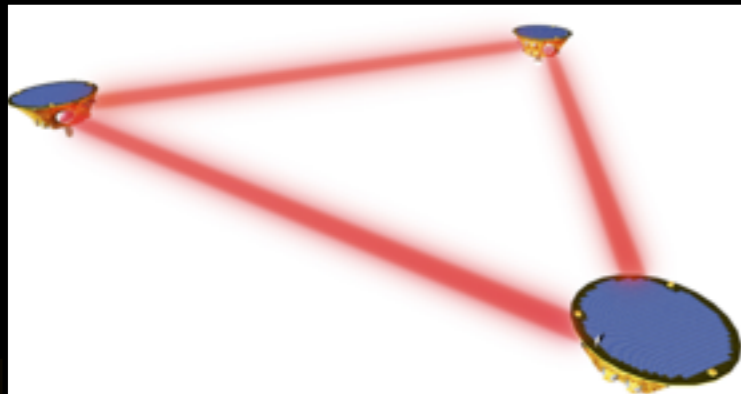




LISA Pathfinder

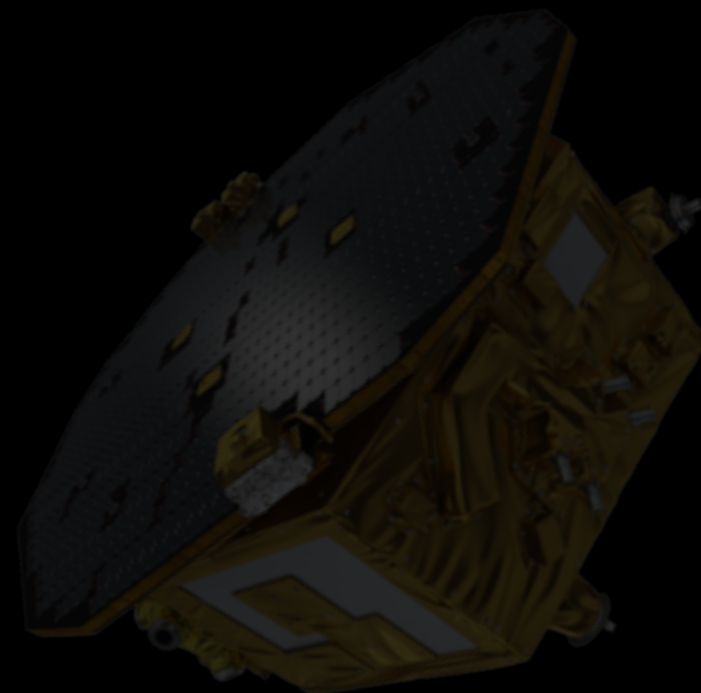
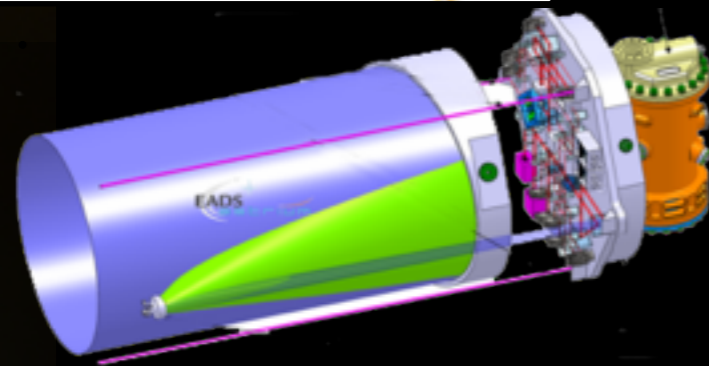


- ▶ Technological demonstrator for LISA



LISA :

- ▶ 3 spacecraft separated by millions of km
- ▶ Role of each spacecraft is to protect the fiducial test masses from external forces

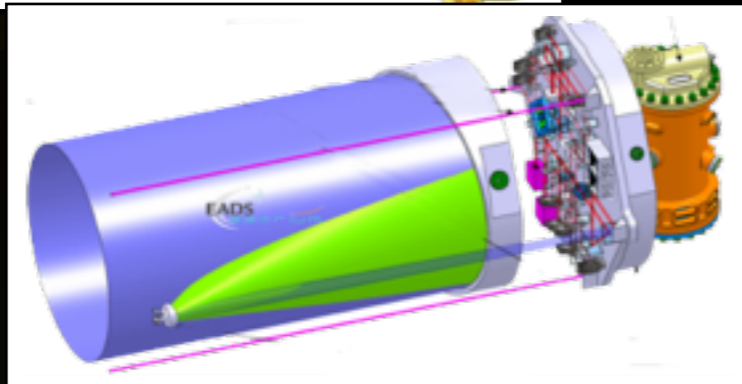
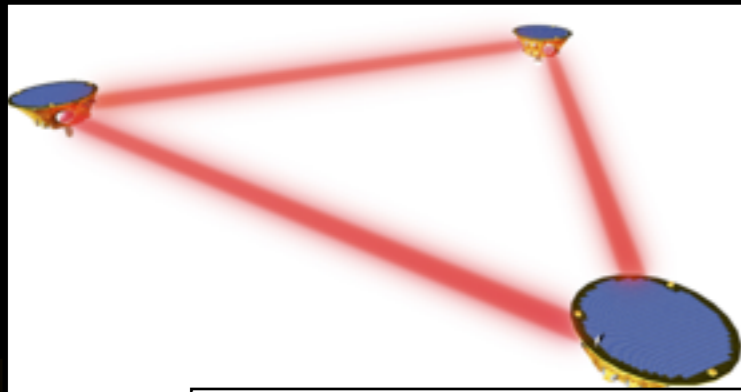




LISA Pathfinder



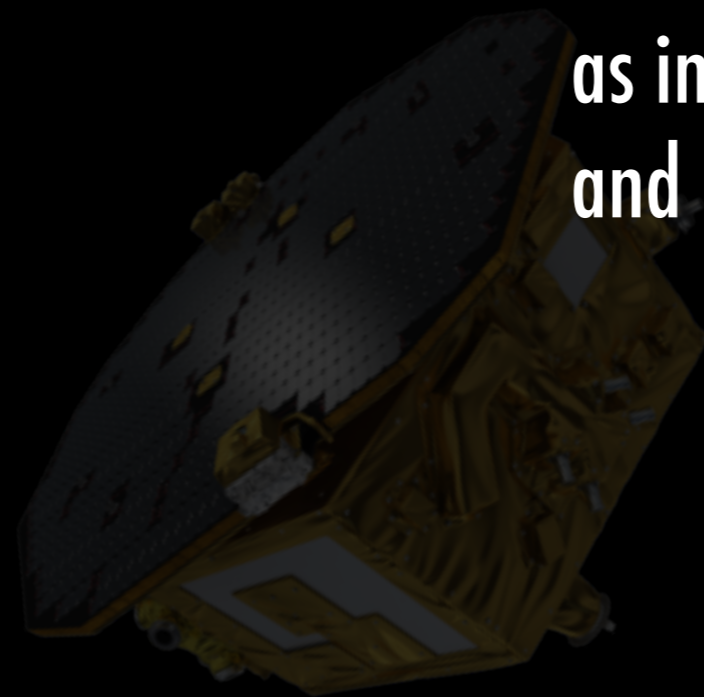
▶ Technological demonstrator for LISA



LISA :

Locally measure distance from TM to SC using:

- ▶ Laser interferometry along sensitive axis (between SC)
- ▶ Capacitive sensing on orthogonal axes
- ▶ TM displacement measurements are used as input to DFACS which controls position and attitude of SC respect to the TM

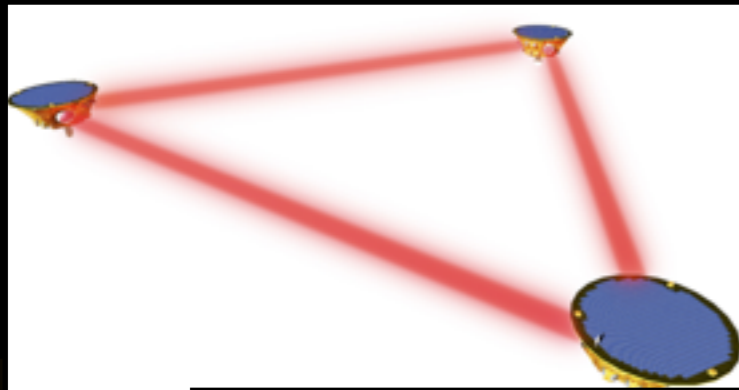




LISA Pathfinder



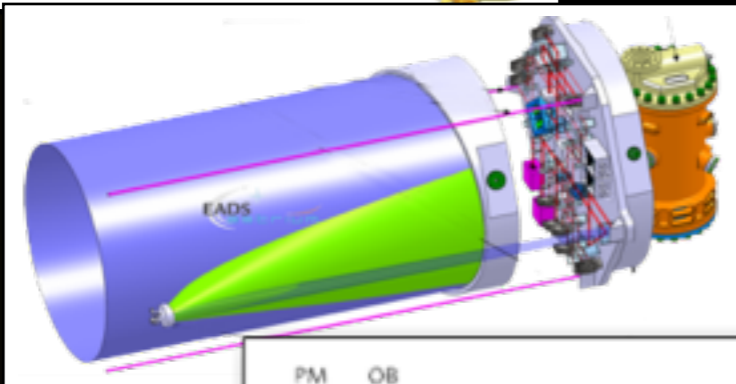
▶ Technological demonstrator for LISA



LISA :

▶ Measure distance along using laser interferometry

$(TM1 \rightarrow SC1) + (SC1 \rightarrow SC2) + (SC2 \rightarrow TM2)$



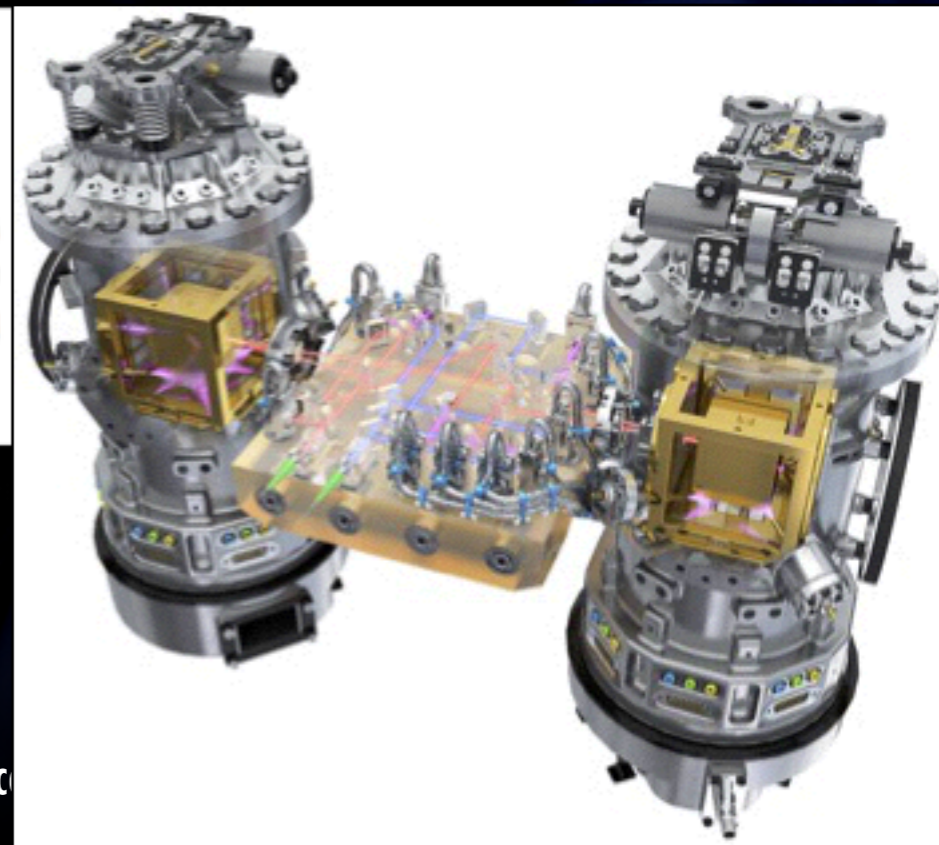
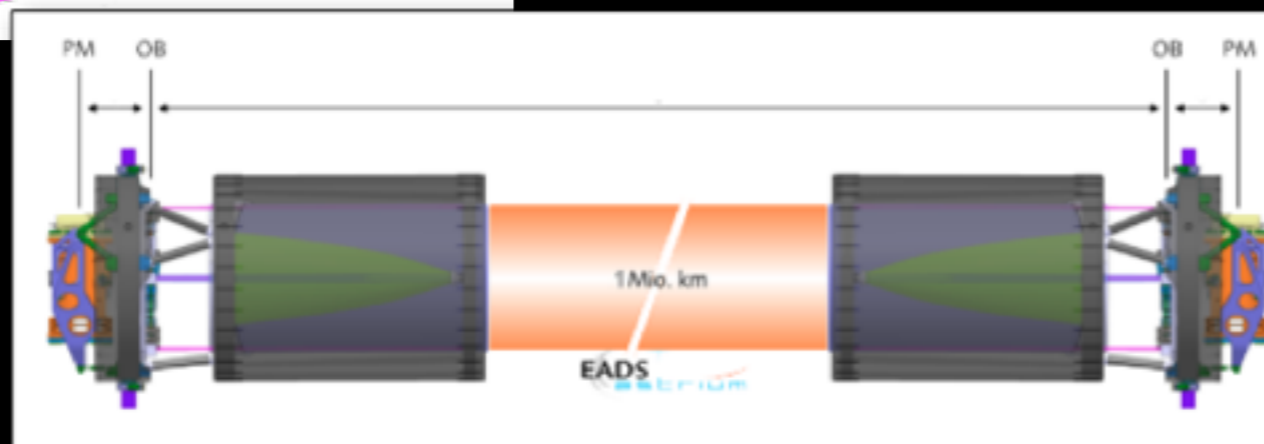
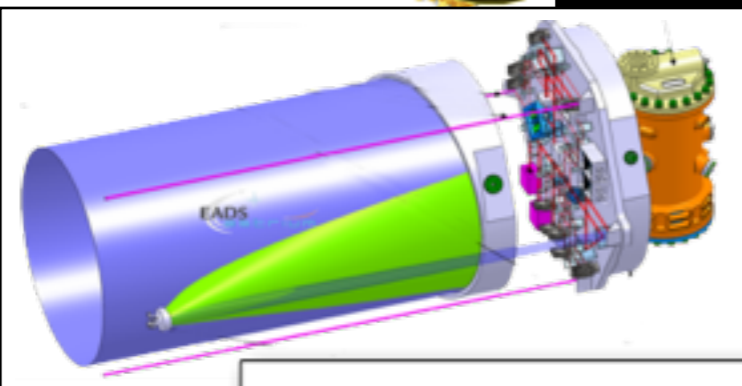
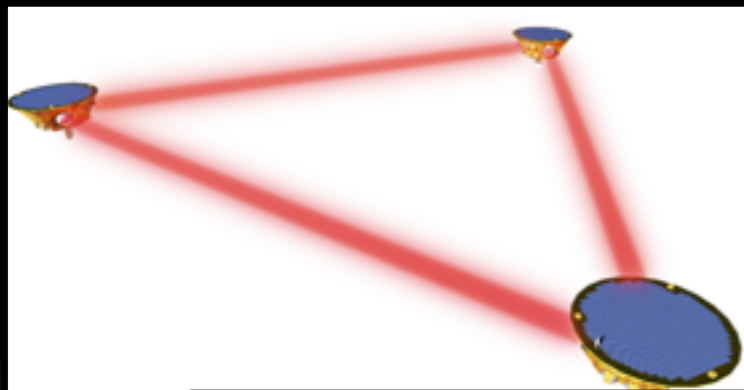


LISAPathfinder

▶ Technological demonstrator for LISA

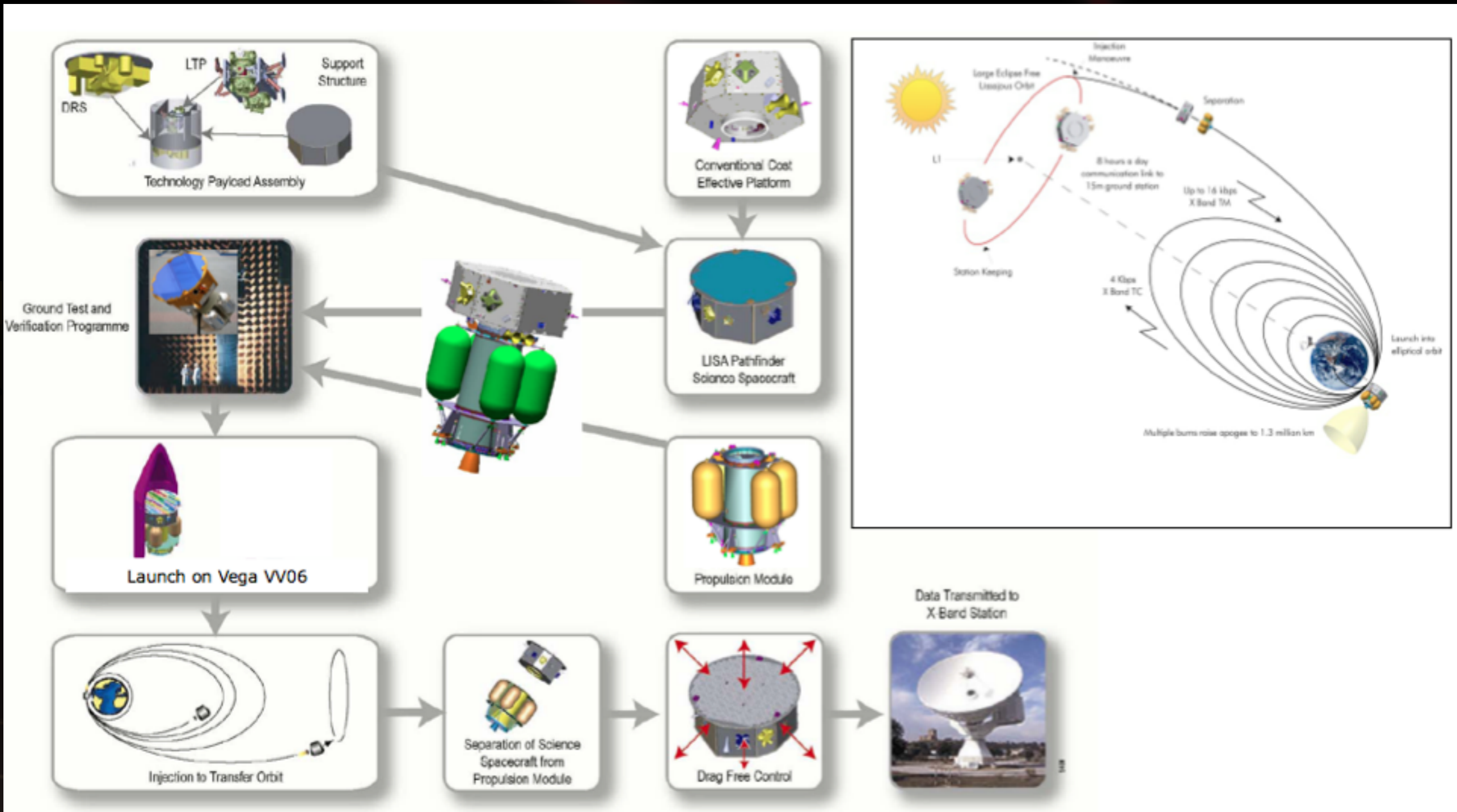
LISAPathfinder:

- ▶ 2 test masses / 2 inertial sensors
- ▶ Laser readout of TM1 → SC and TM1 → TM2
- ▶ Capacitive readout of all 6 d.o.f. of TM
- ▶ Drag-Free and Attitude Control System
- ▶ Micro-newton thrusters



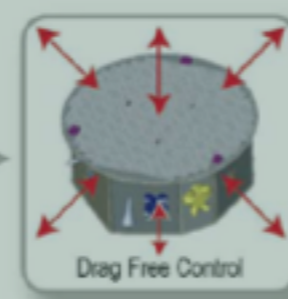
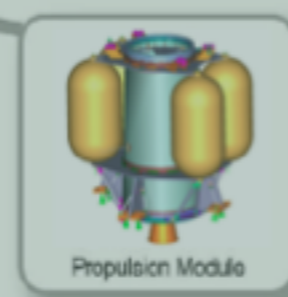
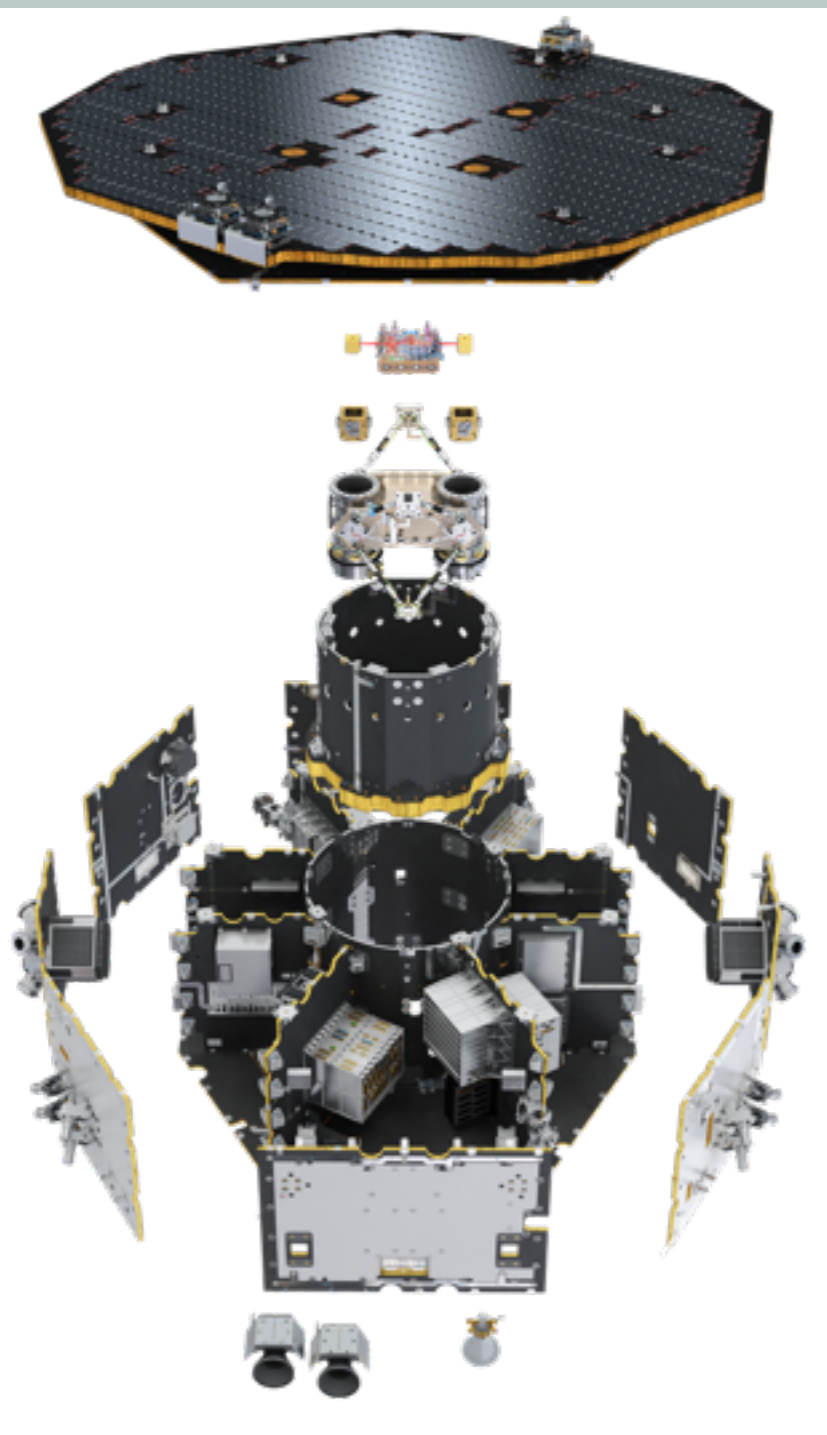


LISA Pathfinder timeline





LISA Pathfinder timeline



Ground Verification

Launch into elliptical orbit



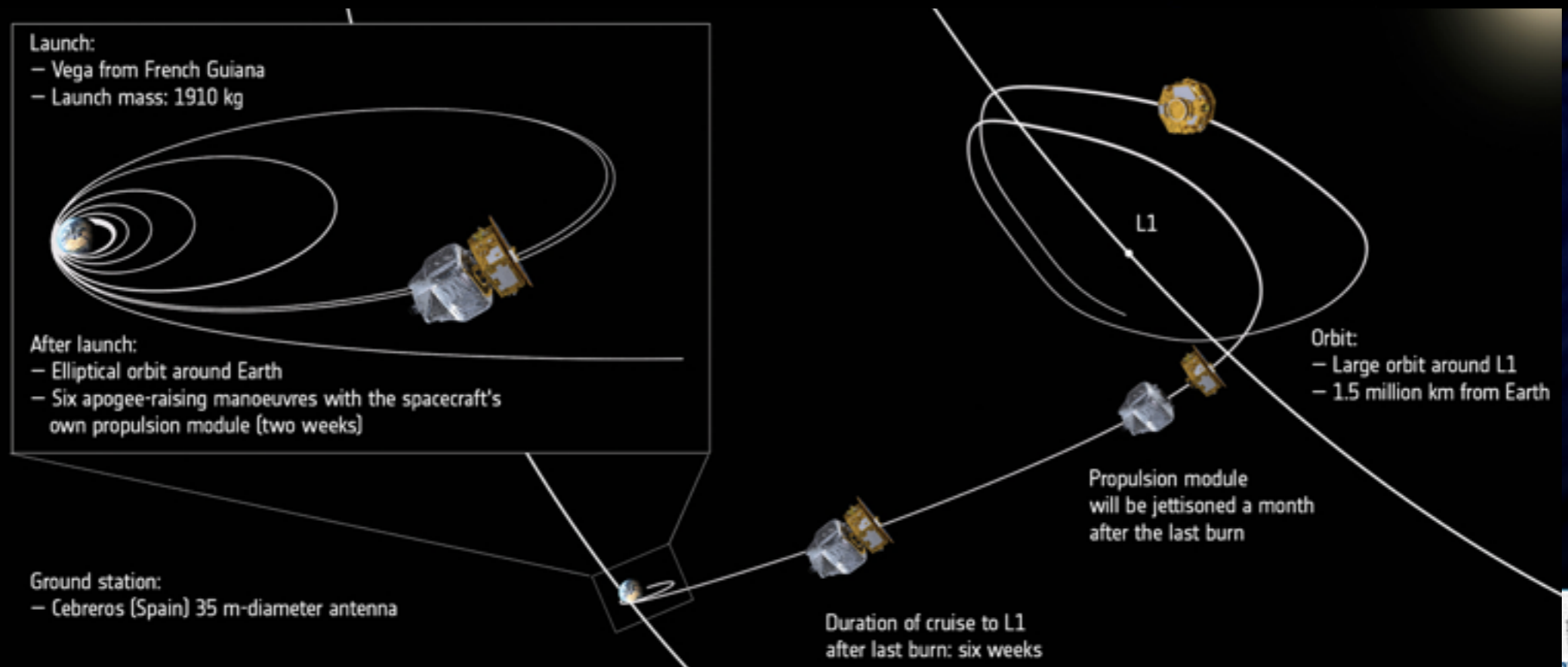
LISAPathfinder timeline





LISA Pathfinder timeline

- ▶ 3/12/2015: Launch from Kourou
- ▶ 22/01/2016: arrived on final orbit & separation of propulsion module
- ▶ 17/12/2015 → 01/03/2016: commissioning
- ▶ 01/03/2016 → 27/06/2016: LTP operations (Europe)
- ▶ 27/06/2016 → 11/2016: DRS operations (US) + few LTP weeks
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Last command: 18/07/2017





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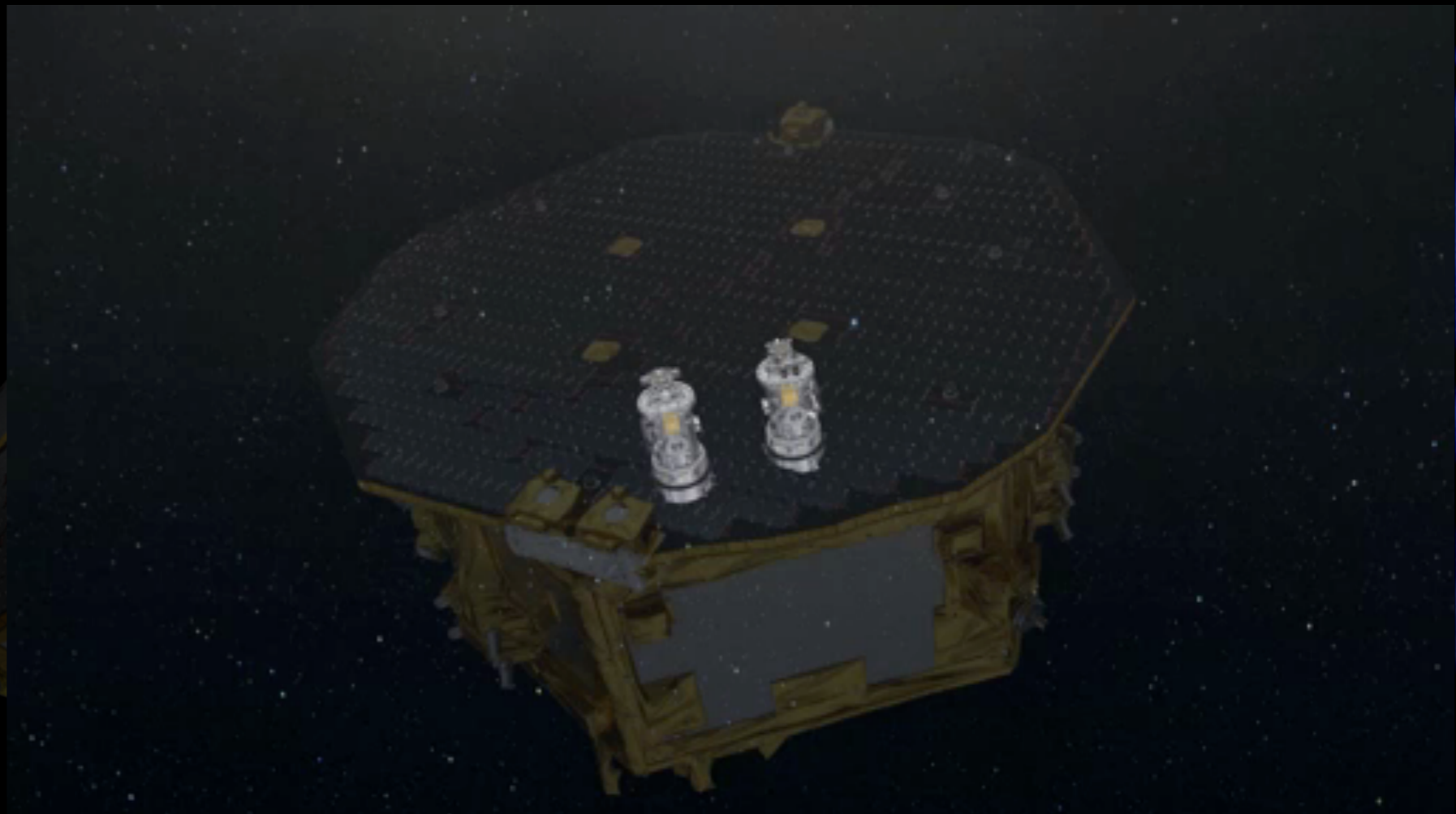
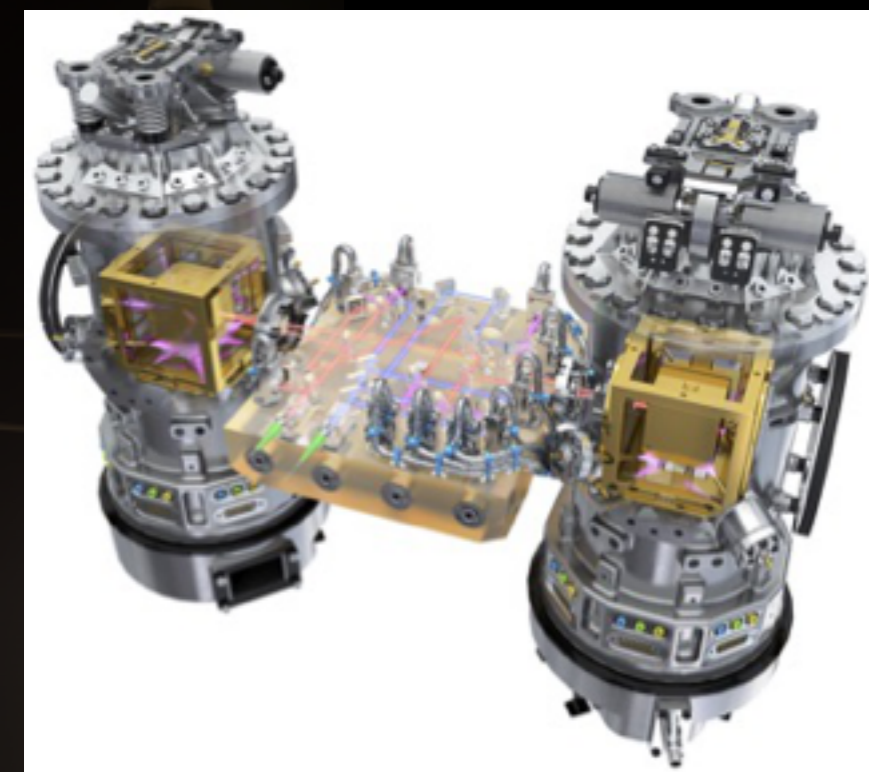
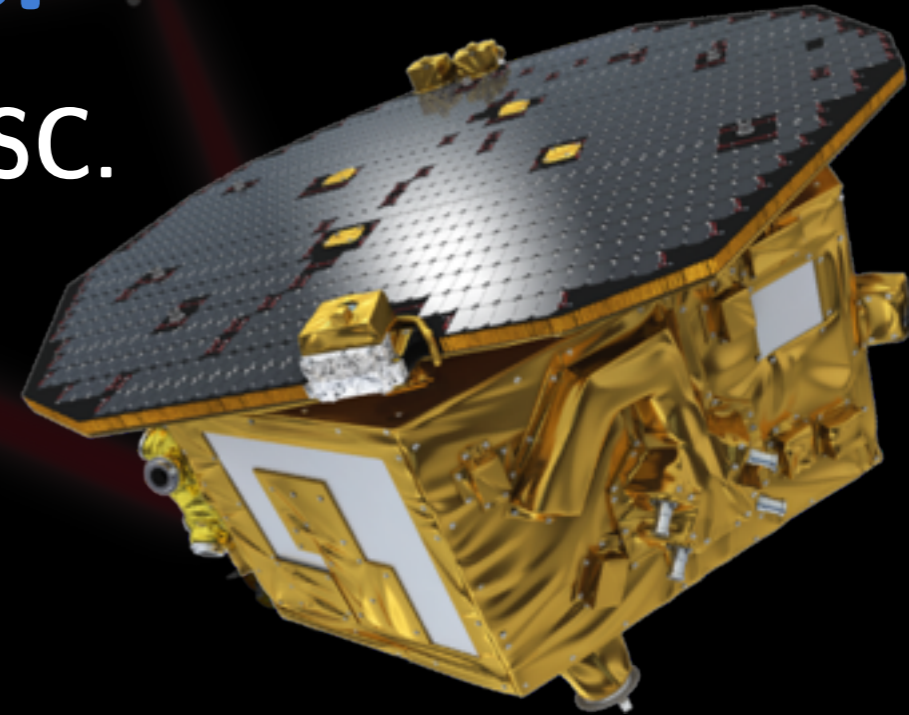
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LISAPathfinder



- ▶ Basic idea: Reduce one LISA arm in one SC.
- ▶ LISAPathfinder is testing :
 - Inertial sensor,
 - Drag-free and attitude control system
 - Interferometric measurement between 2 free-falling test-masses,
 - Micro-thrusters

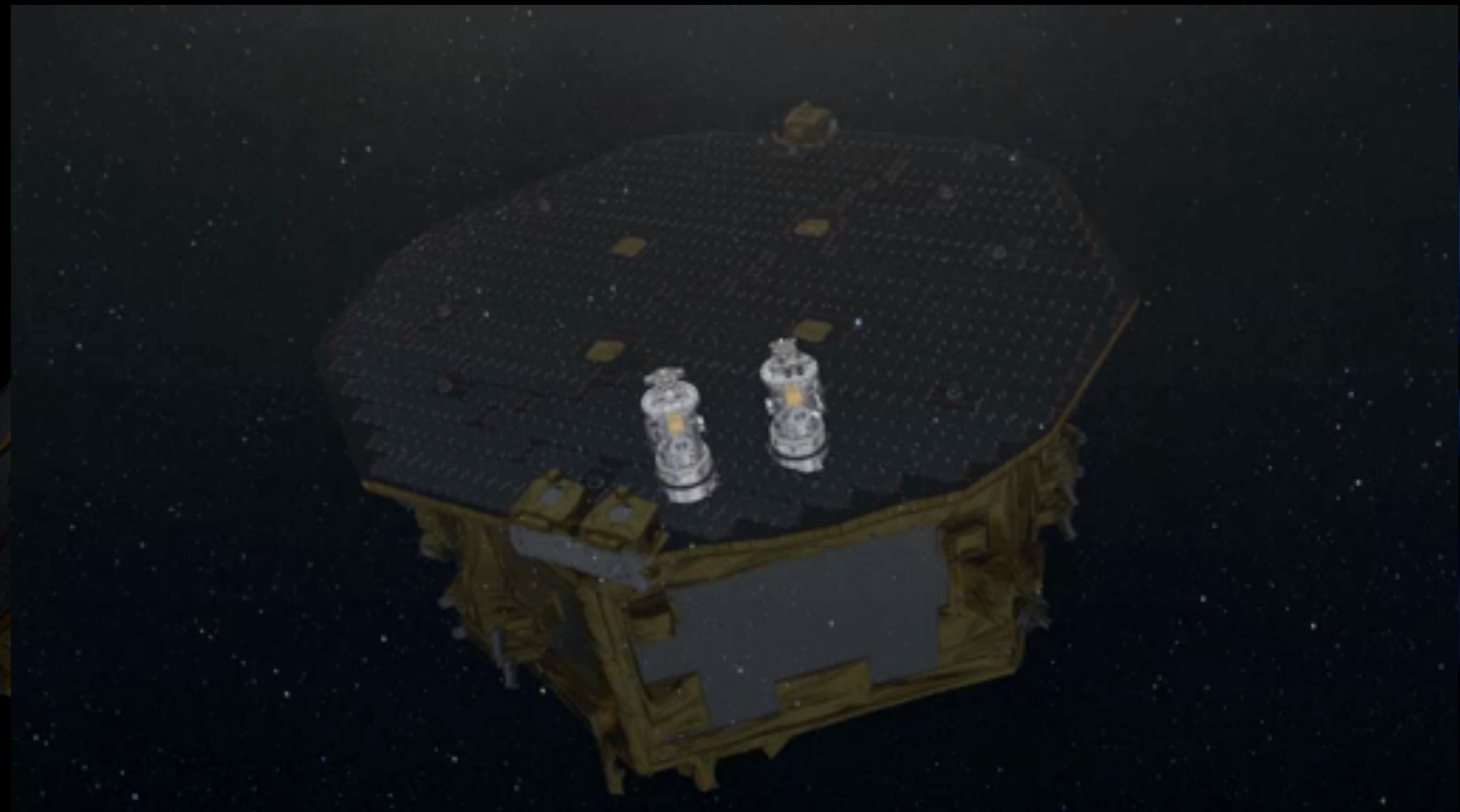
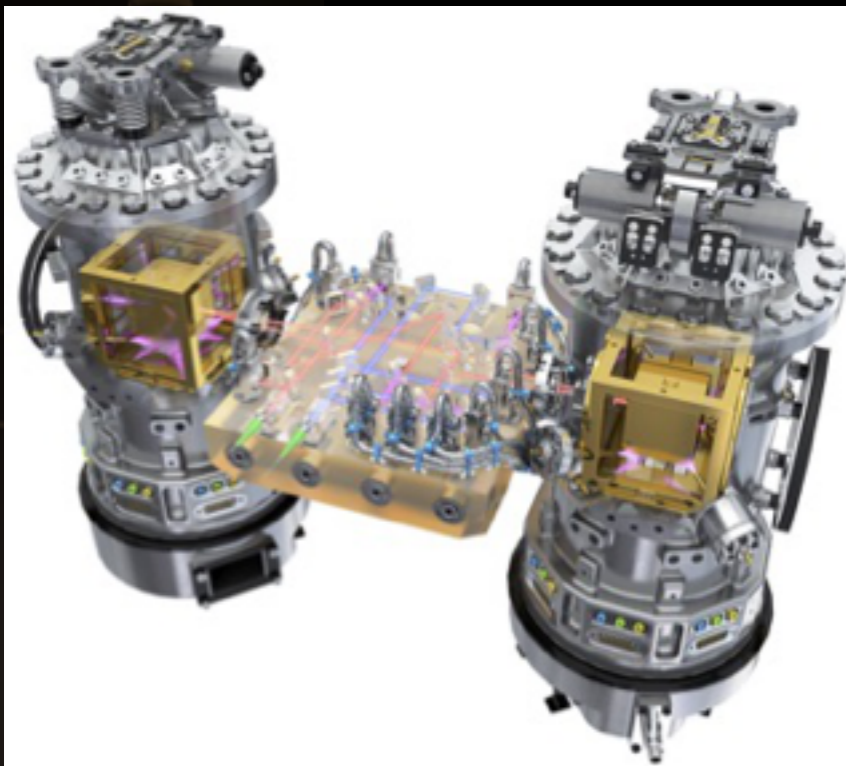
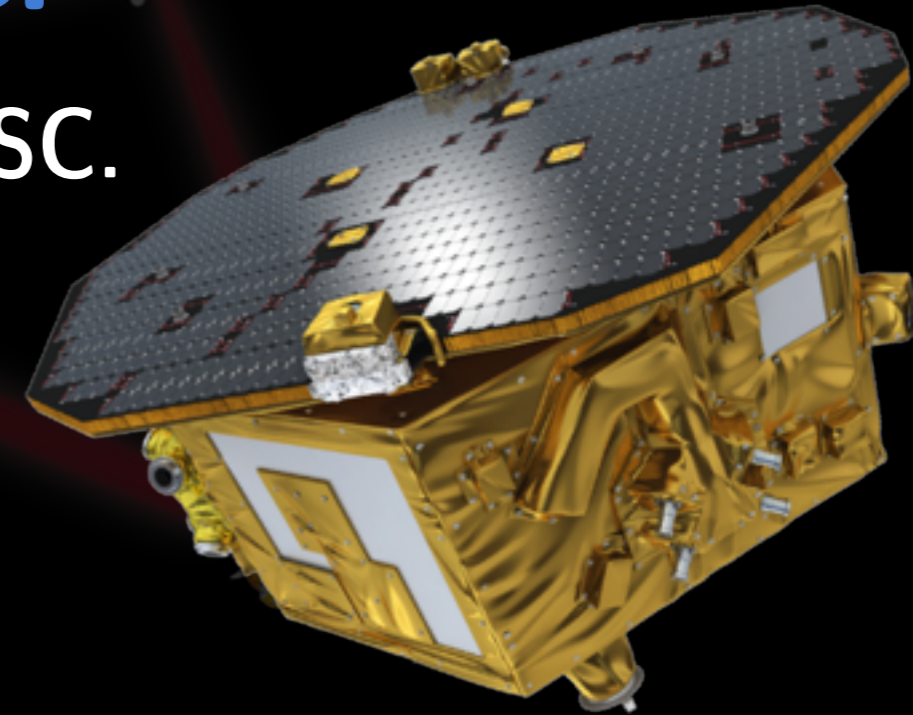




LISAPathfinder

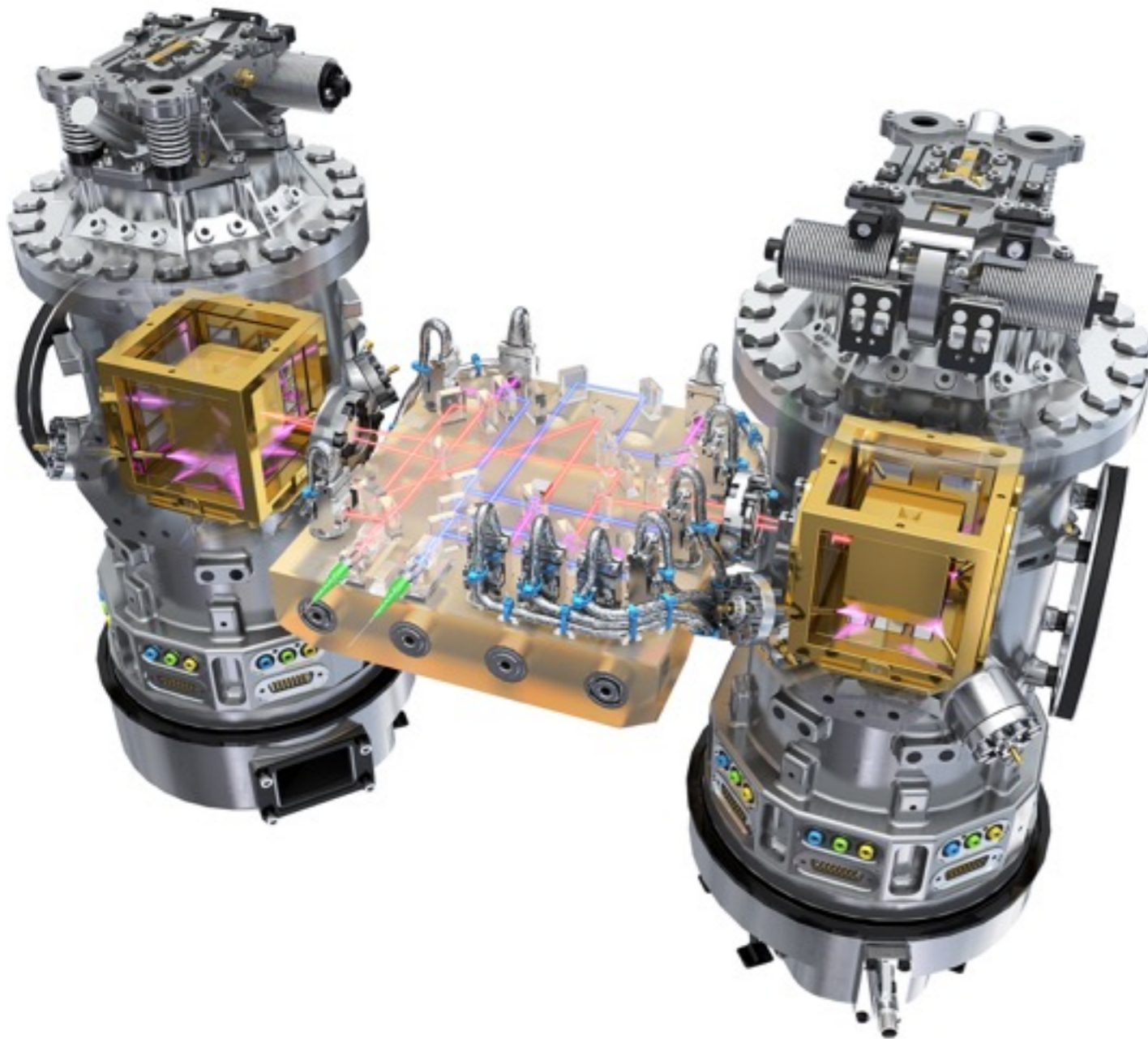


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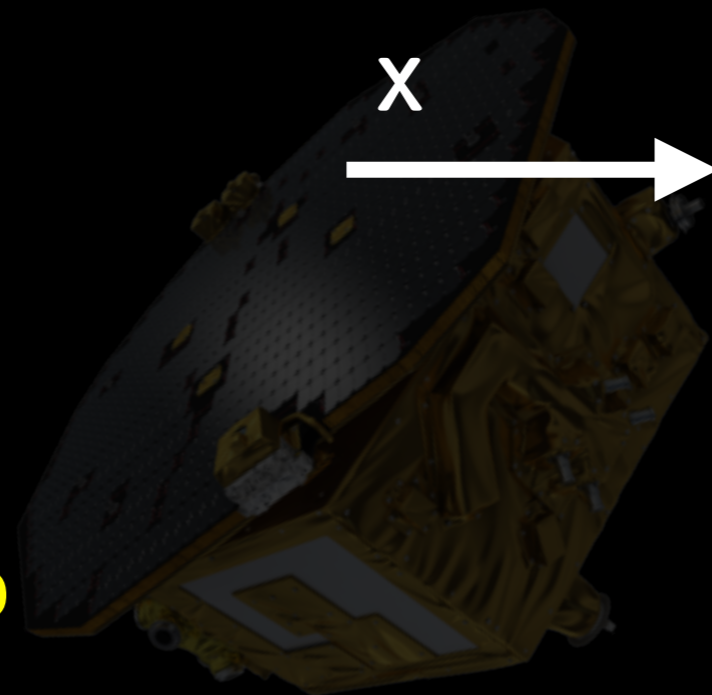
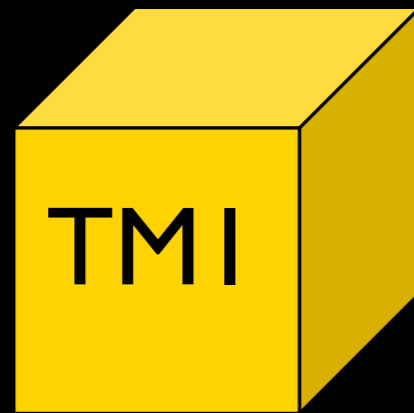
The instrument - LTP



- Gravitational Reference Sensor
- Optical Bench
- Lampe UV
- Laser
- Compensation mass
- Under vacuum
- Caging Mechanism
- Thermal and magnetic monitoring



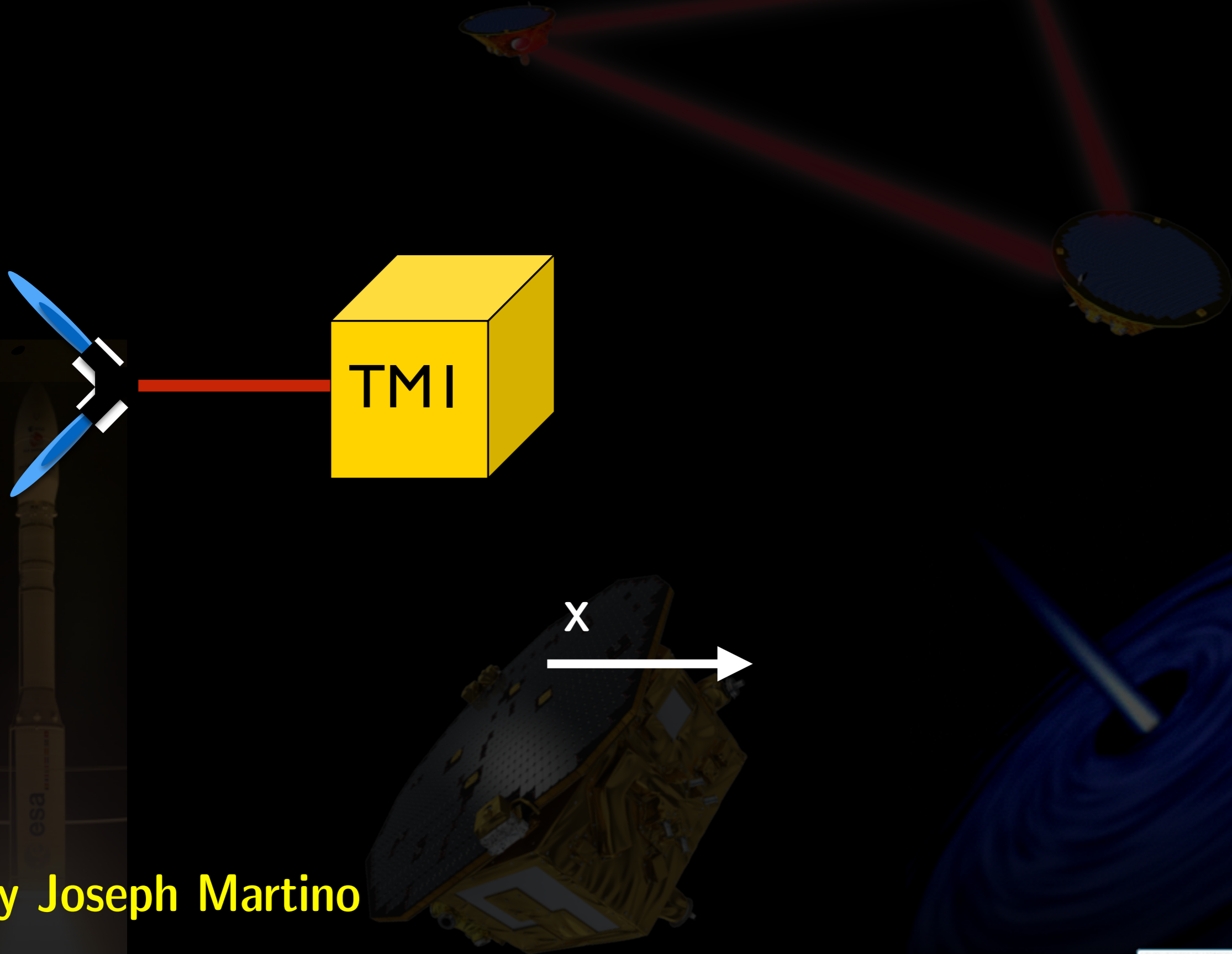
The measurement - deltaG



by Joseph Martino



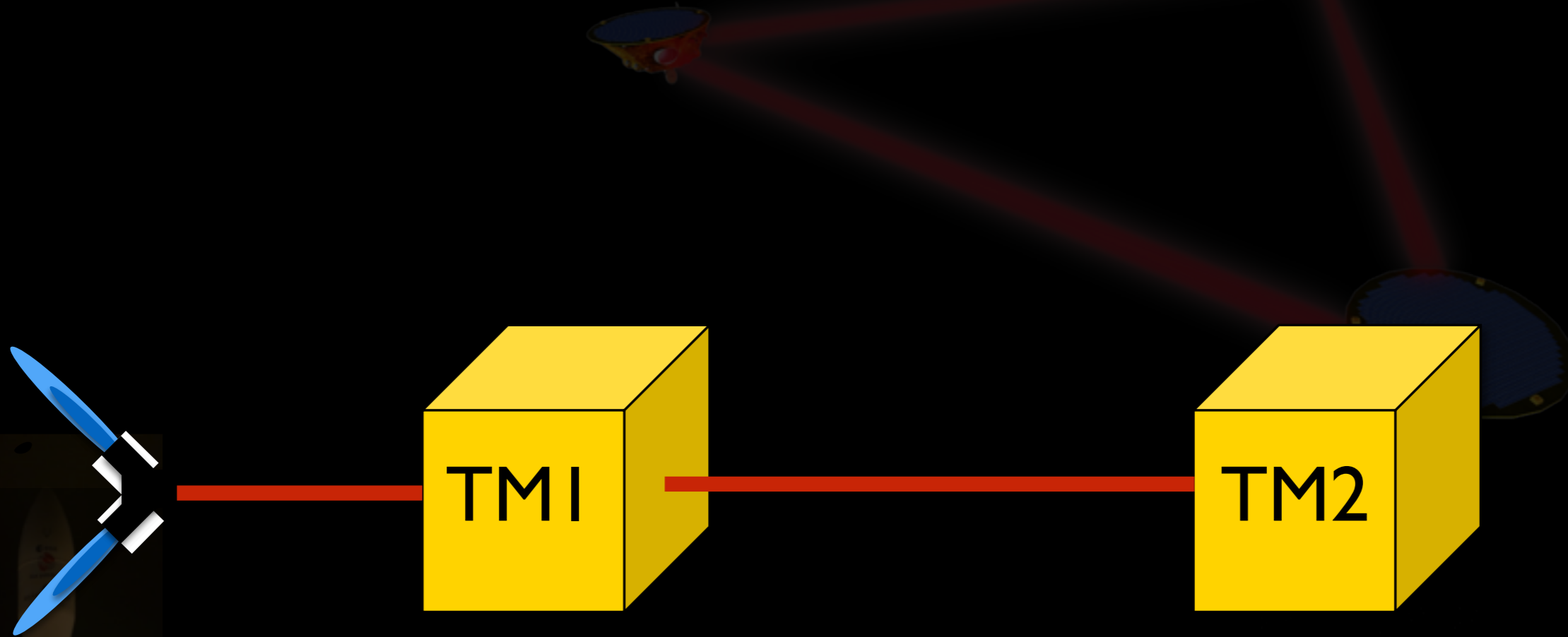
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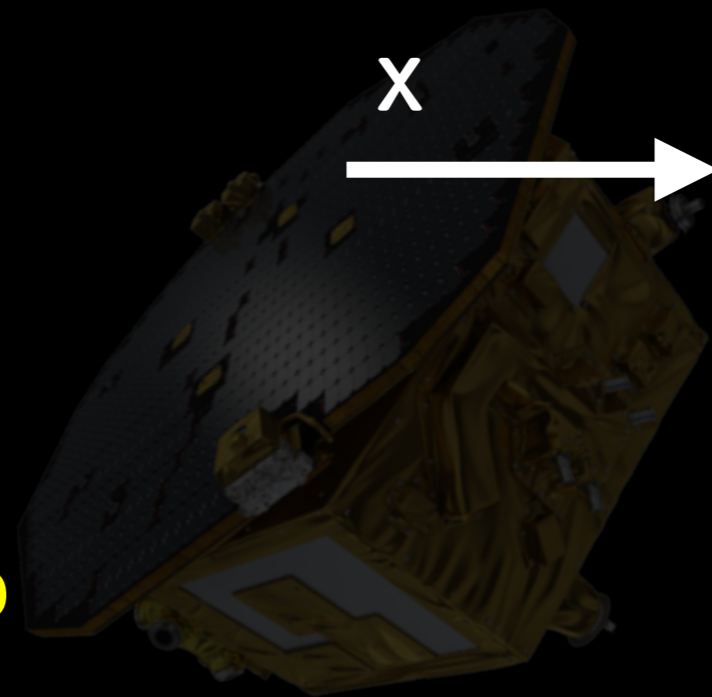
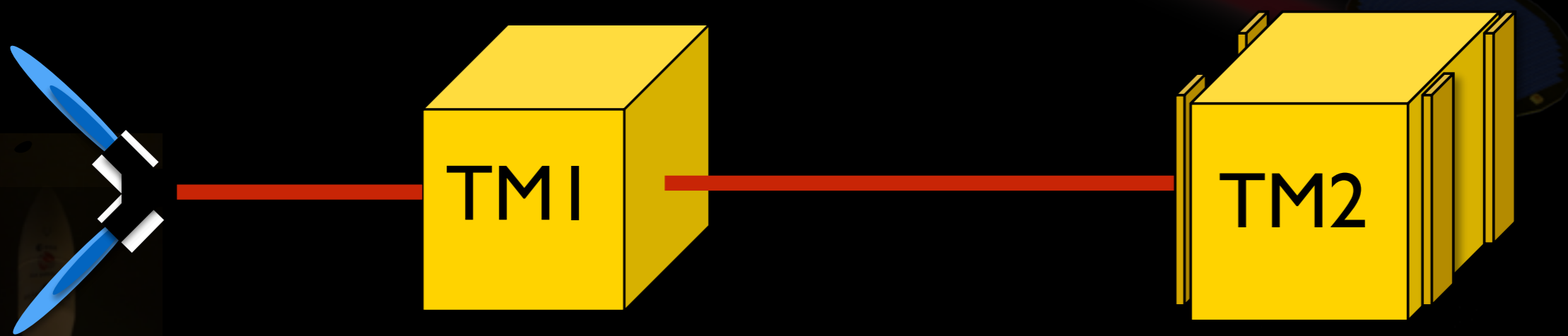
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The measurement - deltaG



Suspension ($f < 1\text{mHz}$)



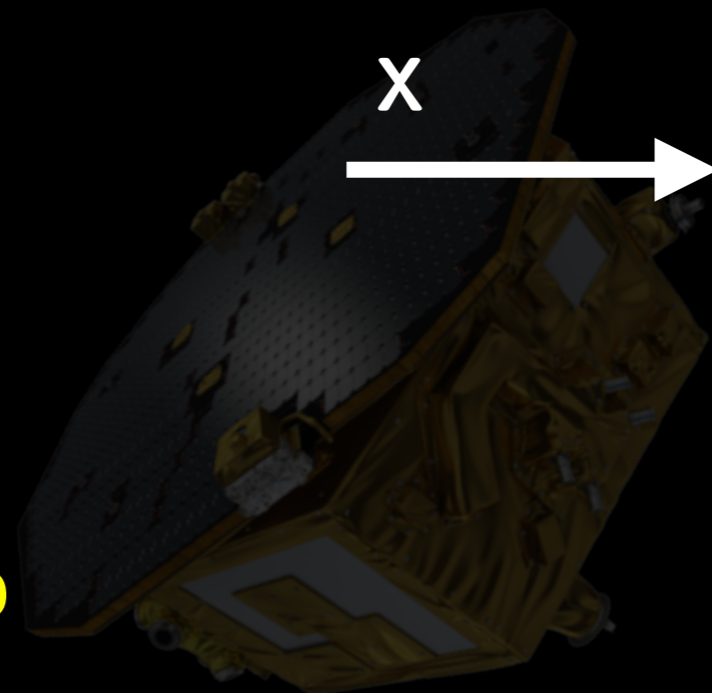
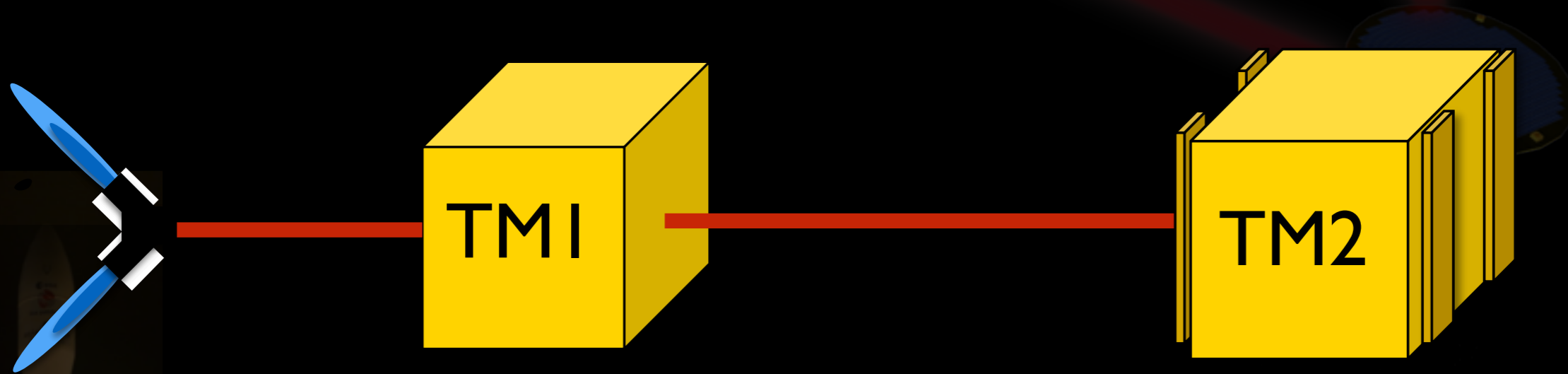
by Joseph Martino



The measurement - deltaG

$$\text{deltaG} = d^2(\text{o12})/dt^2 - \text{Stiff} * \text{o12} - \text{Gain} * \text{Fx2}$$

Suspension ($f < 1\text{mHz}$)

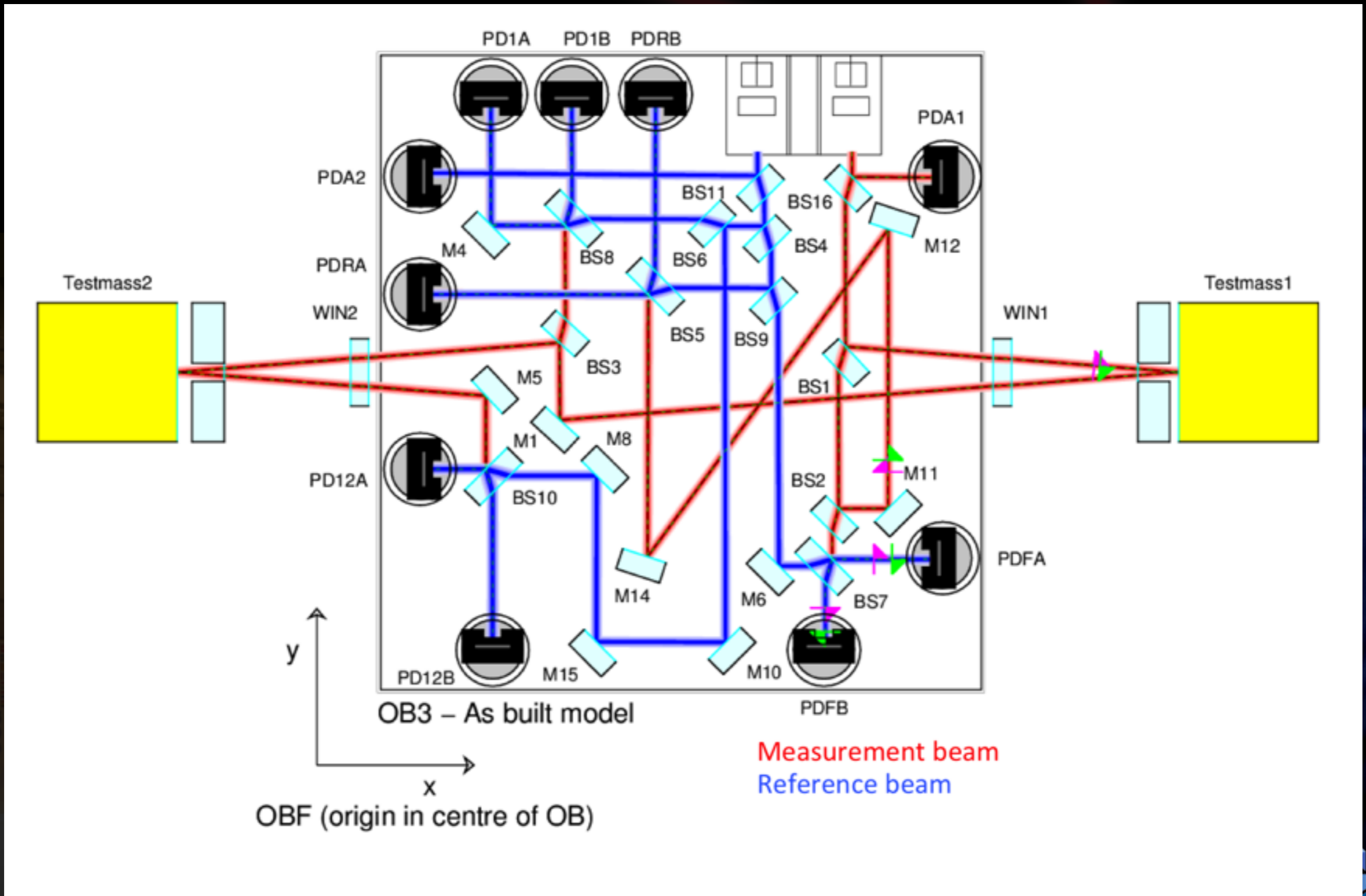


by Joseph Martino



Optical bench

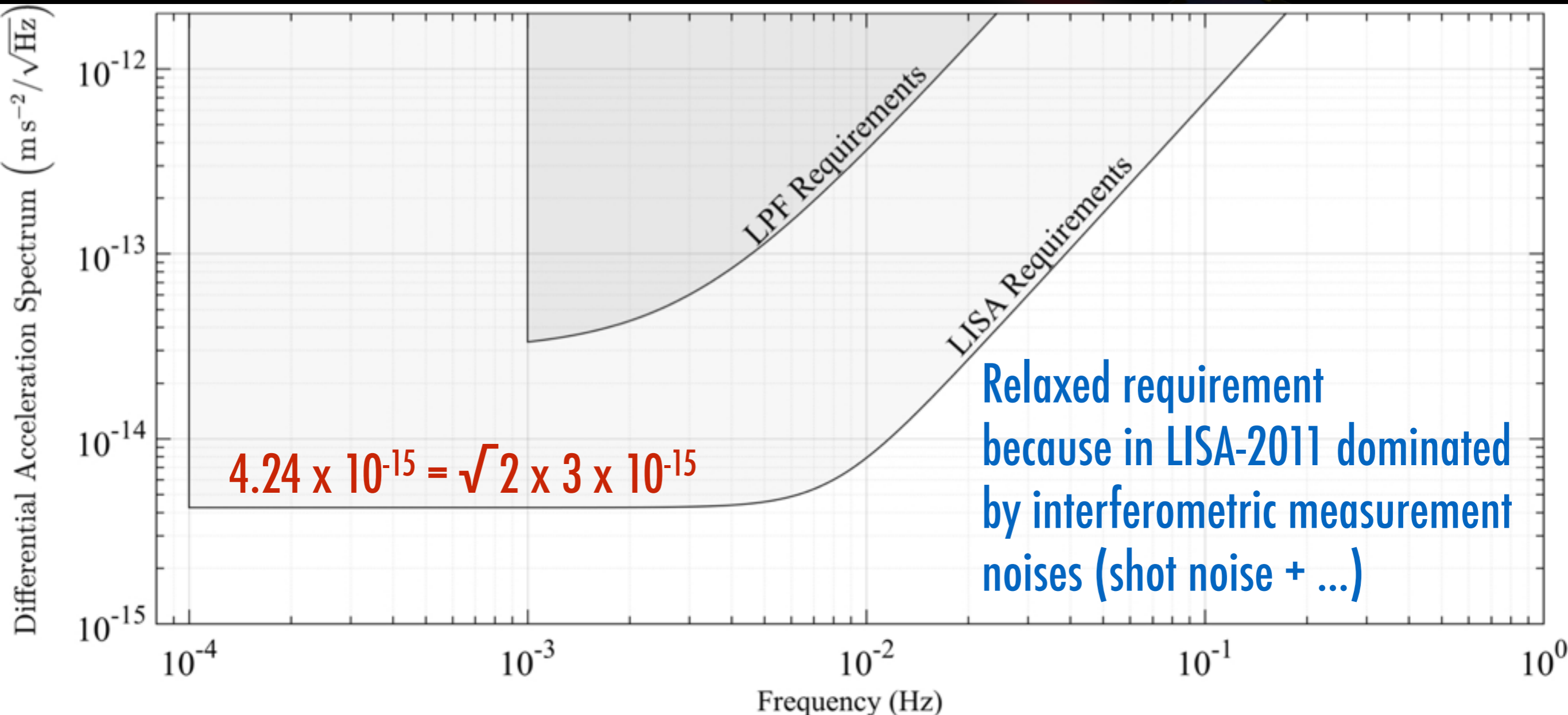
$$\Delta G = d^2(o_{12})/dt^2 - \text{Stiff} * o_{12} - \text{Gain} * F_{x2}$$





Requirements: LPF vs LISA

- ▶ Main LISAPathfinder (LPF) measurement : Δg : differential acceleration between the 2 test-masses





Requirements: LPF vs LISA



Why the LISAPathfinder requirements are restricted compare to LISA ones ?

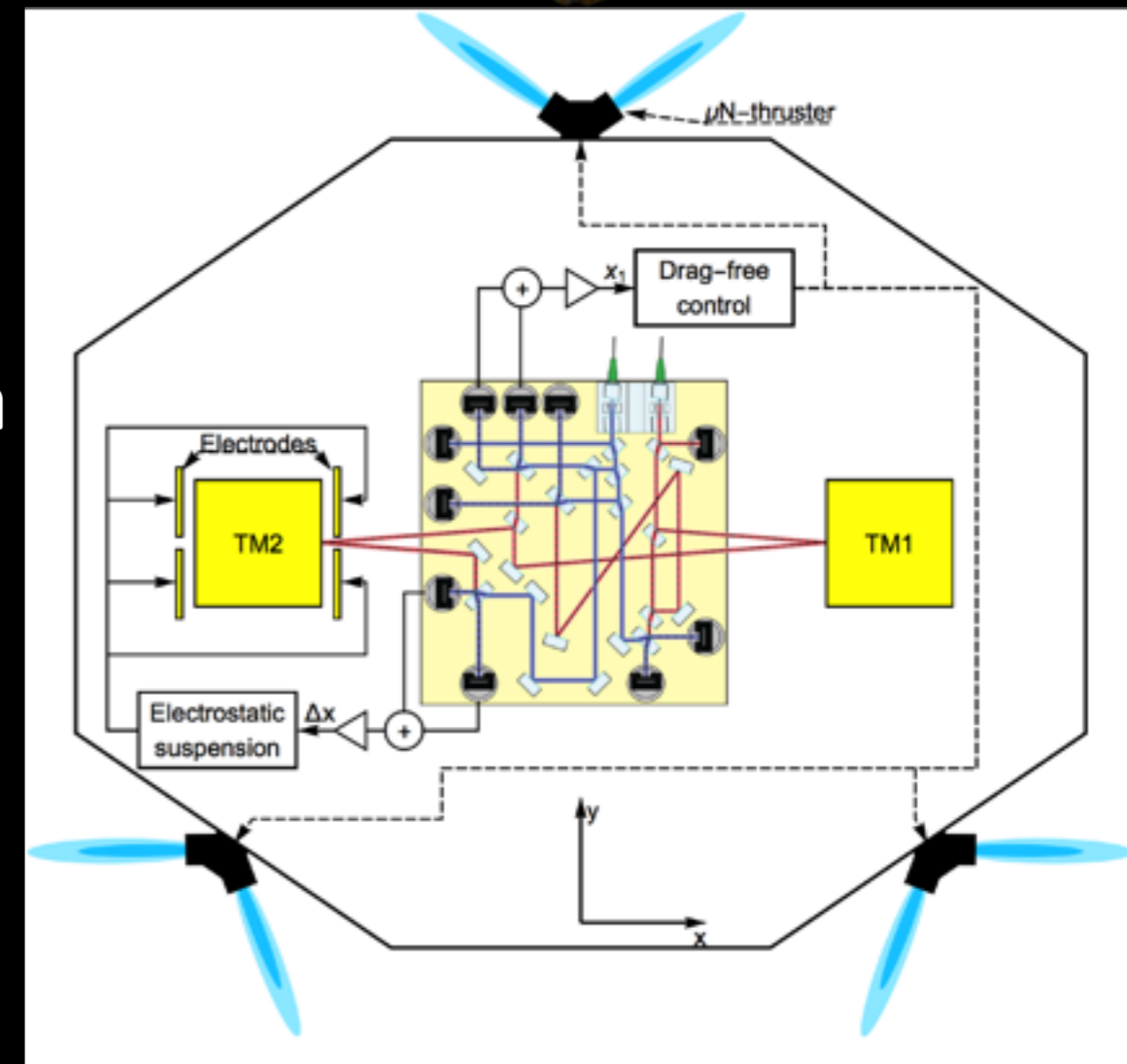
▶ We understand limitations with LISAPathfinder and correct for them in LISA

▶ Short arm limitation :

- Gravitational field not perfectly flat
=> constant electrostatic actuation on test- mass 2

▶ $f > 1$ mHz : limit duration of industrial testing

▶ Industrial margin





Angle Decorrelation - Euler Forces



$$\begin{aligned}\Delta \vec{g}_{\text{tang}} &= \vec{g}_{\text{tang},2} - \vec{g}_{\text{tang},1} \\ &= (\vec{r}_2 - \vec{r}_1) \times \dot{\vec{\Omega}}\end{aligned}$$

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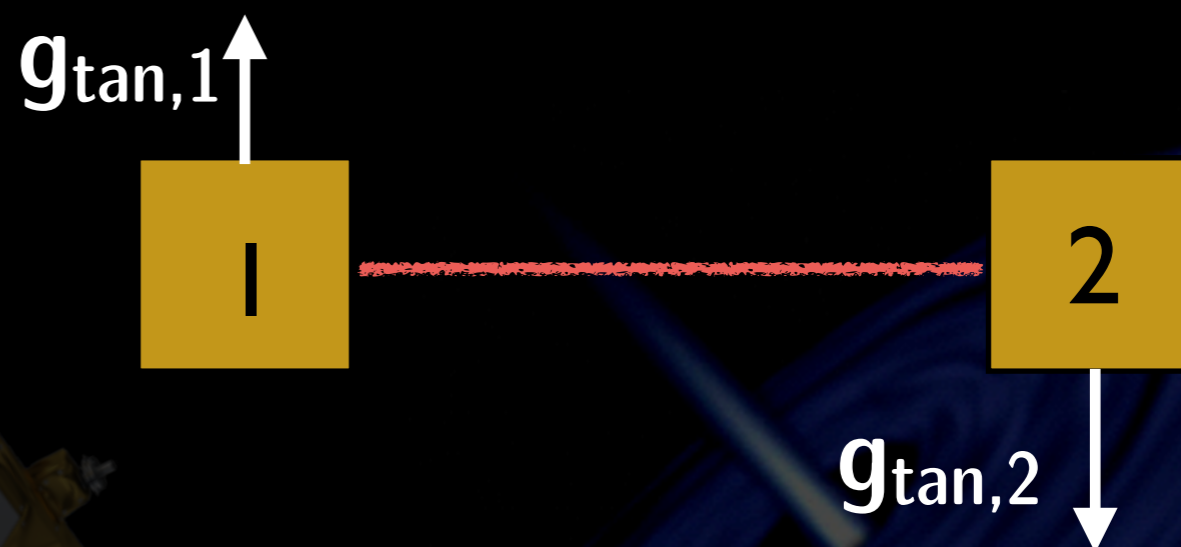


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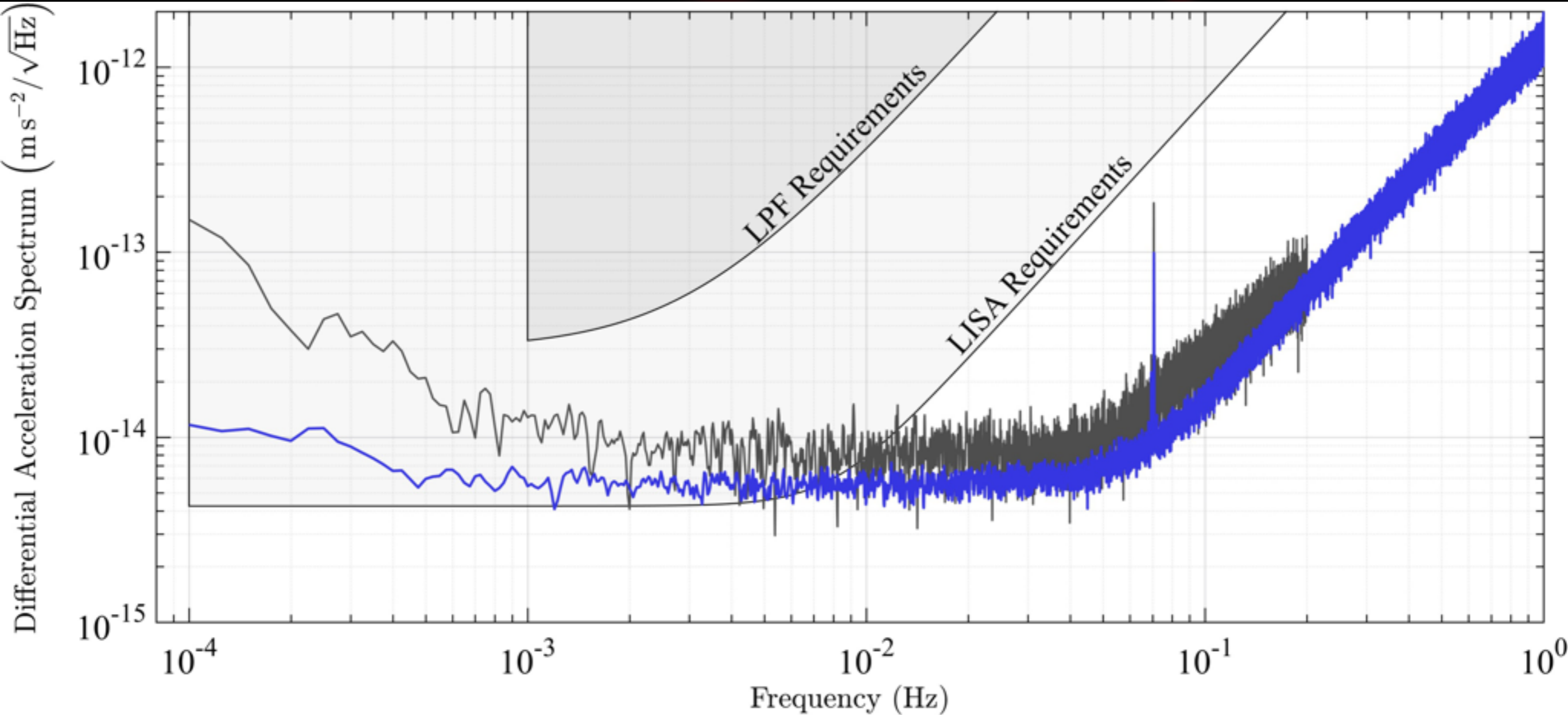
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First results

M. Armano et al. PRL 116, 231101 (2016)

► Results

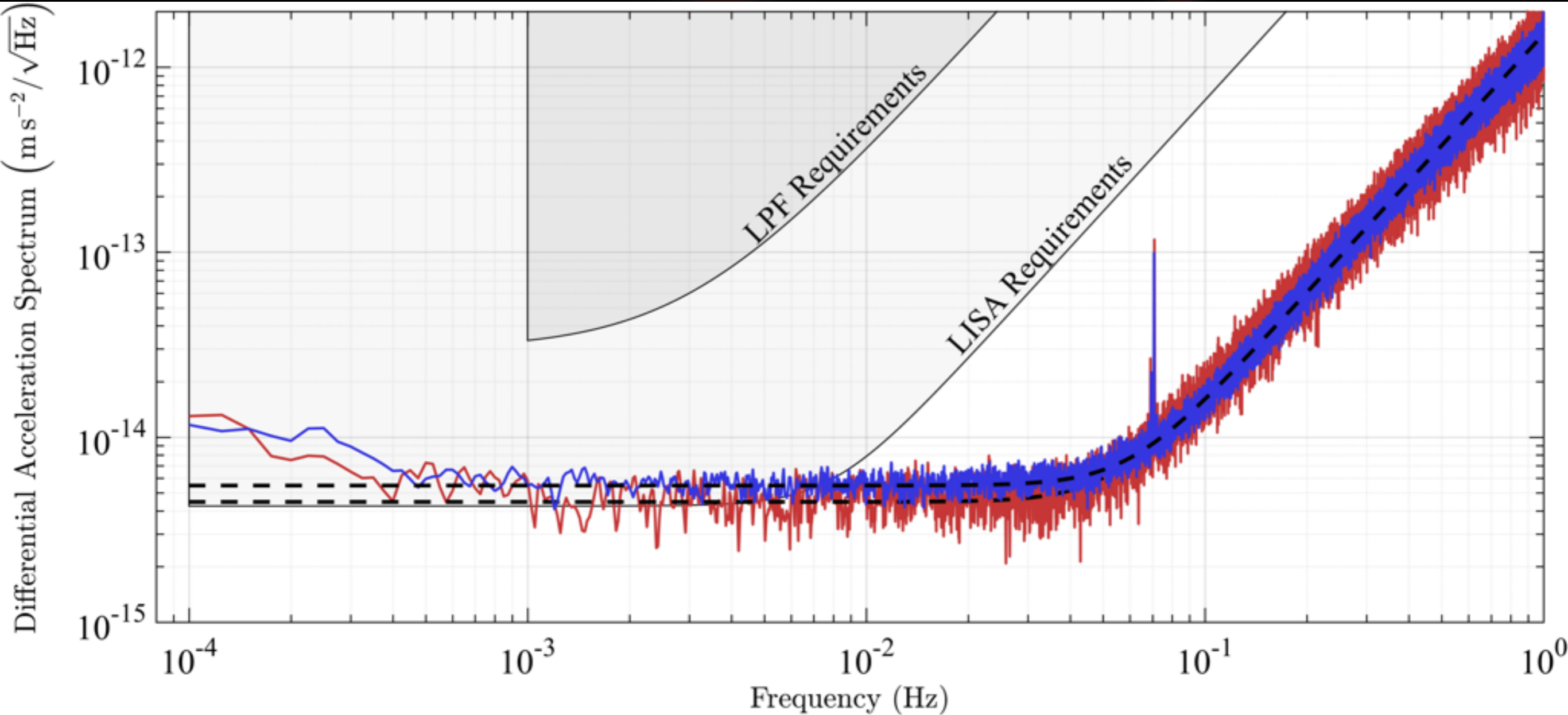




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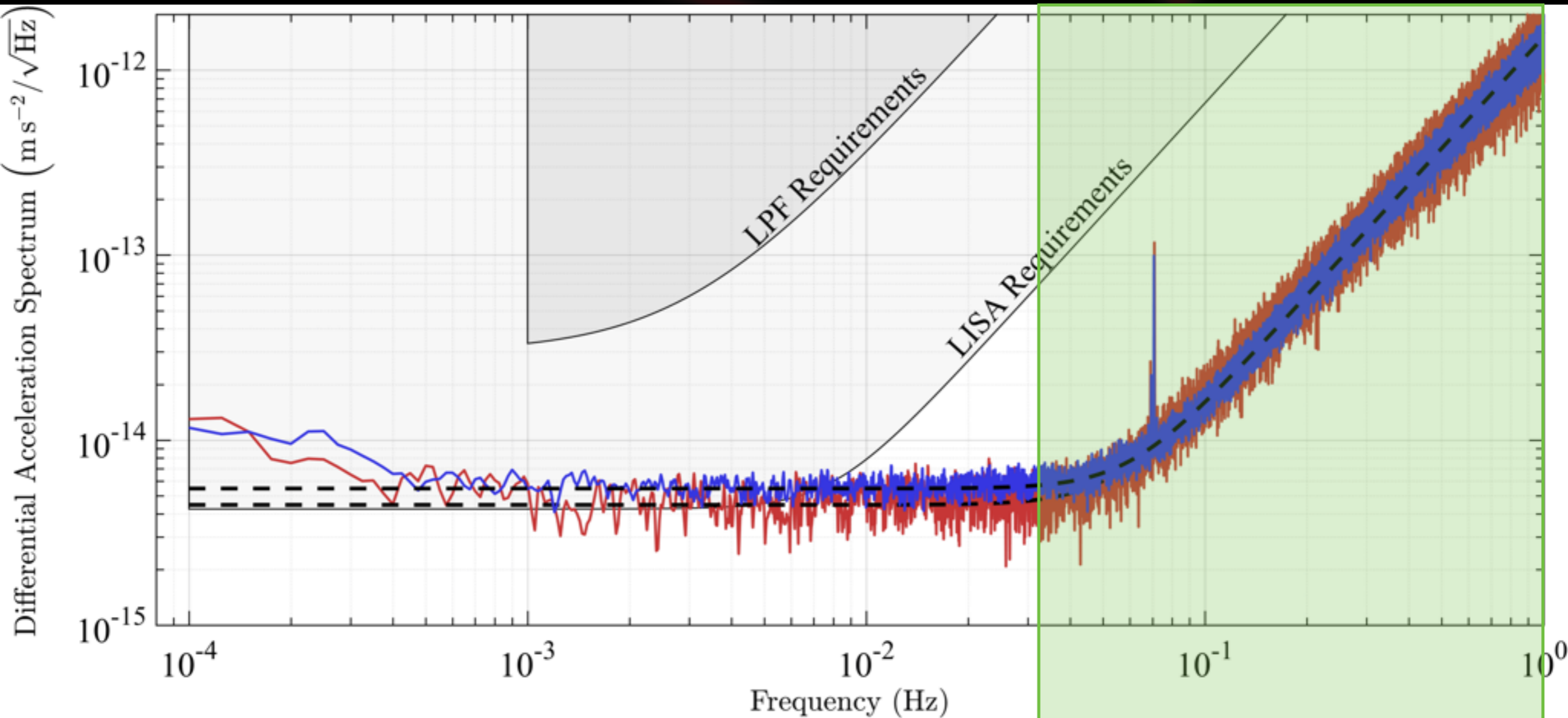




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Interferometric noise
Not real test-mass motion



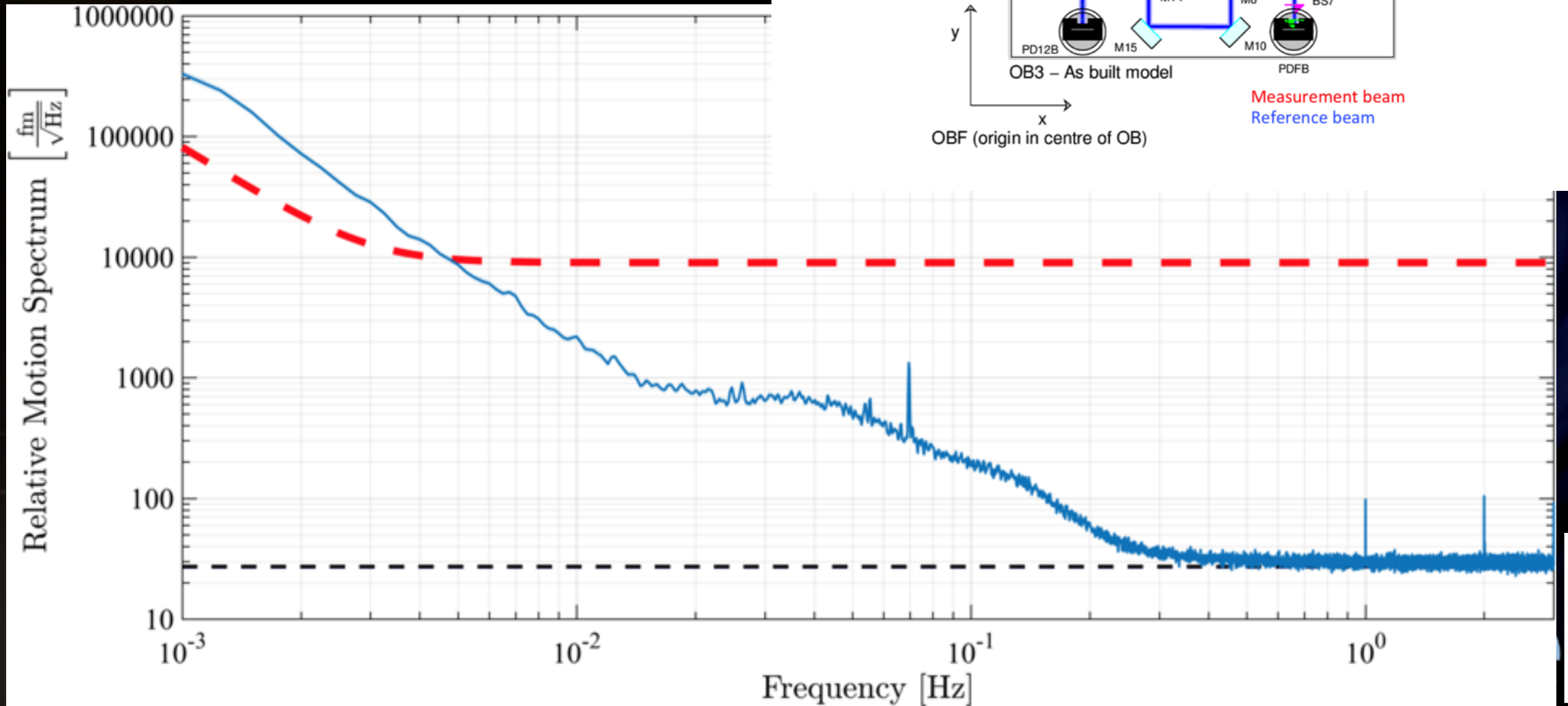
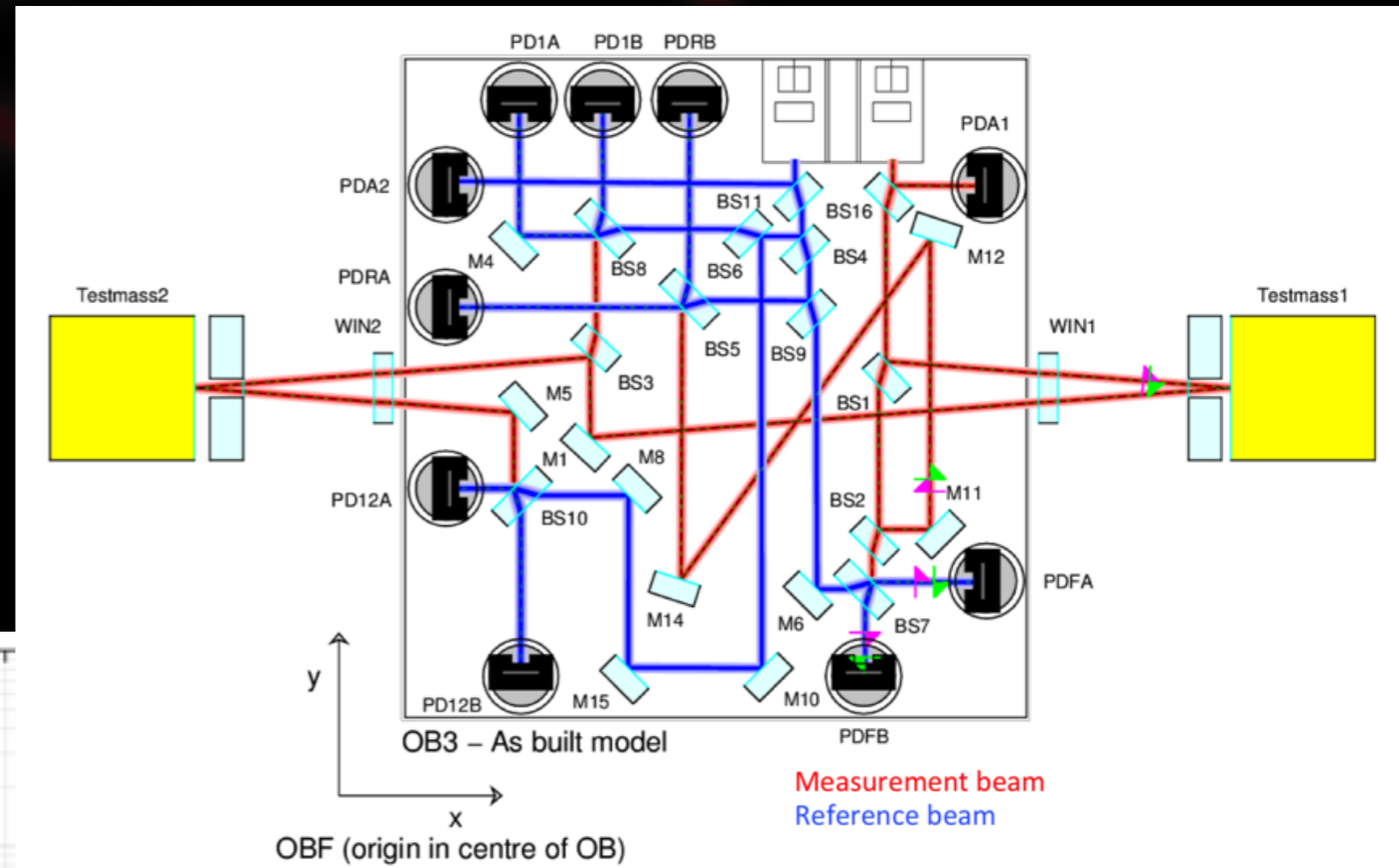


High frequency limit



Optical measurement system:

- Interferometric precision:
 $30 \text{ fm}\cdot\text{Hz}^{-1/2}$
- Orientation of test-masses

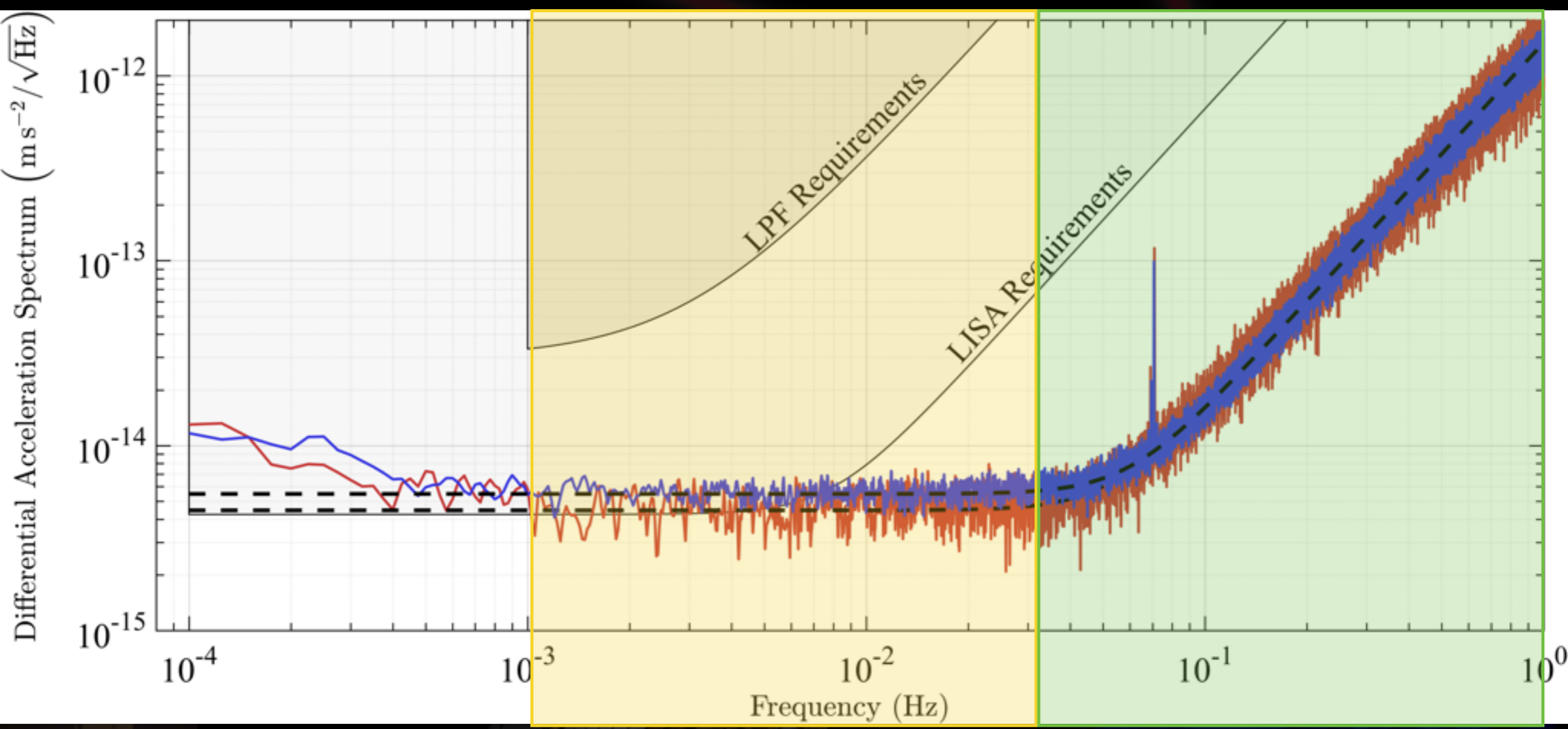




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M. Armano et al. PRL 116, 231101 (2016)

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Brownian noise
 Molecules within the noise
 hit test-masses

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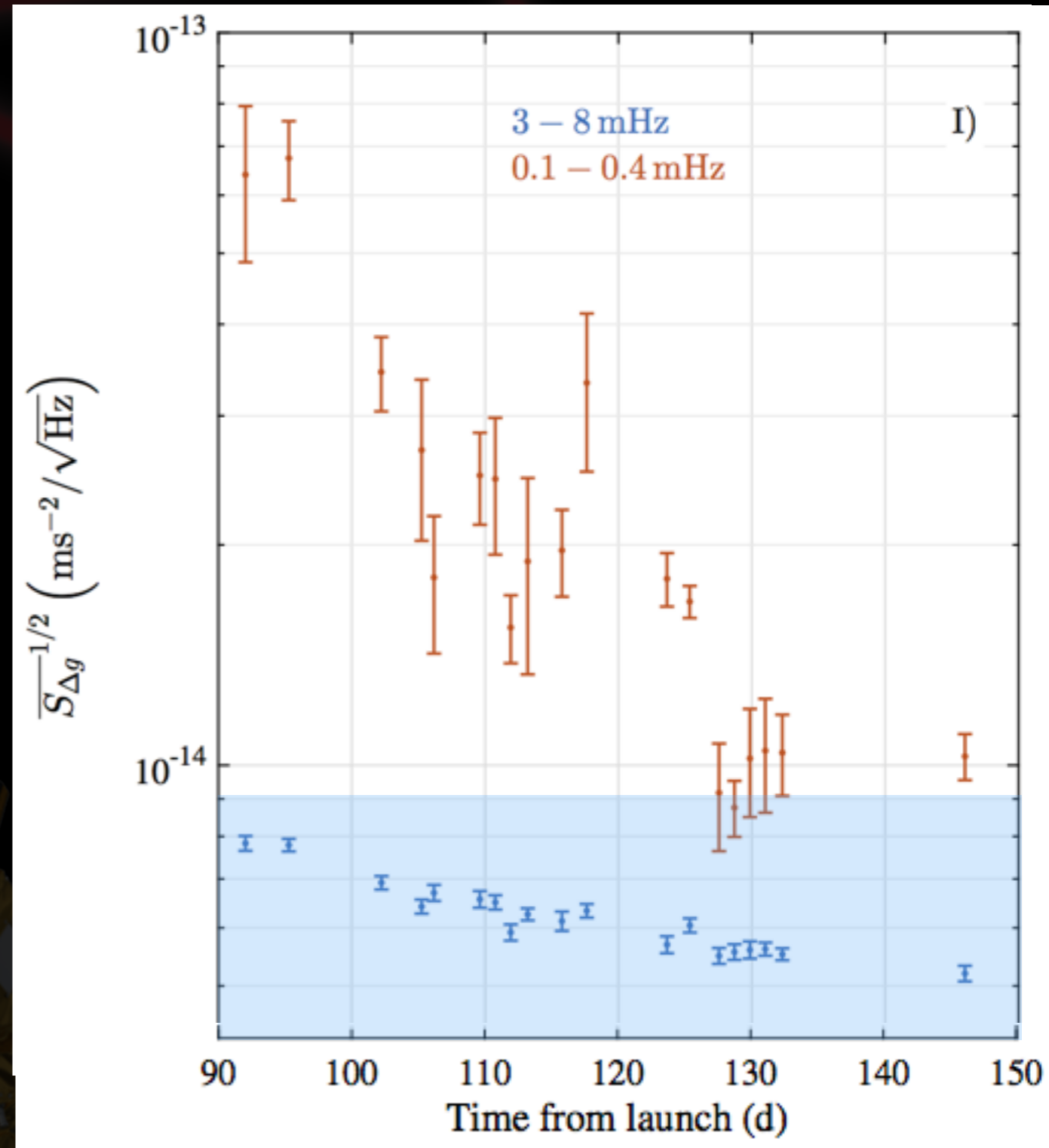




Mid-frequency limit



- ▶ Noise in 1–10 mHz: brownian noise due to residual pressure:
 - Molecules within the housing hitting the test-masses
 - Possible residual outgassing
- ▶ Evolution:
 - Pressure decreases with time
- ▶ For LISA:
 - Better evacuation system ... pump ?



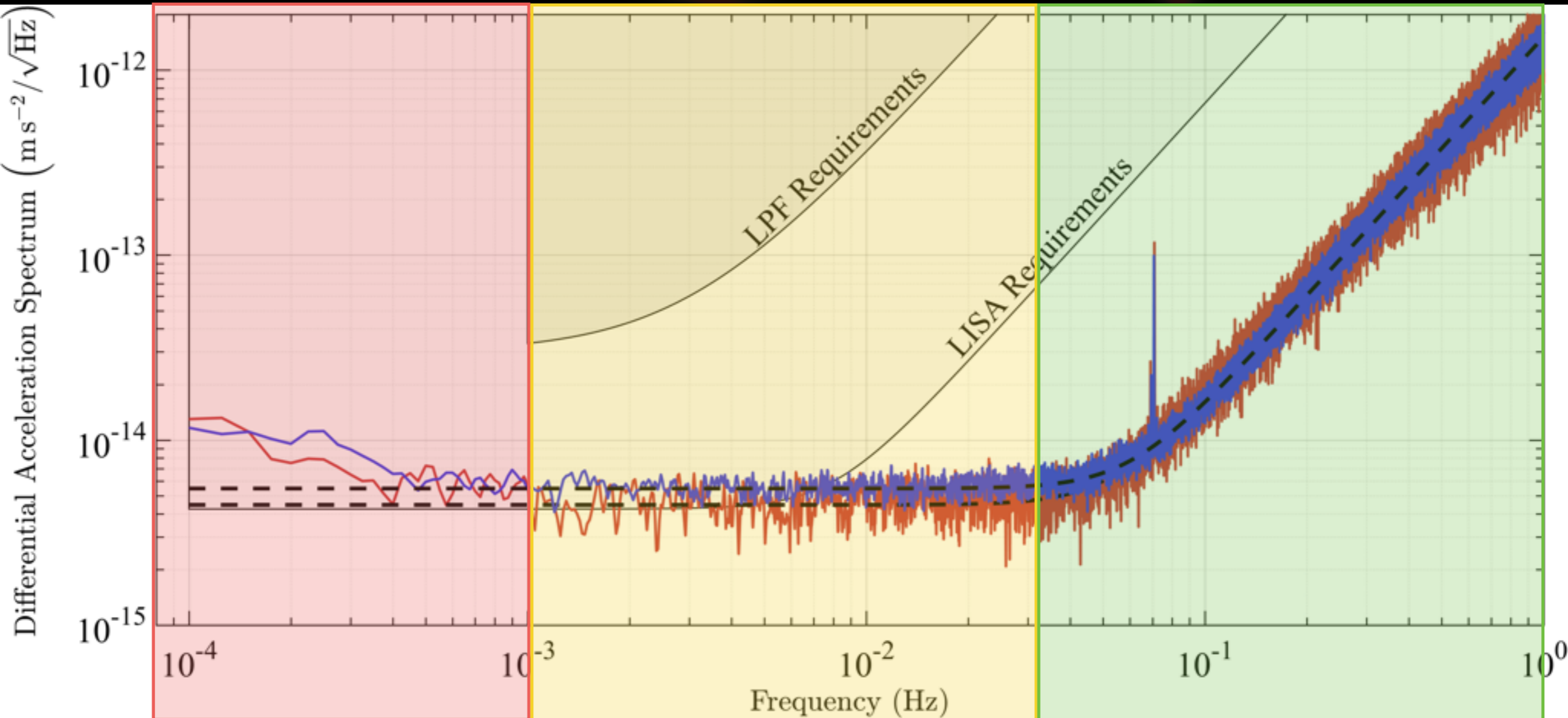
M. Armano et al. PRL 116, 231101 (2016)



First results

M. Armano et al. PRL 116, 231101 (2016)

► Results



Low frequency noise:
actuation noise + ...

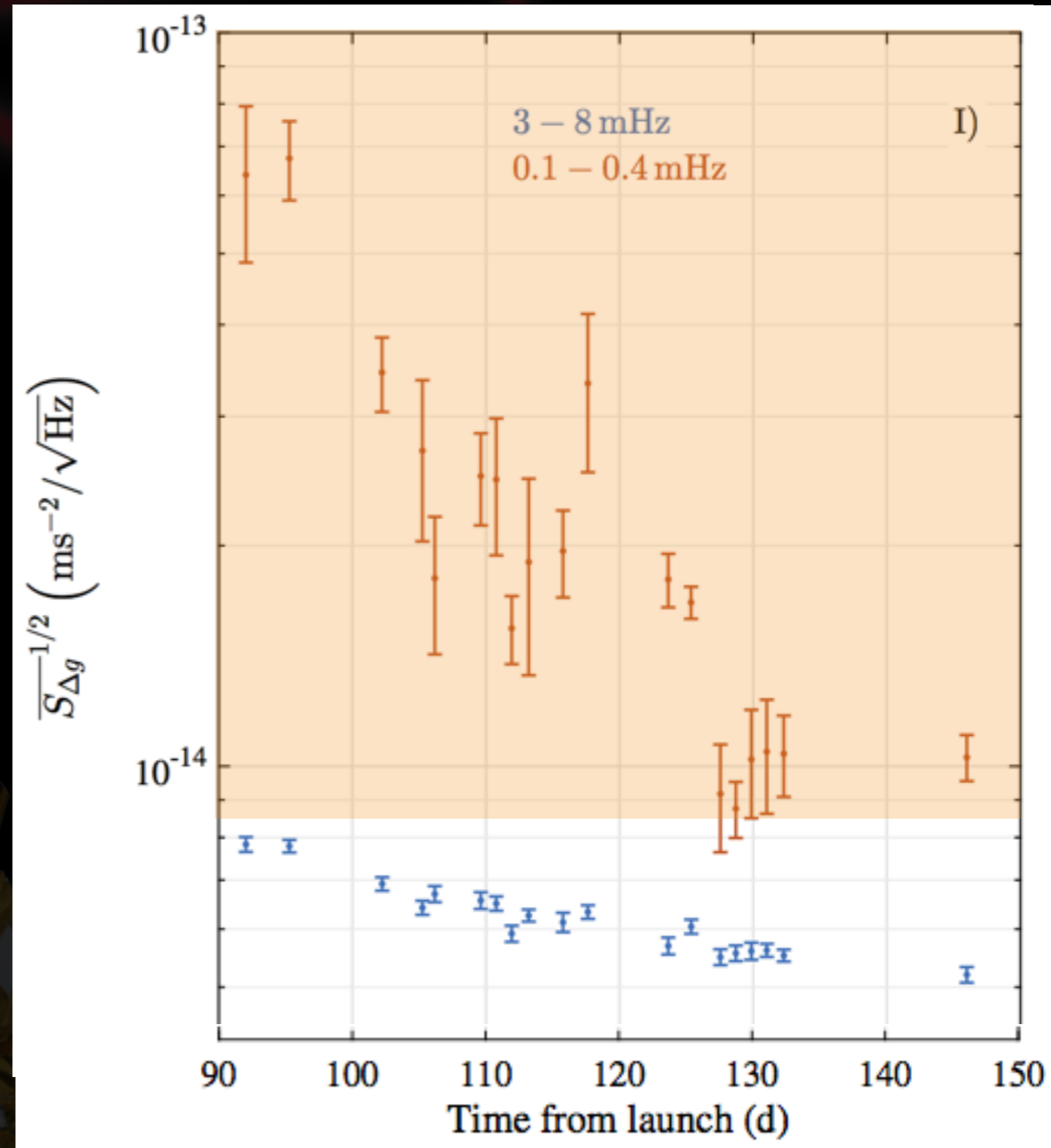
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Interferometric noise
Not real test-mass motion



Low-frequency limit

- ▶ Noise in 0.1 – 1 mHz: not yet completely understood but seems:
 - to evolve with time
 - to have 1/f slope ?
 - Temperature ? Actuation ?
- ▶ Work in progress ...
- ▶ For $f < 0.1$ mHz:
 - just few long noise measurements

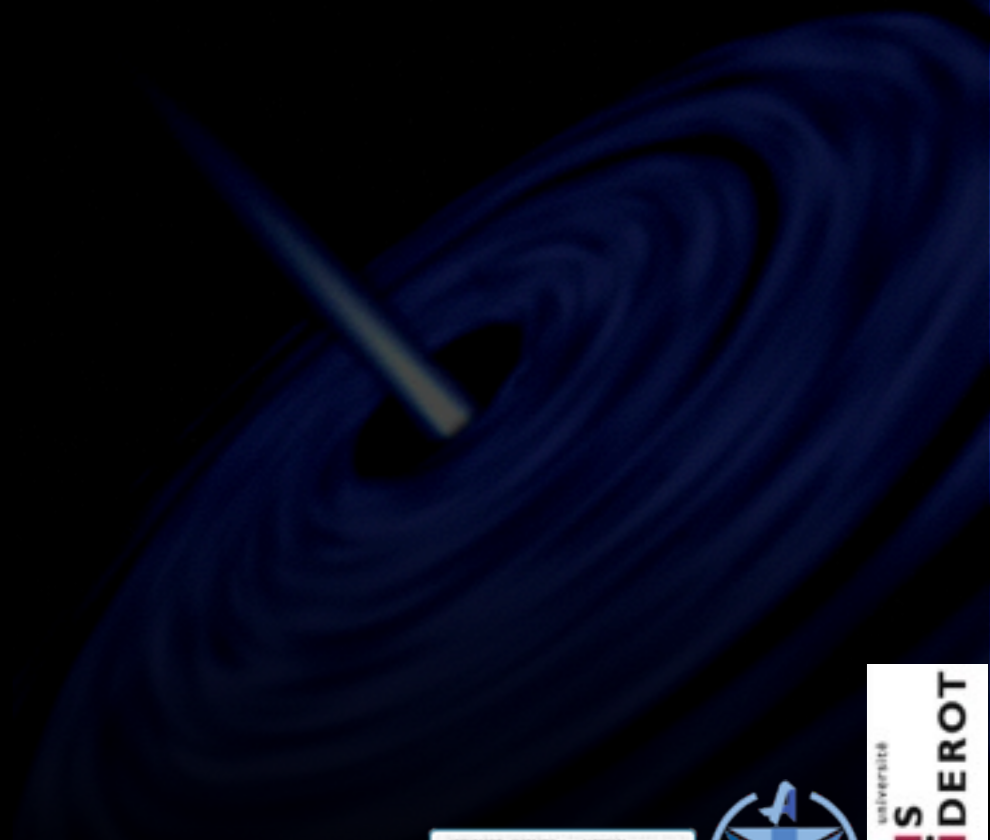


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Overview

- ▶ Introduction
- ▶ Gravitational wave sources around the mHz
- ▶ Free fall in space: LISA Pathfinder
- ▶ Long arm interferometry: Time Delay Interferometry
- ▶ Noise sources
- ▶ Sensitivity
- ▶ Response to GW and orbital motion
- ▶ Data analysis
- ▶ Conclusion

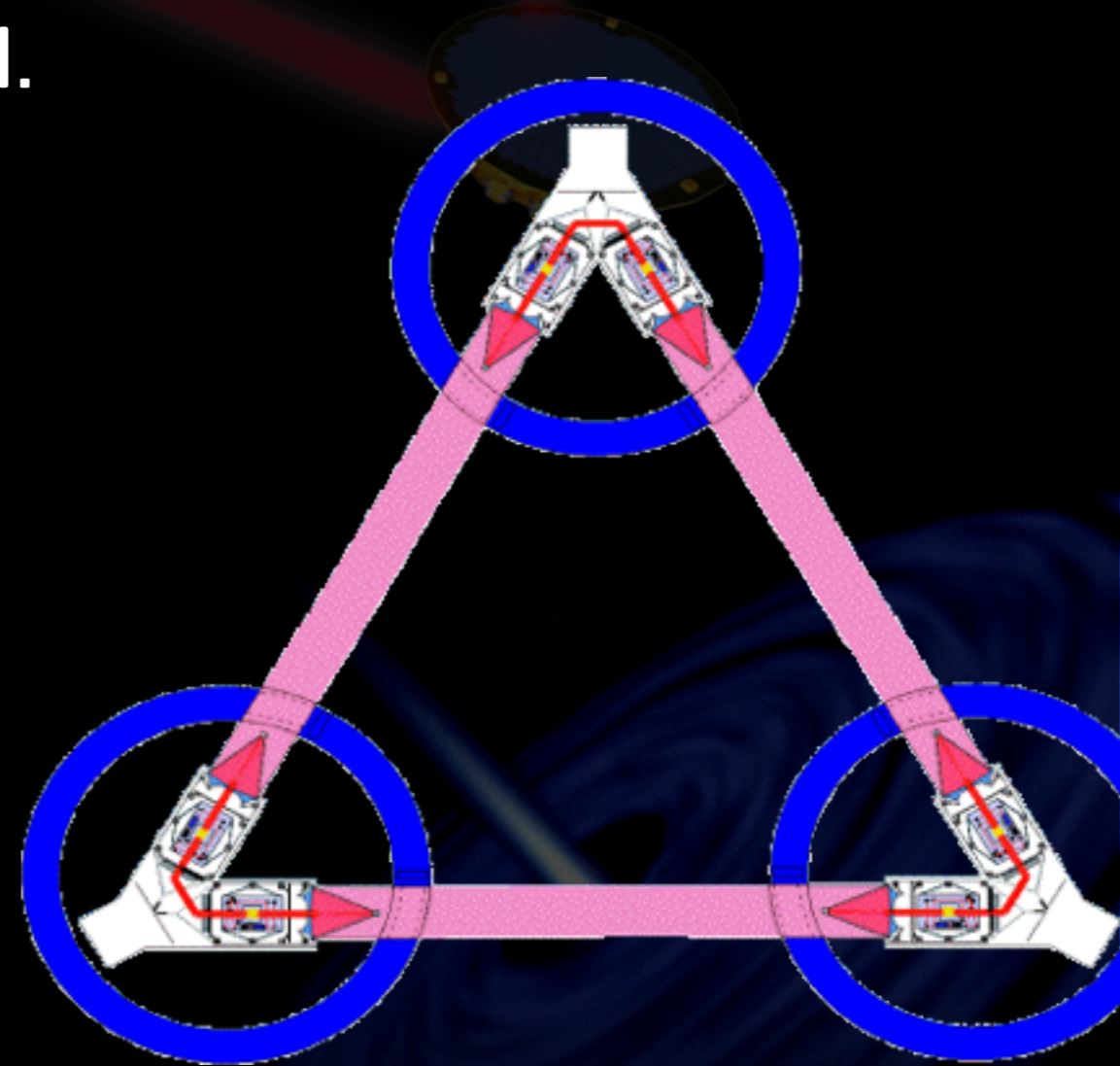




LISA : Measurements



- ▶ Problem with 2.5×10^9 m : A laser beam cannot make a round trip because too much intensity is lost.
 - 100pW received for 1 Watt emitted.
- ▶ Measurement with one arm and interference between two incoherent lasers in phase :
 - Distant laser
 - Local laser.
- ▶ 6 measurements ... at least!





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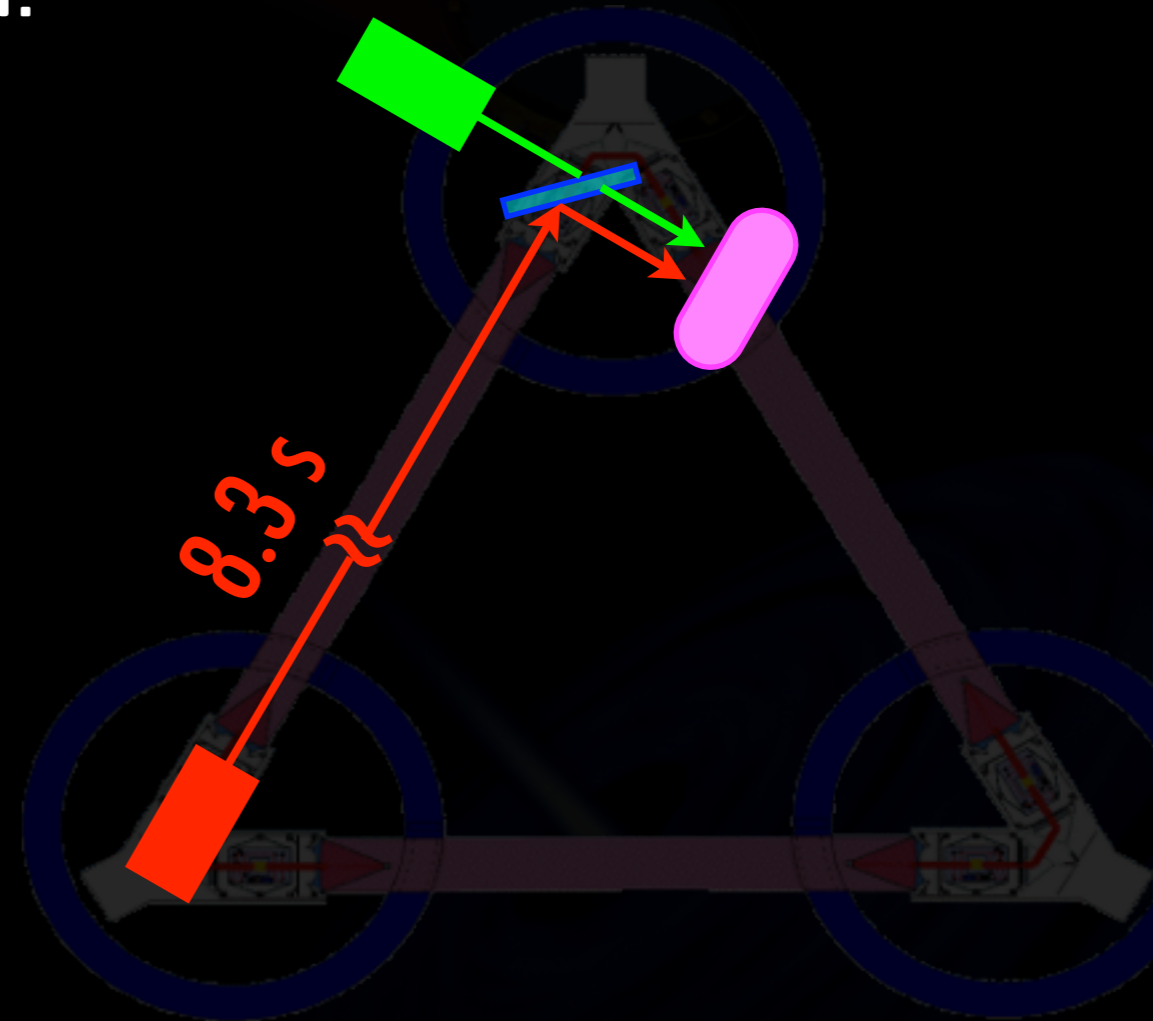
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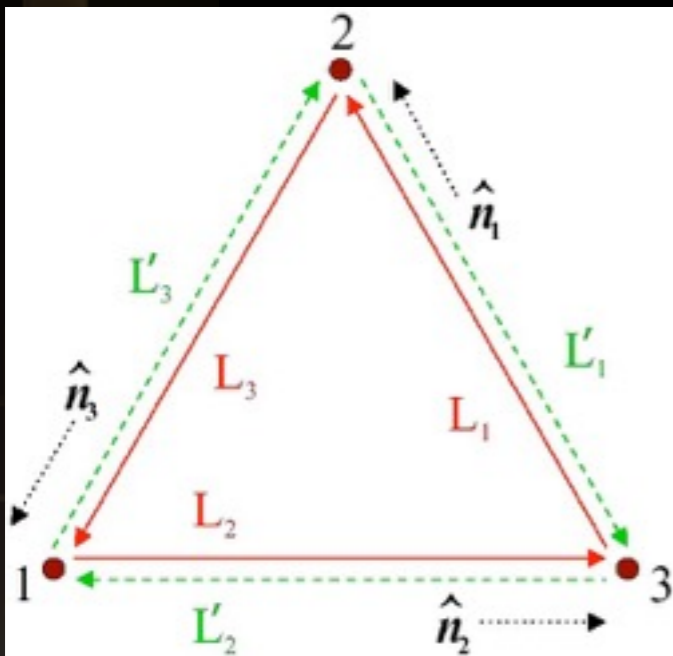


LISA : Measurements (old design)



- ▶ Phase shift between the two beams measured by phasemeter.
- ▶ Beams from an external spacecraft, are delayed :
 - delay operator $D_i^{real} : D_i^{real} x(t) = x\left(t - \frac{L_i^{real}}{c}\right)$.
- ▶ The measurement :

$$s_1 = s_1^{GW} + s_1^{ShotNoise} + D_3^{real} p_2^{lasernoise} - p_1^{lasernoise} - 2\delta^{Acc.Noise}$$



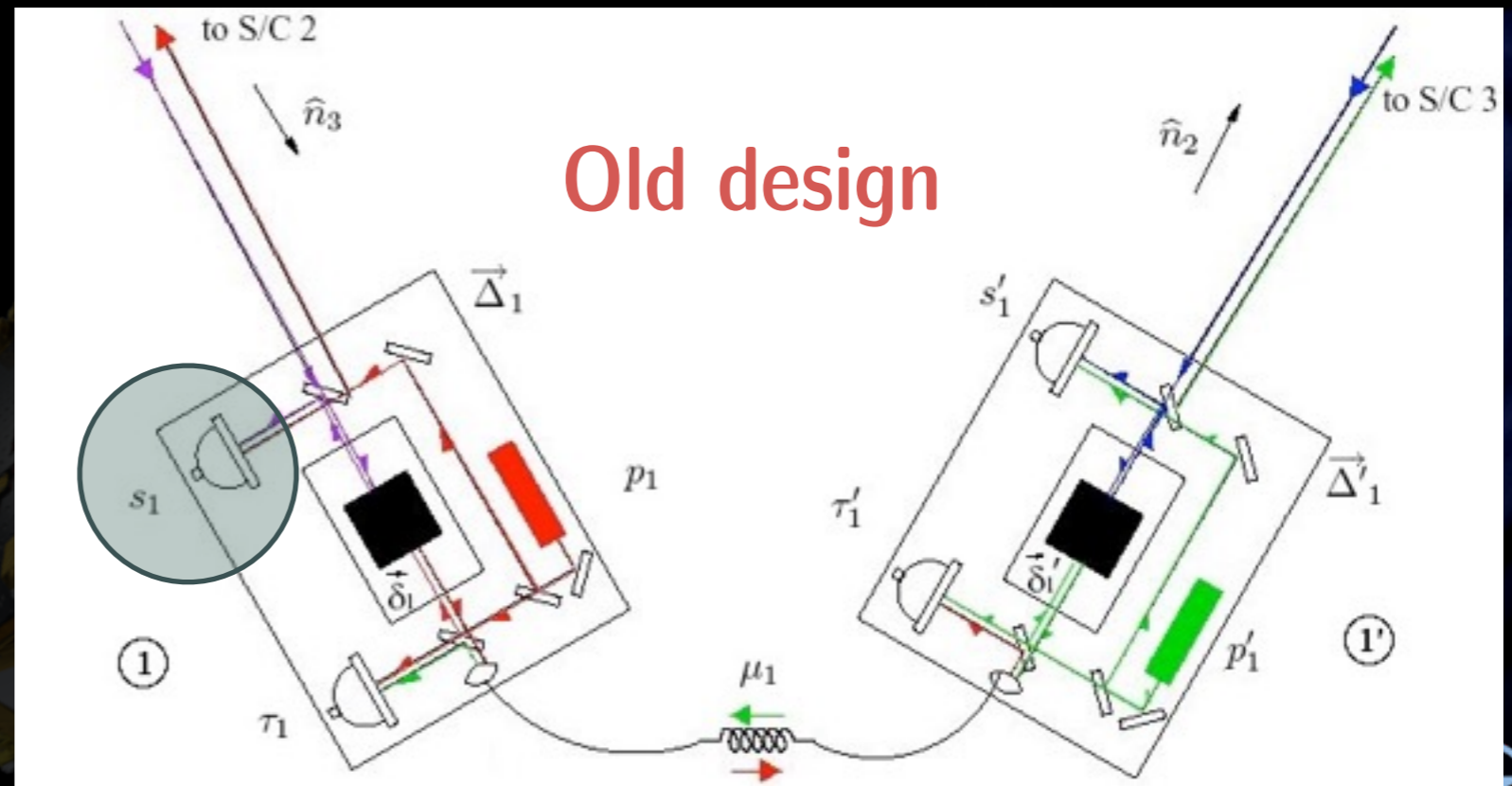
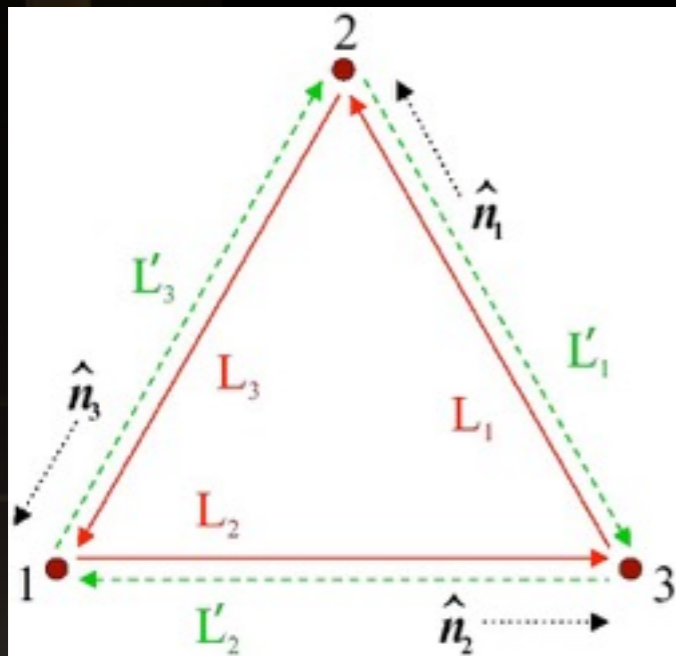


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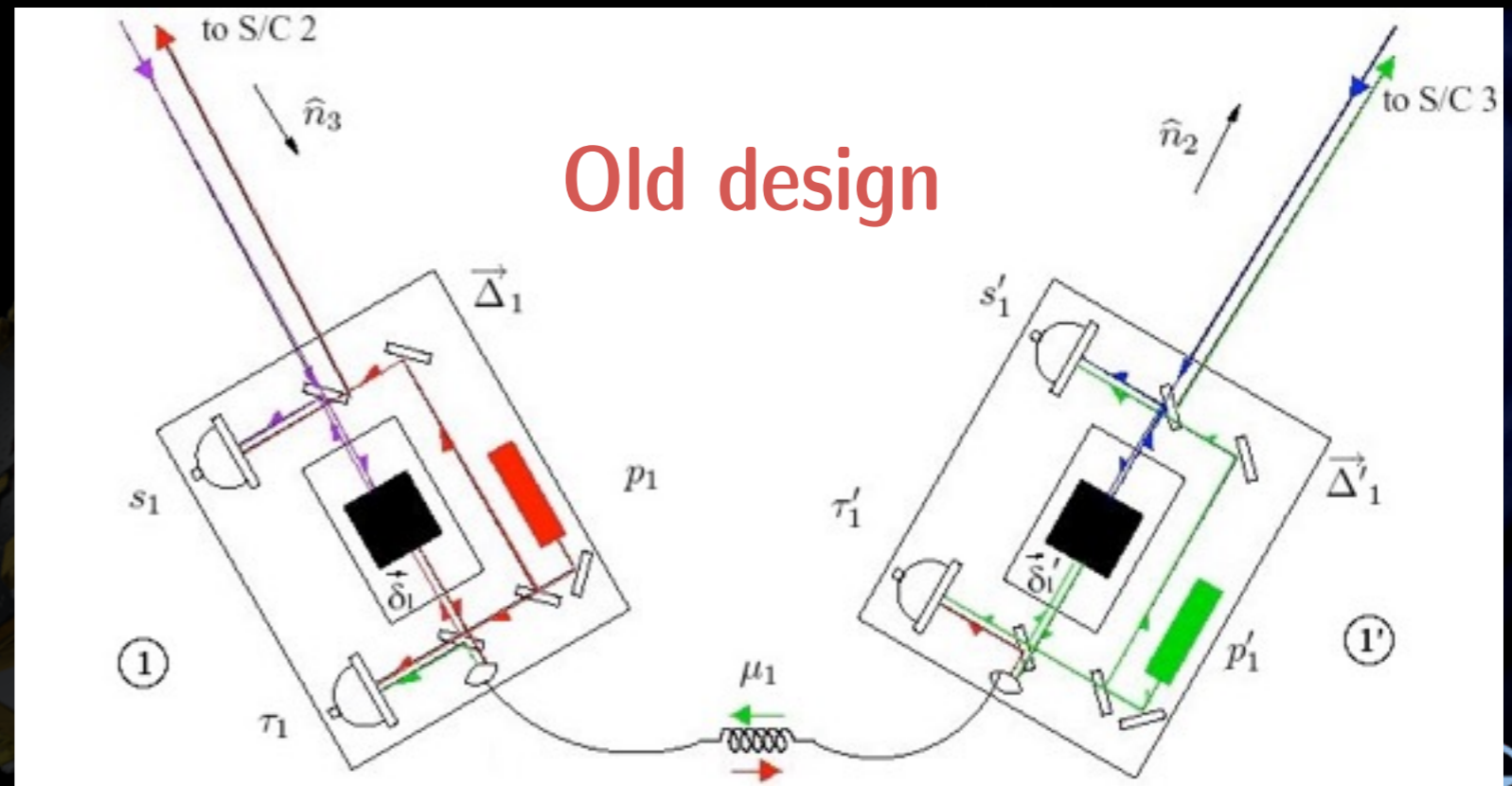
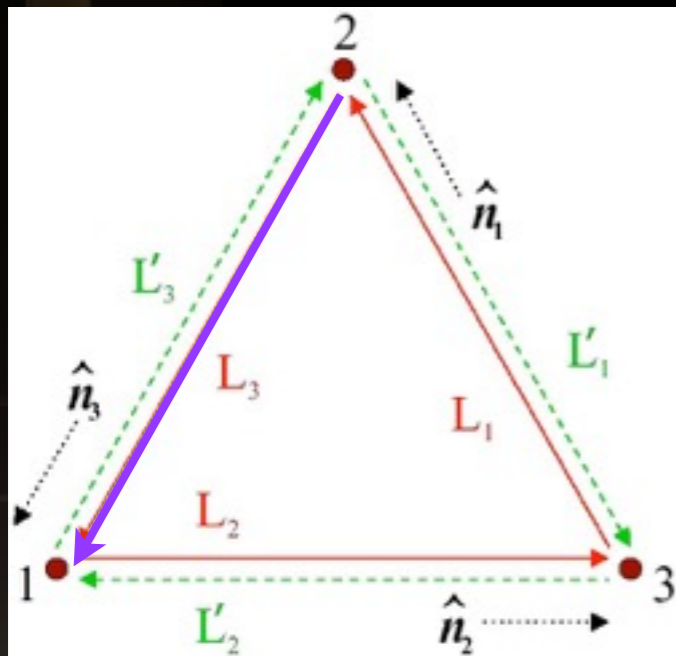
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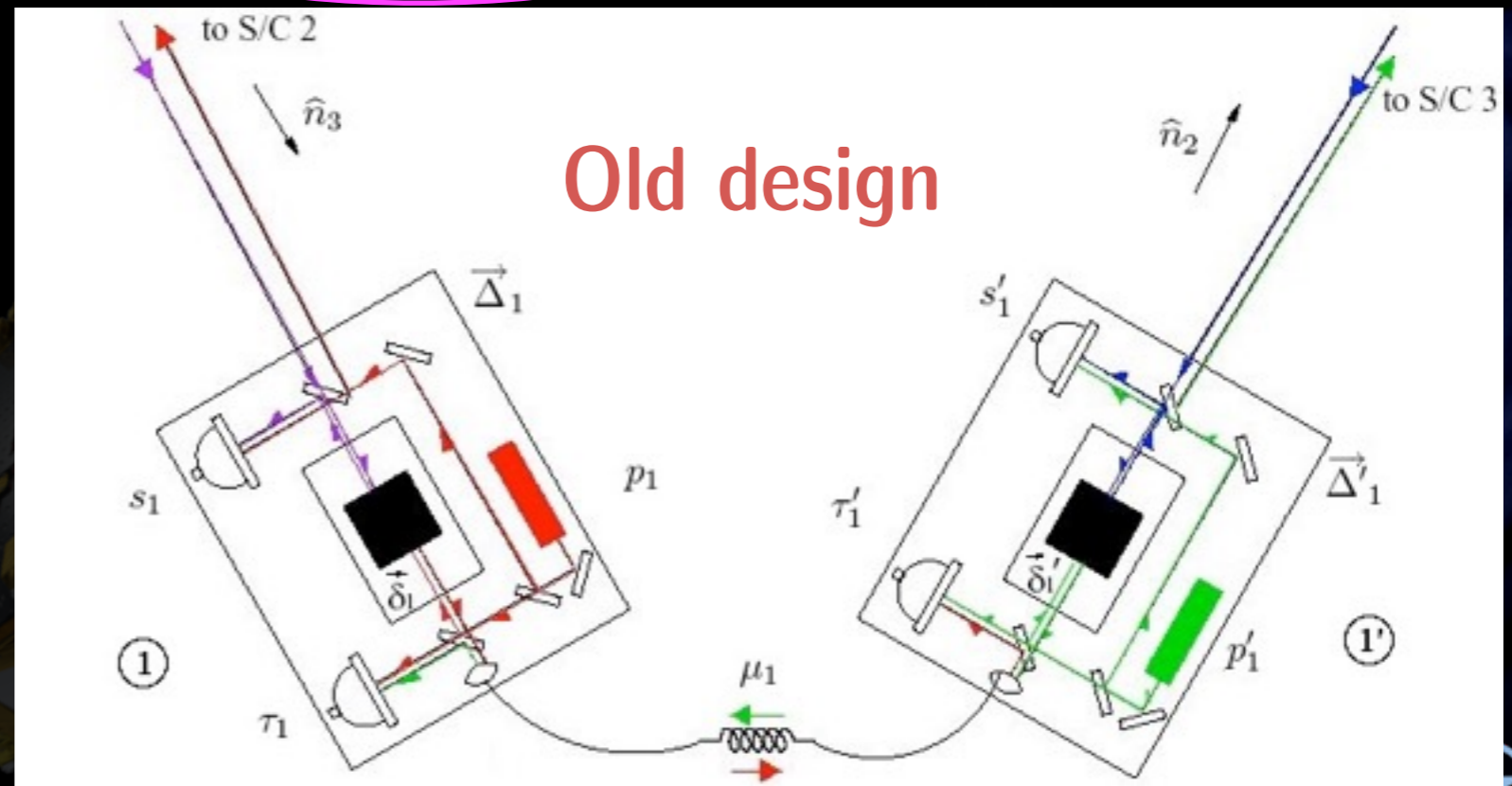
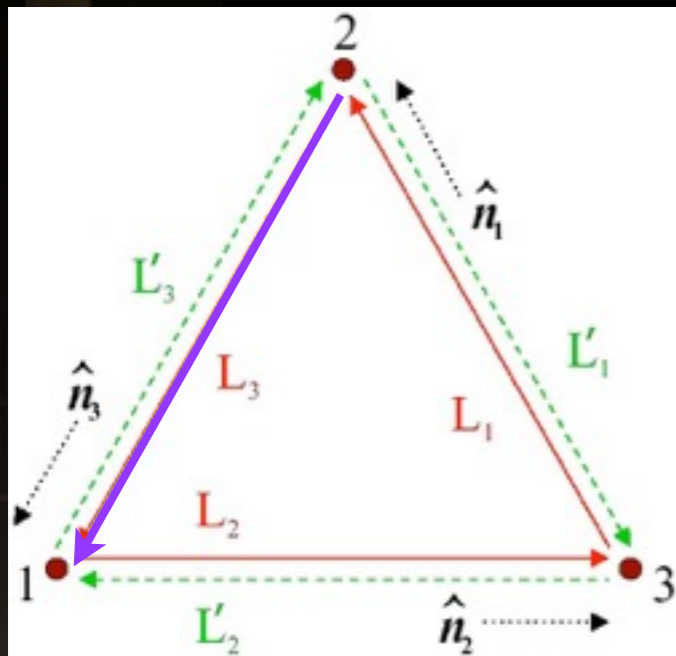
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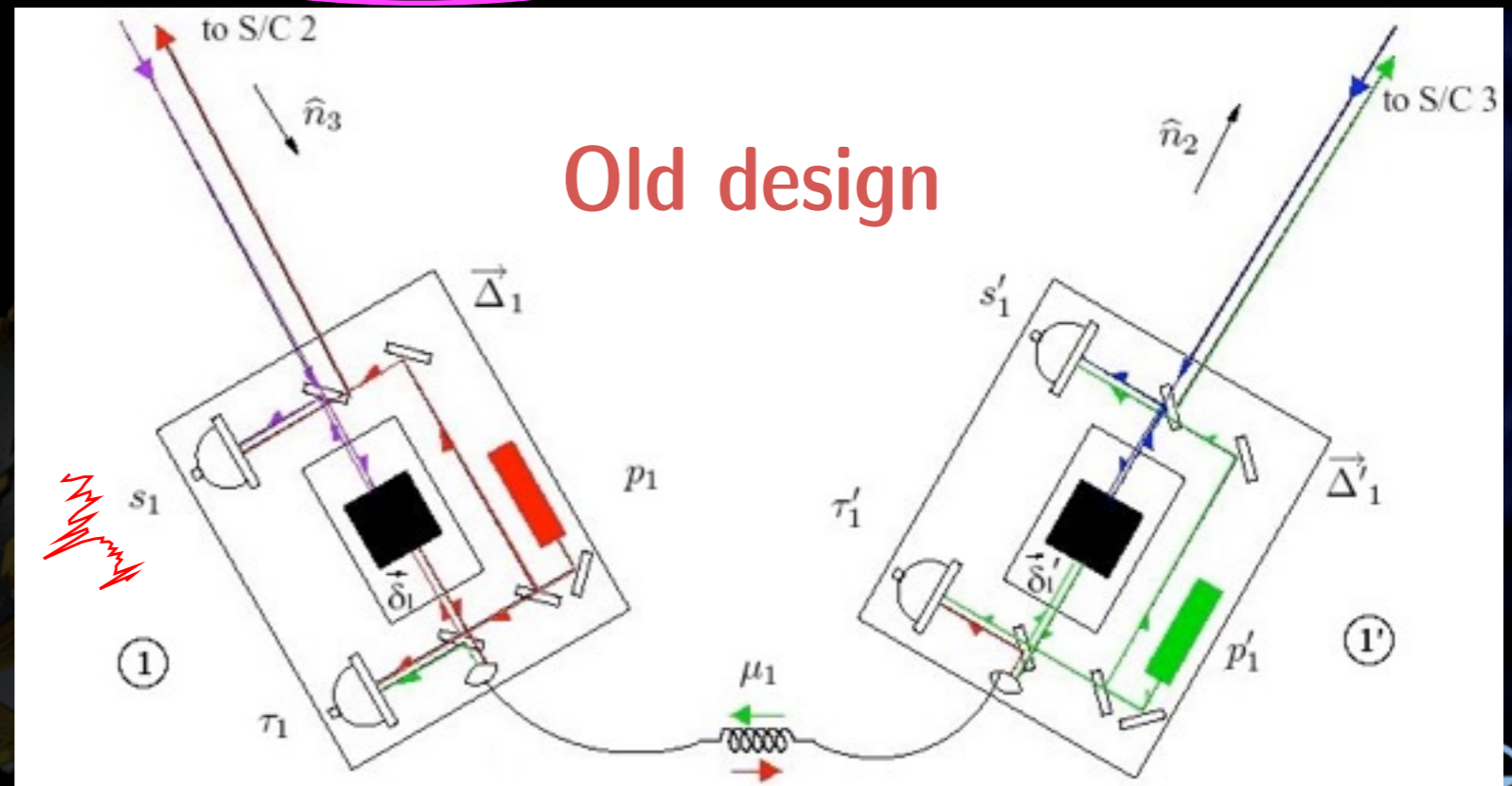
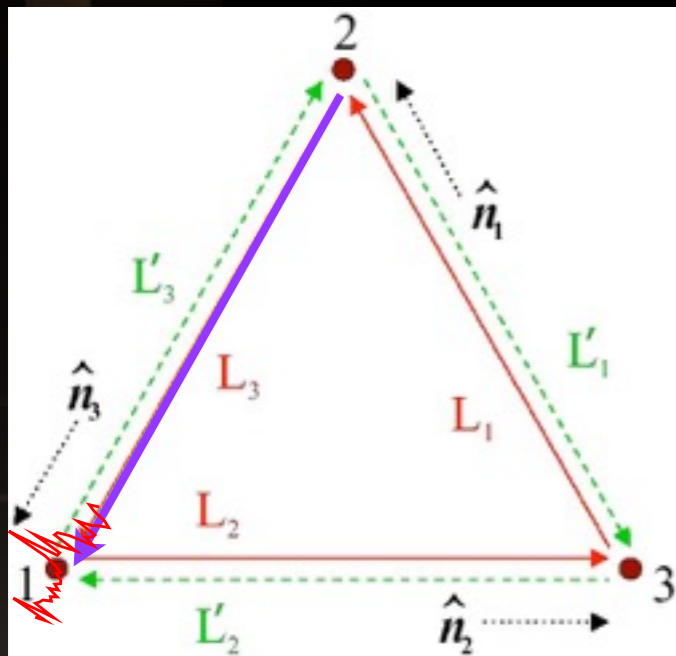


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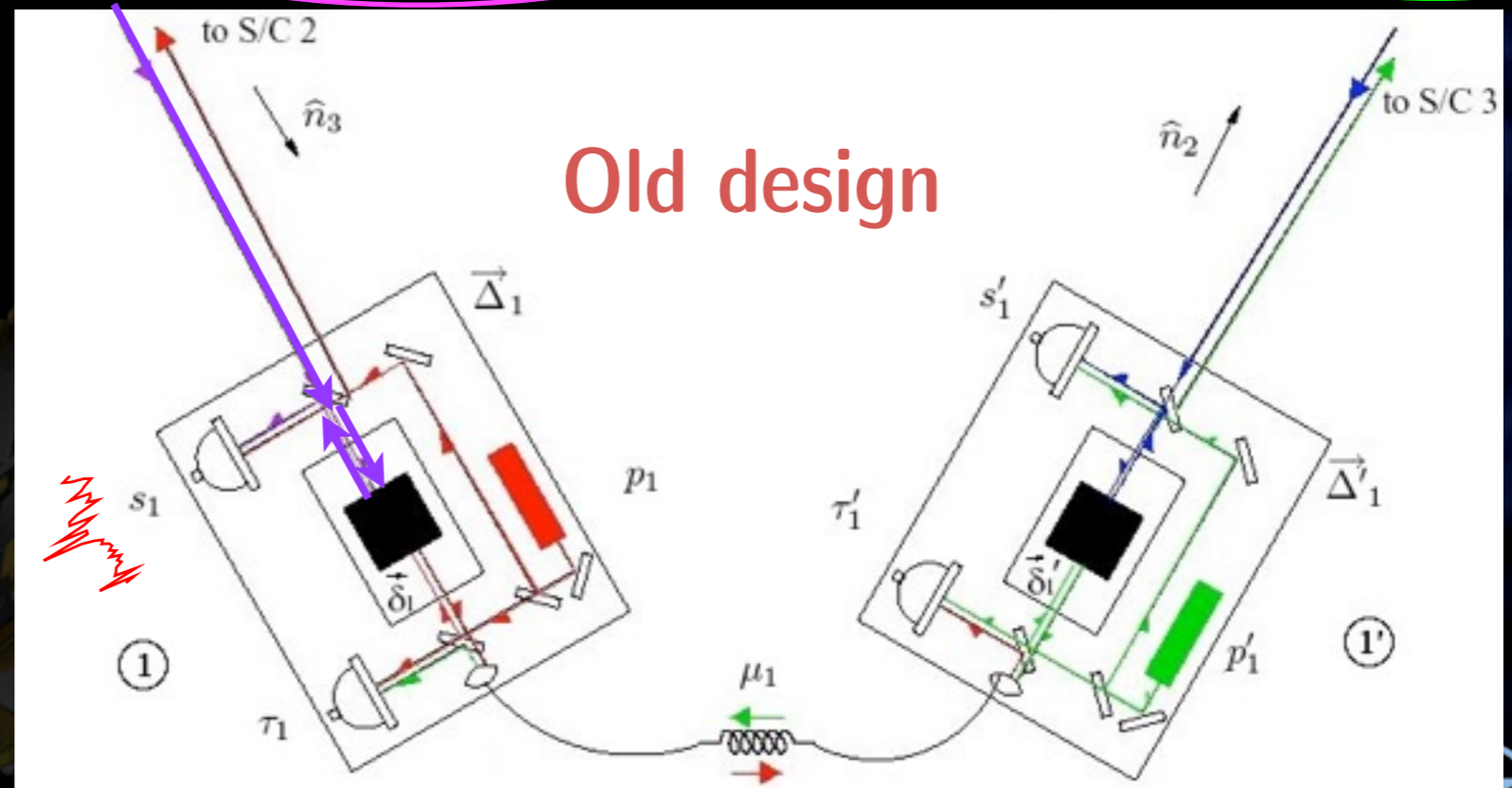
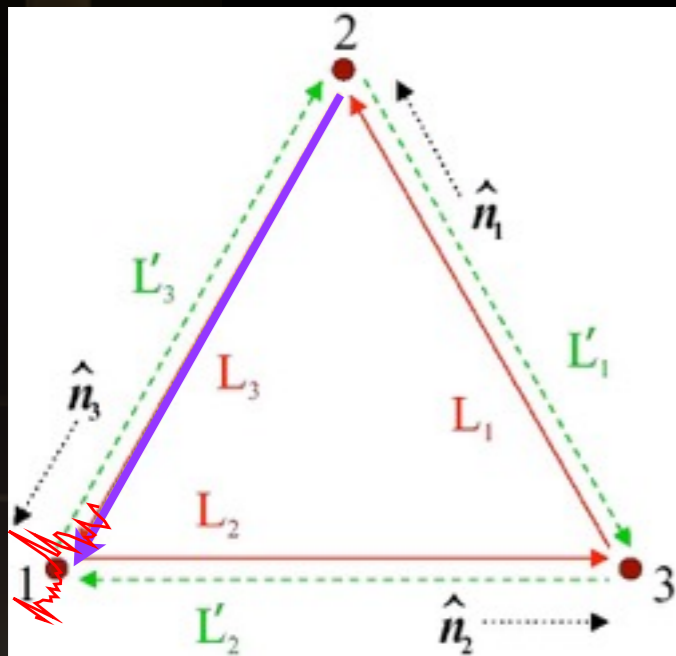
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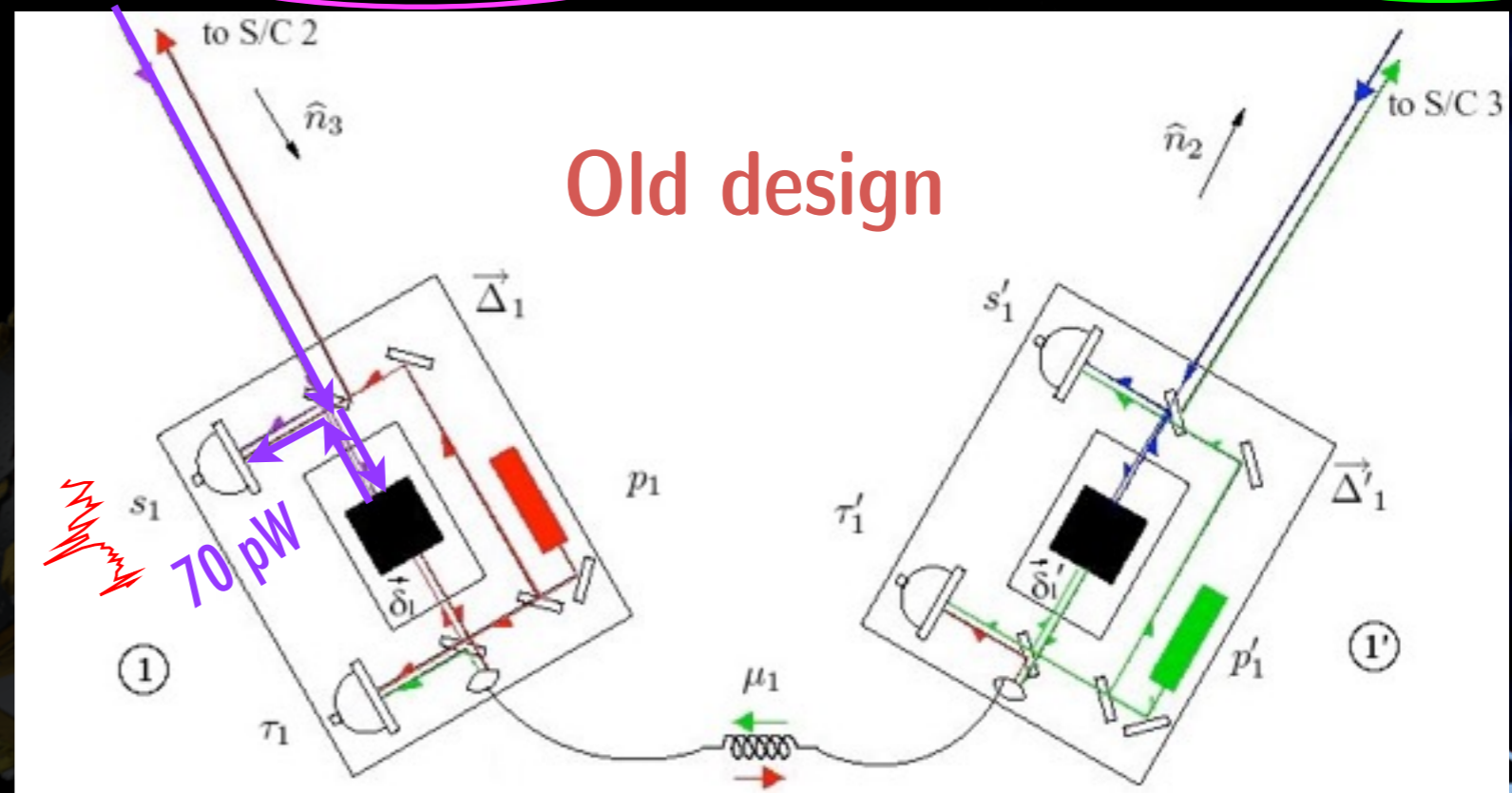
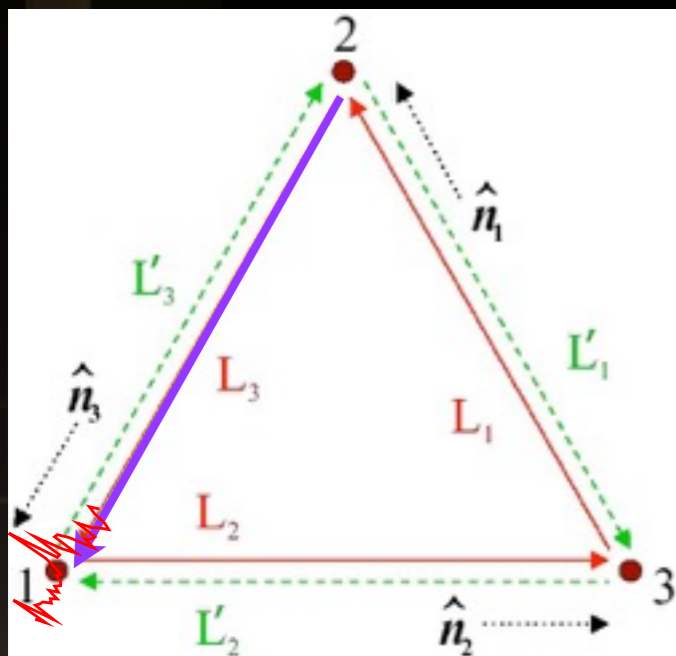


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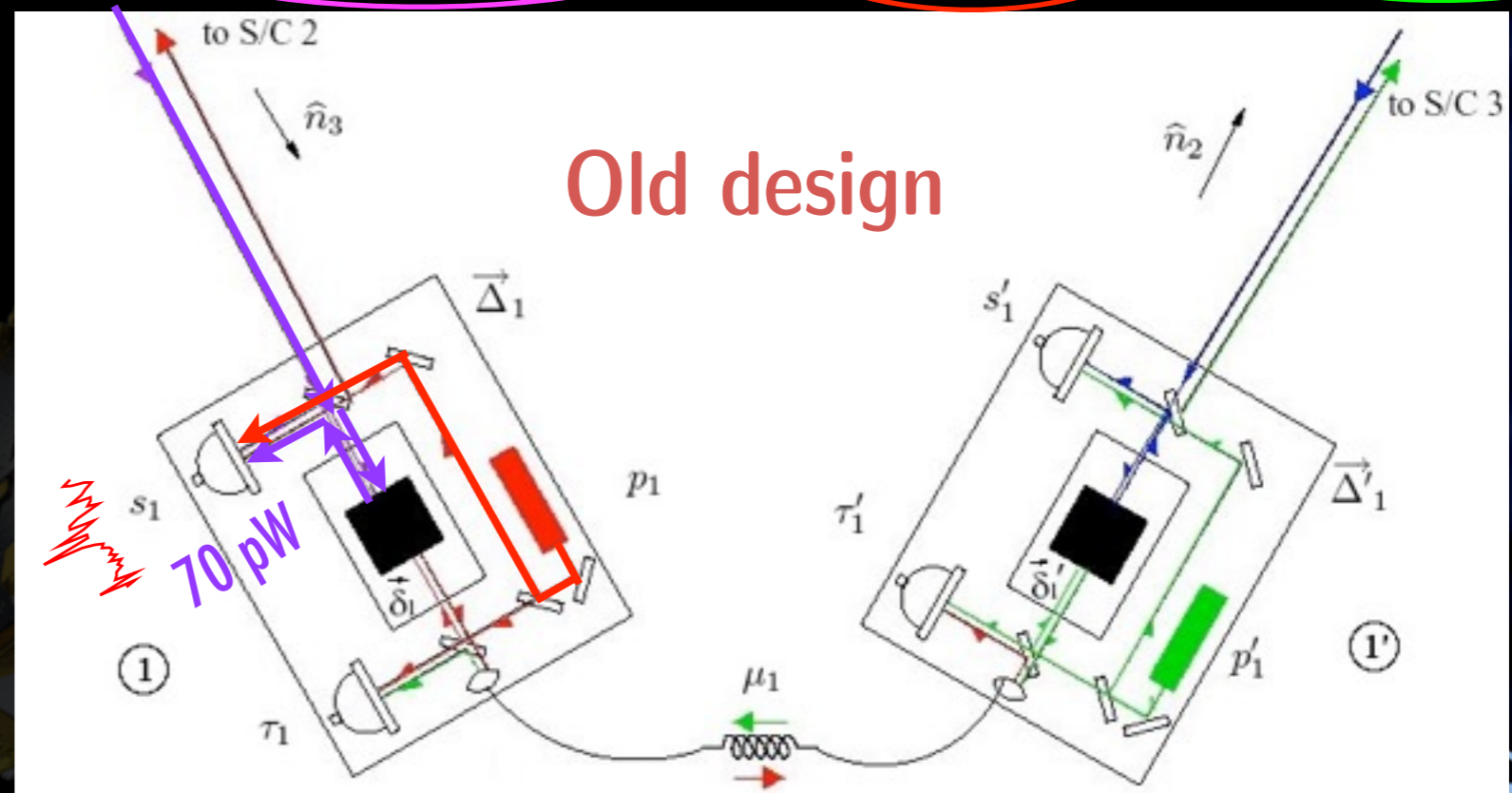
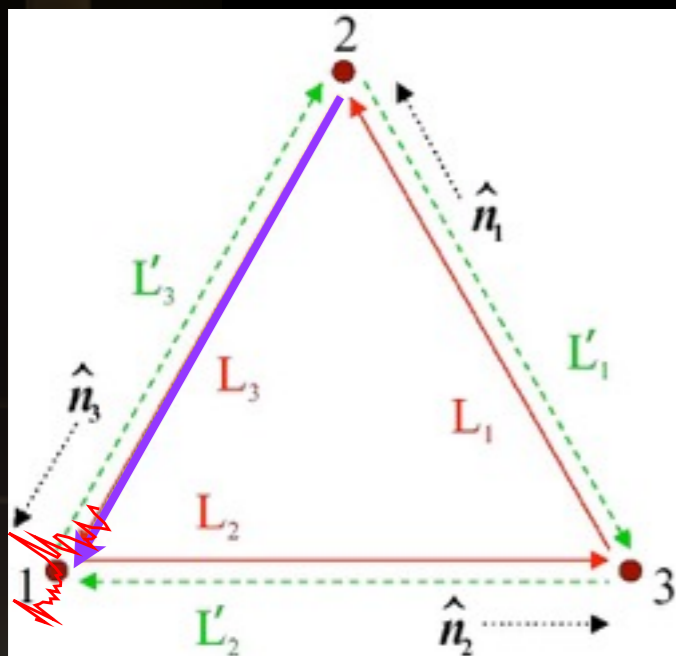


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Time Delay Interferometry

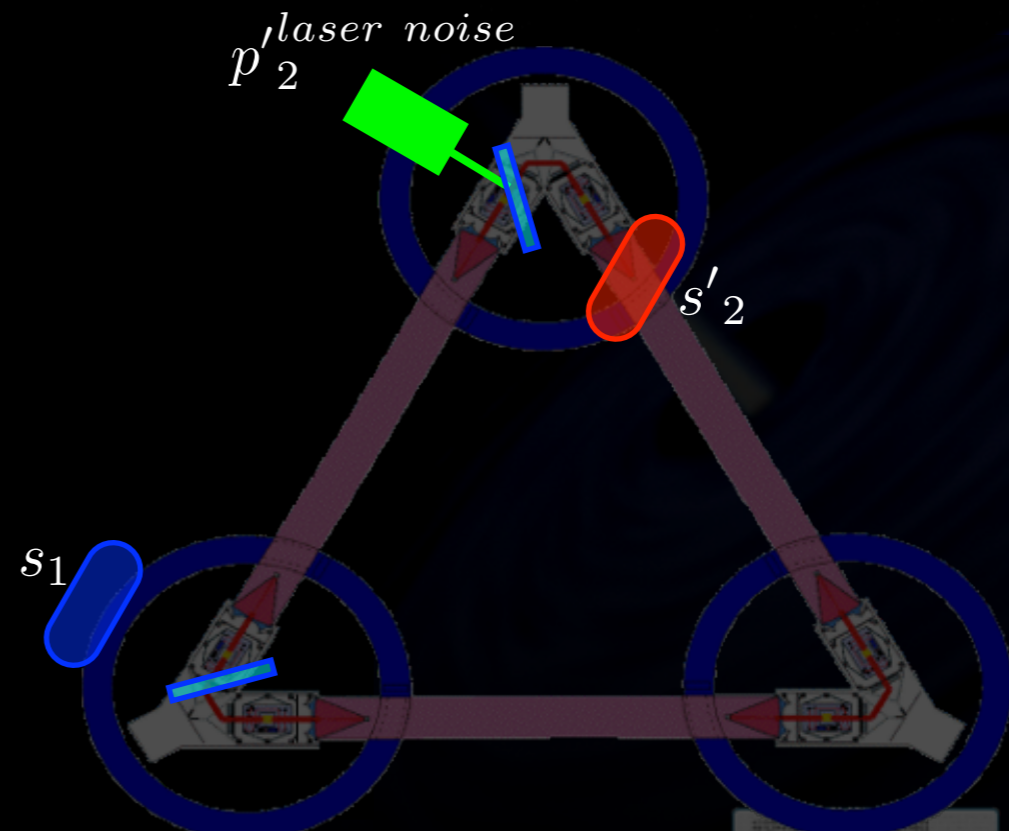
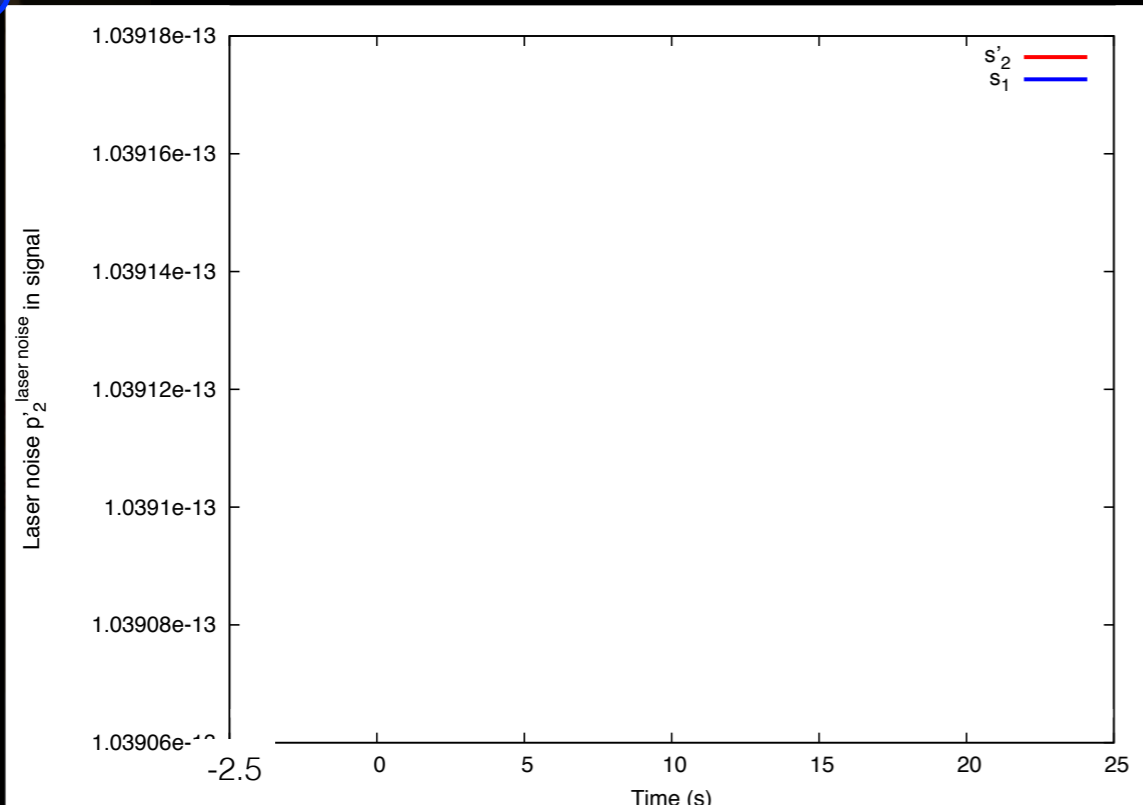


- ▶ Pre-processing of the science data,
- ▶ Combinations of delayed measurements to reduce laser noise:

Tinto & Durandhar, *Revue Living Rev. Rel.* 8 p 4 (2005)
 Durandhar, Nayak & Vinet, *PRD* 65 102002 (2002)

 $-s'_2(t) = p'_2(t)$

 $s_1(t) = D_3^{real} p'_2(t) = p'_2 \left(t - \frac{L_3^{real}}{c} \right)$





Time Delay Interferometry





▶ Pre-processing of the science data,

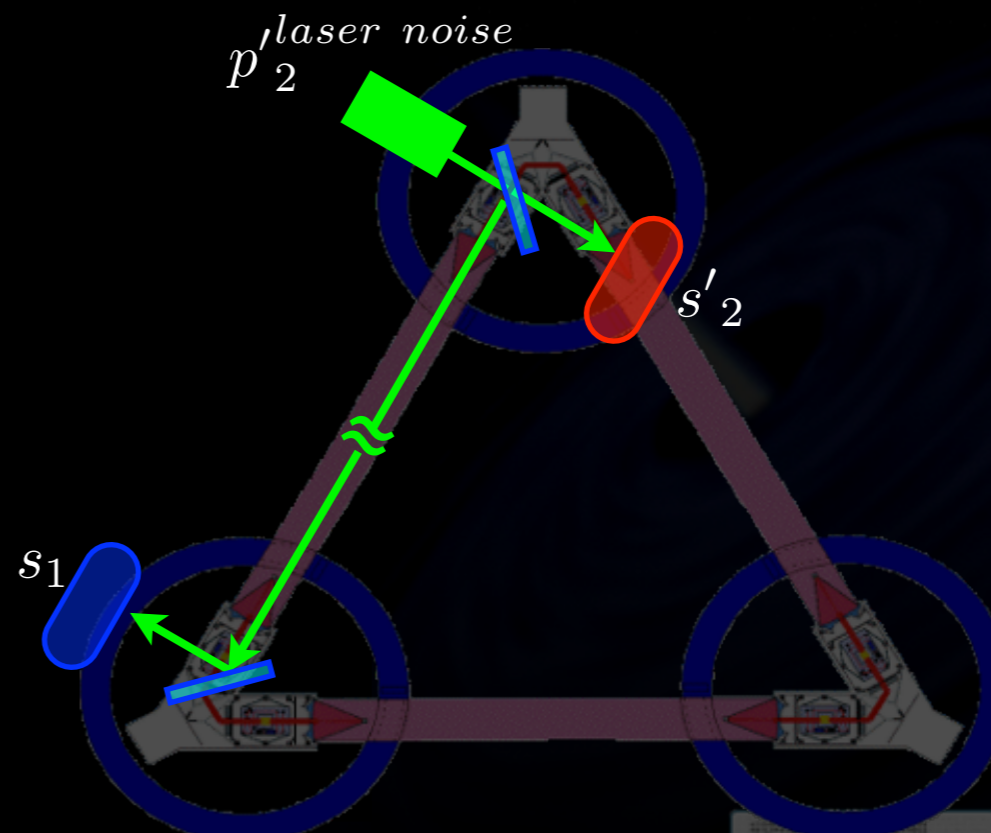
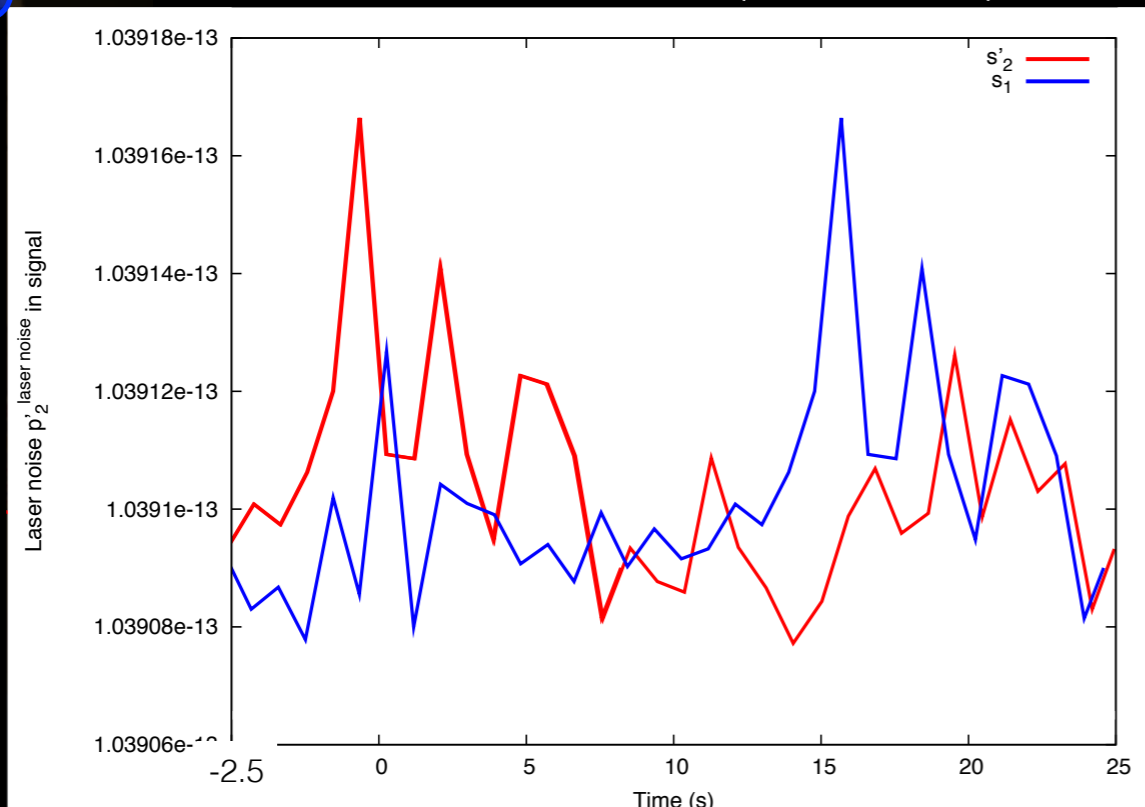
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▶ Combinations of delayed measurements to reduce laser noise:

 $-s'_2(t) = p'_2(t)$

 $s_1(t) = D_3^{real} p'_2(t) = p'_2 \left(t - \frac{L_3^{real}}{c} \right)$





Time Delay Interferometry



▶ Pre-processing of the science data,

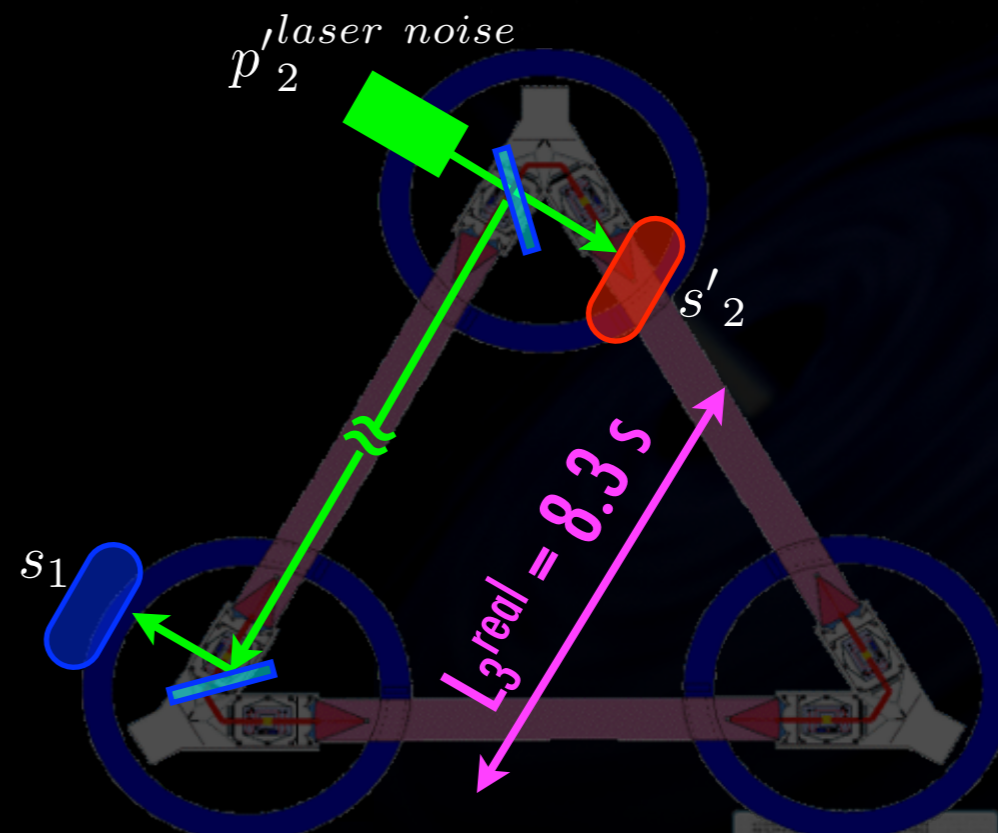
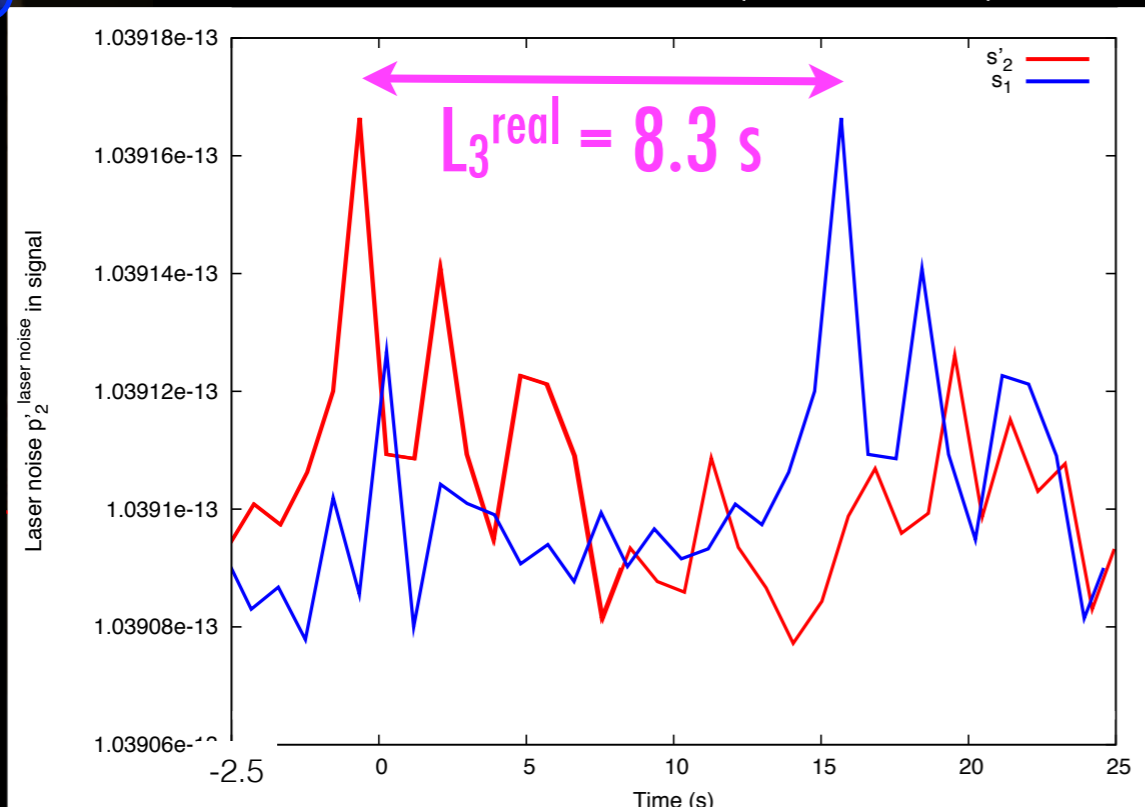
Tinto & Durandhar, *Revue Living Rev. Rel.* 8 p 4 (2005)

Durandhar, Nayak & Vinet, *PRD* 65 102002 (2002)

▶ Combinations of delayed measurements to reduce laser noise:

Red oval icon: $-s'_2(t) = p'_2(t)$

Blue oval icon: $s_1(t) = D_3^{real} p'_2(t) = p'_2\left(t - \frac{L_3^{real}}{c}\right)$





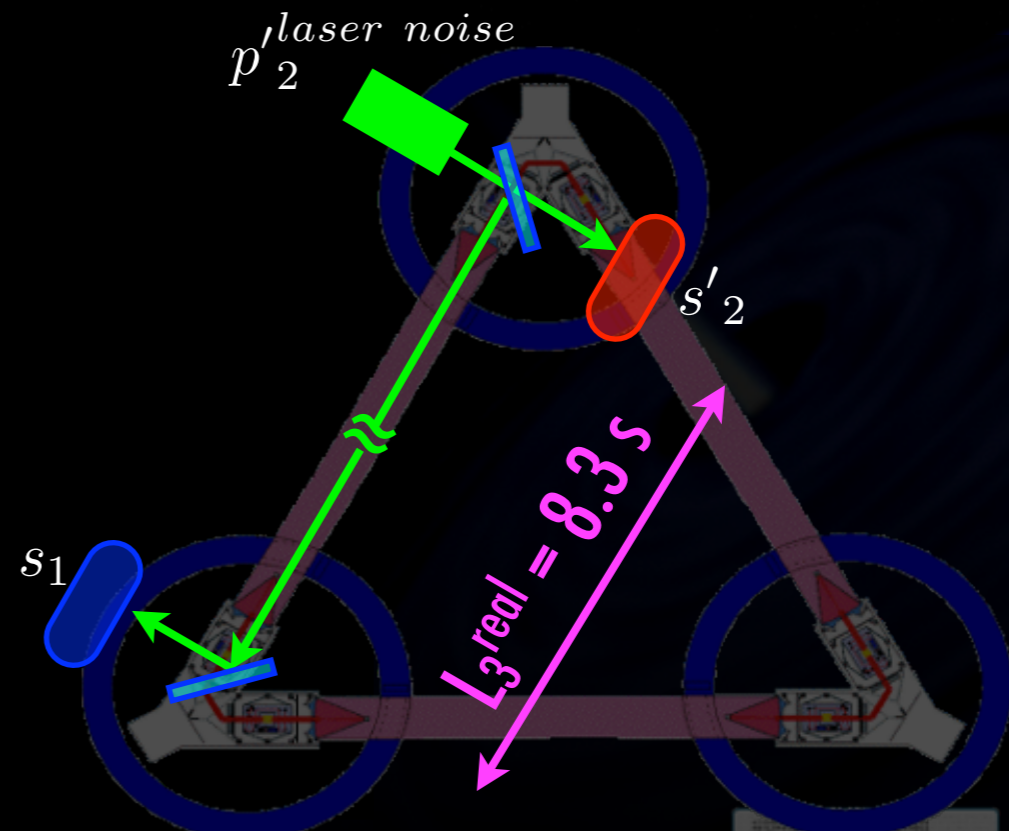
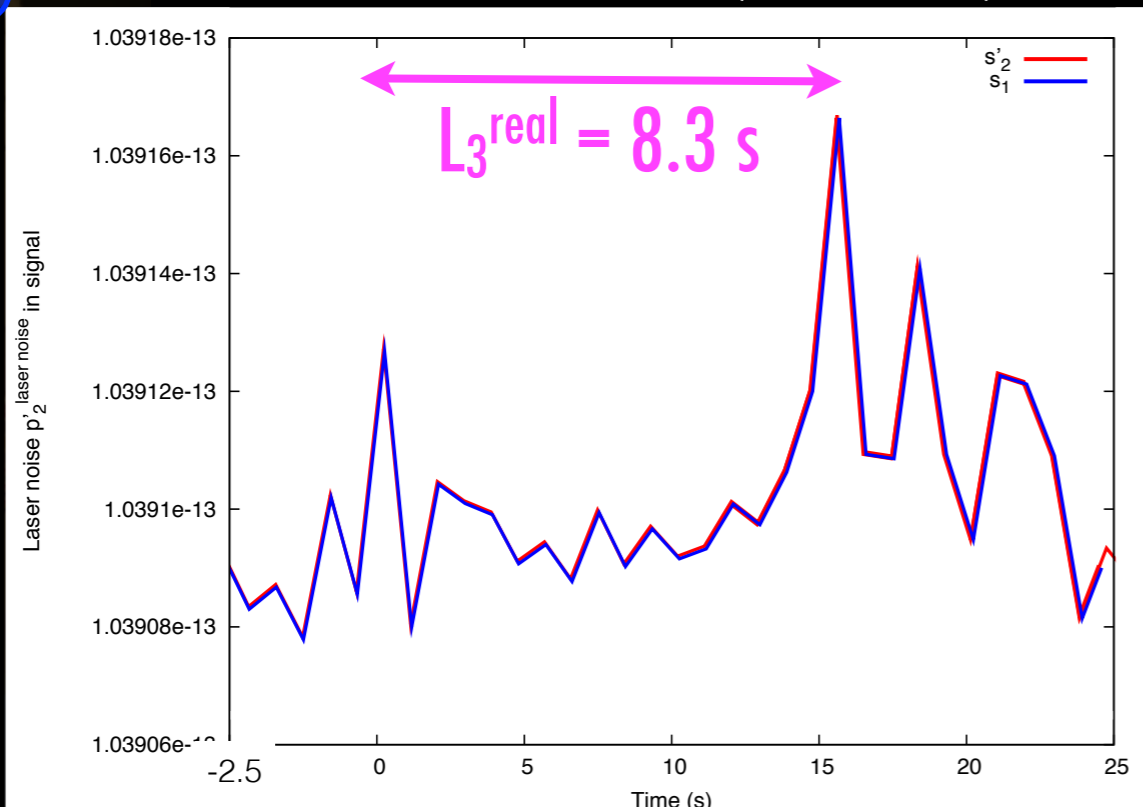
Time Delay Interferometry

- ▶ Pre-processing of the science data,
- ▶ Combinations of delayed measurements to reduce laser noise:

Tinto & Durandhar, *Revue Living Rev. Rel.* 8 p 4 (2005)
 Durandhar, Nayak & Vinet, *PRD* 65 102002 (2002)

$$-D_3^{TDI} s'_2(t) = -D_3^{TDI} s'_2 = p'_2 \left(t - \frac{L_3^{TDI}}{c} \right)$$

$$s_1(t) = D_3^{real} p'_2(t) = p'_2 \left(t - \frac{L_3^{real}}{c} \right)$$



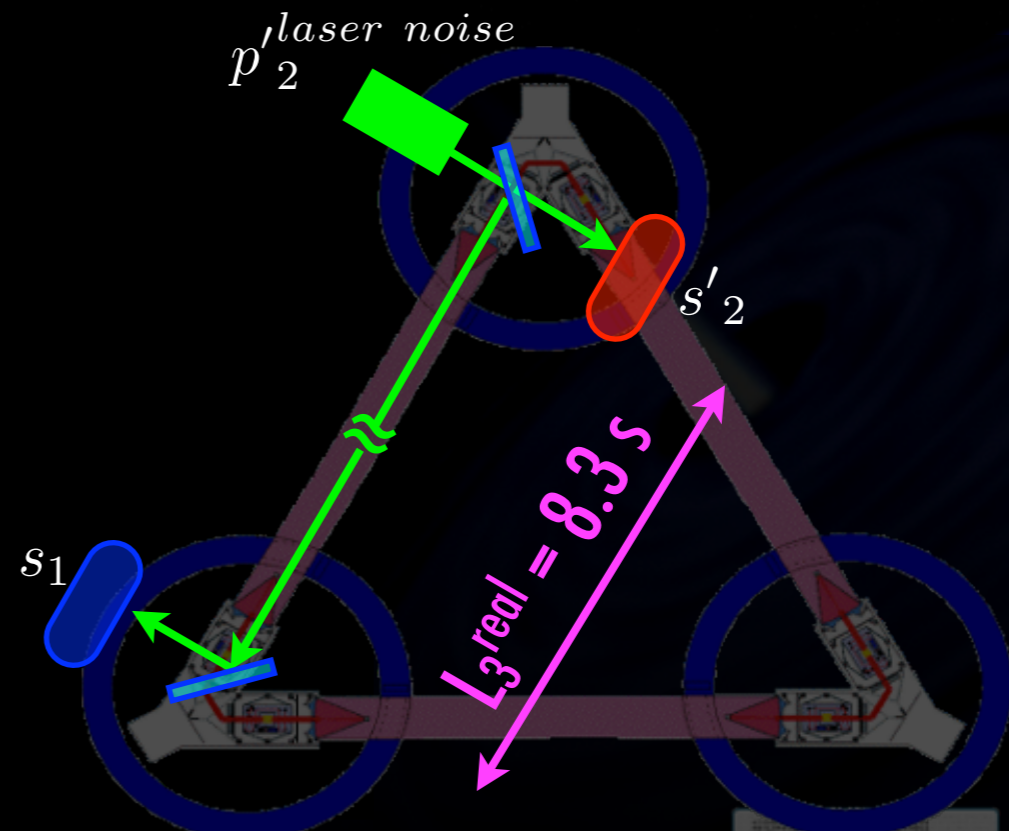
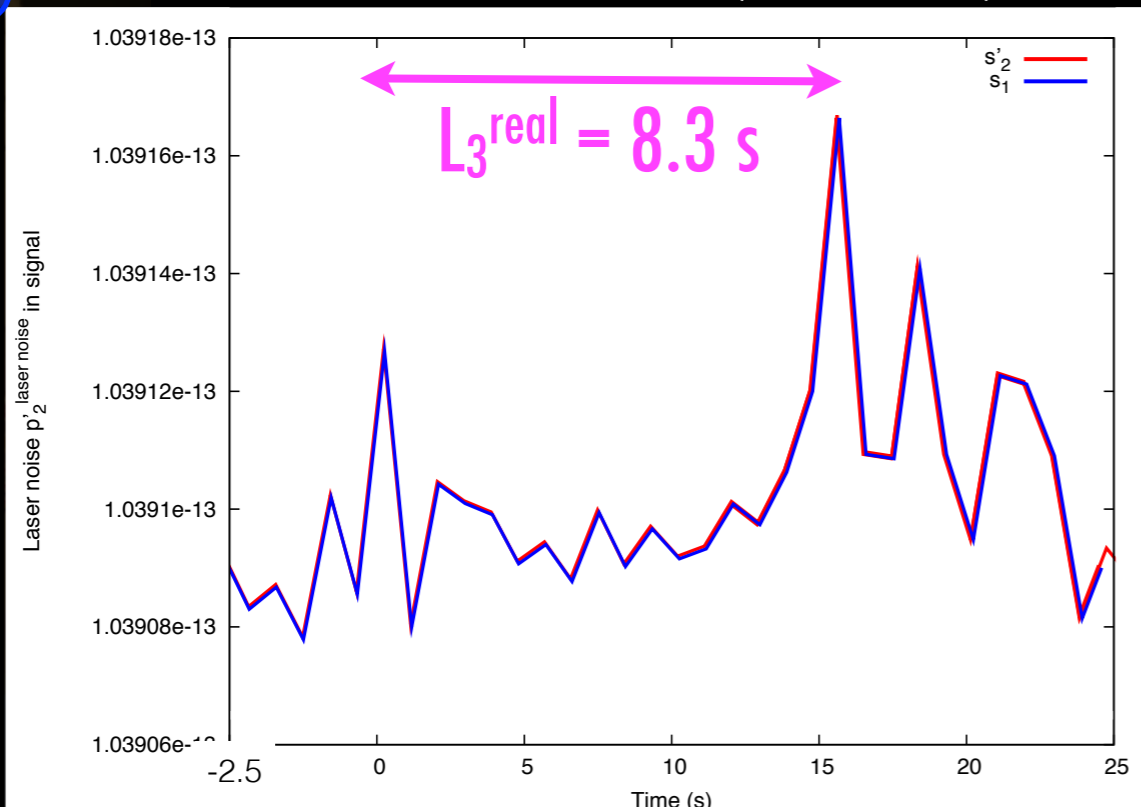


Time Delay Interferometry

- ▶ Pre-processing of the science data,
- ▶ Combinations of delayed measurements to reduce laser noise:

Tinto & Durandhar, *Revue Living Rev. Rel.* 8 p 4 (2005)
 Durandhar, Nayak & Vinet, *PRD* 65 102002 (2002)

$$\begin{aligned}
 -D_3^{TDI} s'_2(t) &= -D_3^{TDI} s'_2 = p'_2 \left(t - \frac{L_3^{TDI}}{c} \right) \\
 s_1(t) &= D_3^{real} p'_2(t) = p'_2 \left(t - \frac{L_3^{real}}{c} \right)
 \end{aligned}
 \Rightarrow
 \begin{aligned}
 s_1(t) + D_3^{TDI} s'_2(t) &= D_3^{real} p'_2 - D_3^{TDI} p'_2 \\
 &\approx 0
 \end{aligned}$$





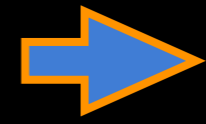
Time Delay Interferometry

TDI requires :

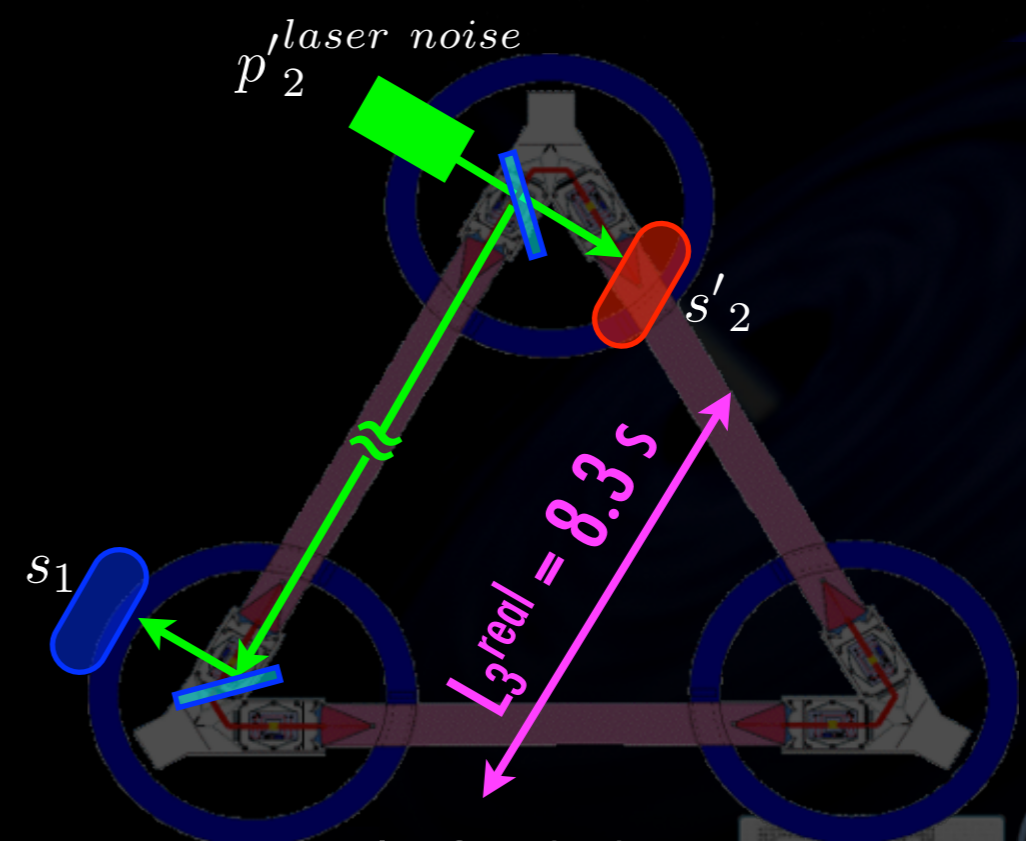
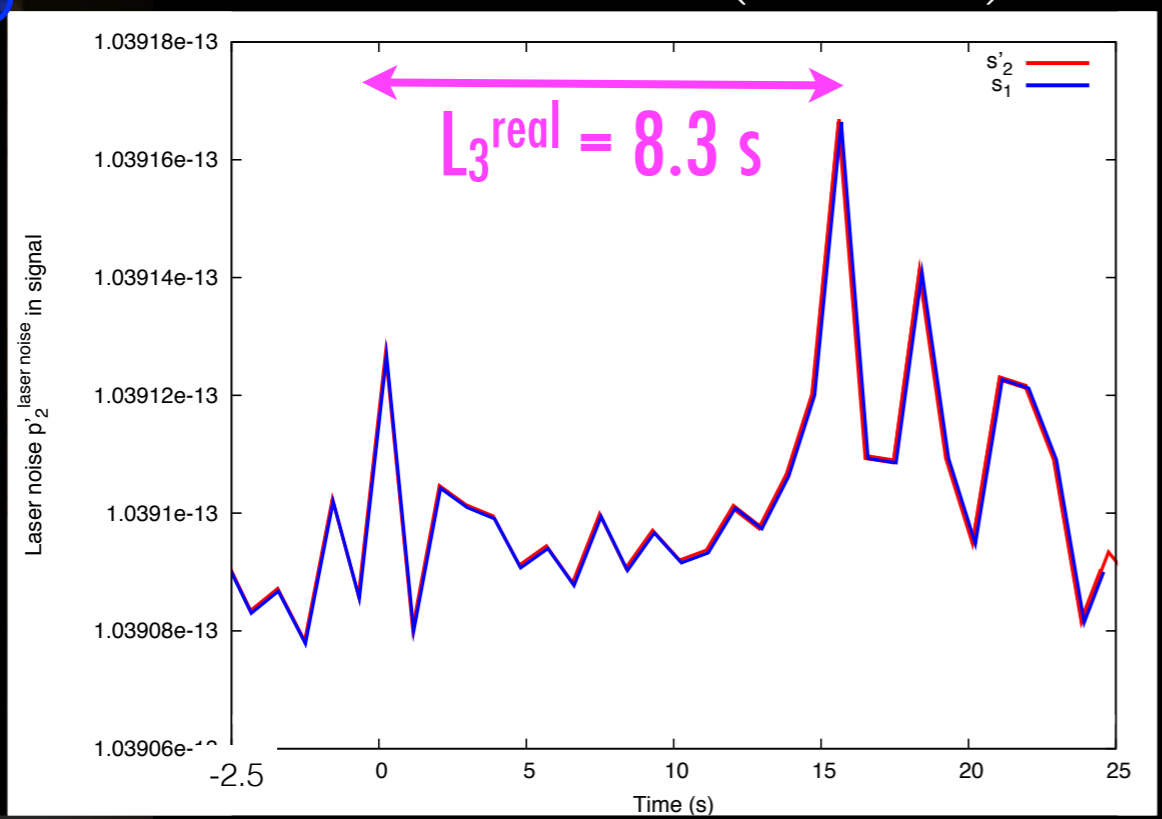
- ▶ knowledge of delays : $L_i^{TDI} = L_i^{real}$
- ▶ interpolation due to the sampling of phasemeter signal

$$-D_3^{TDI} s'_2(t) = -D_3^{TDI} s'_2 = p'_2 \left(t - \frac{L_3^{TDI}}{c} \right)$$

$$s_1(t) = D_3^{real} p'_2(t) = p'_2 \left(t - \frac{L_3^{real}}{c} \right)$$



$$s_1(t) + D_3^{TDI} s'_2(t) = D_3^{real} p'_2 - D_3^{TDI} p'_2 \approx 0$$





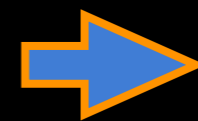
Time Delay Interferometry

TDI requires :

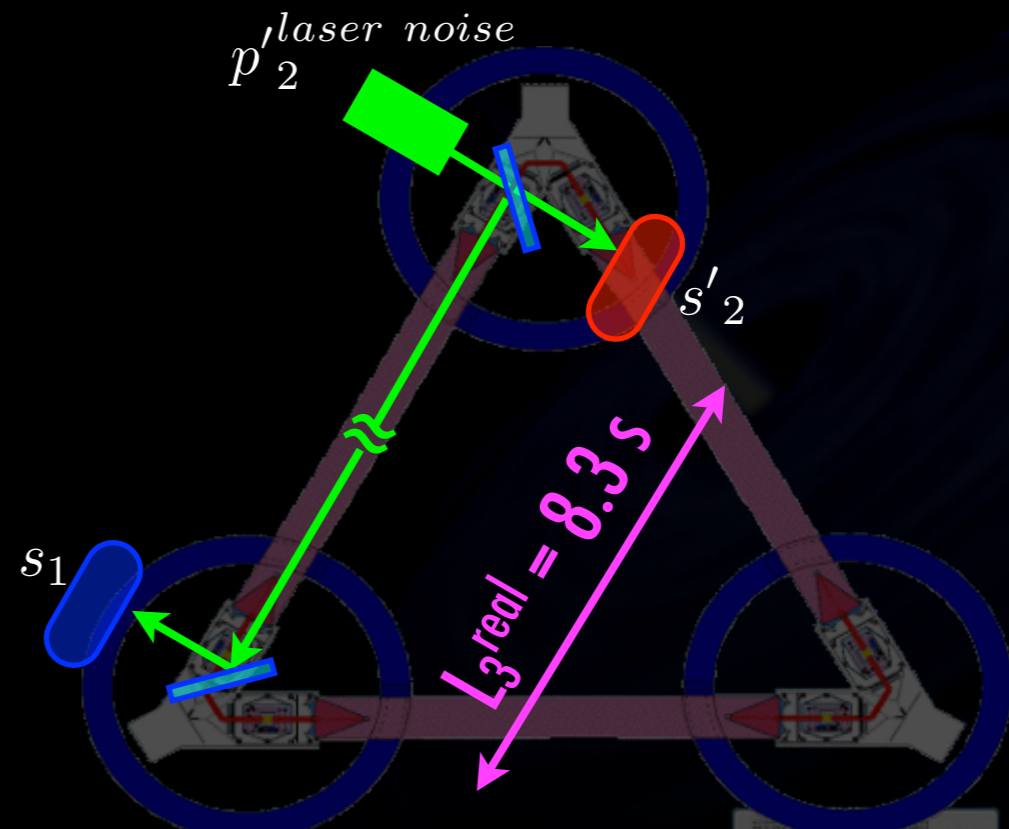
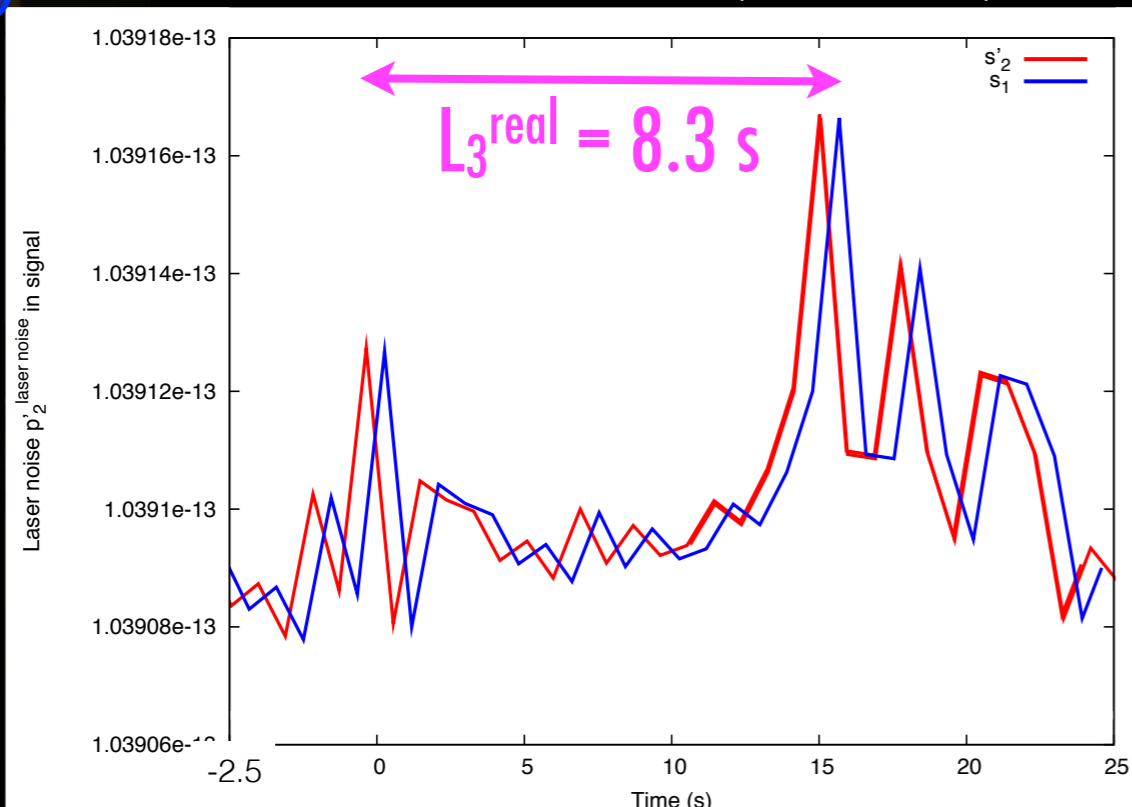
- ▶ knowledge of delays : $L_i^{TDI} = L_i^{real}$
- ▶ interpolation due to the sampling of phasemeter signal

$$-D_3^{TDI} s'_2(t) = -D_3^{TDI} s'_2 = p'_2 \left(t - \frac{L_3^{TDI}}{c} \right)$$

$$s_1(t) = D_3^{real} p'_2(t) = p'_2 \left(t - \frac{L_3^{real}}{c} \right)$$



$$s_1(t) + D_3^{TDI} s'_2(t) = D_3^{real} p'_2 - D_3^{TDI} p'_2 \simeq \text{residual laser noise}$$





Time Delay Interferometry



▶ Time Delay Interferometry:

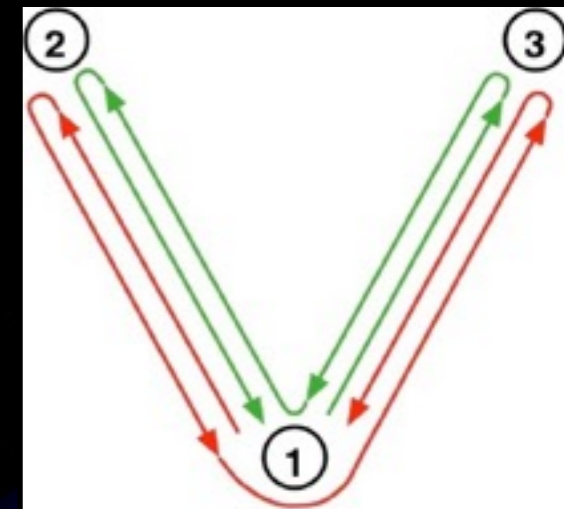
Tinto & Durandhar, *Revue Living Rev. Rel.* 8 p 4 (2005)

Durandhar, Nayak & Vinet, *PRD* 65 102002 (2002)

Vallisneri, *gr-qc/0504145* (2005)

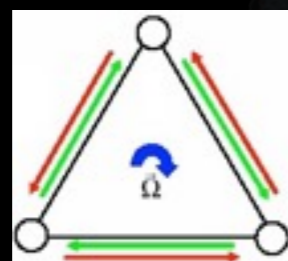
- Combine delayed measurements to reduce laser noises, optical bench noises, ... ?
- Algebraic development: many combinations (generators)

$$\begin{aligned}
 X &= -s_1 - D_3 s'_2 - D_3 D_{3'} s'_1 - D_3 D_{3'} D_{2'} s_3 \\
 &\quad + s'_1 + D_{2'} s_3 - D_{2'} D_2 s_1 - D_{2'} D_2 D_3 s_3 \\
 &\approx 0
 \end{aligned}$$



• Different precisions level

- 1st generation: rigid formation of LISA : $D_{i'} s = D_i s$,
- generation 1.5: Sagnac effect : $D_{i'} s \neq D_i s$ but $D_j D_i s = D_i D_j s$,
- 2nd generation: flexing and Sagnac effect : $D_j D_i s \neq D_i D_j s$





Time Delay Interferometry

▶ TDI generation 1

$$X_{1st} = (1 - D_2^2, 0, -D_2 + D_2 D_3^2, -1 + D_3^2, D_3 - D_2^2 D_3, 0)$$

▶ TDI generation 1.5

$$X_{1.5} = (1 - D_2 D'_2, 0, -D'_2 + D'_2 D'_3 D_3, -1 + D'_3 D_3, D_3 - D_2 D'_2 D_3, 0)$$

▶ TDI 2nd generation: until 7 delay operators combined

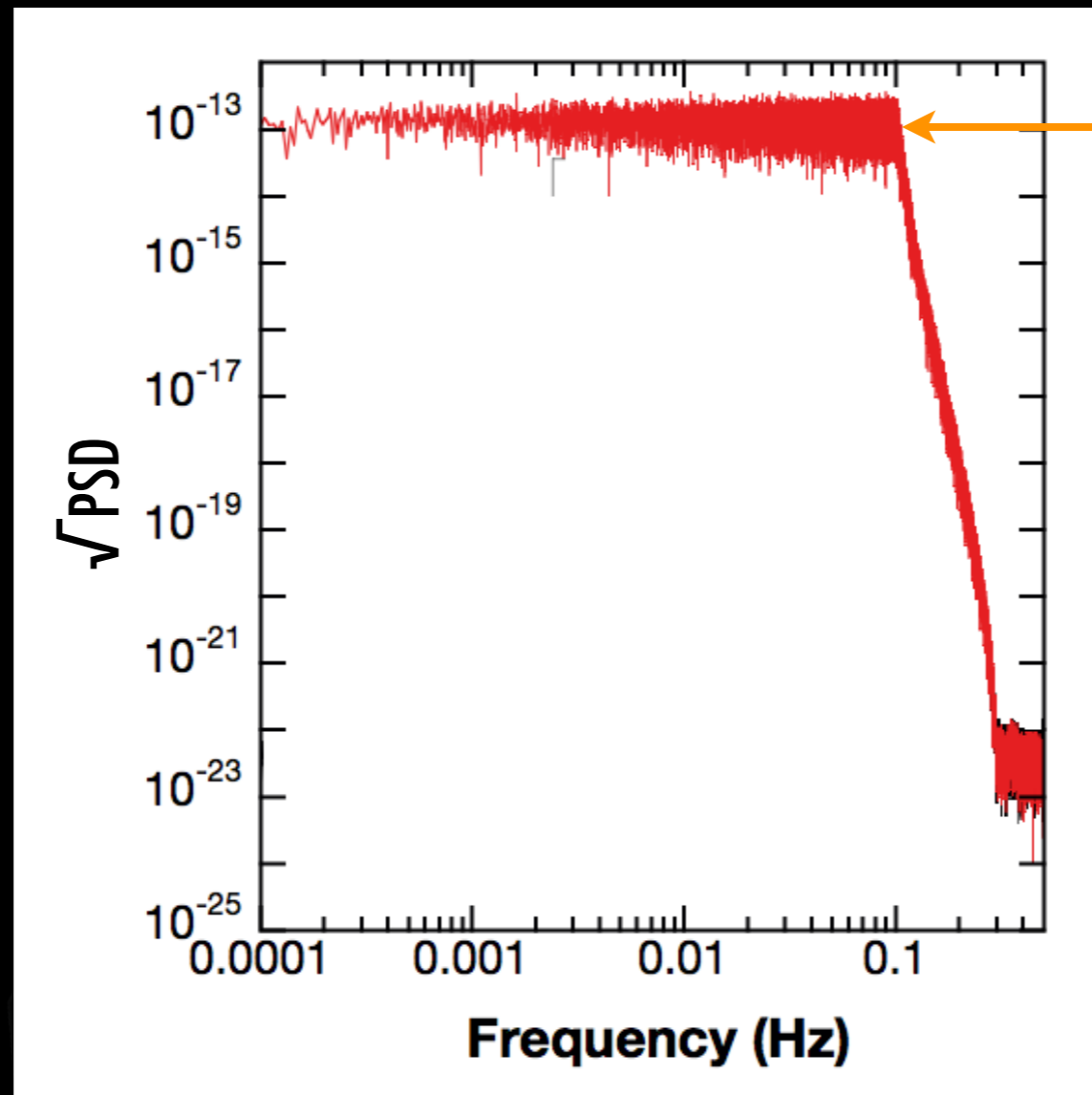
$$\begin{aligned} X_{2nd} = & (1 + D_3 D'_3 D'_2 D_2 D'_2 D_2 - D'_2 D_2 - D'_2 D_2 D_3 D'_3, \\ & 0, \\ & D_3 D'_3 D'_2 + D_3 D'_3 D'_2 D_2 D'_2 - D'_2 - D'_2 D_2 D_3 D'_3 D_3 D'_3 D'_2, \\ & D_3 D'_3 + D_3 D'_3 D'_2 D_2 - 1 - D'_2 D_2 D_3 D'_3 D_3 D'_3, \\ & D_3 + D_3 D'_3 D'_2 D_2 D'_2 D_2 D_3 - D'_2 D_2 D_3 - D'_2 D_2 D_3 D'_3 D_3, \\ & 0) \end{aligned}$$



Time Delay Interferometry

- ▶ Reduction of laser noises by 8 orders of magnitude !

A GW is hidden here !



Phasemeter
(cut off due to the filter required for digitalization of signal)

Petiteau & al, Phys. Rev. D (2008)

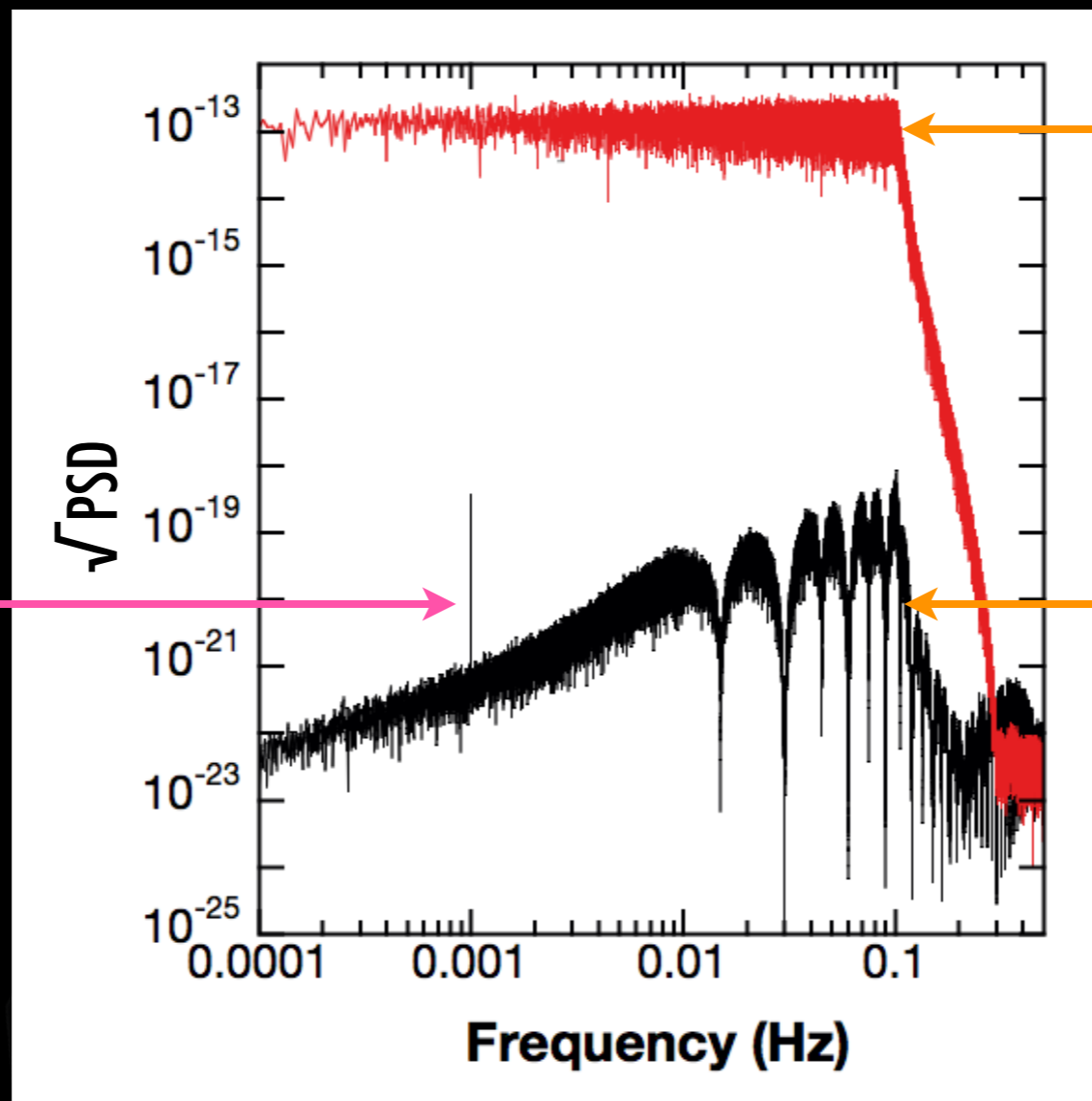


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TDI Michelson

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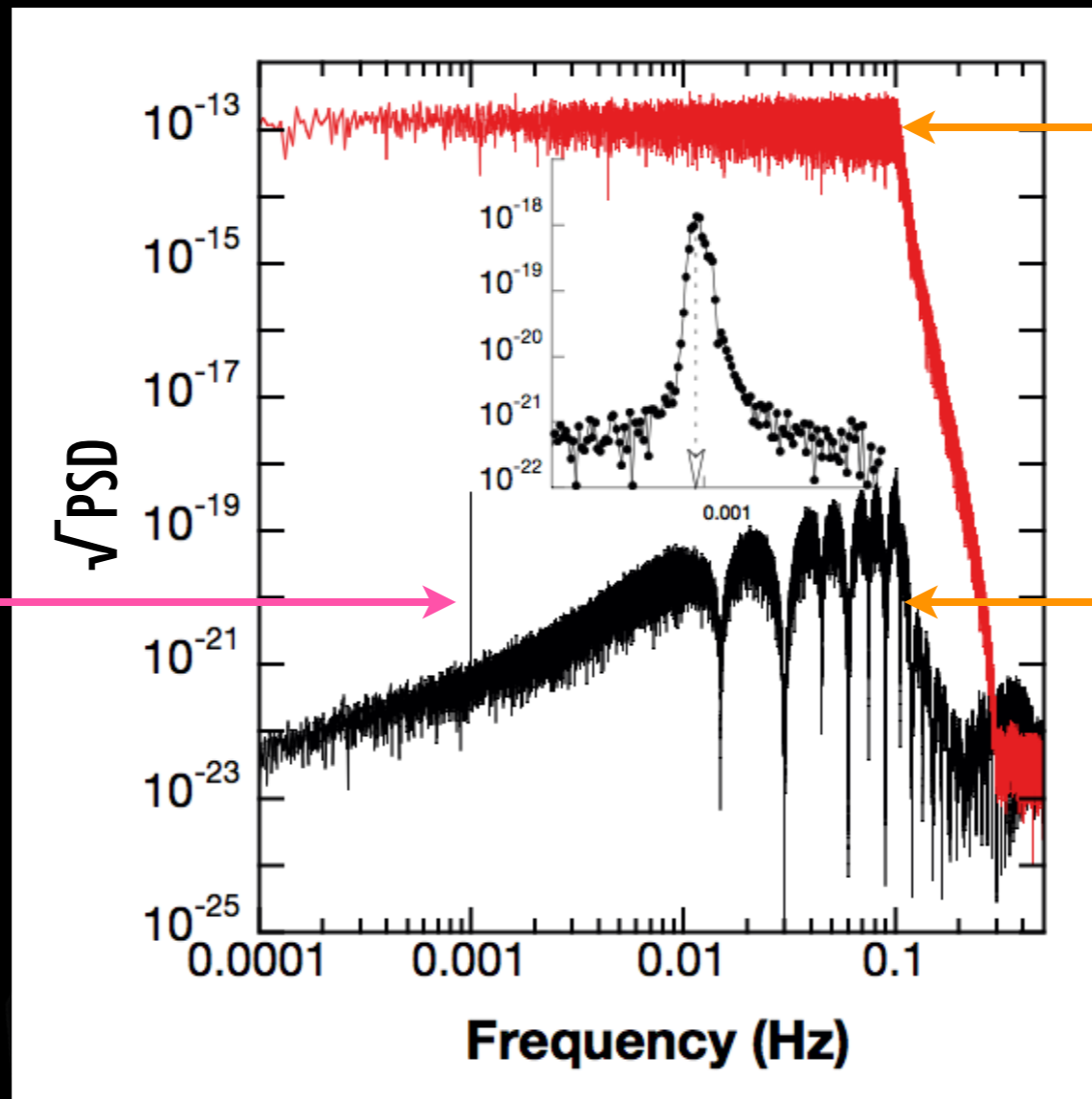


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Current design

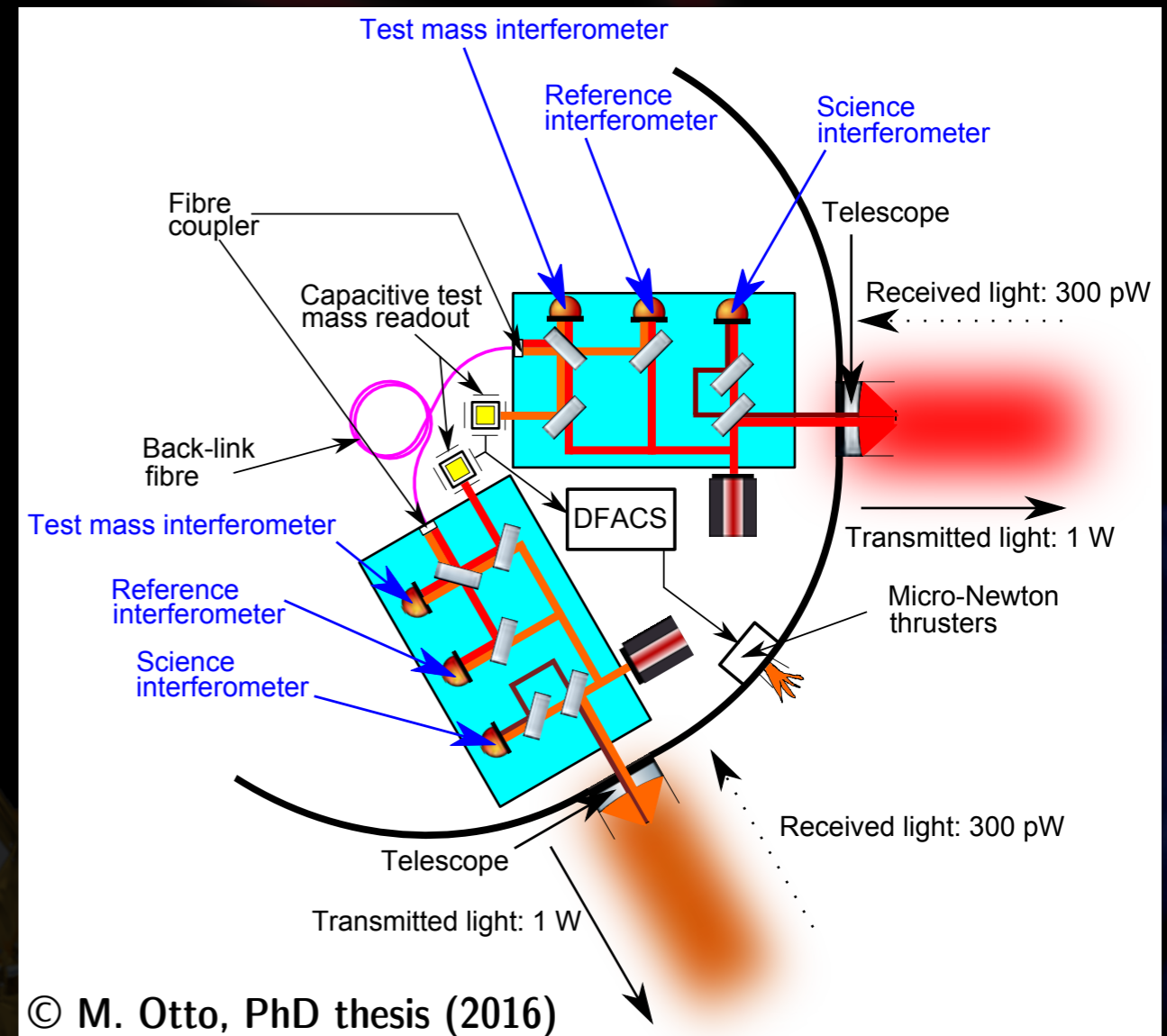


- ▶ Exchange of laser beam to form **several interferometers**
- ▶ **Phasemeter measurements** on each of the 6 Optical Benches:

- Distant OB vs local OB
- Test-mass vs OB
- Reference using adjacent OB
- Transmission using sidebands
- Distance between spacecrafts

▶ **Noises sources:**

- Laser noise : 10^{-13} (vs 10^{-21})
- Clock noise (3 clocks)
- Acceleration noise (see LPF)
- Read-out noises





Current design

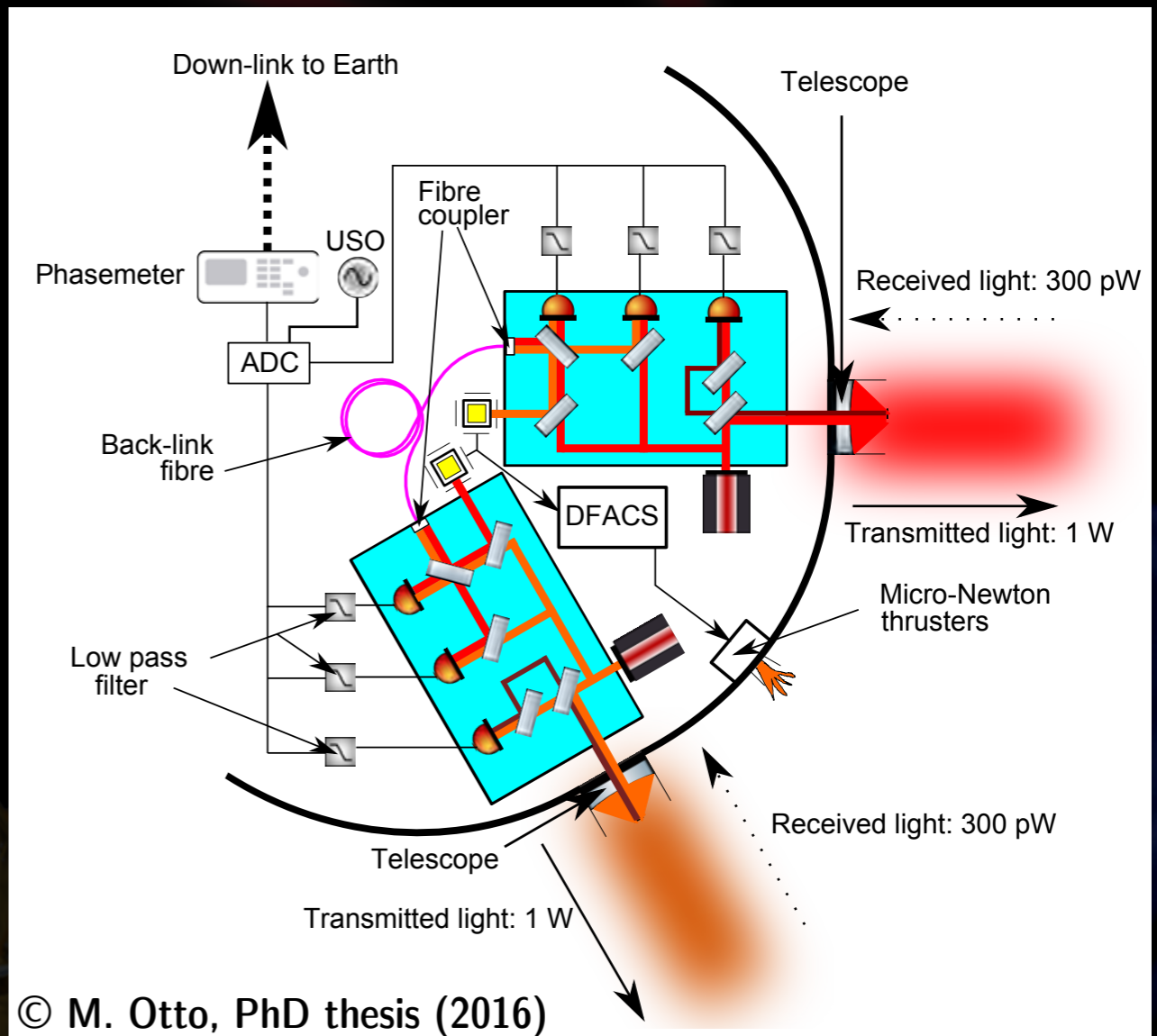


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- ▶ **Noises sources:**

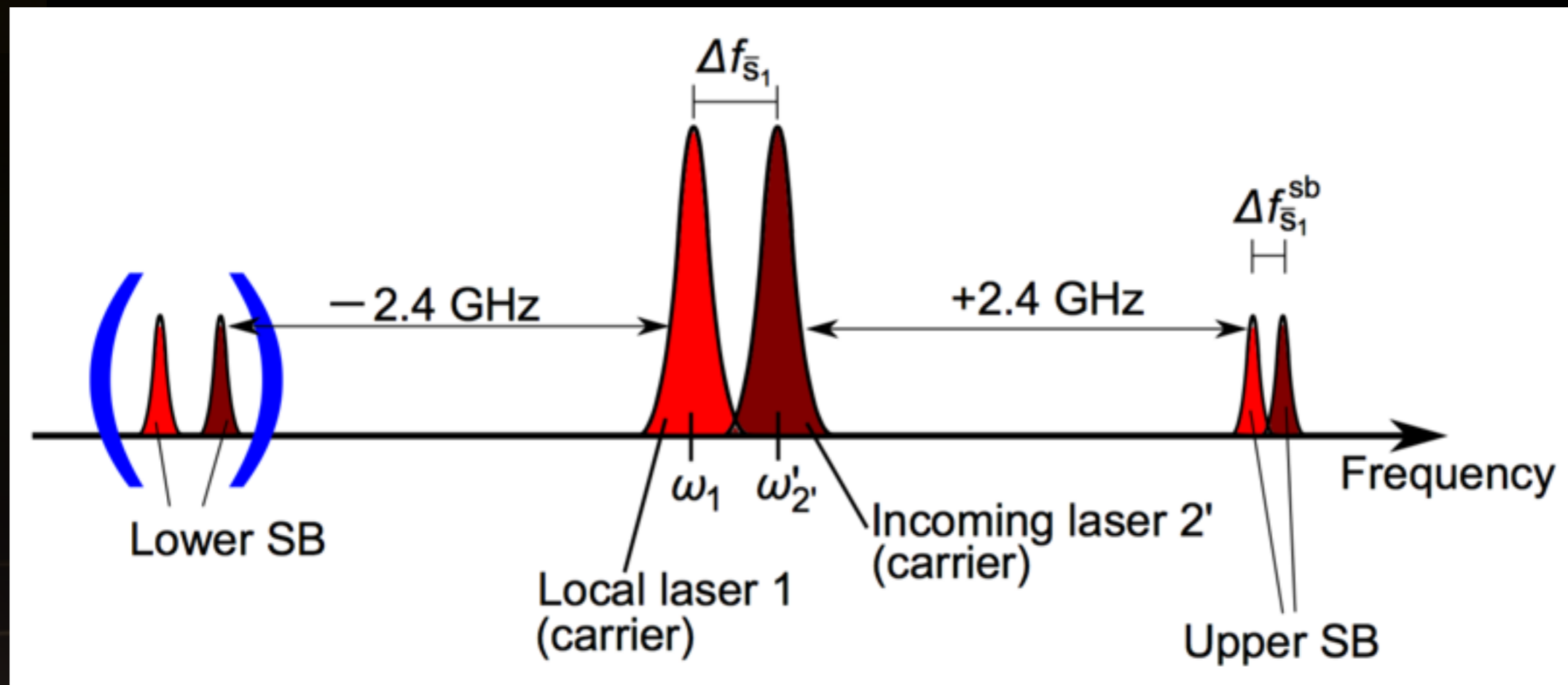
- Laser noise : 10^{-13} (vs 10^{-21})
- Clock noise (3 clocks)
- Acceleration noise (see LPF)
- Read-out noises





Current design

- ▶ Use sidebands to transfer clock noise
- ▶ Modulation of laser with a pseudo-random code to measure the absolute distance at a precision of 30cm.



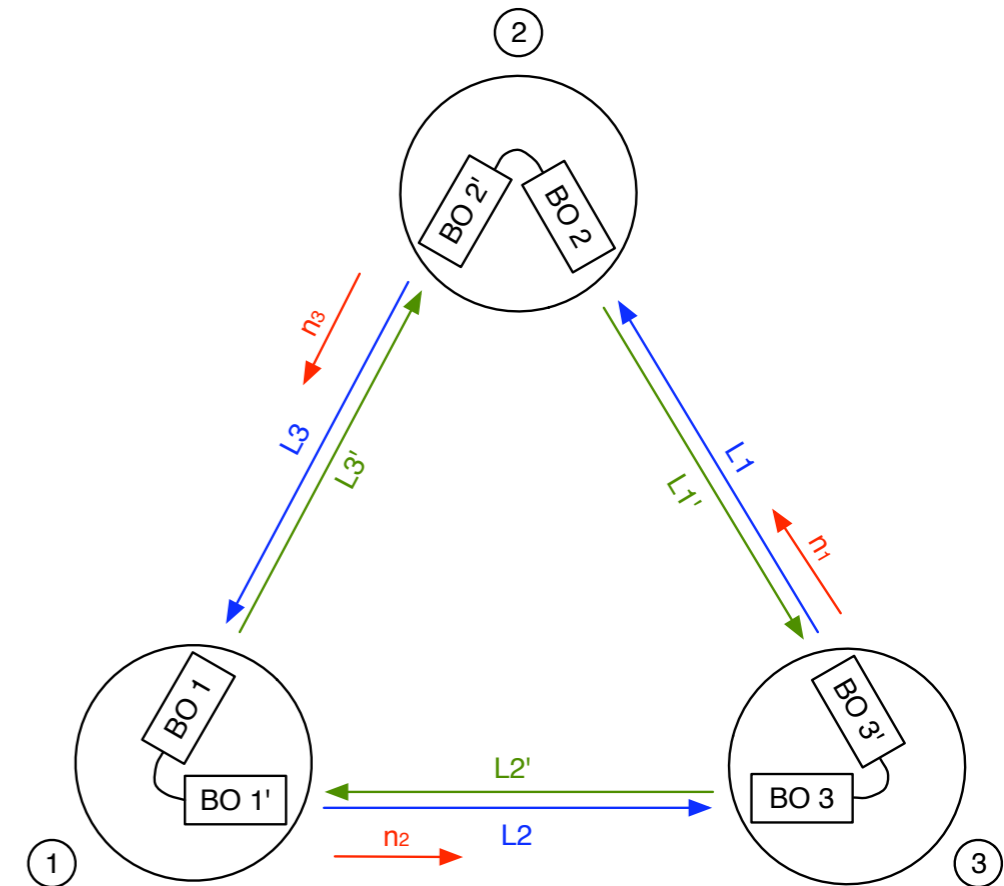
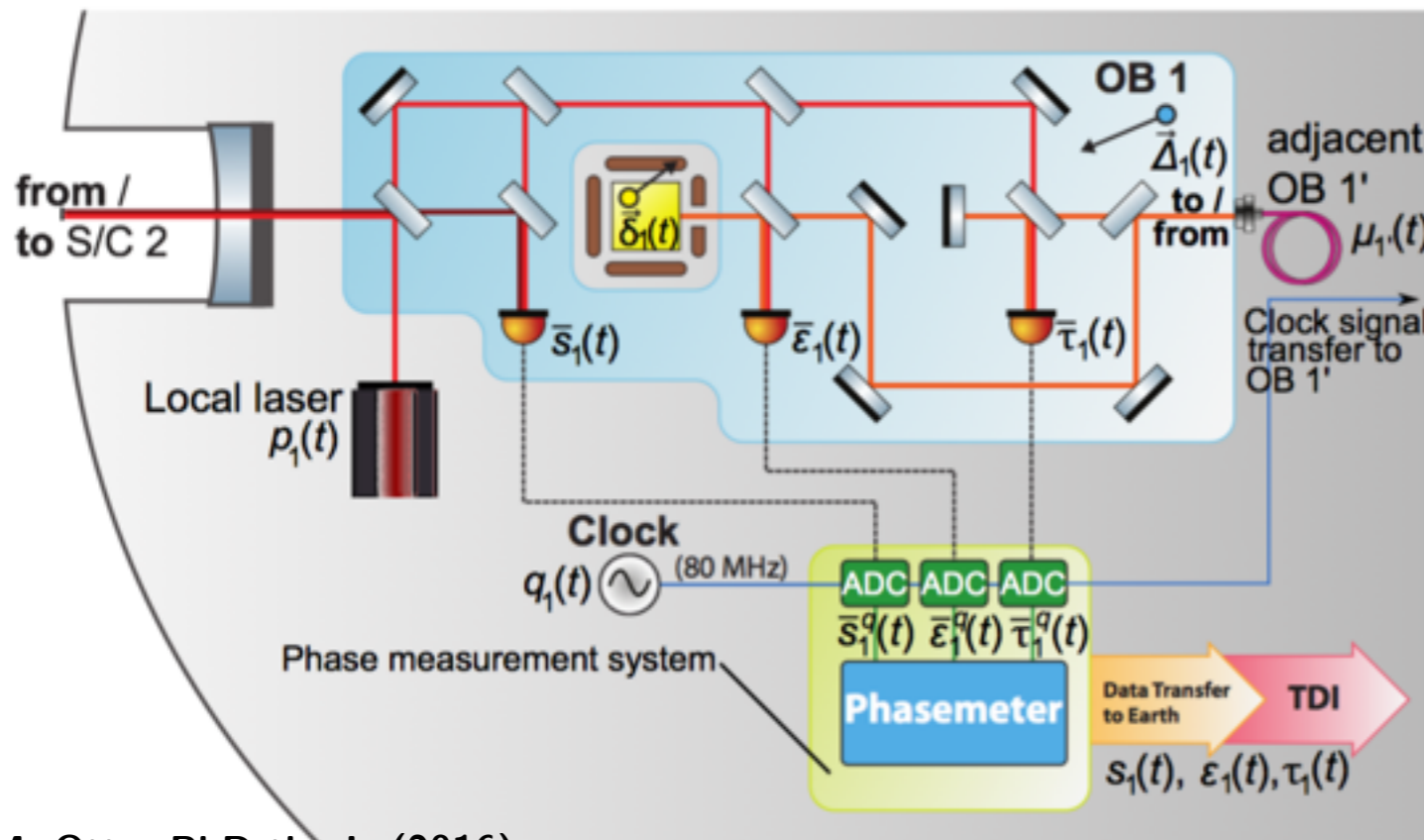


Current design



- s_i^c : scientific interferometer measurement at the carrier frequency,
- s_i^{sb} : scientific interferometer measurement at the sideband frequency,
- τ_i : reference interferometer measurement
- ϵ_i : test-mass interferometer measurement
- θ_i^j : factor to track of the sign of the phasemeter input
- h_i : gravitational wave signal on the link
- p_i : laser noise
- Δ_i : optical bench displacement noise projected along the arm :

$$\Delta_i = 2\pi \vec{\Delta}_i \cdot \vec{n}_{i'+2} \quad \text{and} \quad \Delta_{i'} = -2\pi \vec{\Delta}_{i'} \cdot \vec{n}_{i+1}$$





Current design



- δ_i : test mass displacement noise projected along the arm (acceleration noise):

$$\delta_i = 2\pi\vec{\delta}_i \cdot \vec{n}_{i'+2} \quad \text{and} \quad \delta_{i'} = -2\pi\vec{\delta}_{i'} \cdot \vec{n}_{i+1} \quad (10)$$

- $N_i^{ro,s}, N_i^{ro,\tau}, N_i^{ro,\epsilon}, N_i^{ro,sb}$: Read-out noises, i.e. all noises from the photodiode to the output of the phasemeter
- $N_i^{opt,s}, N_i^{opt,\tau}, N_i^{opt,\epsilon}, N_i^{opt,sb}$: Optical noise, i.e. all noises on the two interfering beams before the photodiode of the scientific interferometer at the carrier frequency
- μ_i : noise of the back link optical fiber from optical bench i to optical bench i'
- q_i : noise of the clock of spacecraft i
- a_i : translation factor of the clock noise for the scientific interferometer at the carrier frequency :

$$a_i = \frac{|f_{i'+1 \rightarrow i} - f_i|}{f_{PT,i}} \quad \text{and} \quad a_{i'} = \frac{|f_{i+2 \rightarrow i'} - f_{i'}|}{f_{PT,i}} \quad (11)$$

- b_i : translation factor of the clock noise for the reference interferometer and the test-mass interferometer :

$$b_i = \frac{|f_{i'} - f_i|}{f_{PT,i}} \quad \text{and} \quad b_{i'} = \frac{|f_i - f_{i'}|}{f_{PT,i}} \quad (12)$$

- c_i : translation factor of the clock noise for the scientific interferometer at the sideband frequency :

$$c_i = \frac{|f_{i'+1 \rightarrow i}^{sb} - f_i^{sb}|}{f_{PT,i}} \quad \text{and} \quad c_{i'} = \frac{|f_{i+2 \rightarrow i'}^{sb} - f_{i'}^{sb}|}{f_{PT,i}} \quad (13)$$

The $x_{;i}$ correspond to the application of a **real** delay :

$$x_{;i}(t) \equiv \mathcal{D}_i x(t) \equiv x \left(t - \frac{\mathbf{L}_i(t)}{c} \right) \quad (14)$$



Current design



► Optical bench 1 :

$$s_1^c(t) = \theta_1^{2'} \left[h_1 + p_{2';3} - p_1 + \frac{\Delta_{2';3}}{\lambda_{2'}} - \frac{\Delta_1}{\lambda_{2'}} + N_1^{opt,s} \right] + a_1 q_1 + N_1^{ro,s}$$

$$\tau_1(t) = \theta_1^{1'} \left[p_{1'} - p_1 + \mu_{1'} + N_1^{opt,\tau} \right] + b_1 q_1 + N_1^{ro,\tau}$$

$$\epsilon_1(t) = \theta_1^{1'} \left[p_{1'} - p_1 + 2 \left(\frac{\delta_1}{\lambda_{1'}} - \frac{\Delta_1}{\lambda_{1'}} \right) + \mu_{1'} + N_1^{opt,\epsilon} \right] + b_1 q_1 + N_1^{ro,\epsilon}$$

$$s_1^{sb}(t) = \theta_1^{2'} \left[h_{1'} + p_{2';3} - p_1 + m_{2'} q_{2;3} - m_1 q_1 + \frac{\Delta_{2';3}}{\lambda_{2'}} - \frac{\Delta_1}{\lambda_{2'}} + N_1^{opt,sb} \right] + c_1 q_1 + N_1^{ro,sb}$$

► Optical bench 1'

$$s_{1'}^c(t) = \theta_{1'}^3 \left[h_{1'} + p_{3;2'} - p_{1'} - \frac{\Delta_{3;2'}}{\lambda_3} + \frac{\Delta_{1'}}{\lambda_3} + N_{1'}^{opt,s} \right] + b_{1'} q_1 + N_{1'}^{ro,s}$$

$$\tau_{1'}(t) = \theta_{1'}^1 \left[p_1 - p_{1'} + \mu_1 + N_{1'}^{opt,\tau} \right] + b_{1'} q_1 + N_{1'}^{ro,\tau}$$

$$\epsilon_{1'}(t) = \theta_{1'}^1 \left[p_1 - p_{1'} + 2 \left(-\frac{\delta_{1'}}{\lambda_1} + \frac{\Delta_{1'}}{\lambda_1} \right) + \mu_1 + N_{1'}^{opt,\epsilon} \right] + b_{1'} q_1 + N_{1'}^{ro,\epsilon}$$

$$s_{1'}^{sb}(t) = \theta_{1'}^3 \left[h_{1'} + p_{3;2'} - p_{1'} + m_3 q_{3;2'} - m_{1'} q_1 - \frac{\Delta_{3;2'}}{\lambda_3} + \frac{\Delta_{1'}}{\lambda_3} + N_{1'}^{opt,sb} \right] + c_{1'} q_1 + N_{1'}^{ro,sb}$$



TDI with current design



M. Otto, PhD thesis (2016)

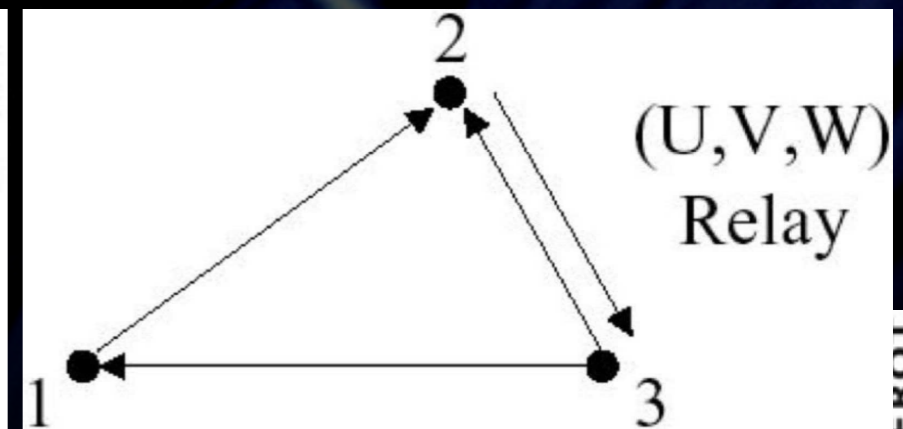
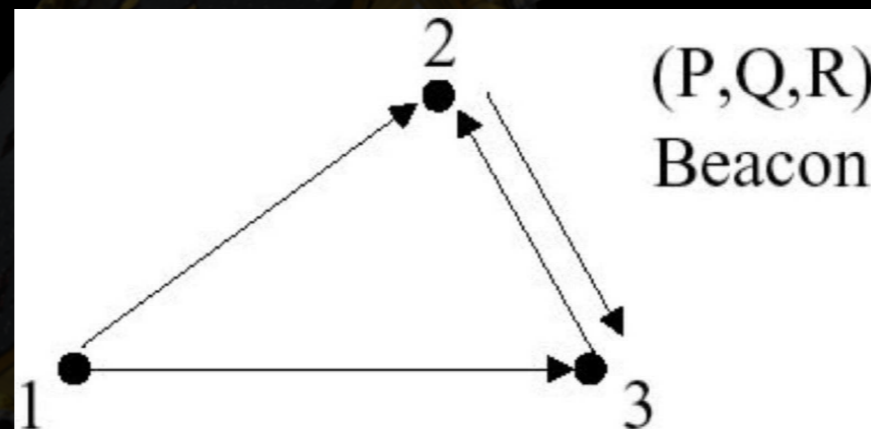
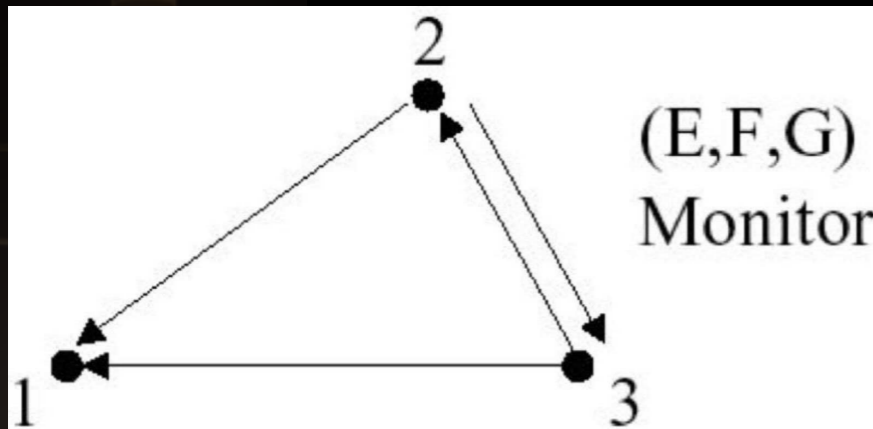
- ▶ Intermediate TDI: first step
 - Step 1: Combine science and test mass interferometers
 - => Suppression of **optical bench displacement noises**
 - Step 2: Combine with reference interferometers
 - => Suppression of 3 free running laser noises
 - Step 3: Combine with sidebands
 - => Clock noise removable
- ▶ Then apply on the results of step 3 the regular TDI combination. With real orbits you need at least the generation 2



TDI generators



- ▶ With 6 links, there is a large numbers of possible TDI combinations: generators
- ▶ Usual ones:
 - X, Y, Z: Michelson equivalent
 - A, E: the 2 noises uncorrelated channel = equivalent to 2 independent detectors
 - T: “Sagnac” or “null channel”: very weak response to GW





Overview

- ▶ Introduction
- ▶ Gravitational wave sources in the mHz band
- ▶ LISA: Laser Interferometer Space Antenna
- ▶ Free fall in space: LISAPathfinder
- ▶ Long arm interferometry: Time Delay Interferometry
- ▶ **Noise sources**
- ▶ Sensitivity
- ▶ Response to GW and orbital motion
- ▶ Data analysis



Acceleration noise

- ▶ Due to residual forces acting on the test-mass
- ▶ Obtain via LISAPathfinder measurements

$$S_{acc}(f) = S_{acc,unmodelled} + S_{acc,brownian}$$

with

$$S_{acc,brownian} = \text{constant}$$

$$S_{acc,unmodelled}(f) = (c_{acc,red})^2 \left(\left(\frac{2 \times 10^{-5}}{f} \right)^{10} + \left(\frac{1 \times 10^{-4}}{f} \right)^2 \right) + (c_{acc,flat})^2 \left(1 + \left(\frac{f}{8 \times 10^{-3}} \right)^4 \right)$$



Readout noise

► Composition of a number of effects:

$$S_{ro,k,m} = \left(\frac{\lambda}{2\pi}\kappa_k\right)^2 \left(\langle\phi_{r/o}^{sn}\rangle^2 + \langle\phi_{r/o}^{rin}\rangle^2 + \langle\phi_{r/o}^{el}\rangle^2 + \langle\phi_{r/o}^{PMc}\rangle^2 + \langle\phi_{r/o}^{PMu}\rangle^2\right) \quad (10)$$

with $k = \{s, sb, \tau, \epsilon\}$ referring to the interferometers. κ_k is the inverse of the fraction of the laser power at frequency of interest :

$$\kappa_s = \frac{1}{J_0(m)^2} \quad \text{science interferometer at the carrier frequency} \quad (11)$$

$$\kappa_{sb} = \frac{1}{\sqrt{2}} \frac{f_{het}}{f_{mod}} \frac{1}{J_1(m)^2} \quad \text{science interferometer at the sideband frequency}$$

$$\kappa_\epsilon = 1 \quad \text{test-mass interferometer} \quad (12)$$

$$\kappa_\tau = 1 \quad \text{reference interferometer} \quad (13)$$

- If $k = s$ or $sb \Rightarrow P_1 = P_{rec}$ and $P_2 = P_{local,1}$
- If $k = \tau$ or $\epsilon \Rightarrow P_2 = P_{local,1}$ and $P_2 = P_{local,2}$



Readout: shot noise

▶ Due to the small number of photons in the incoming beam

- Emitted laser power: $P_{tel} = \eta_{TX} P_{laser}$

- Received laser intensity:

$$I_{red} = \frac{\pi P_{tel} d_{tel}^2}{2 L_{arm}^2 \lambda_{laser}^2} \times \alpha^2 e^{-\frac{2}{\alpha^2}} \left(e^{\frac{1}{\alpha^2}} - 1 \right)^2$$

- Received laser power on the optical bench:

$$P_{red} = \pi \left(\frac{d_{tel}}{2} \right)^2 \eta_{opt} I_{rec}$$

- Shot noise:

$$\langle \phi_{r/o}^{sn} \rangle = M_{IMS}(f) \sqrt{\frac{q_e (P_1 + P_2)}{R_{pd} \eta_{het} P_1 P_2}}$$

- P_{laser} : P_laser : laser power output
- η_{TX} : eta_TX : transmission from laser to telescope
- d_{tel} : d_tel : telescope diameter
- L_{arm} : L_arm : armlength
- λ_{laser} : lambda_laser : laser wavelength
- η_{opt} : eta_opt : optical efficiency



Readout: electronic noise

▶ Electronic noise associated to the photodiode

$$\left\langle \phi_{r/o}^{el} \right\rangle = M_{IMMS}(f) \frac{\sqrt{N_{seg} N_{pd}}}{R_{pd} \sqrt{2}} \sqrt{\frac{\frac{4k_B T}{R_{FB}} + I_{pd}^2 + \left(\frac{U_{pd}}{Z_{pd}}\right)^2}{\eta_{het} P_2 P_1}}$$

$$Z_{pd} = \frac{1}{2\pi C_{pd} f_{het}}$$

- R_{FB} : Rfb : feedback resistor
- T : T_preamplifier : temperature at the photodiode preamplifier
- I_{pd} : I_pd : input current noise
- U_{pd} : U_pd : intrinsic voltage noise
- C_{pd} : C_pd : photodiode capacitance
- f_{het} : f_het : heterodyne maximal frequency



Readout: RIN & phase meter

► RIN: Relative Intensity Noise:

- For a balanced detection, the phase noise contribution from RIN is

$$\left\langle \phi_{r/o}^{rin} \right\rangle = \phi_{r/o}^{rin} = M_{IMS}(f) \frac{RIN_{laser}}{\sqrt{2}} \frac{\sqrt{1 + (P_1/P_2)^2}}{1 + P_1/P_2}$$

► Phasemeter measurement noise:

- Correlated term: $\left\langle \phi_{r/o}^{PMc} \right\rangle = M_{IMS}(f) \phi_{r/o}^{PMc}$

- Uncorrelated term: $\left\langle \phi_{r/o}^{PMu} \right\rangle = M_{IMS}(f) \frac{\phi_{r/o}^{PMu}}{\sqrt{N_{pd} N_{seg}}}$



Optical Path Noises

► Noises on the optical path:

$$S_{opt,k,m}(f) = (M(f)x_{opn}^{tel})^2 \dots\dots\dots \blacktriangleright \text{telescope}$$

$$+ (M(f)x_{opn}^{pointing})^2 \dots\dots \blacktriangleright \text{pointing (tilt to length)}$$

$$+ (M(f)x_{opn}^{align})^2 \dots\dots\dots \blacktriangleright \text{line of sight alignment (OB/TM)}$$

$$+ (M(f)x_{opn}^{SLs})^2 \dots\dots\dots \blacktriangleright \text{stray light science interferometer}$$

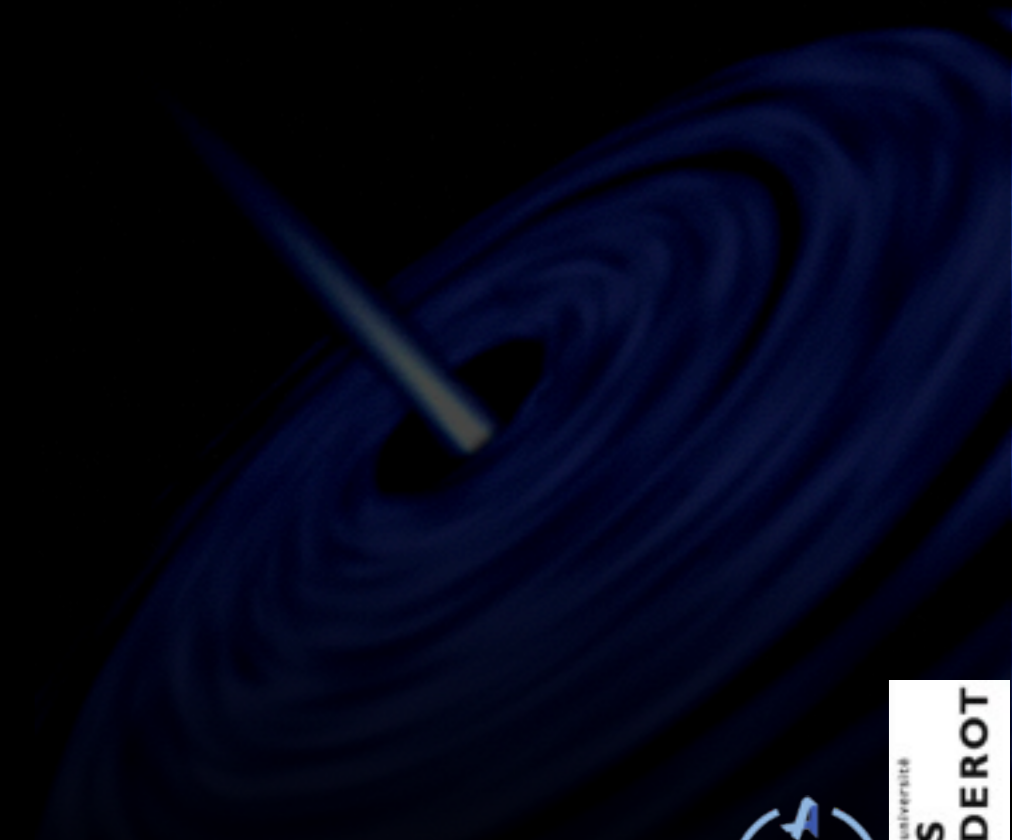
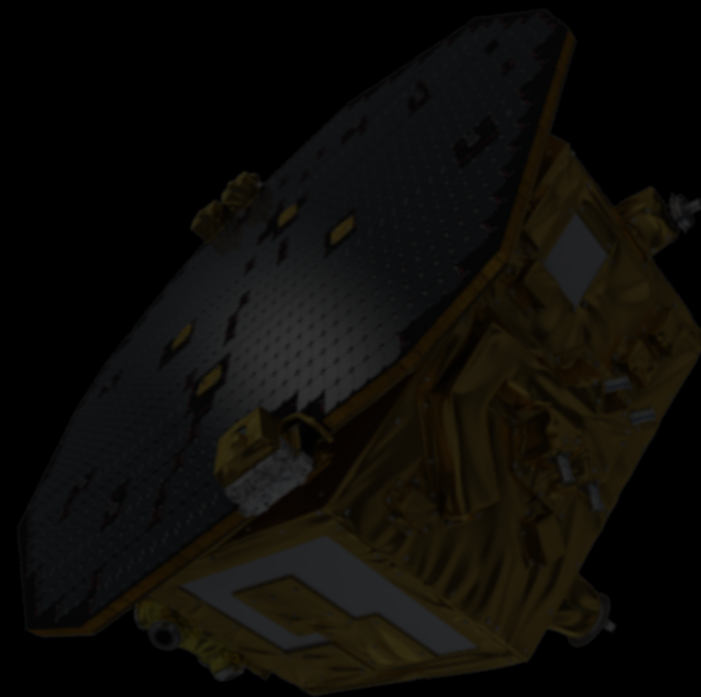
$$+ (M(f)x_{opn}^{PAAM})^2 \dots\dots \blacktriangleright \text{PAAM}$$



Other noises

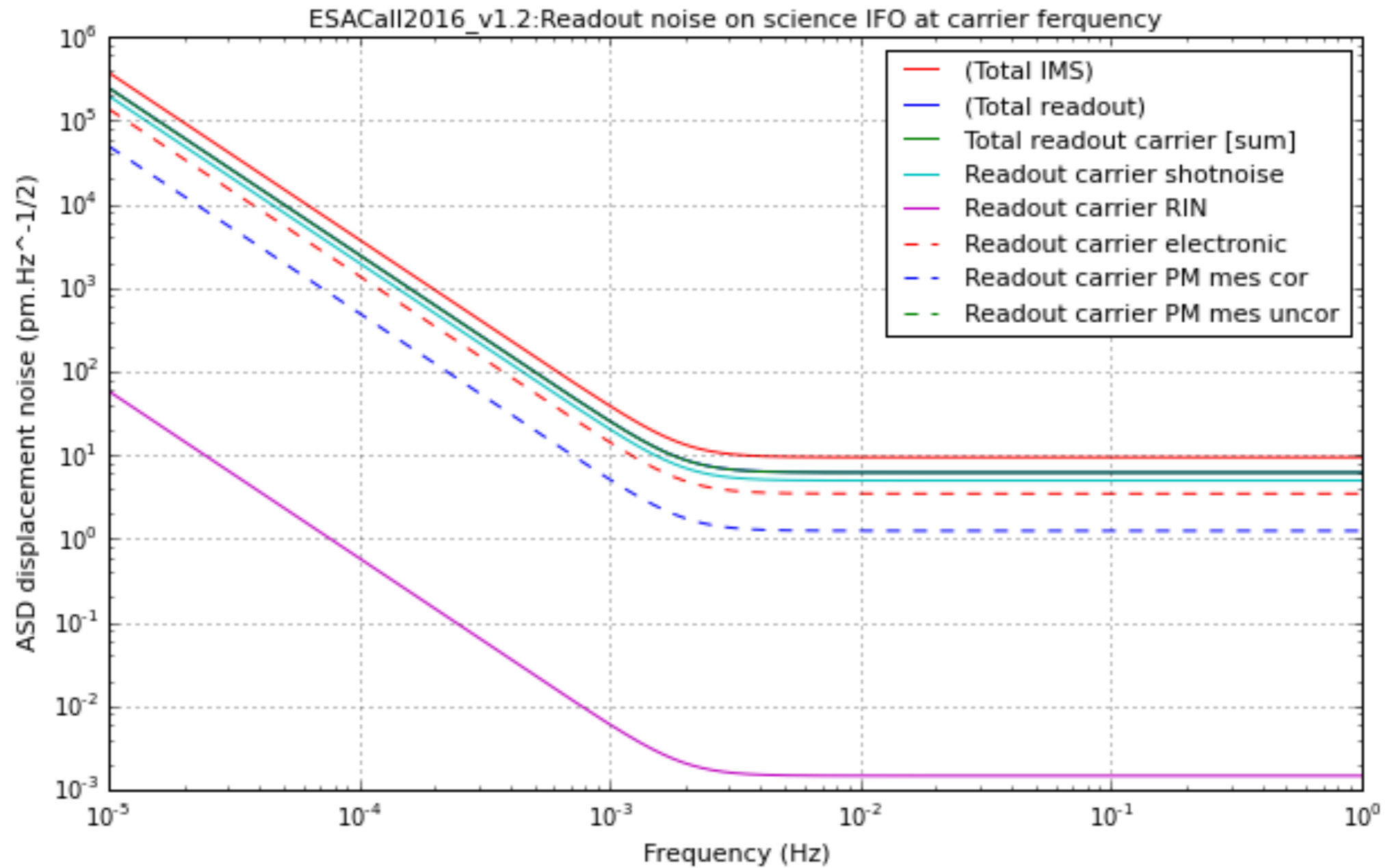


- ▶ Unmodelled interferometer noise
- ▶ Backlink noise
- ▶ Residual laser noise after TDI



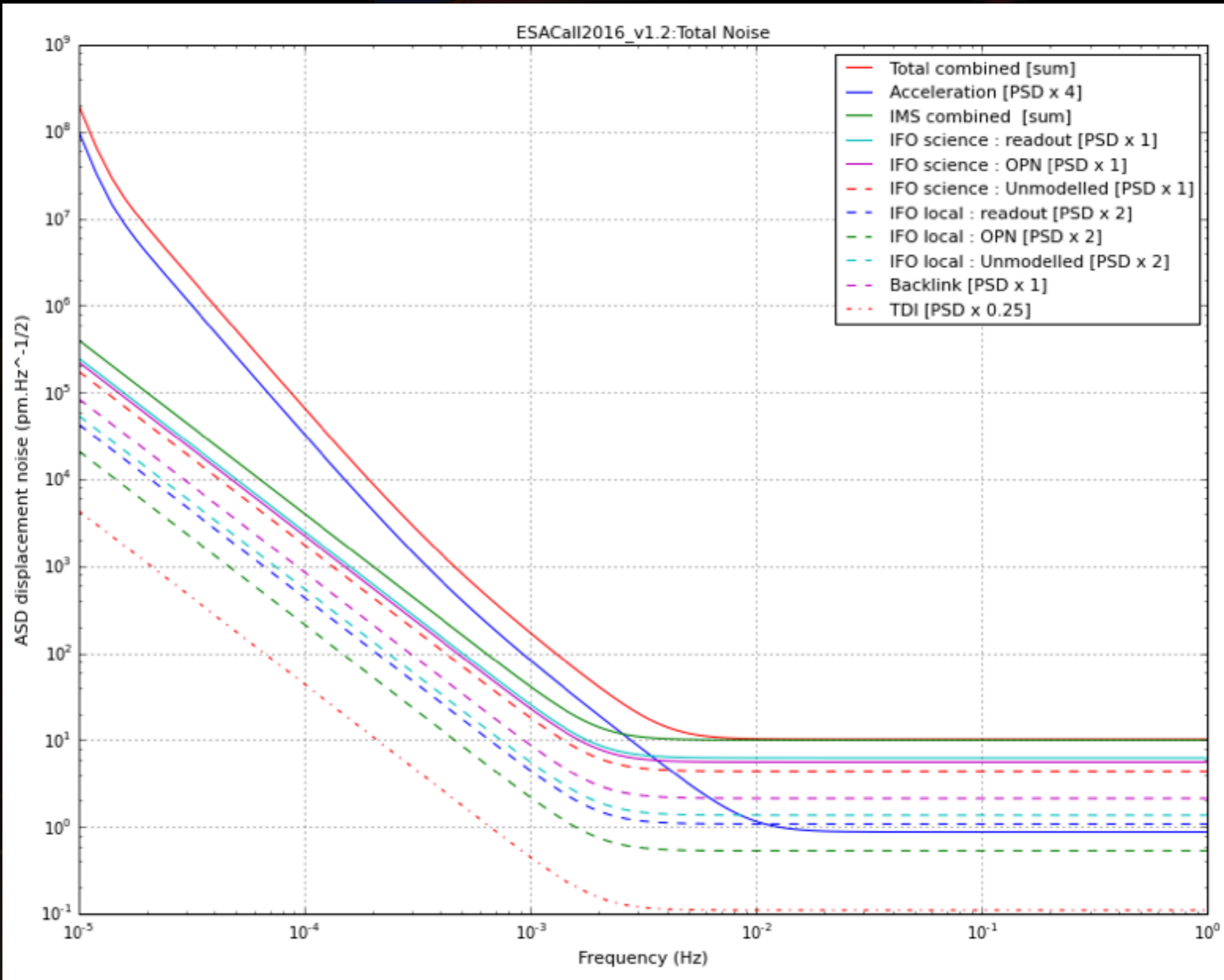


Readout noise budget



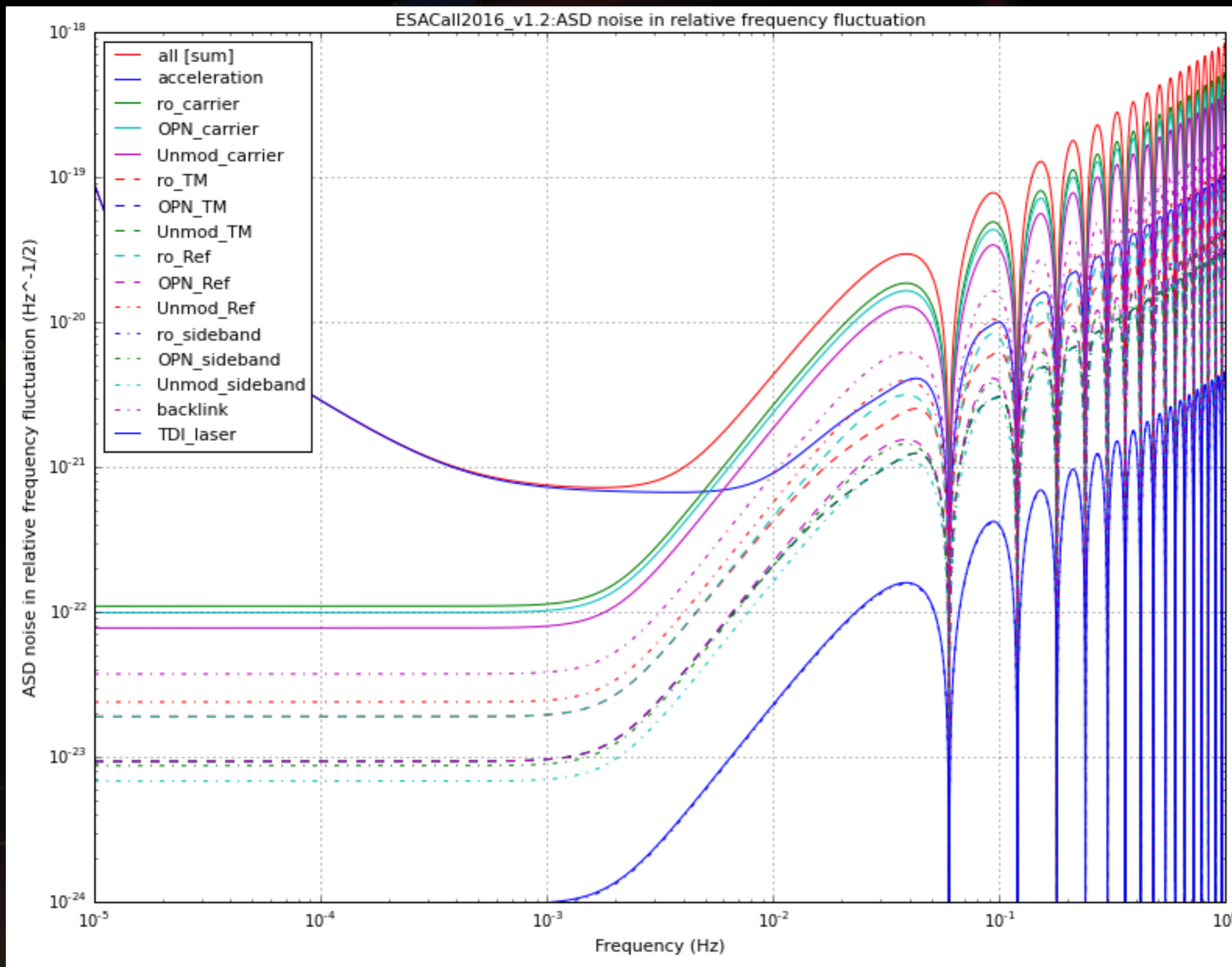


Combined on half round trip





Noise budget in TDI





Overview

- ▶ Introduction
- ▶ Gravitational wave sources in the mHz band
- ▶ LISA: Laser Interferometer Space Antenna
- ▶ Free fall in space: LISAPathfinder
- ▶ Long arm interferometry: Time Delay Interferometry
- ▶ Noise sources
- ▶ **Sensitivity**
- ▶ Response to GW and orbital motion
- ▶ Data analysis
- ▶ Conclusion

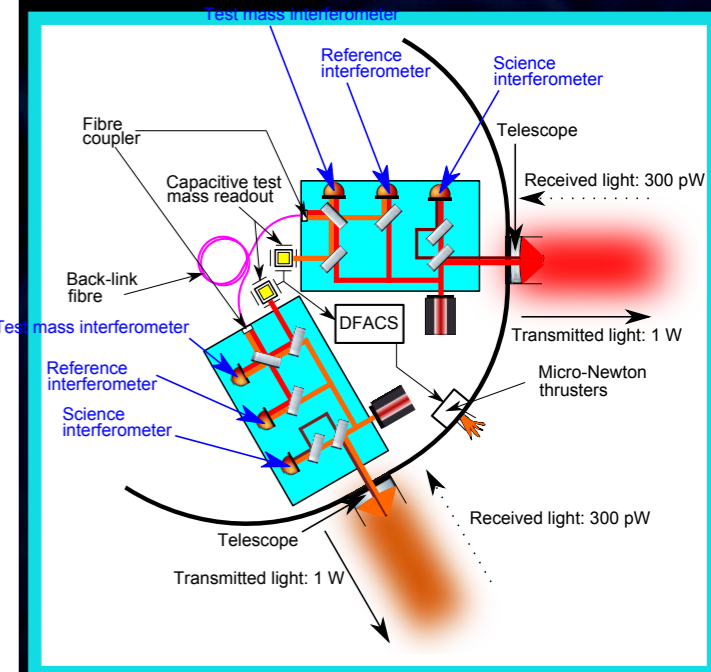
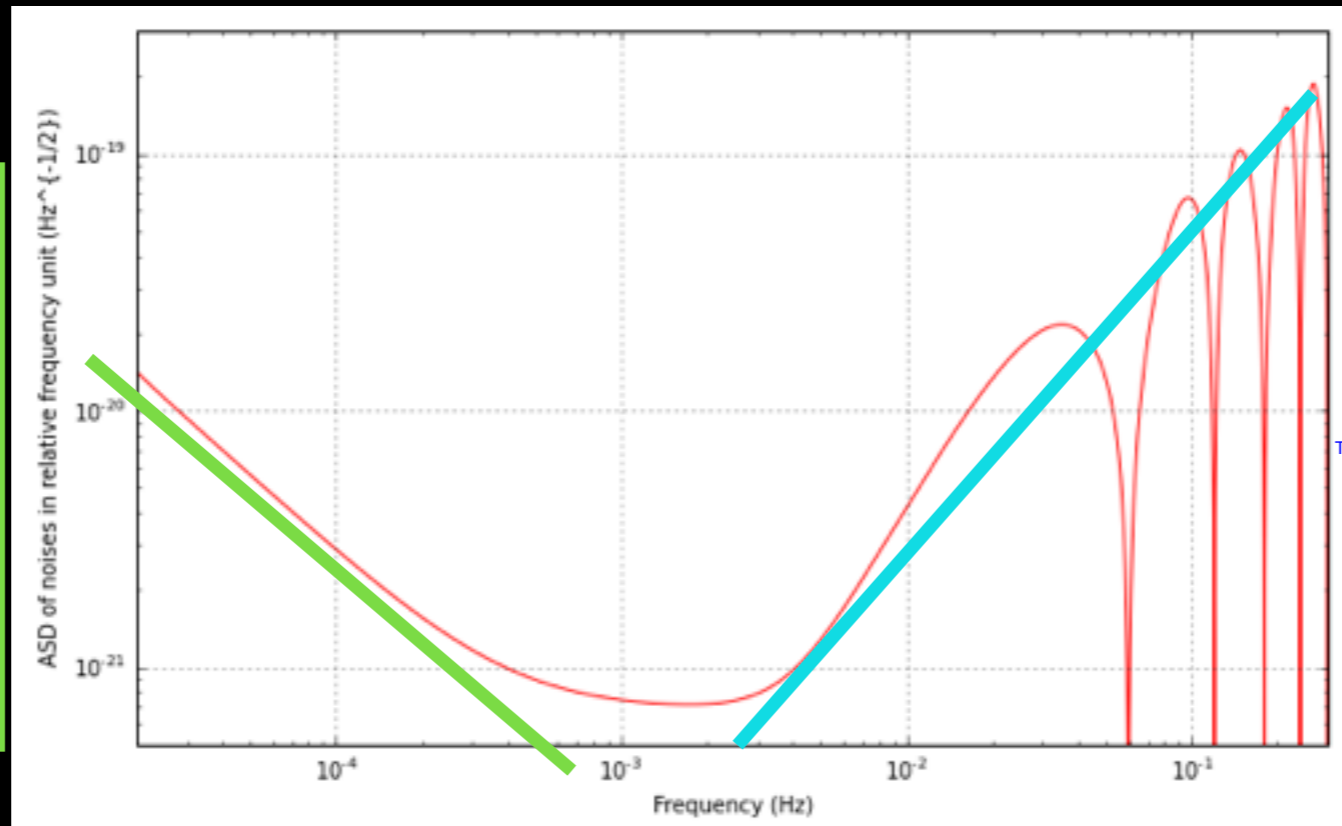


Sensitivity



► Noise budget in 3 points:

- Low frequencies: acceleration noise (unperfect free-falling of the test)
- High frequency: interferometric measurements noise
- Pre-processing pour réduire une partie des bruits (TDI)





Sensitivity

- ▶ Standard sensitivity, so called “strain sensitivity” or “strain linear spectral density” is

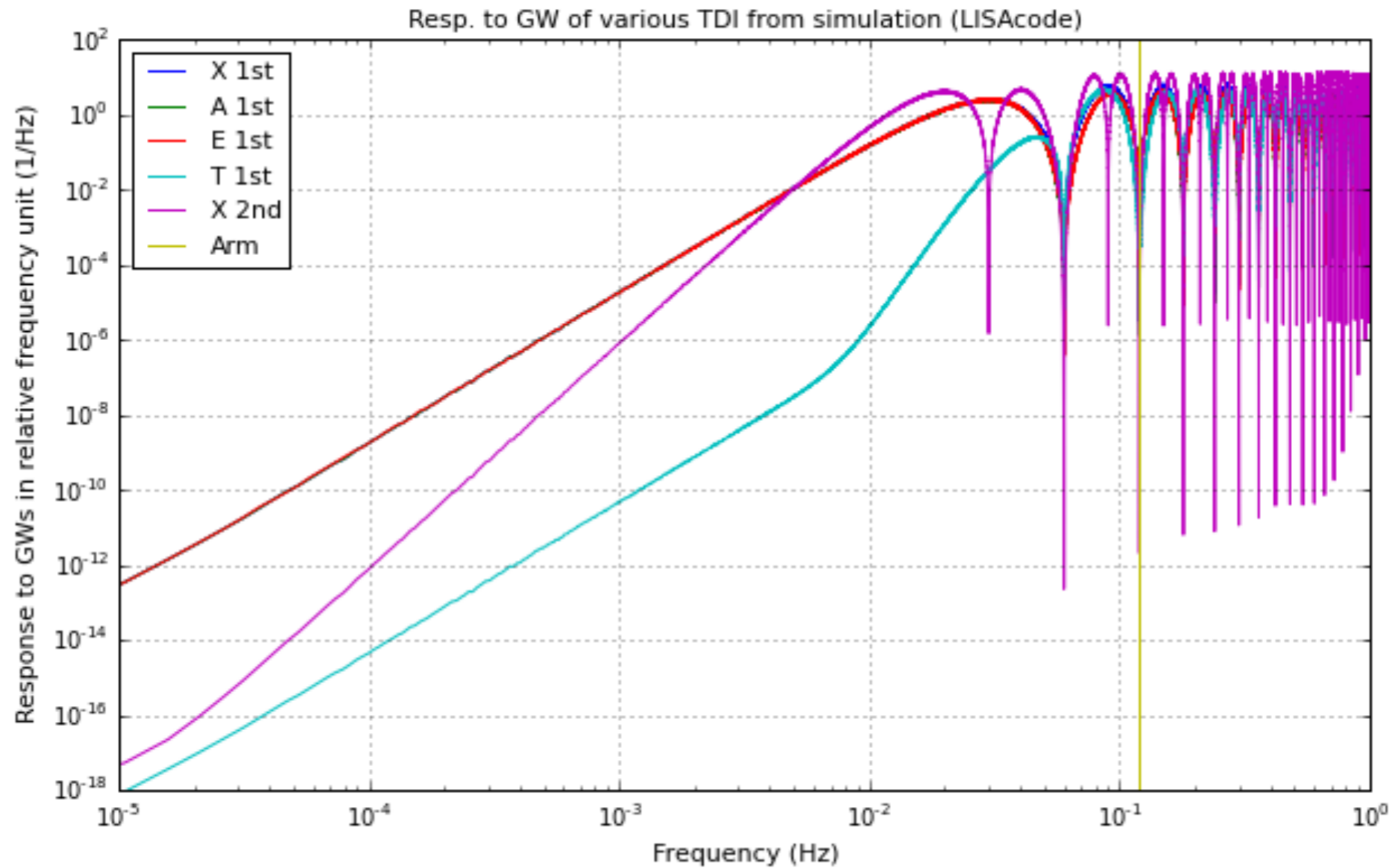
$$S(f) = \frac{Resp_{Noise}}{Resp_{GW}} = \frac{PSD_{Noise}}{PSD_{average\ GW}}$$

- ▶ **Response to GW:**

- Depends on orbits (see later)
- Depends on frequency partially due to TDI
- Computation:
 - Analytic approximation
 - Using simulators: PSD of TDI X with as input 192 white stochastic GWs isotropically distributed on sky



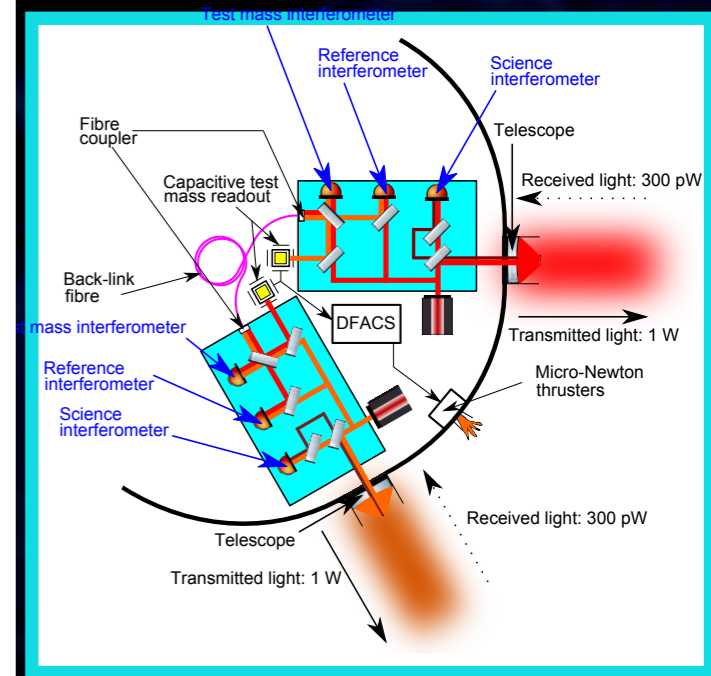
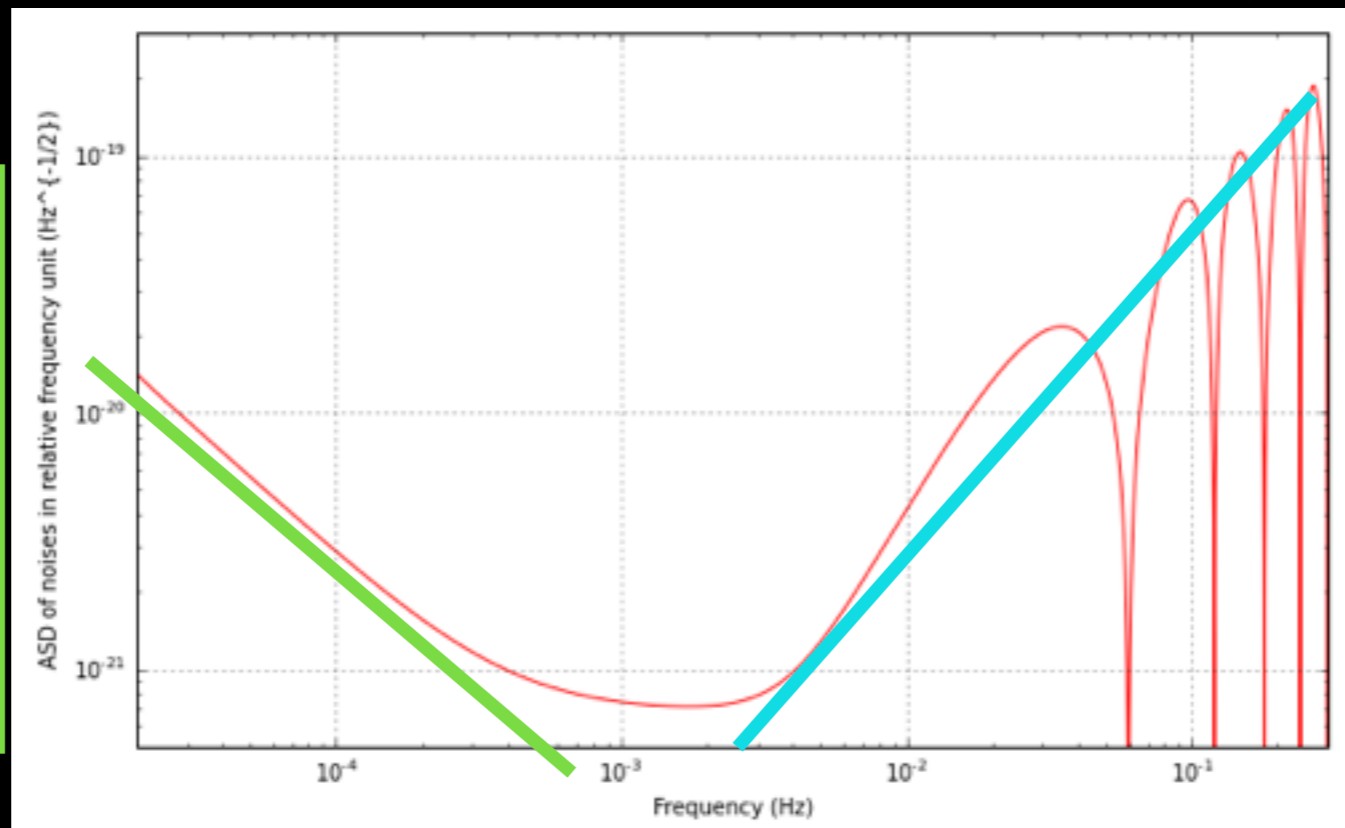
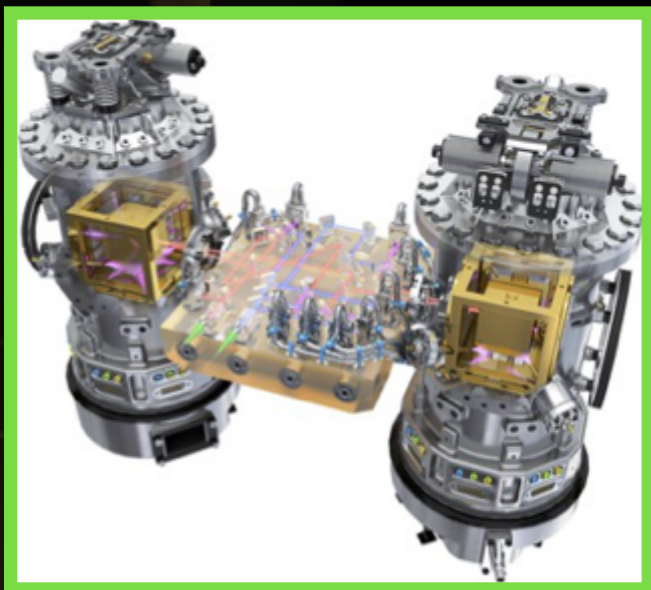
Response to GWs





Sensitivity

Noises

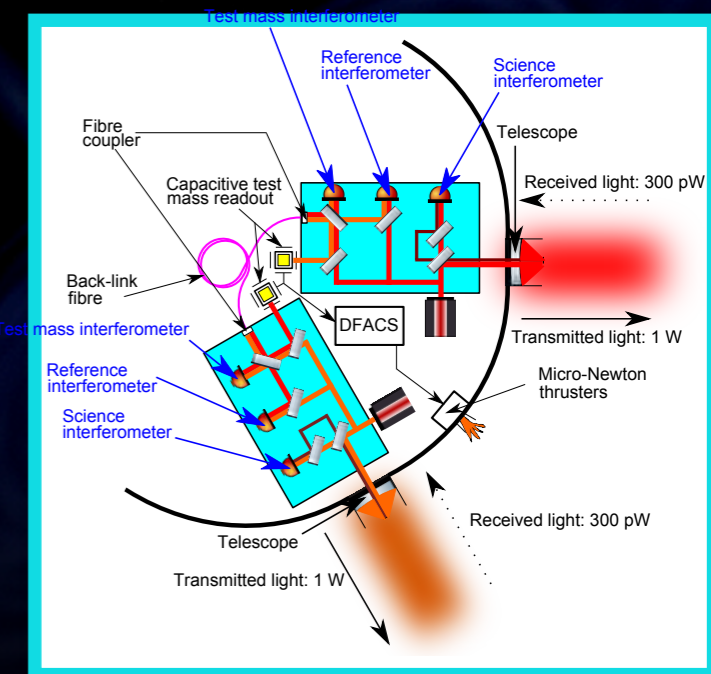
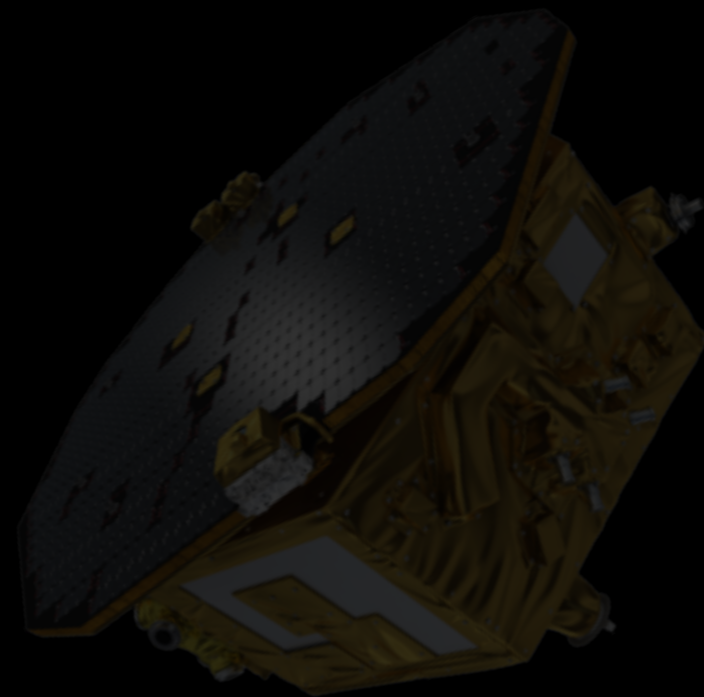
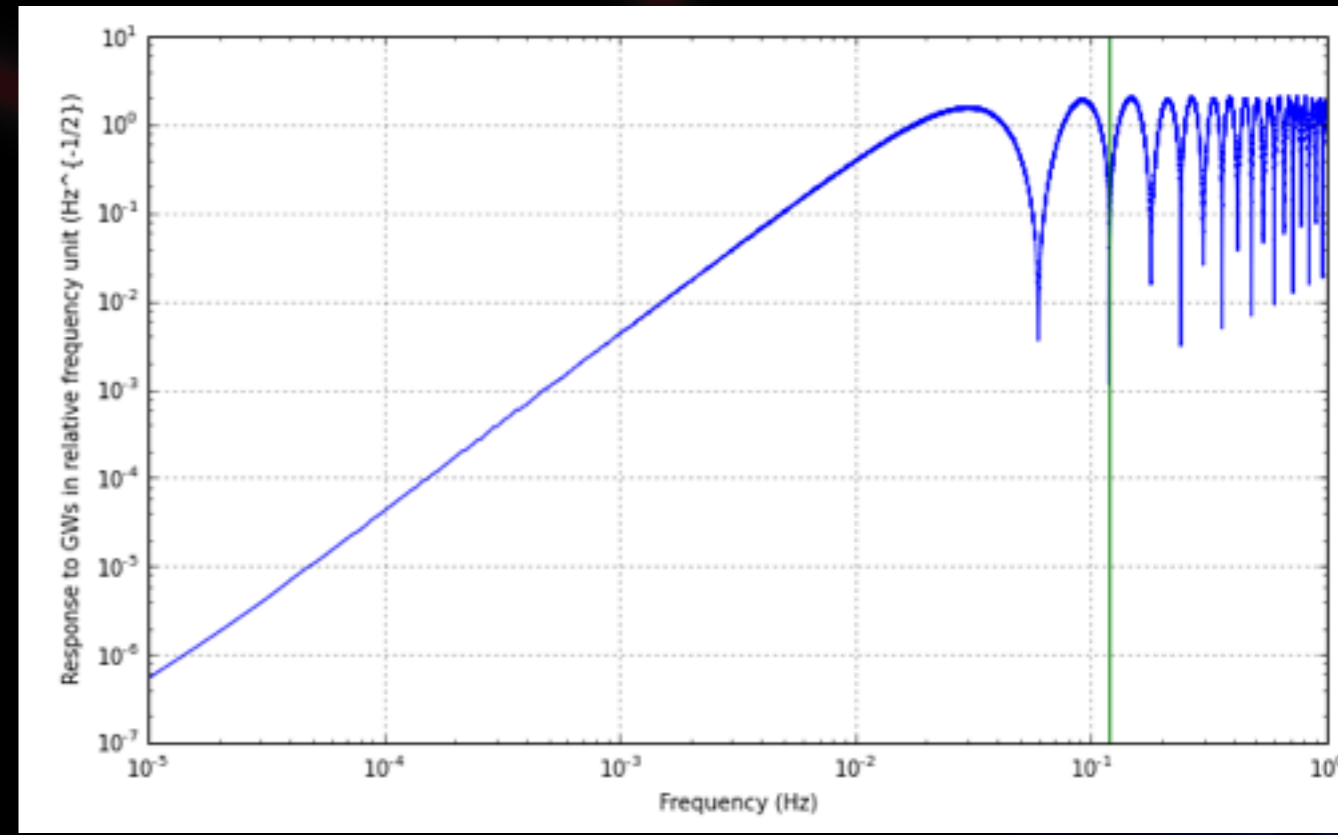
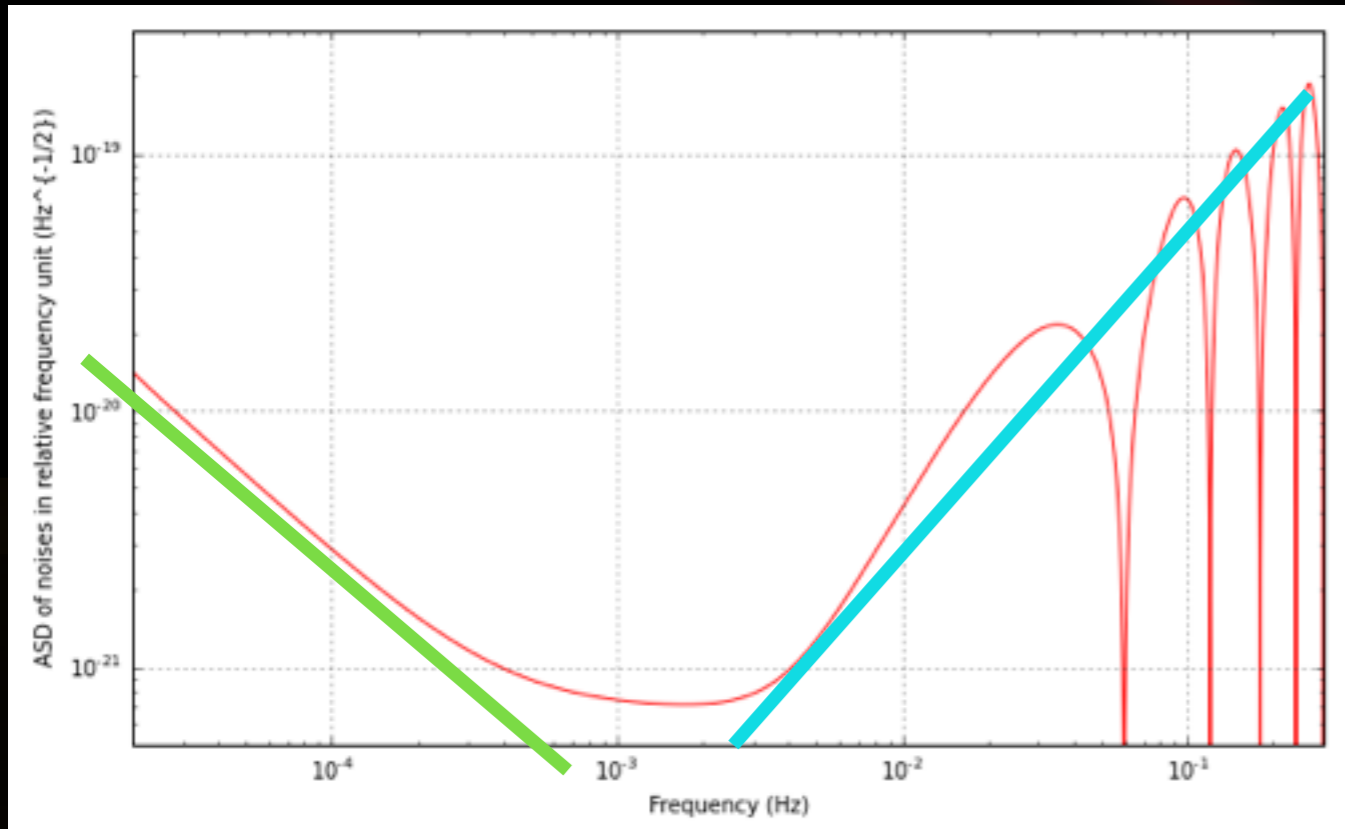




Sensitivity

Noises

Response of the detector to GWs

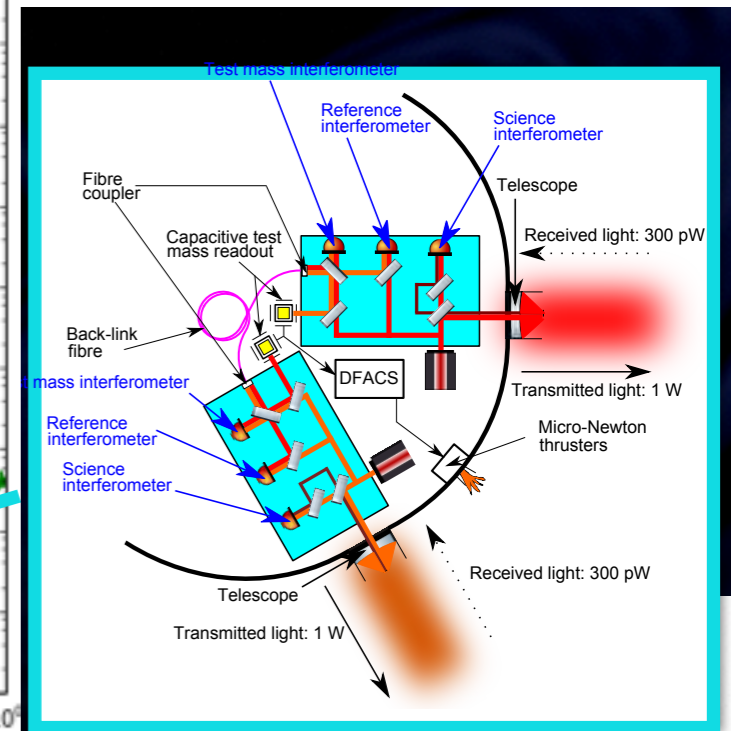
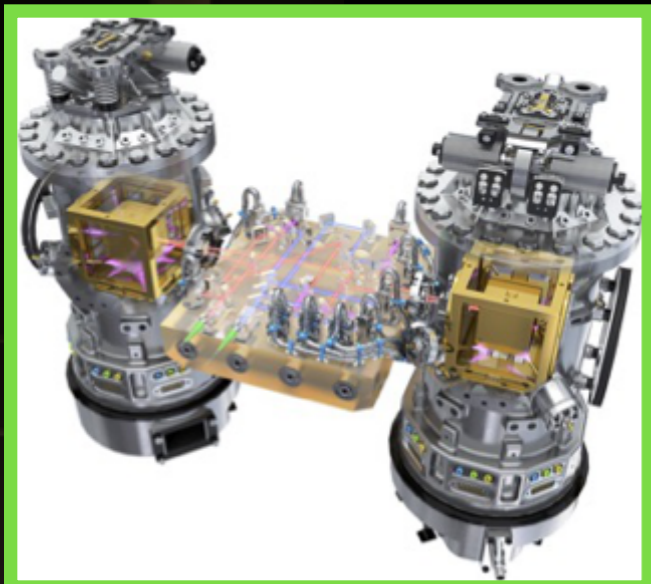
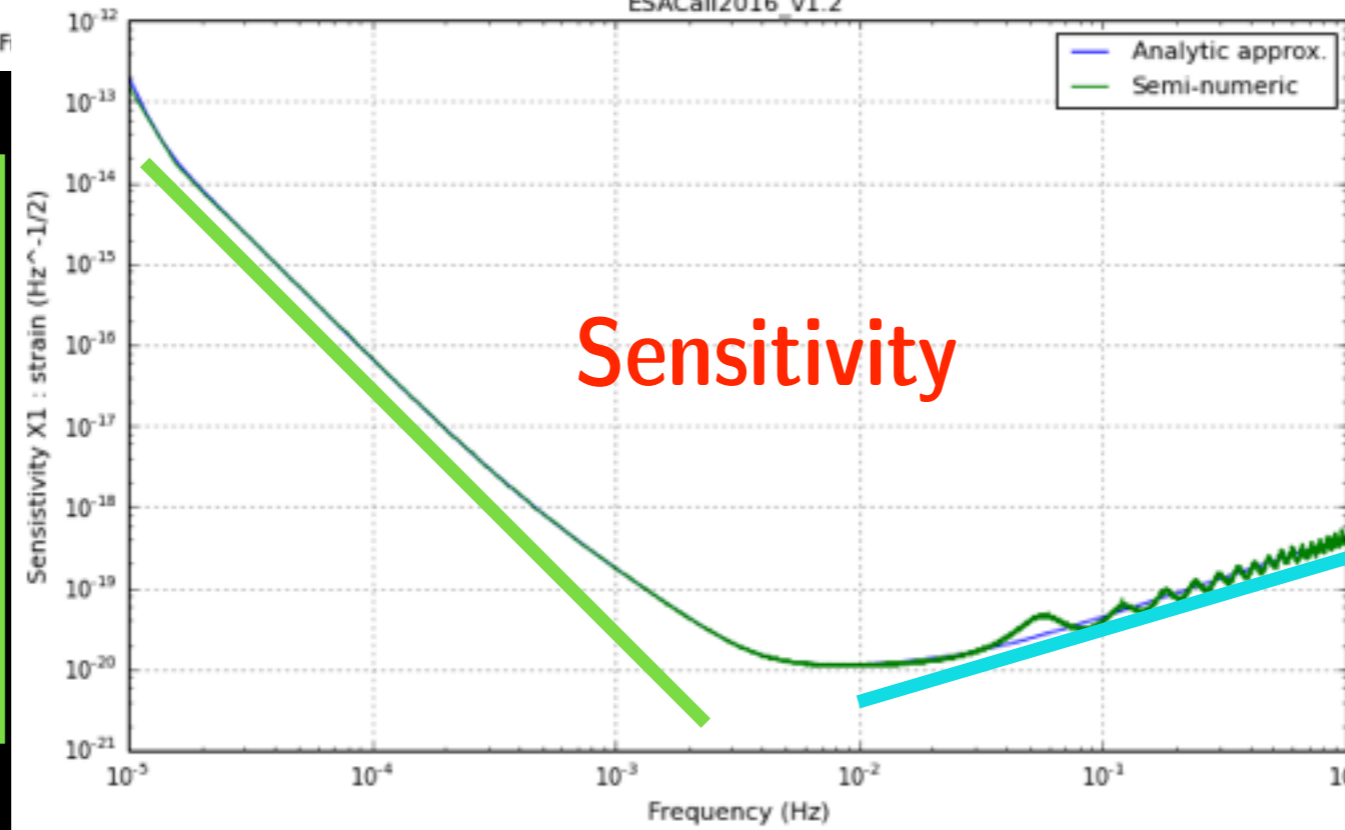
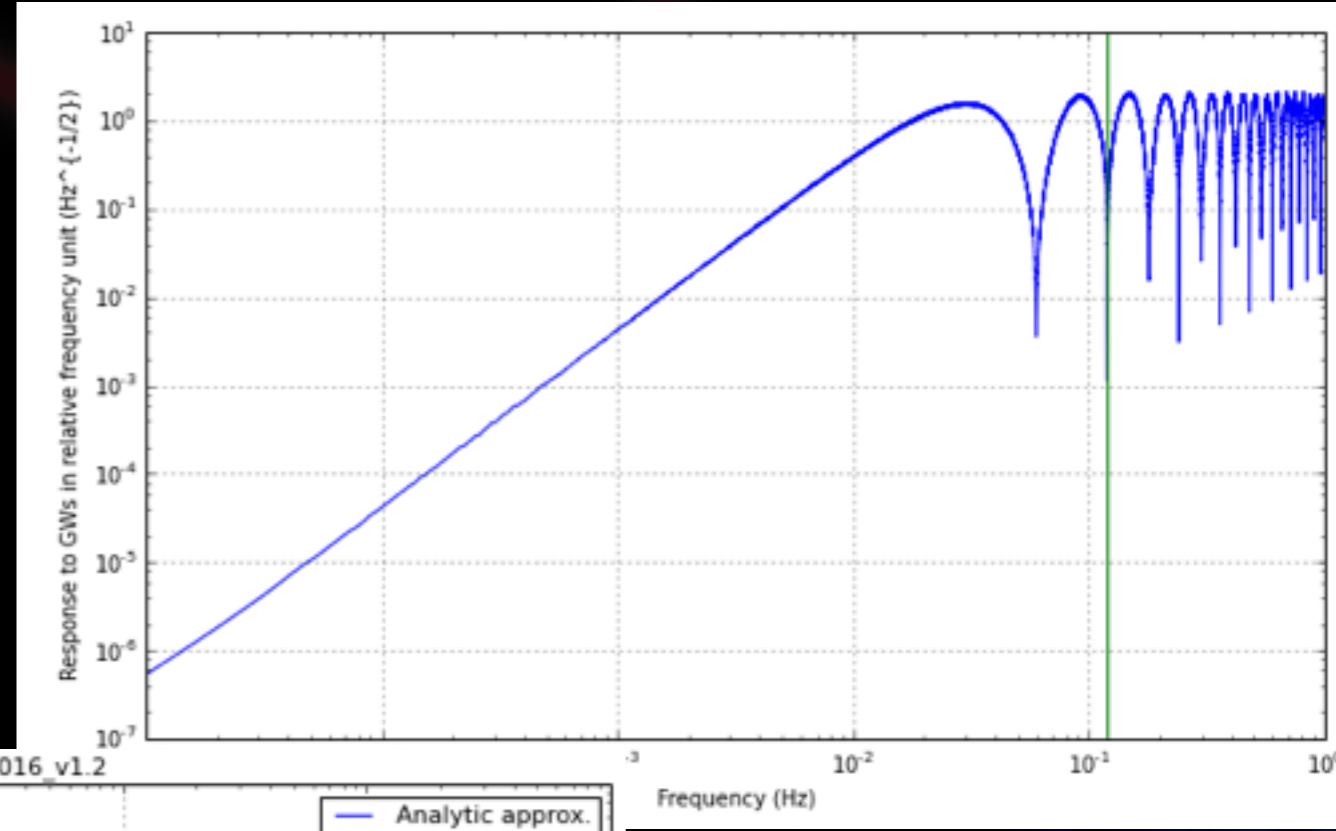
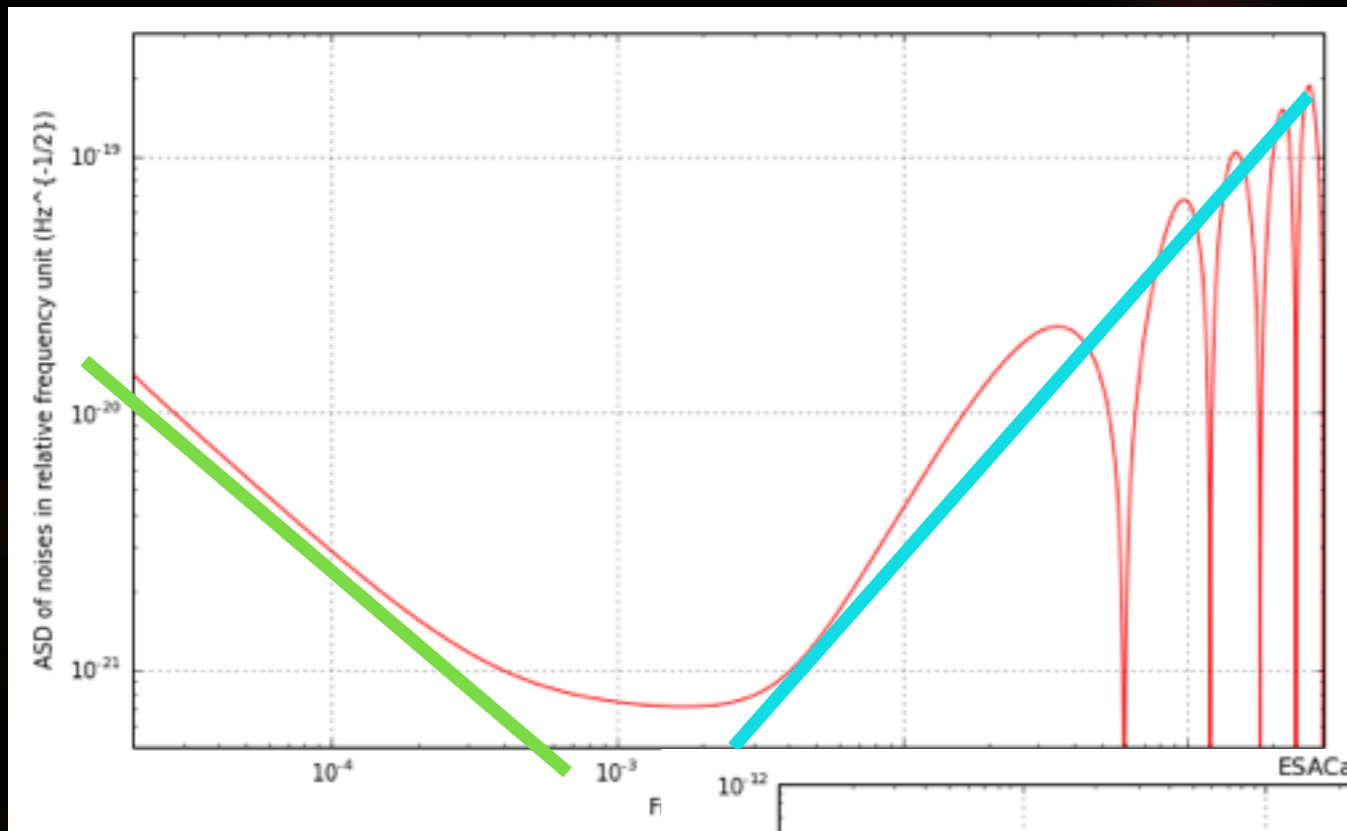




Sensitivity

Noises

Response of the detector to GWs

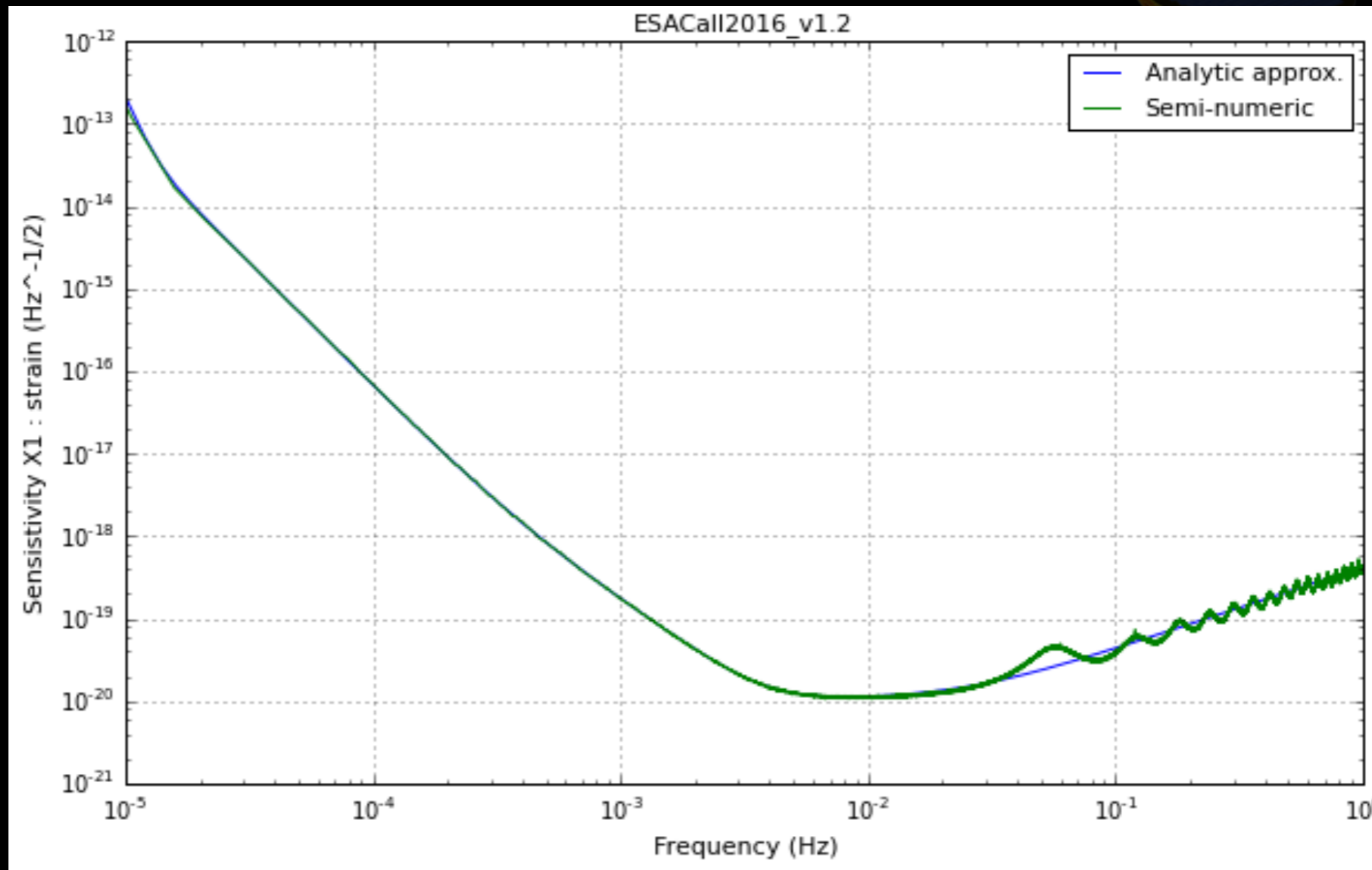




Sensitivity

► Analytic approximation

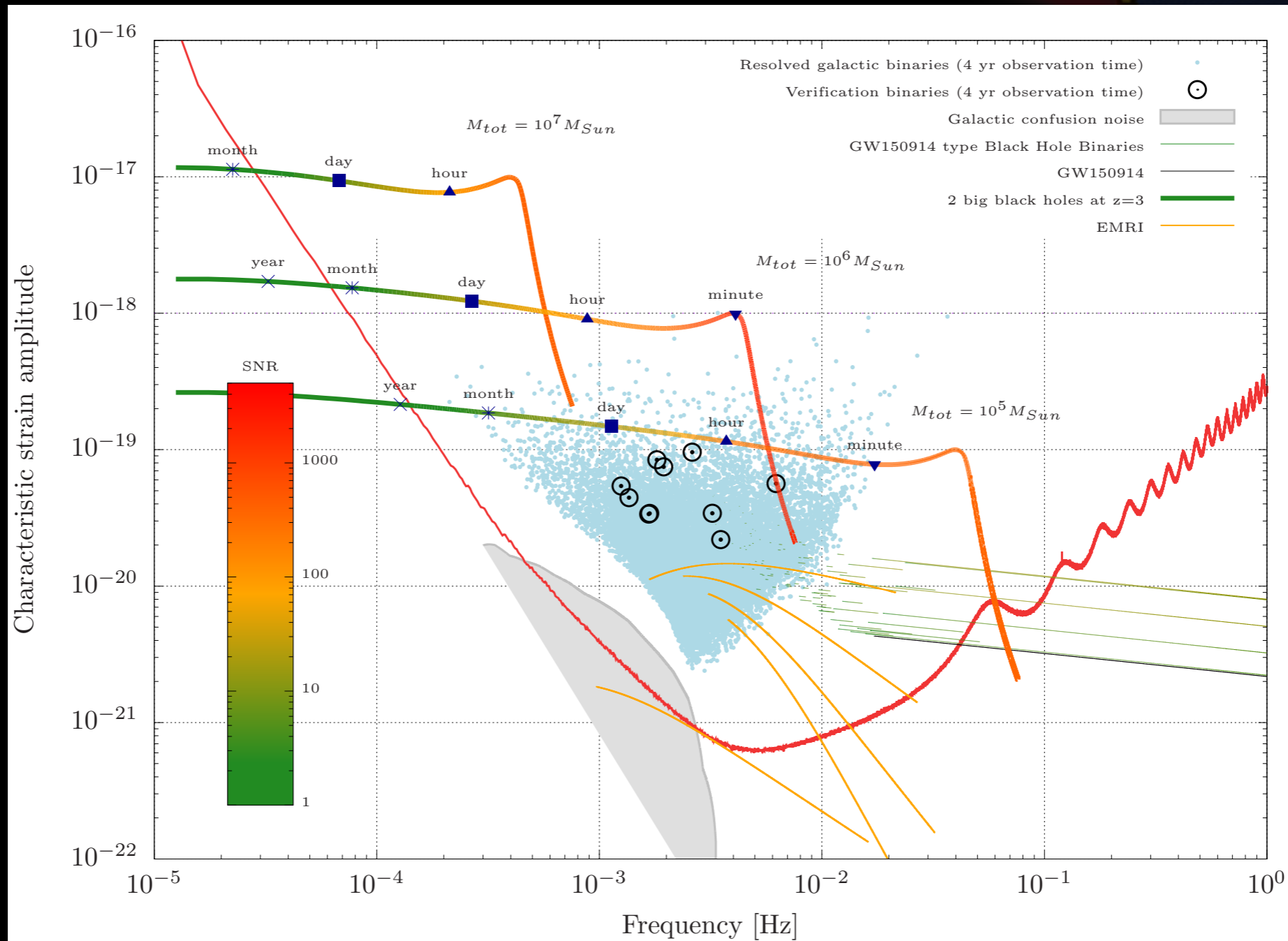
$$\sqrt{S^X}(f) = \sqrt{\frac{20}{3} \left(1 + \left(\frac{f}{0.41 \left(\frac{c}{2L} \right)} \right)^2 \right) \frac{4S_{acc,m} + S_{IMS,m}}{L^2}}$$





Characteristic strain

- ▶ Charceristic strain sensitivity: $S_h(f) = \sqrt{f} S(f)$
- ▶ Useful to compare directly with sources

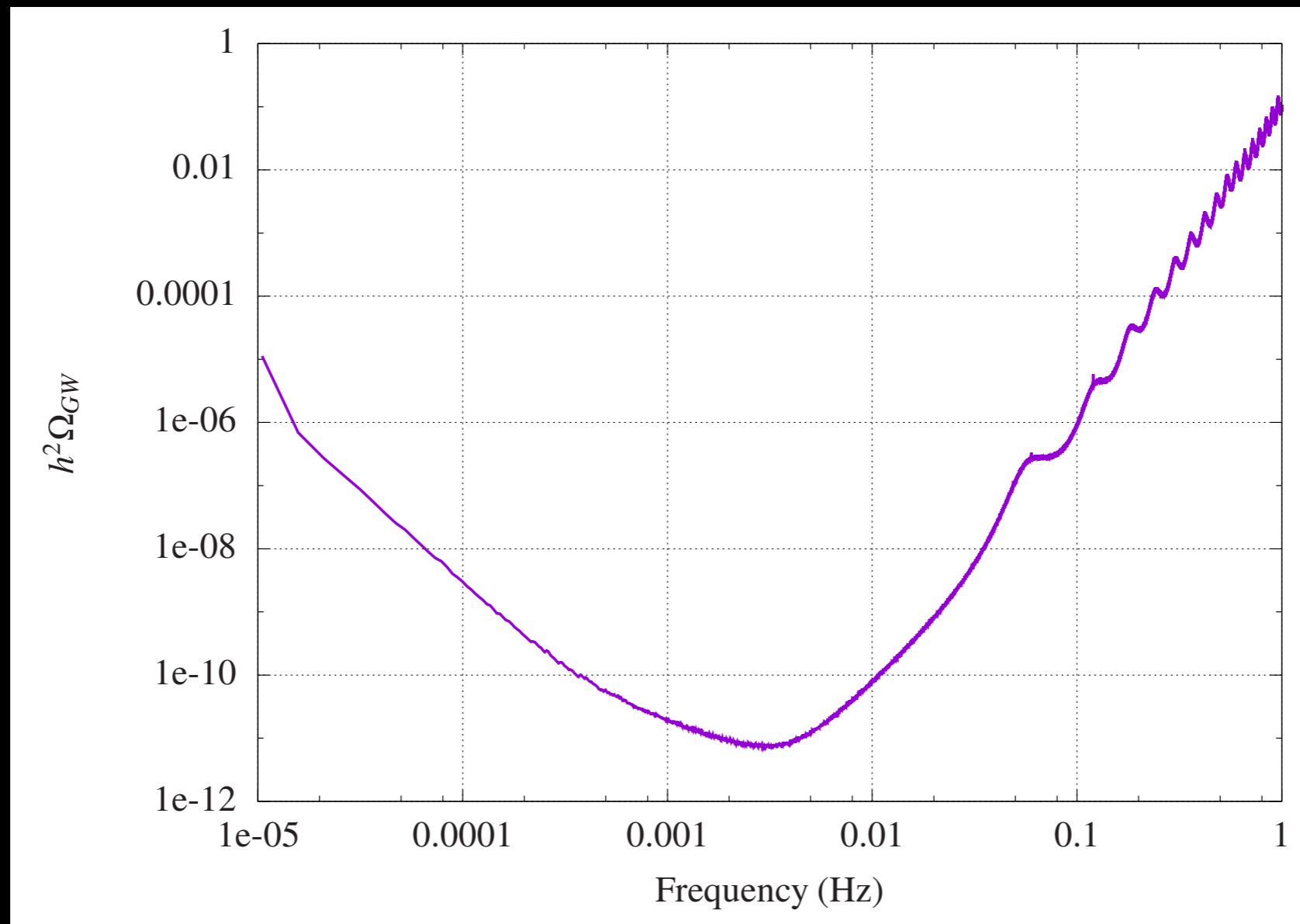




Energy density sensitivity

$$h^2\Omega_{GW}(f) = \frac{4\pi^2}{3H_0^2} f^3 S(f)$$

with $H_0 = h h_0$ with $h_0 = 100 \text{ km.s}^{-1}.\text{Mpc}^{-1} = 3.24 \times 10^{-18} \text{ Hz}$.





Overview

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- ▶ **Response to GW and orbital motion**
- ▶ Data analysis
- ▶ Conclusion



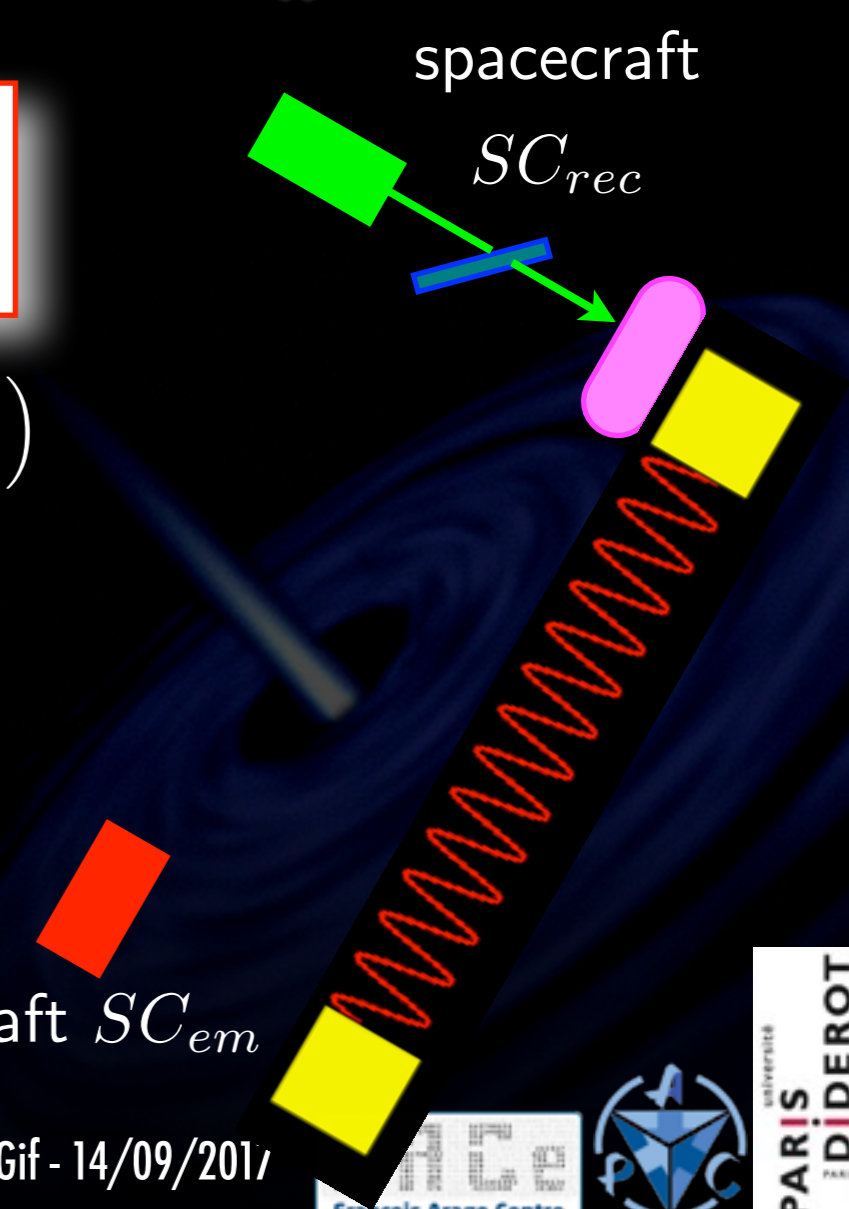
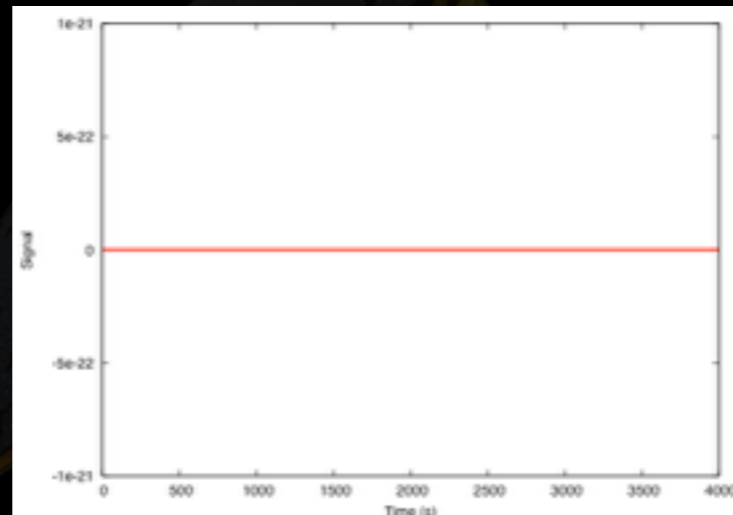
LISA : GWs detection



- ▶ Between spacecraft at r_{em} and spacecraft at r_{rec} (arm unit vector \vec{n}) :
 - GW change phase of received beam
 - This phase is measured by a phasemeter
- ▶ Measurement : relative laser frequency shift :

$$\frac{\delta\nu}{\nu_{laser}}(t) = \frac{1}{2(1 - \vec{k} \cdot \vec{n})} \left[H(t - \vec{k} \cdot \vec{r}_{rec}) - H(t - \vec{k} \cdot \vec{r}_{em} - L) \right]$$

with $H(t) = h_{B+}(t) \xi_+ (\vec{\theta}, \vec{\phi}, \vec{n}(t)) + h_{B\times}(t) \xi_{\times} (\vec{\theta}, \vec{\phi}, \vec{n}(t))$





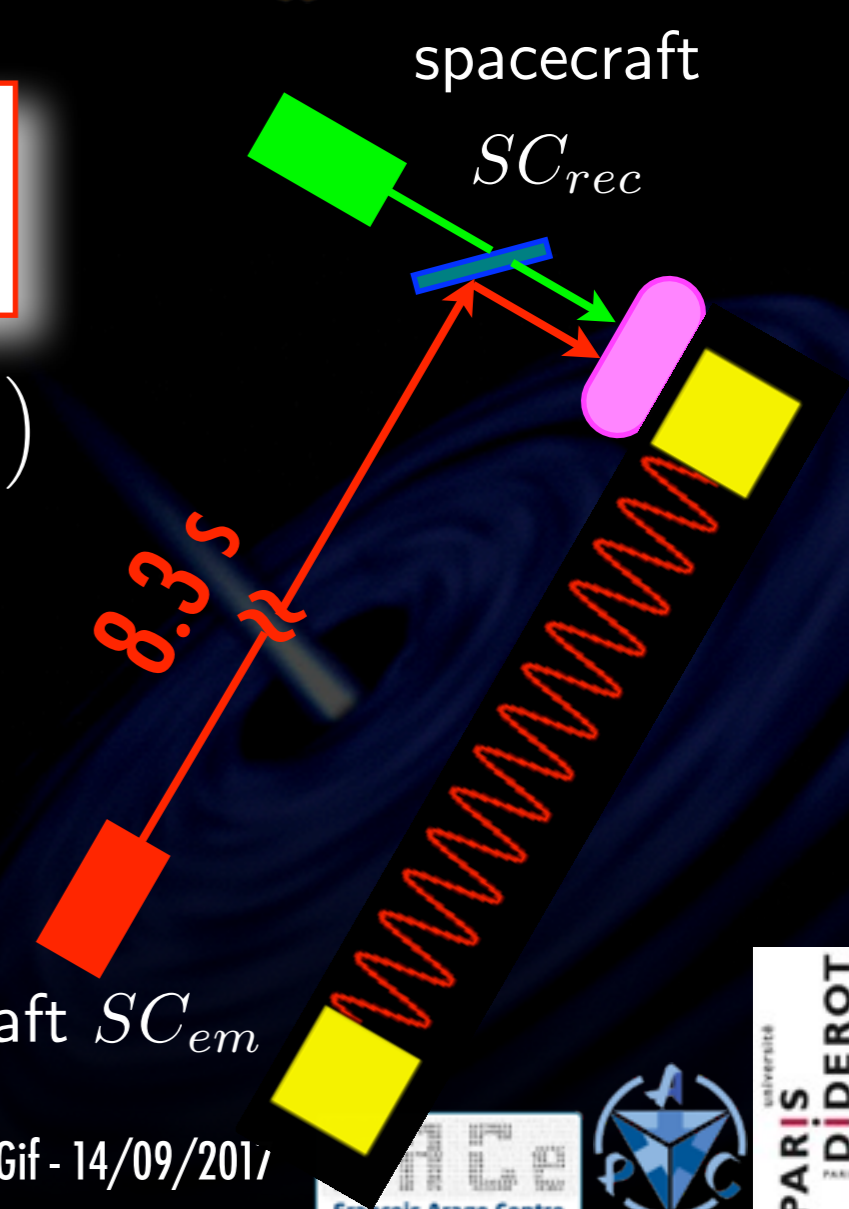
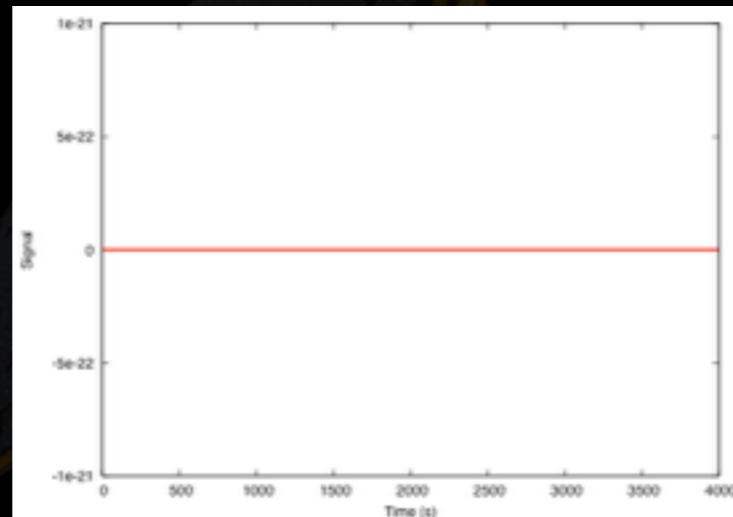
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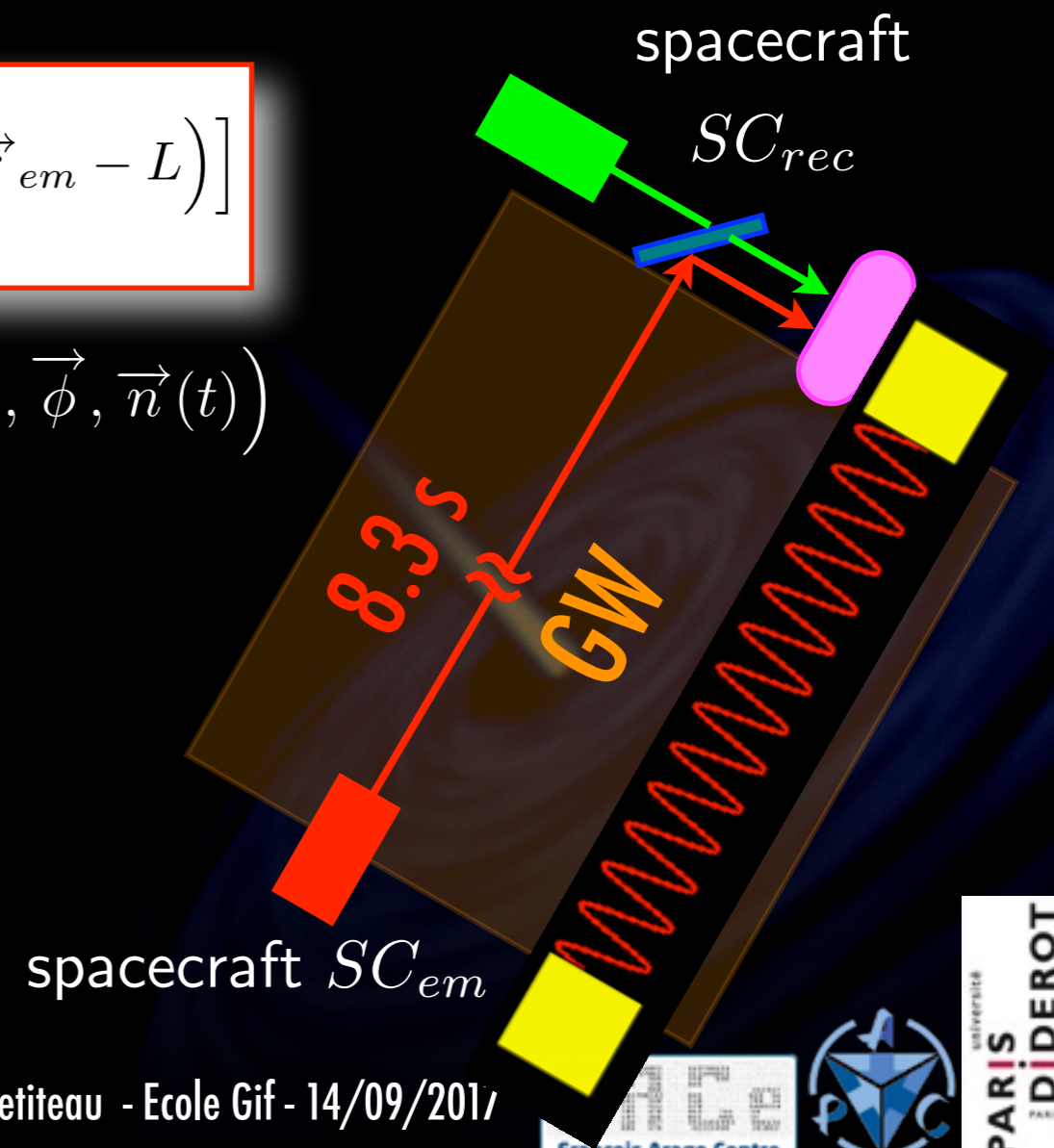
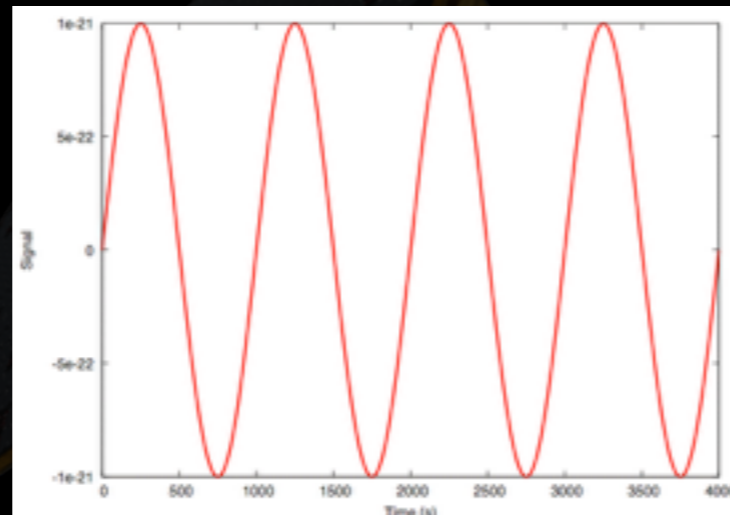


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LISA : GWs detection



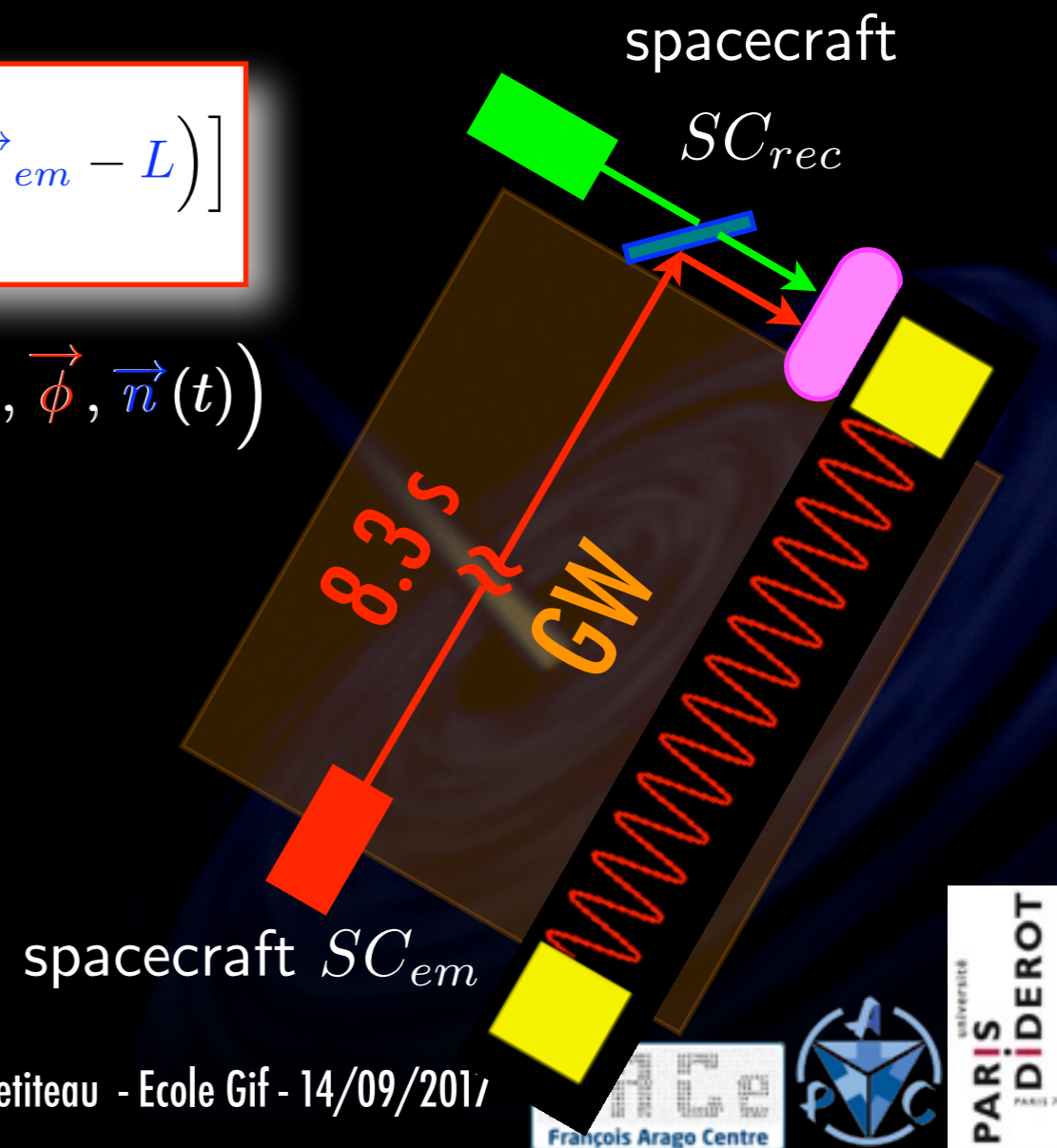
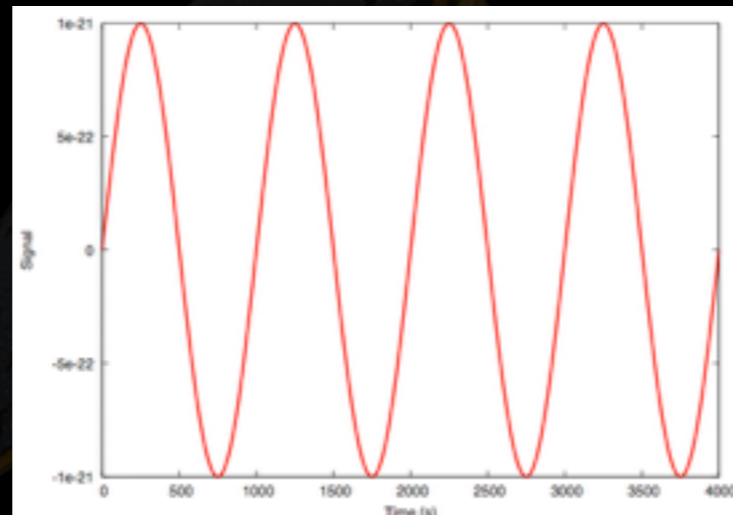
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Dependency :

- amplitude of source
- position of source
- position of arm

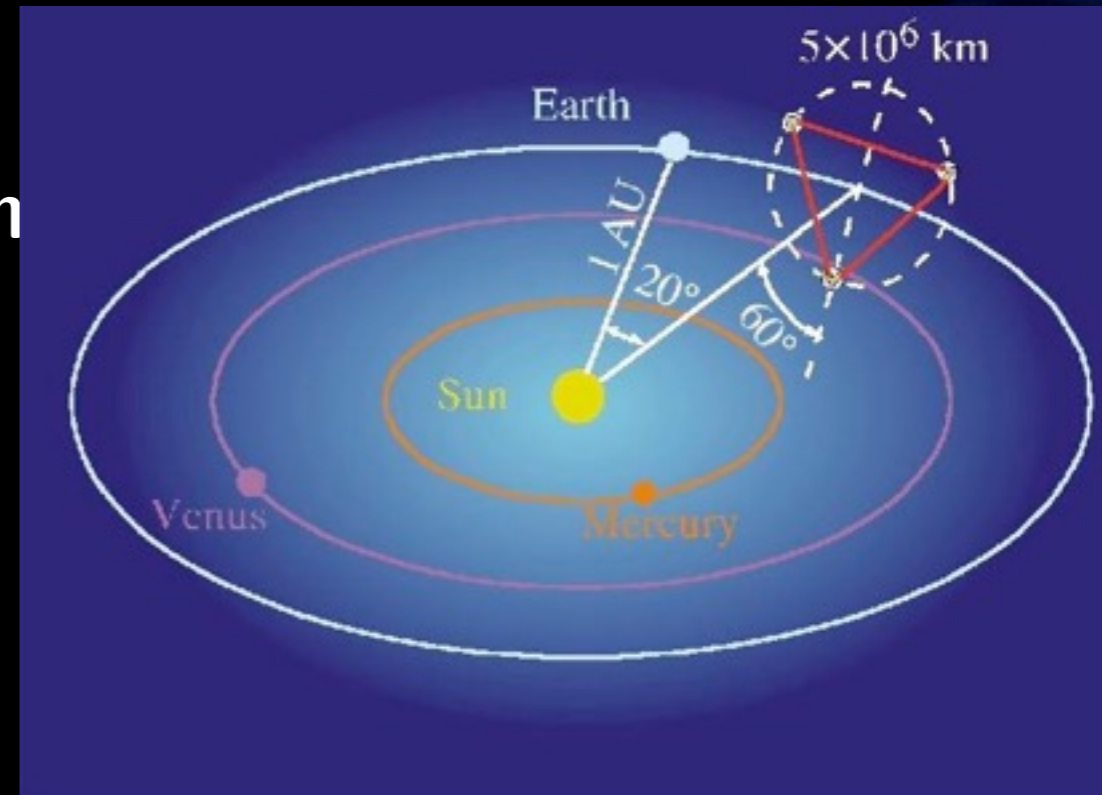
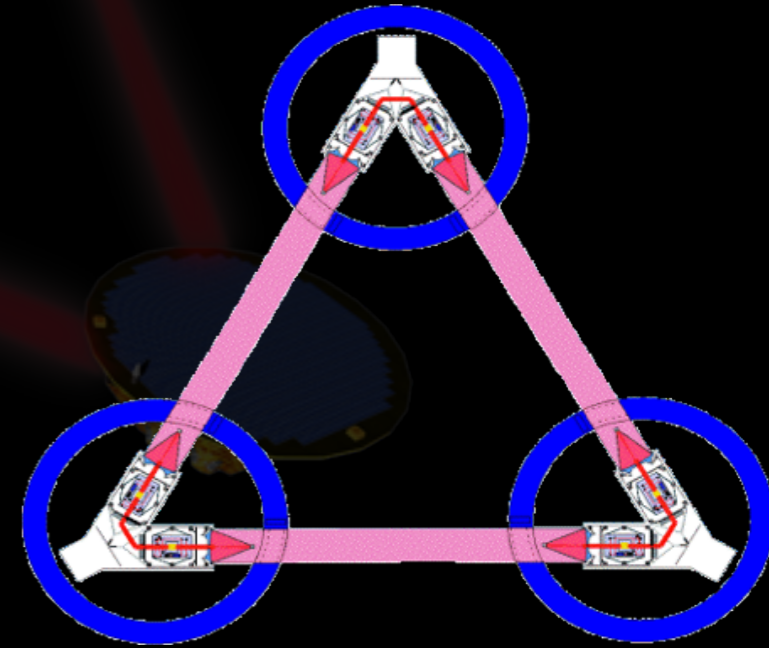




LISA



- ▶ 3 spacecraft and 6 links (2 for each arm)
 - ⇒ 3 interferometers (one redundancy)
- ▶ Armlength = $2.5 \times 10^9 \text{m}$ to detect GWs at $10^{-5} - 1 \text{ Hz}$
- ▶ 3 heliocentric orbits : spacecraft in free fall.
 - LISA centre follows the Earth (-20°).
 - 60° between LISA plane & ecliptic plane
 - Variation of LISA during the year
 - ⇒ **Directional** information of GWs.

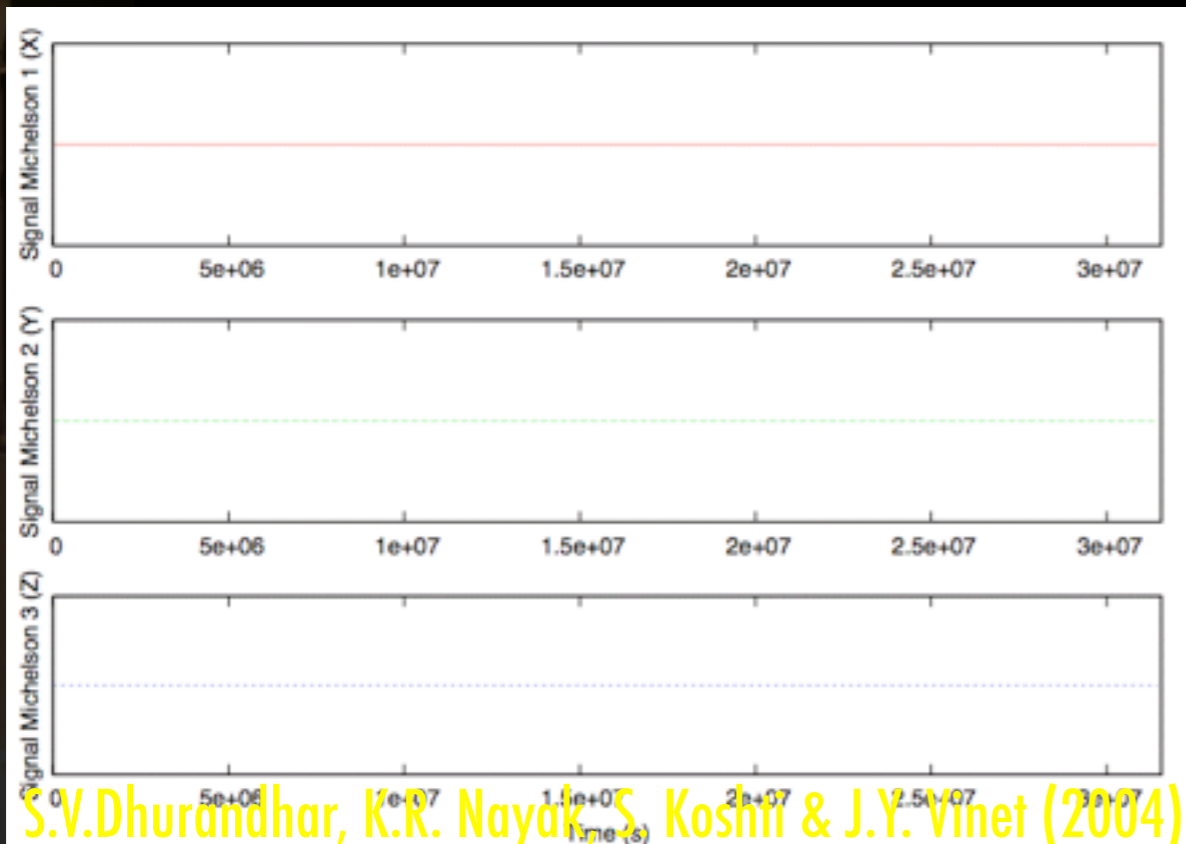
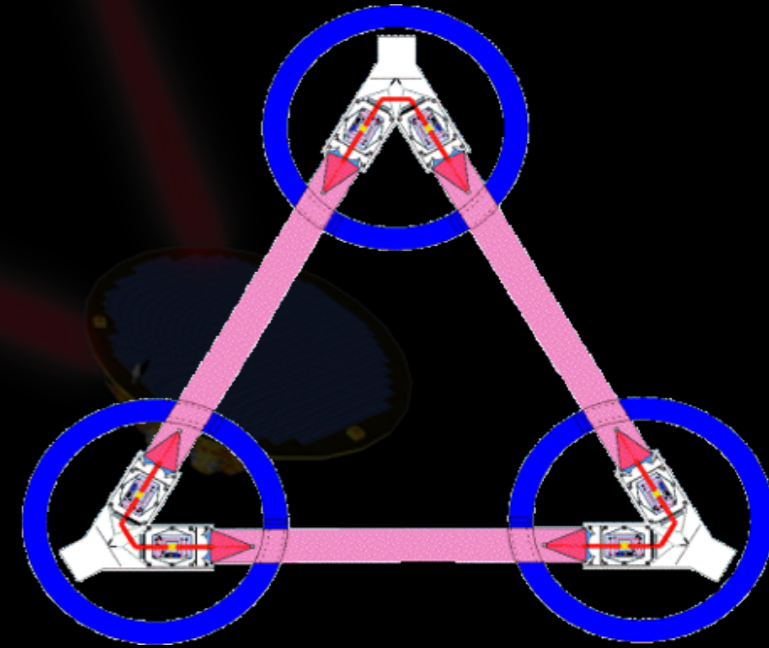




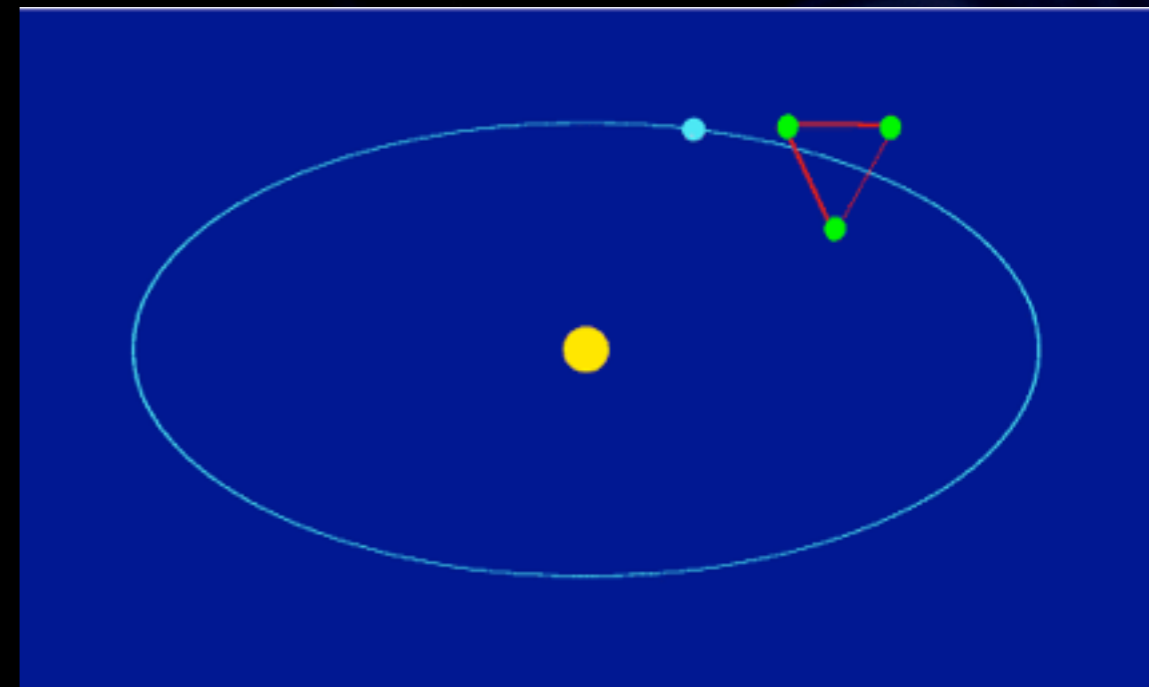
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S.V.Dhurandhar, K.R. Nayak, S. Koshii & J.Y. Vinet (2004)

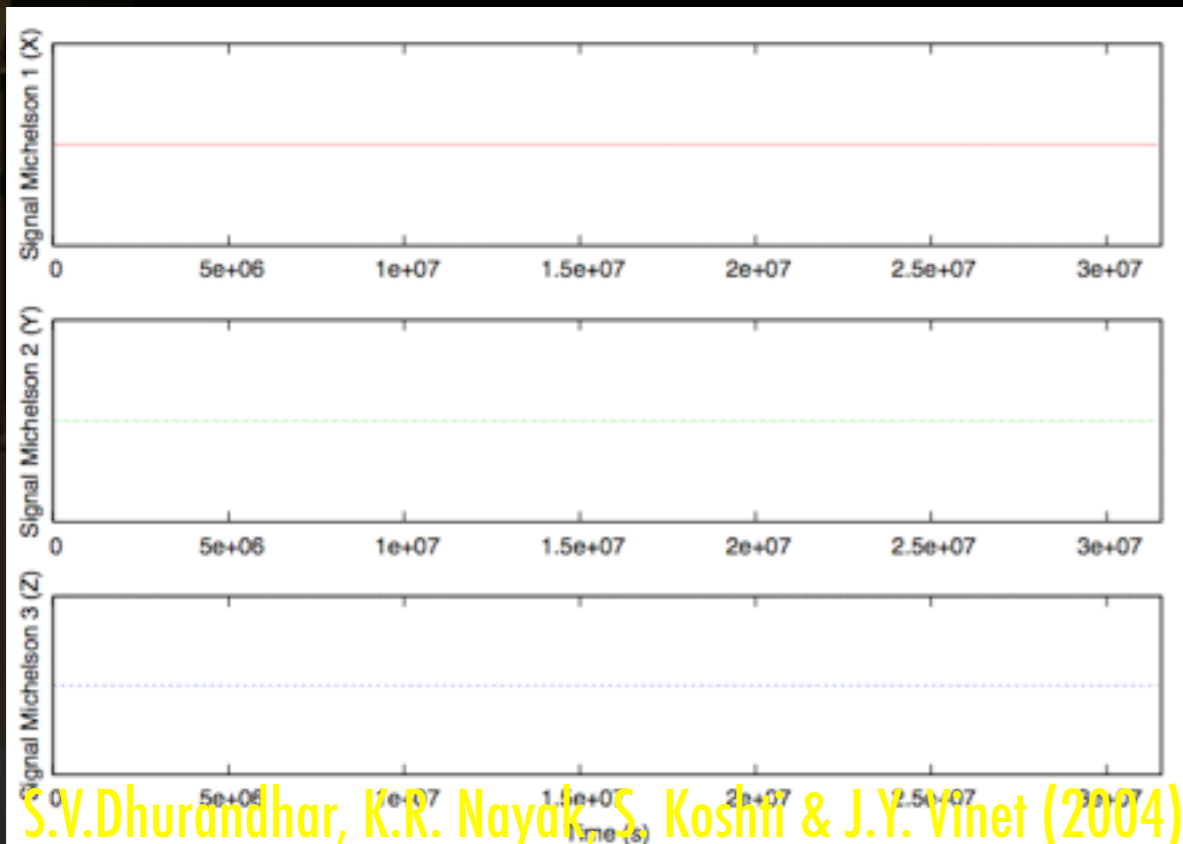
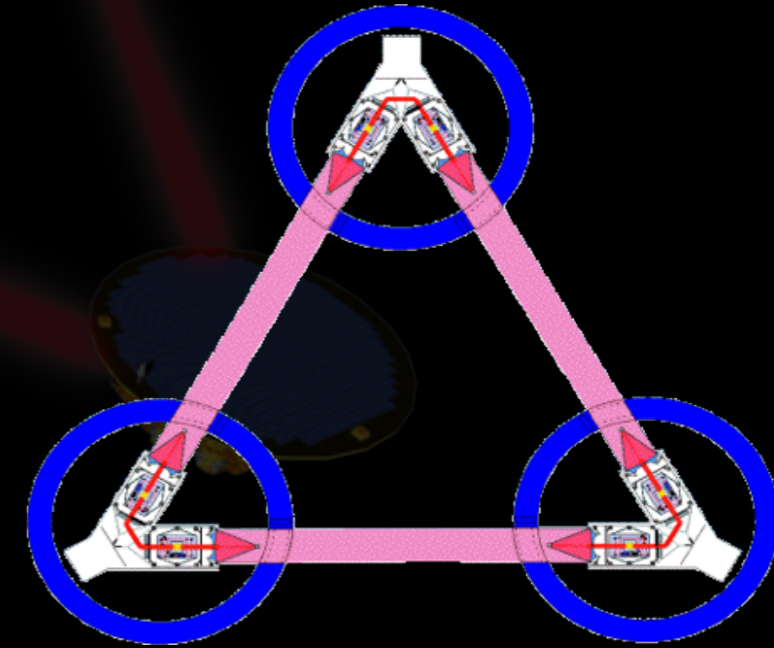




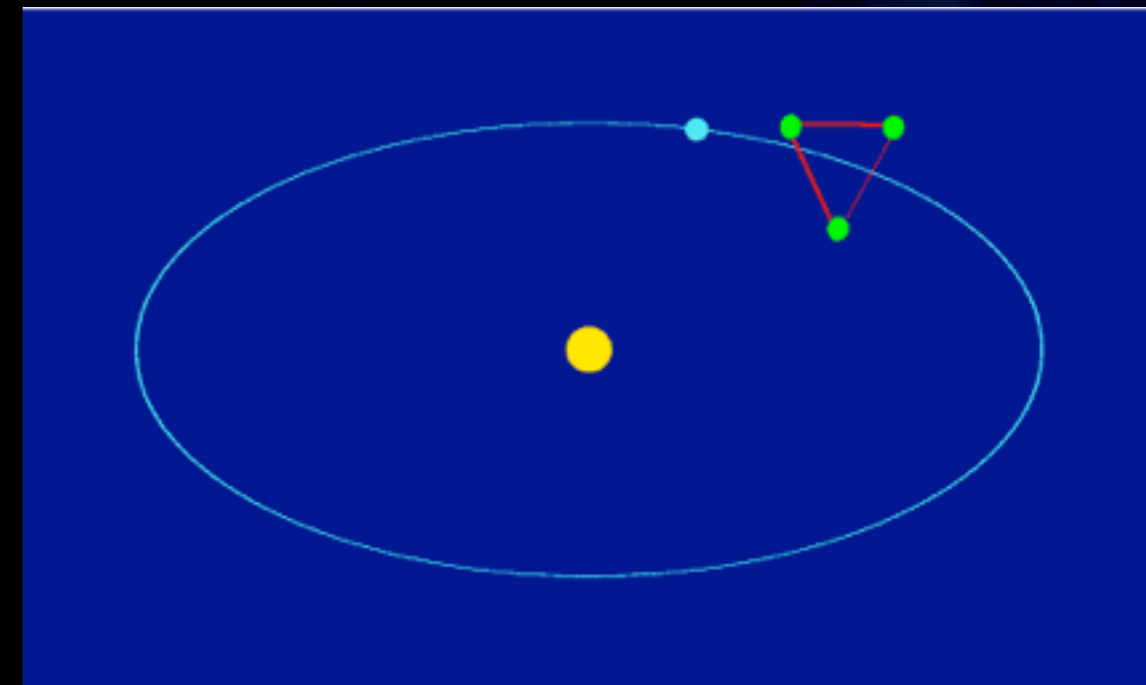
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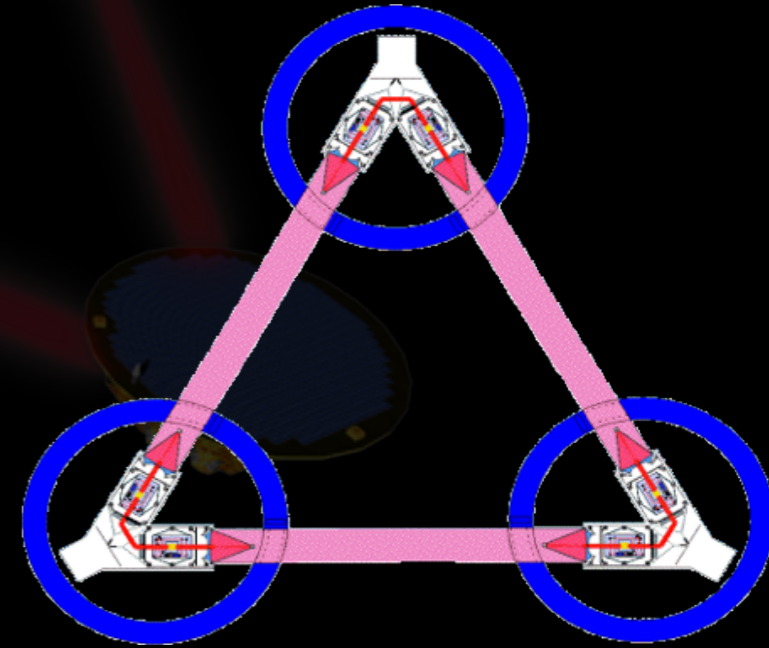




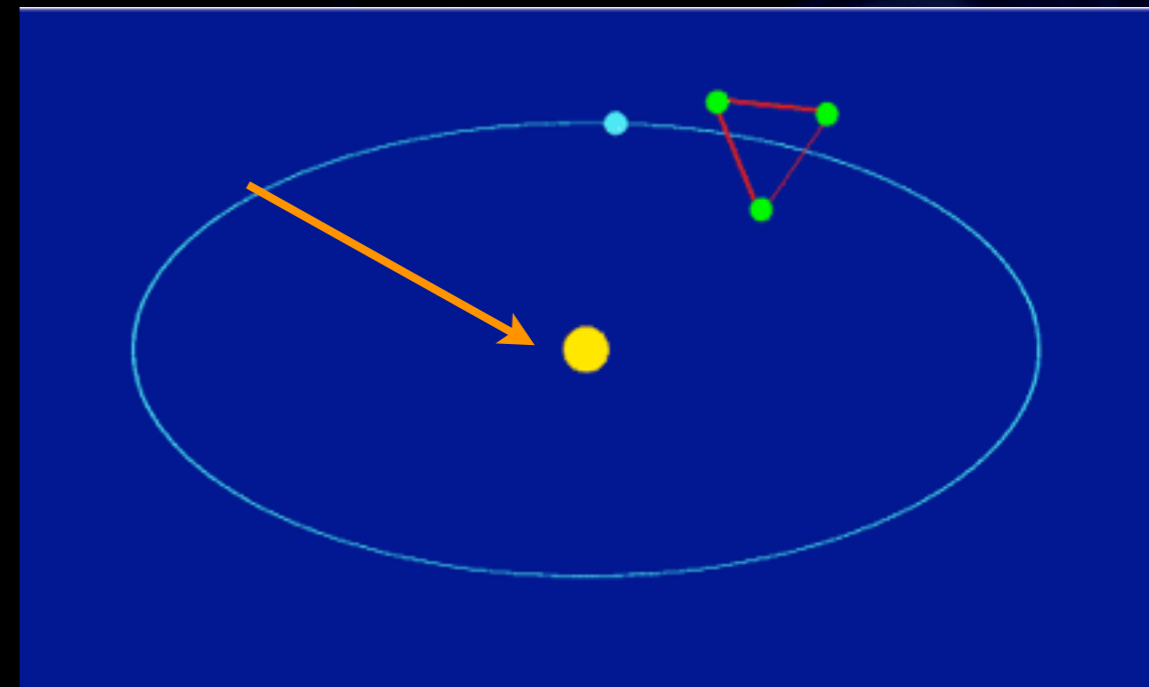
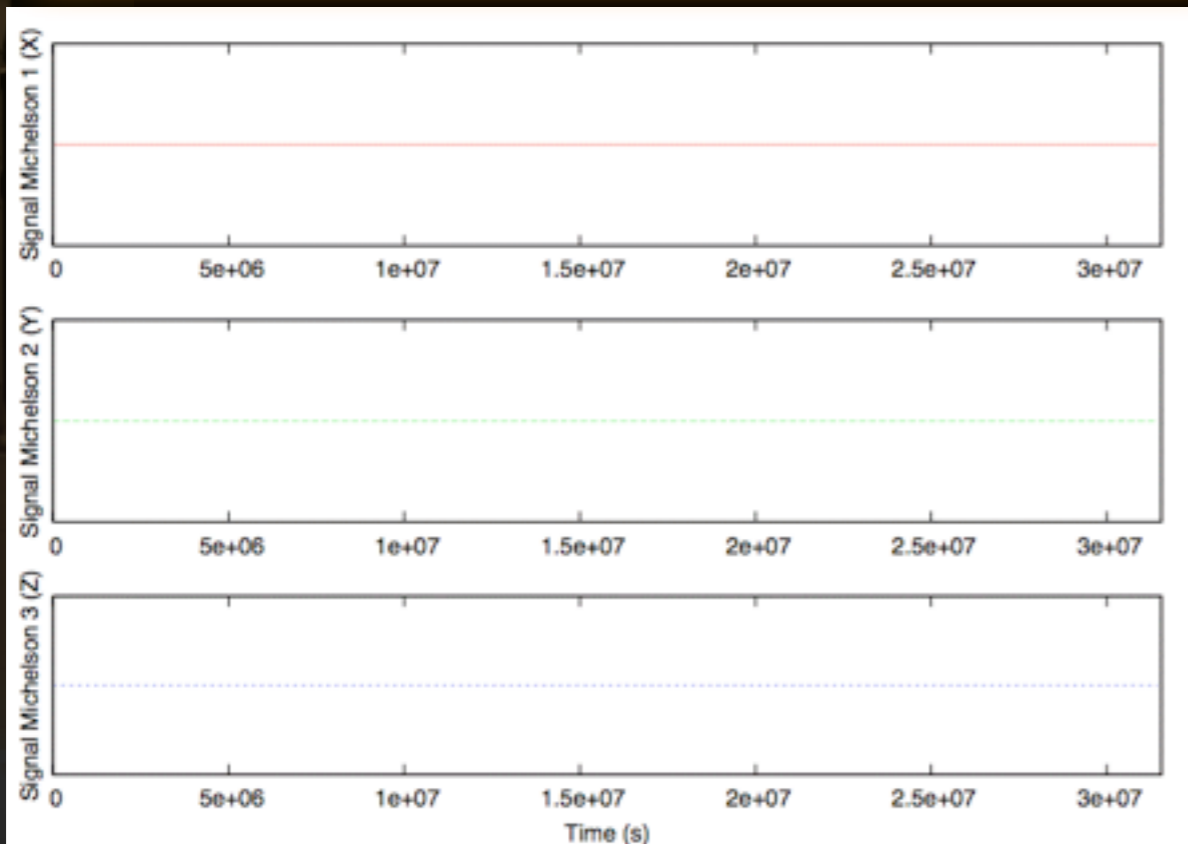
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Modulation of wave amplitude

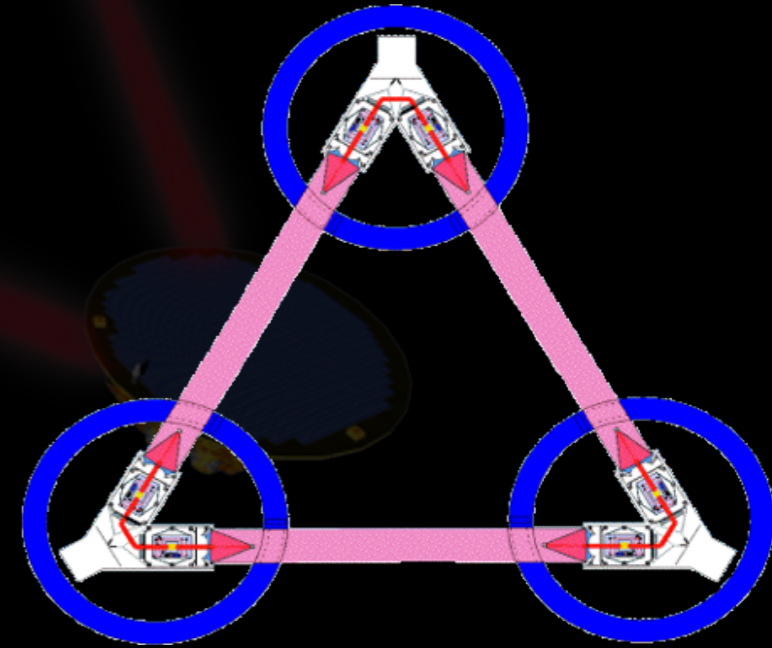




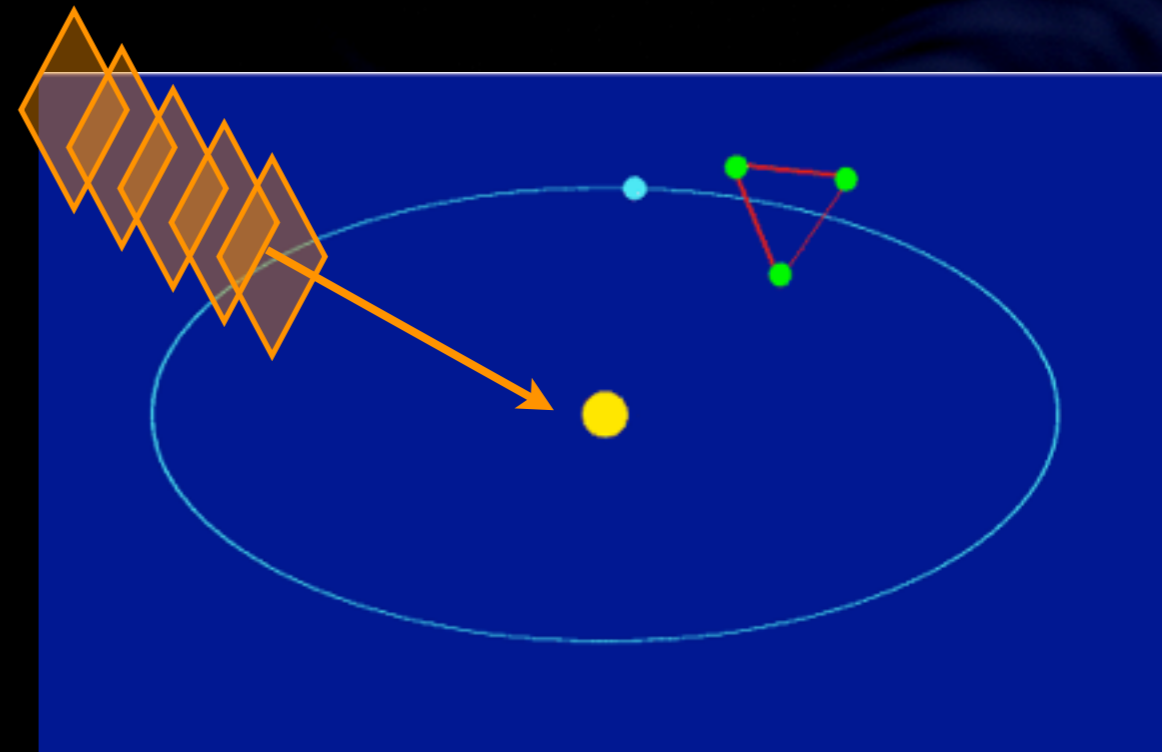
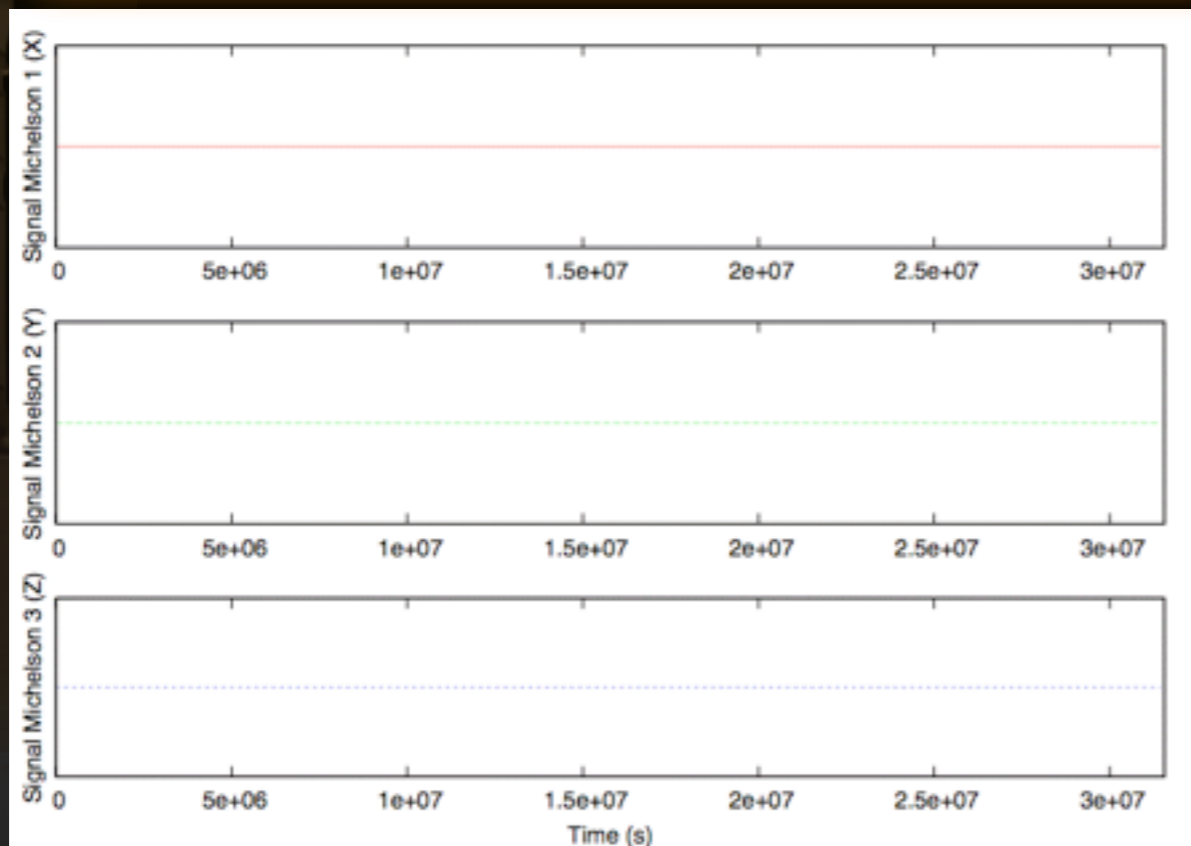
LISA



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Modulation of wave amplitude

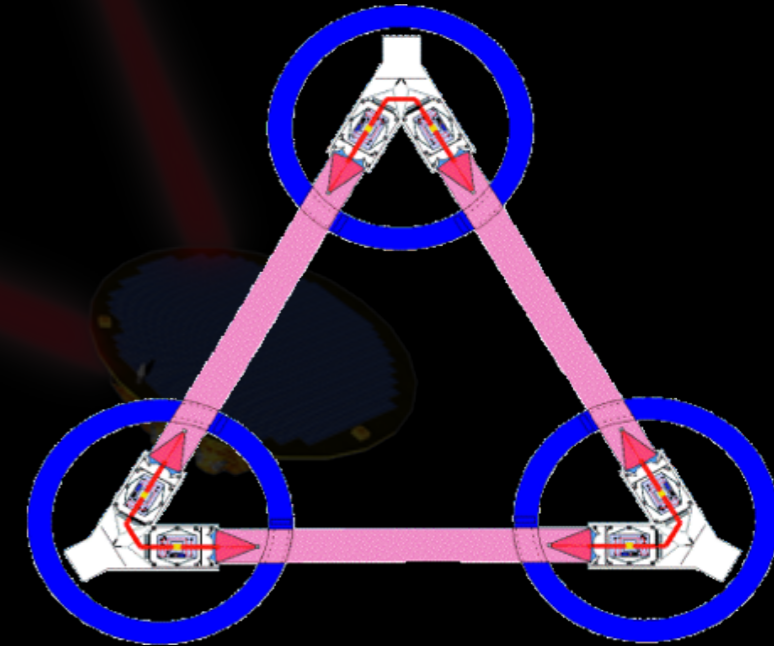




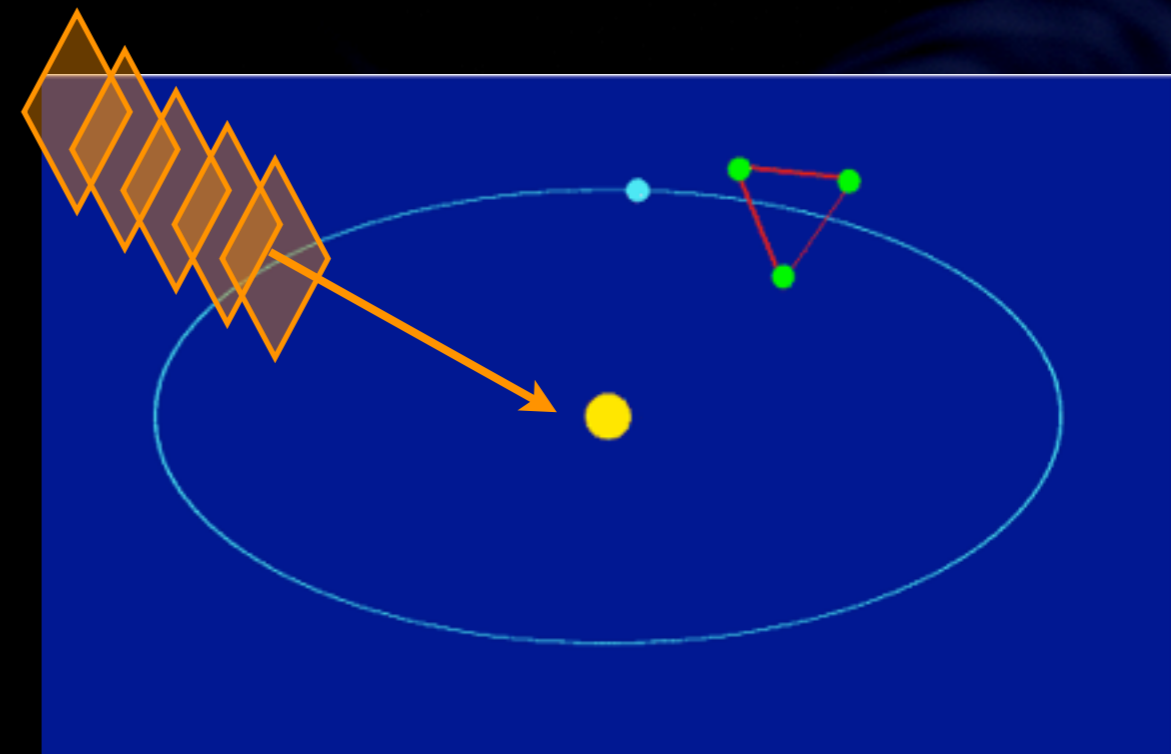
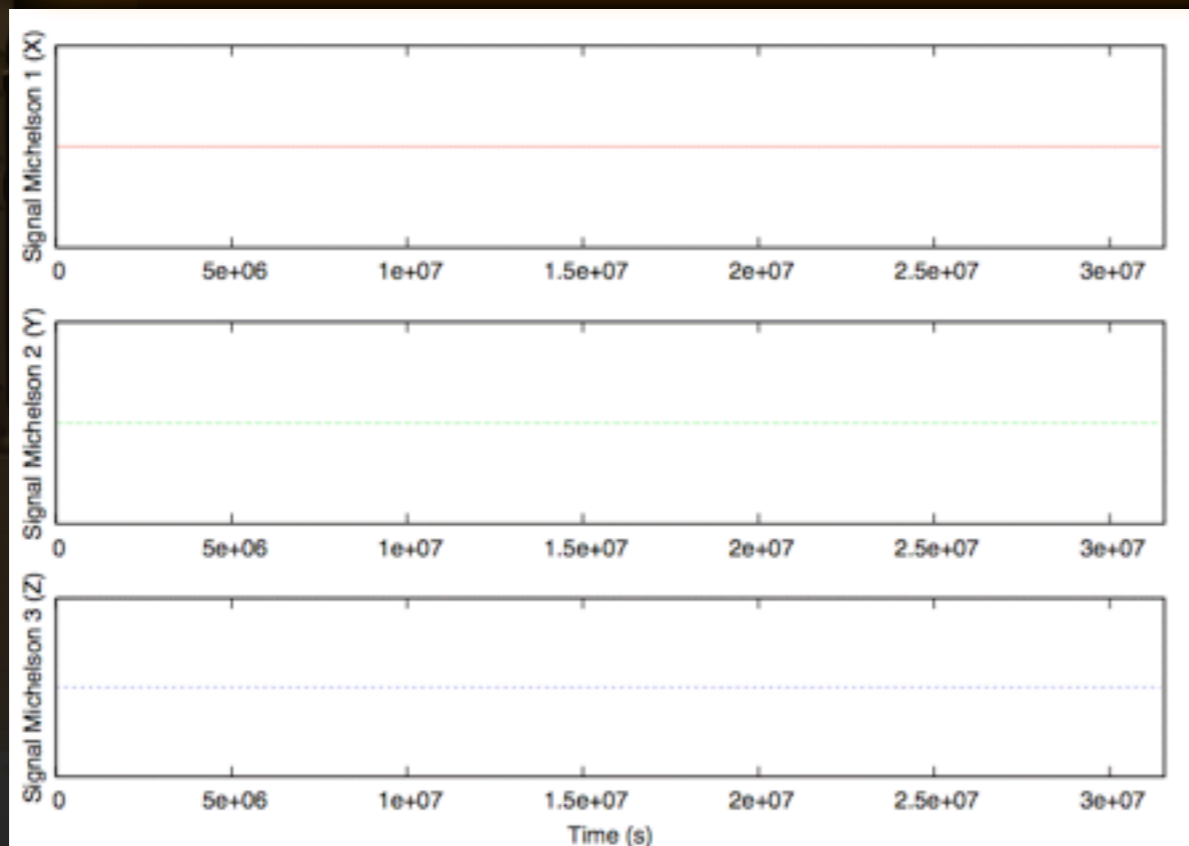
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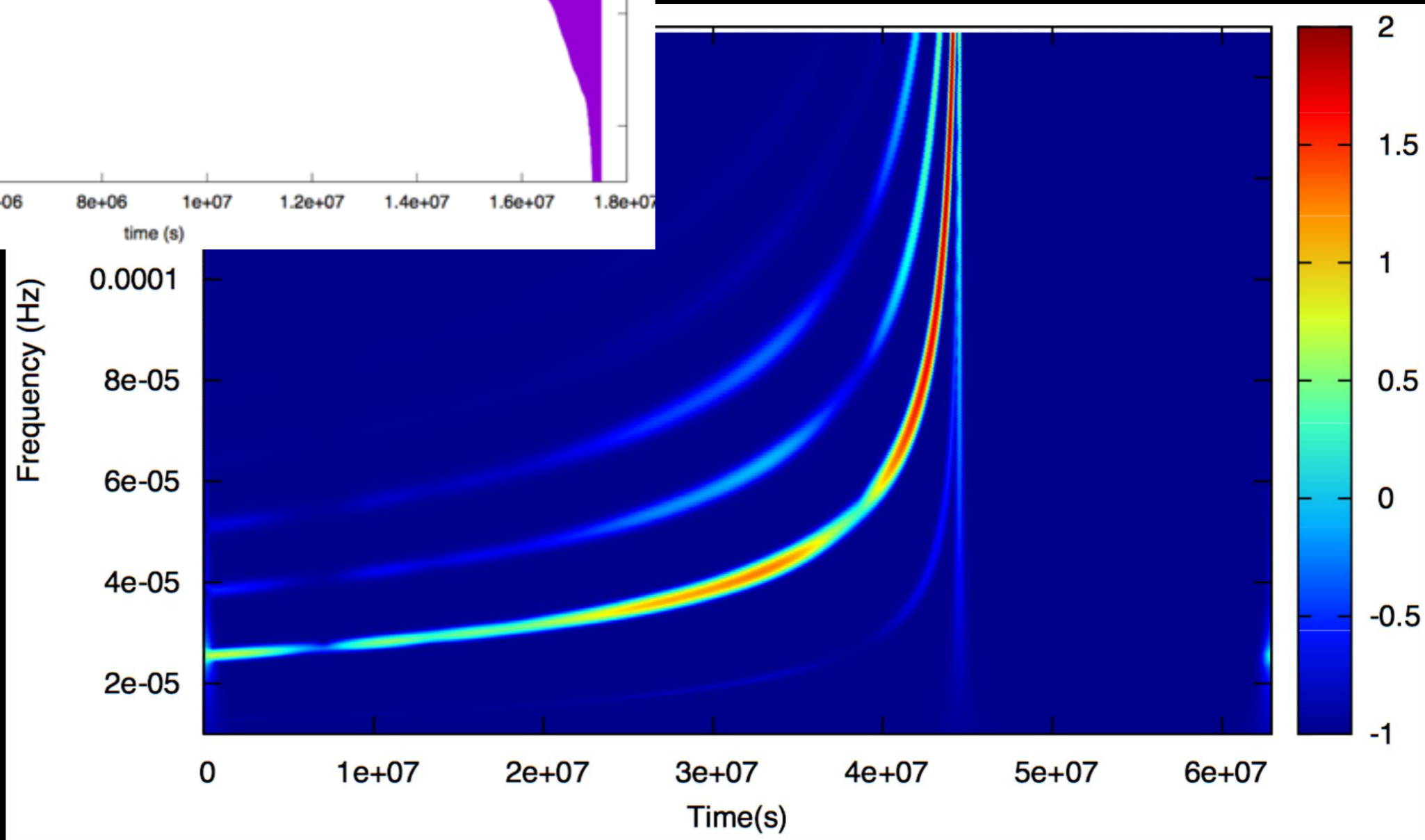
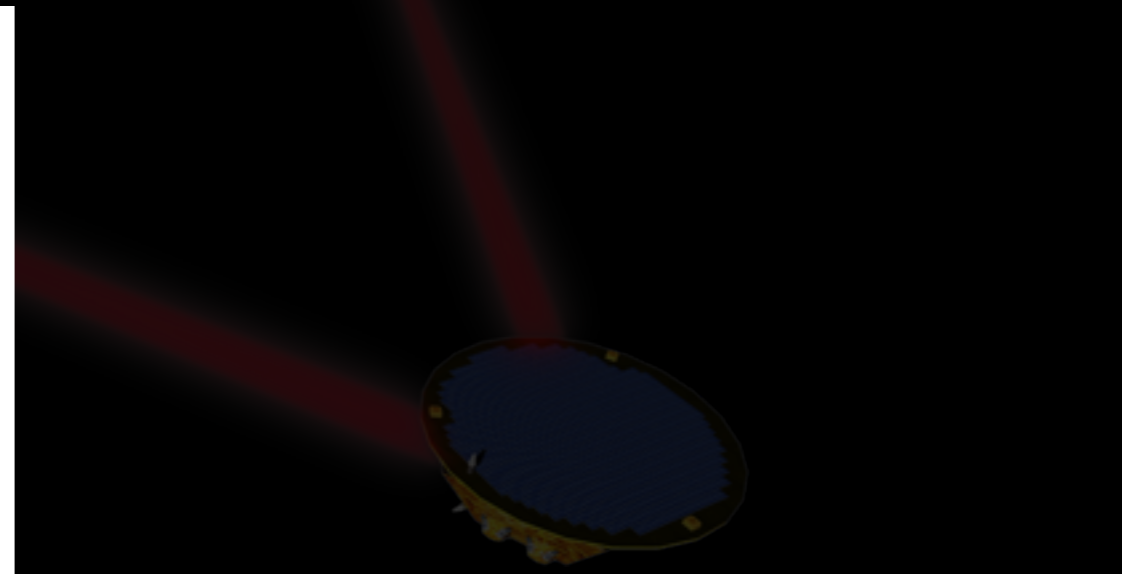
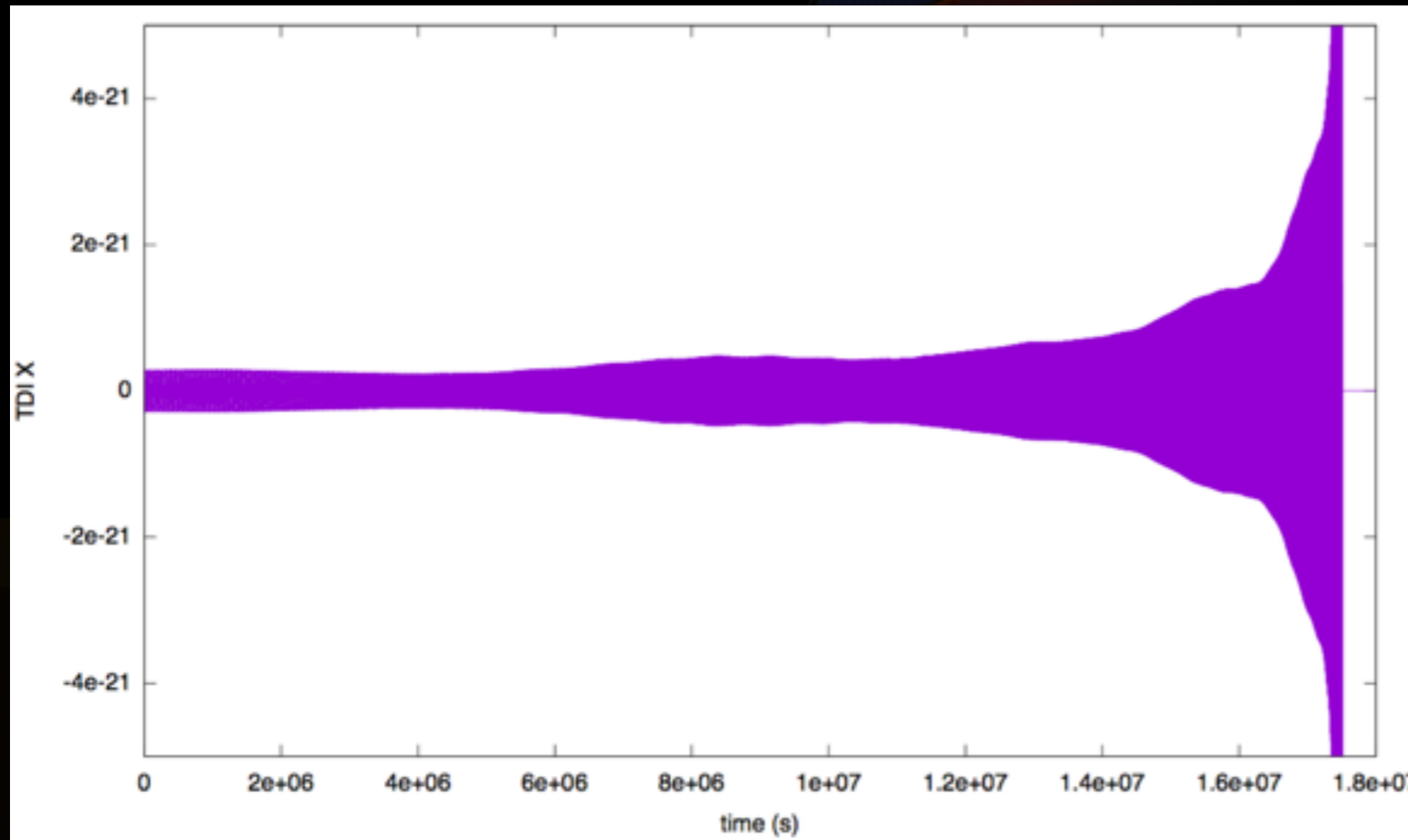


Modulation of wave amplitude





Modulation - sky position

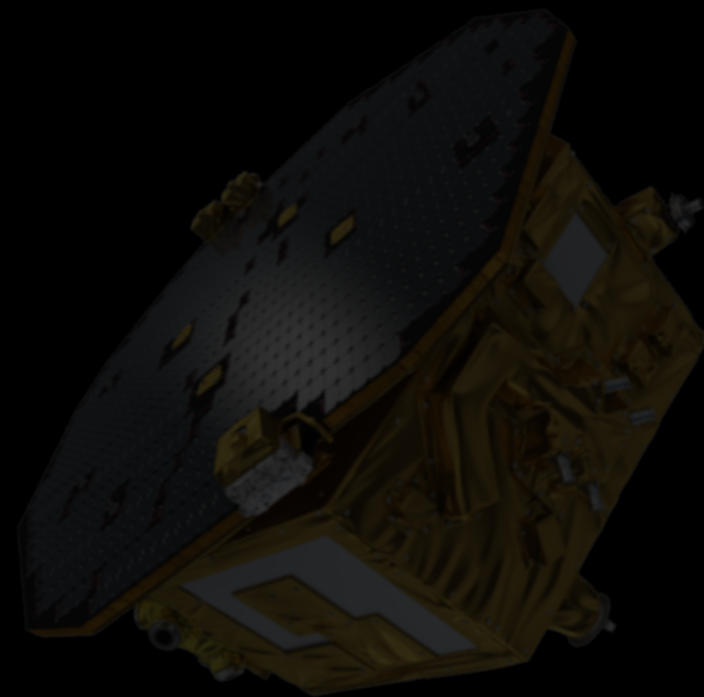




Modulation - sky position



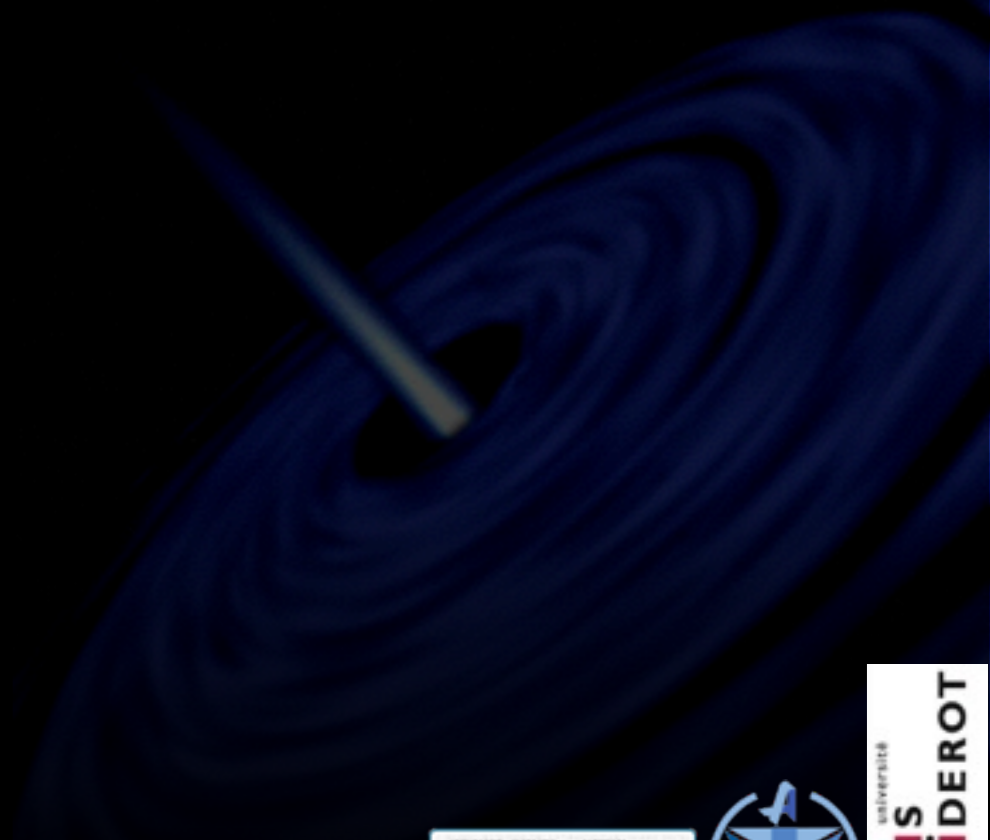
- ▶ Survey type instrument:
 - no pointing
 - observe “all sky every time”
- ▶ Depending on the source power and duration, the angular could go until 1 deg^2
- ▶ ... but better resolution on other parameters!





Overview

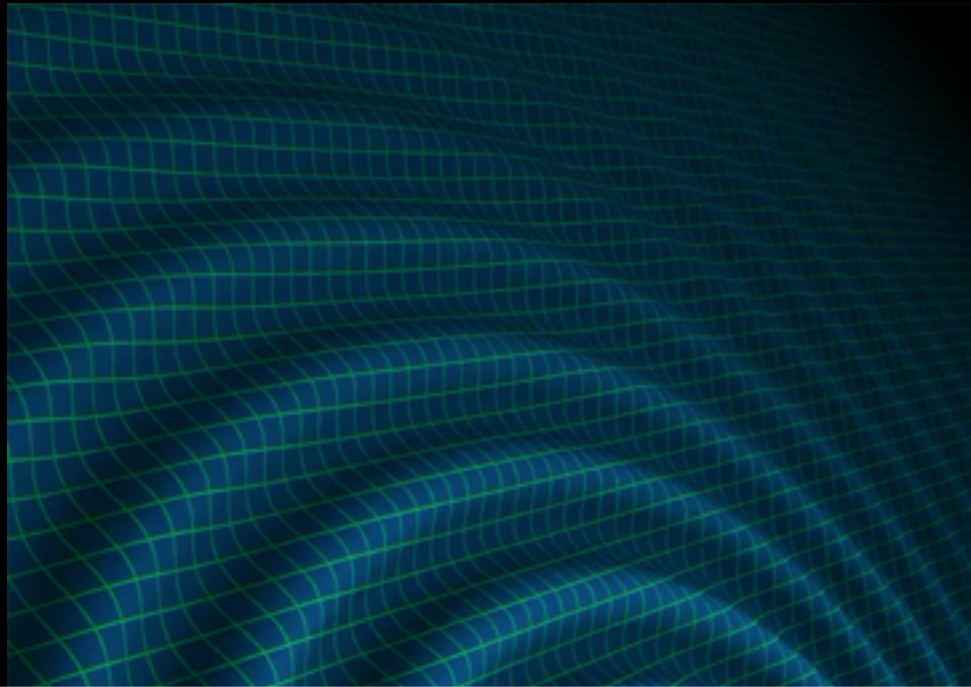
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- ▶ Sensitivity
- ▶ Response to GW and orbital motion
- ▶ **Data analysis**
- ▶ Conclusion



LISA data

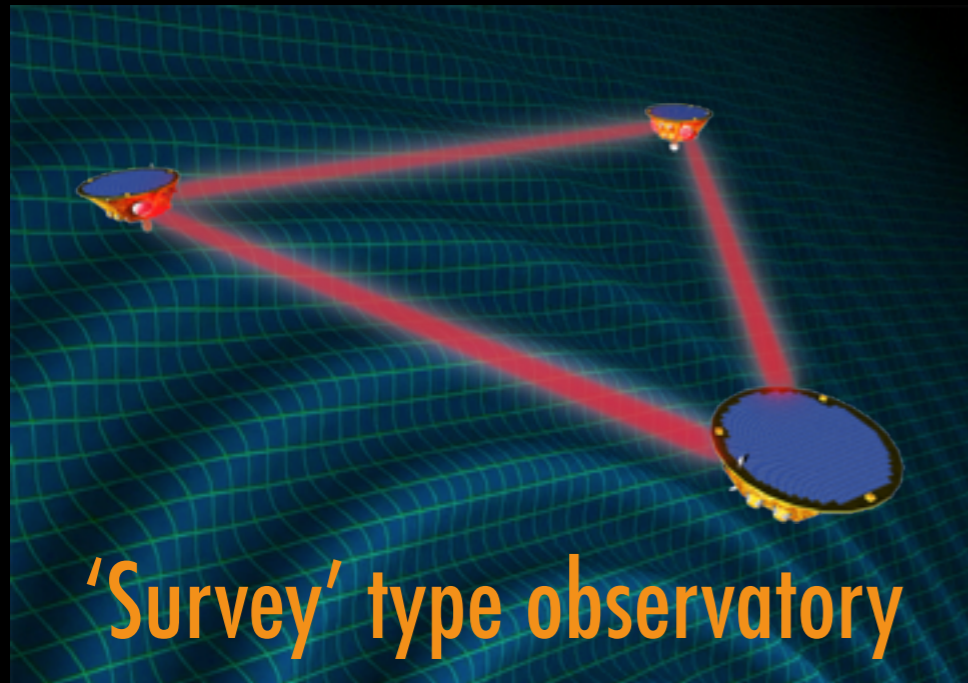
**Gravitational wave sources
emitting between 0.02mHz
and 100 mHz**

LISA data



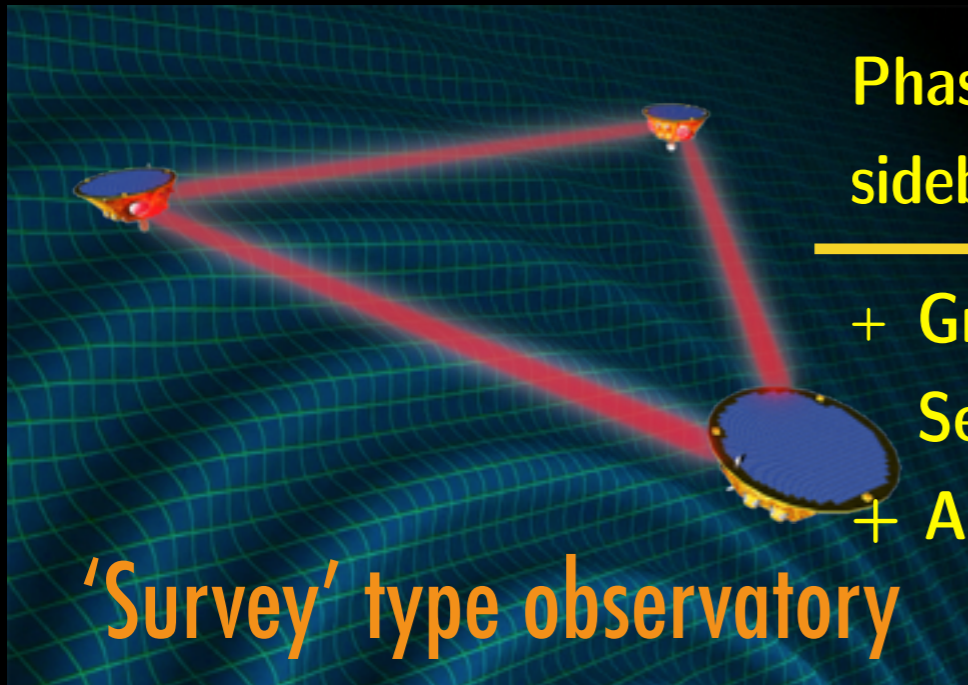
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LISA data



Gravitational wave sources
emitting between 0.02mHz
and 100 mHz

LISA data



Phasemeters (carrier, sidebands, distance)

+ Gravitational Reference Sensor

+ Auxiliary channels



Gravitational wave sources emitting between 0.02mHz and 100 mHz

LISA data

Phasemeters (carrier, sidebands, distance)

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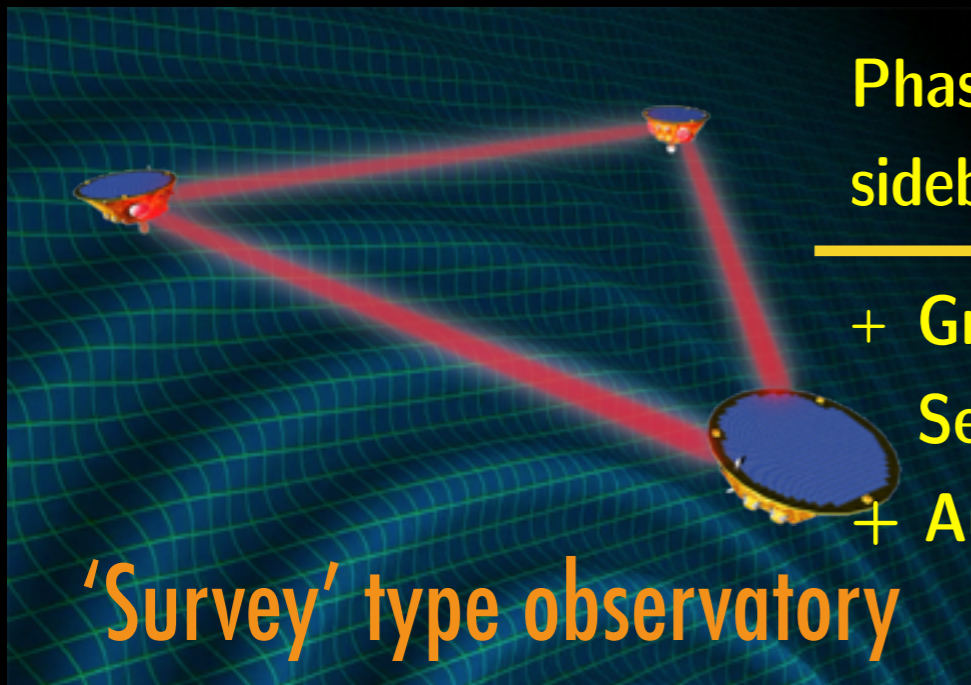
'Survey' type observations



Gravitational waves emitting between 100 and 1000 Hz

Source	Class	Measurement	Count	Sampling Rate [Hz]	Bits / channel	Rate [bits/s]	
Payload							
Phasemeter	IFO Longitudinal	Science IFO	2	3.3	32	213.3	
		Test Mass IFO	2	3.3	32	213.3	
		Reference IFO	2	3.3	32	213.3	
		Clock Sidebands	2	3.3	32	213.3	
	IFO Angular	$S/C \theta, \eta$	4	3.3	32	426.6	
		TM θ, η	4	3.3	32	426.6	
	Anciliary	Time Semaphores	2	3.3	96	639.9	
	Optical Monitoring	PAAM Longitudinal	PAAM Longitudinal	2	3.3	32	213.3
			PAAM Angular	4	3.3	32	426.6
			Optical Truss	6	3.3	32	639.9
GRS FEE	GRS Cap. Sensing	TM x, y, z	6	3.3	24	480.0	
		TM θ, η, ϕ	6	3.3	24	480.0	
Payload Computer	DFACS	TM applied torques	6	3.3	24	480.0	
		TM applied forces	6	3.3	24	480.0	
		S/C applied torques	3	3.3	24	240.0	
		S/C applied forces	3	3.3	24	240.0	
	Payload HK	e.g. Temperature, Power Monitors etc.				2613	
Total Payload						8639	
Platform							
Housekeeping (based on LPF)						1189	
Total Platform						1189	
Totals							
Raw rate per S/C						9828	
Paketisation overhead [10%]						983	
Packaged rate per S/C						10811	
Packaged rate for Constellation						32433	

LISA data



Phasemeters (carrier, sidebands, distance)

+ Gravitational Reference

Sensor

+ Auxiliary channels

'Survey' type observatory



Calibrations corrections

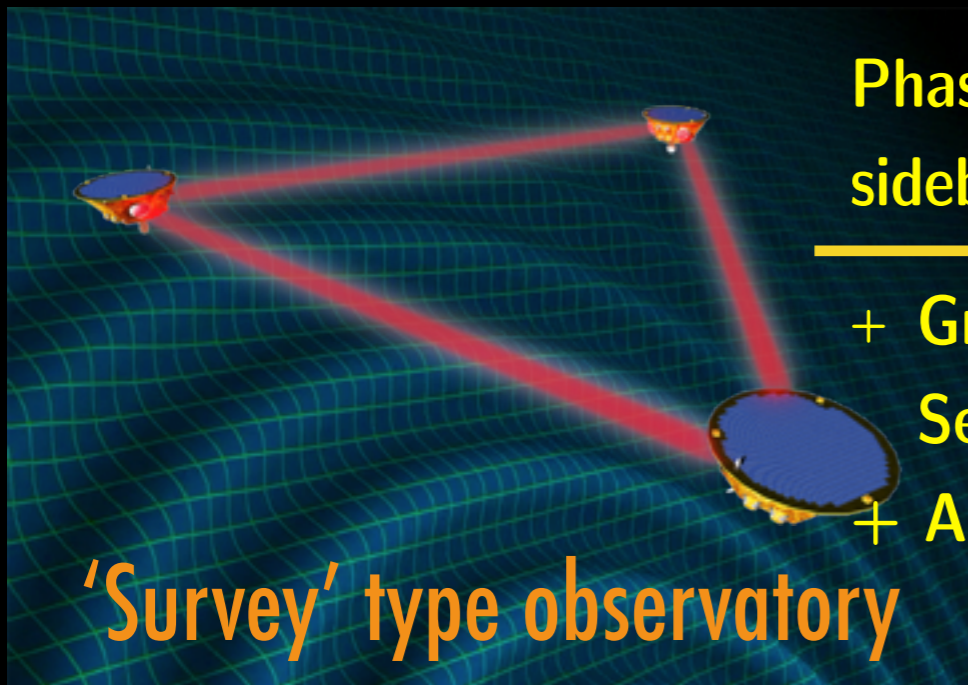
Resynchronisation (clock)

Time-Delay Interferometry
reduction of laser noise

2 data channels TDI non-correlated

Gravitational wave sources
emitting between 0.02mHz
and 100 mHz

LISA data



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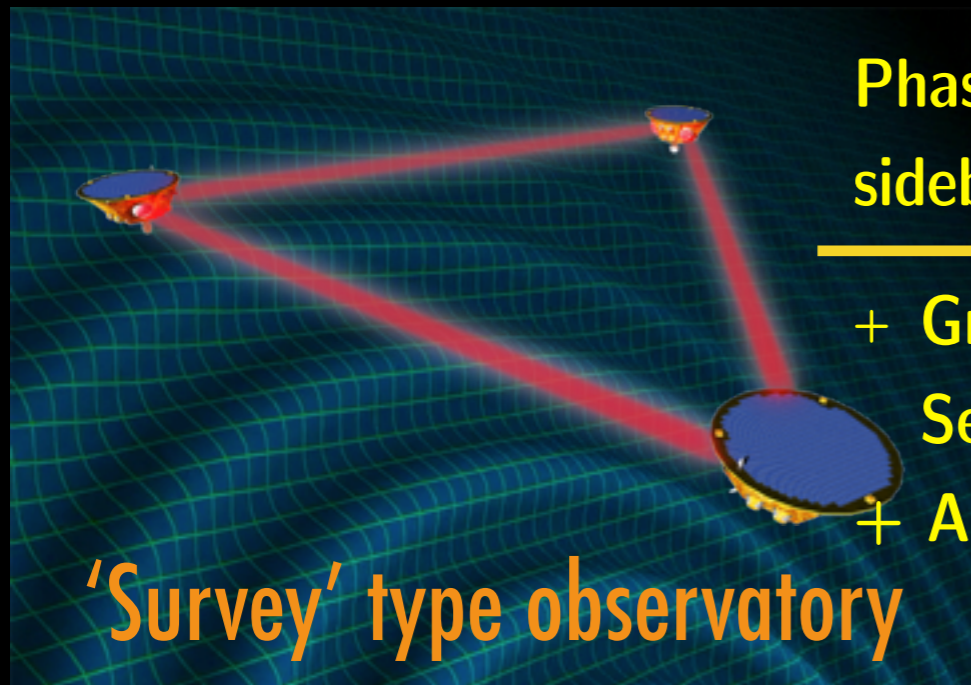
2 data channels TDI non-correlated

Data Analysis of GWs

Catalogs of GWs sources
with their waveform

Gravitational wave sources
emitting between 0.02mHz
and 100 mHz

LISA data



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reduction of laser noise

2 data channels TDI non-correlated

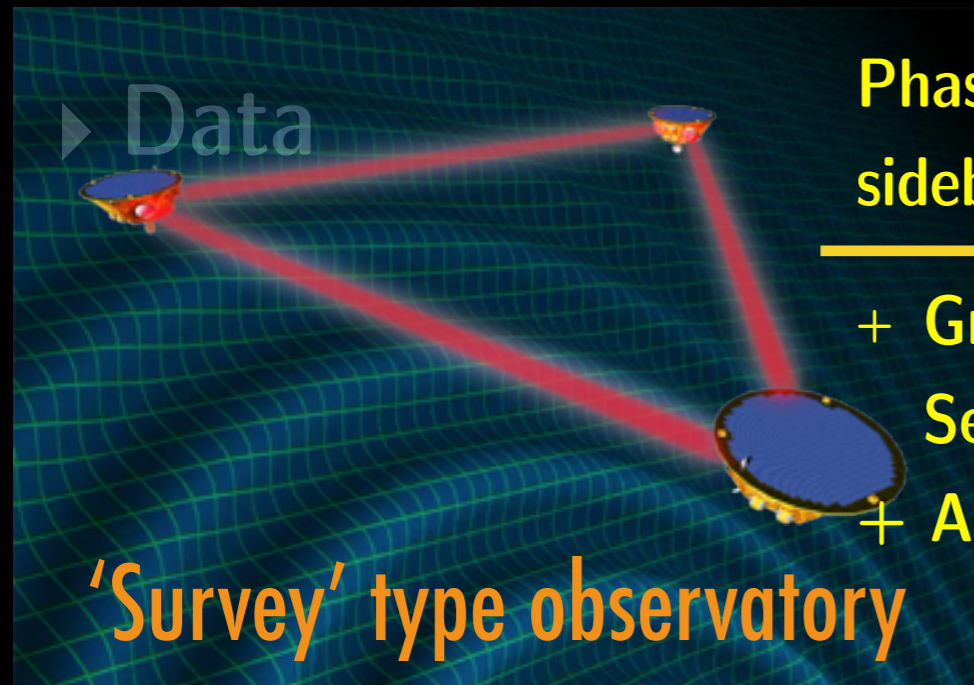
Data Analysis of GWs

Catalogs of GWs sources
with their waveform

Gravitational wave sources
emitting between 0.02mHz
and 100 mHz

= ? =

LISA data



Phasemeters (carrier, sidebands, distance)

+ Gravitational Reference Sensor

+ Auxiliary channels

L0



Calibrations corrections

Resynchronisation (clock)

Time-Delay Interferometry
reduction of laser noise

L1

2 data channels TDI non-correlated

L2

Data Analysis of GWs

L3

Catalogs of GWs sources
with their waveform

Gravitational wave sources emitting between 0.02mHz and 100 mHz

LISA data processing

▶ Data volume to be stored:

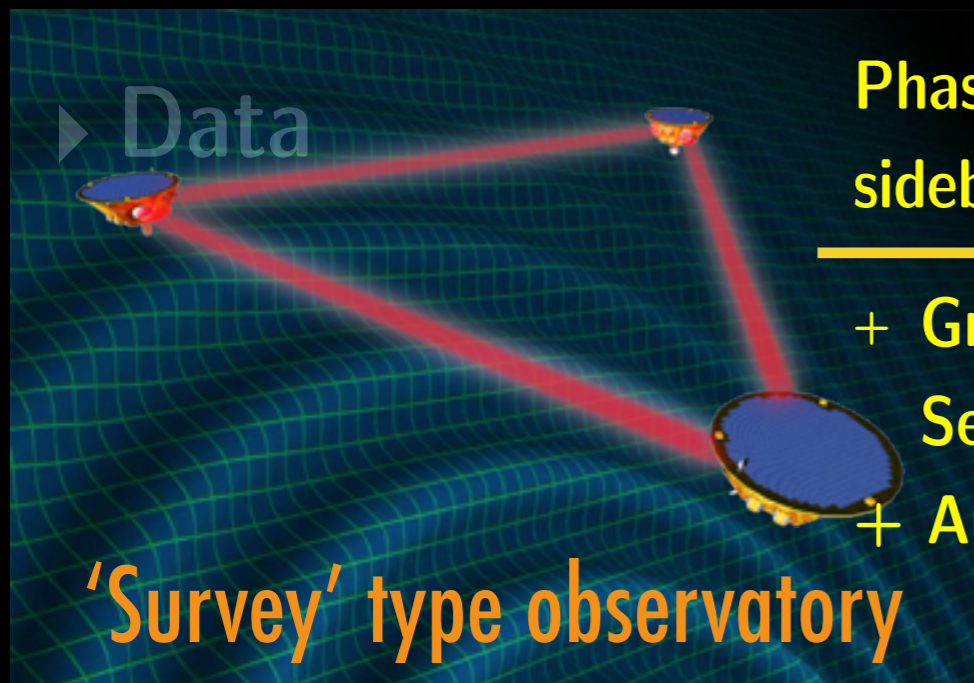
- Level L0: about 300 Mo per day
- Level L1: about 600 Mo per day
- Sub-product of the analysis: fews Go per day
- Level L2 and L3: about 6 Go per day

=> **Storages and archives are not problematic**

▶ The complexity of the data processing is in the analysis

- **all sources together in only 2 independent channels**
- **extract the parameters for a maximum number of sources**
- **could require a large CPU power**

LISA data



Phasemeters (carrier, sidebands, distance)

+ Gravitational Reference Sensor

+ Auxiliary channels

L0



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Data Analysis of GWs

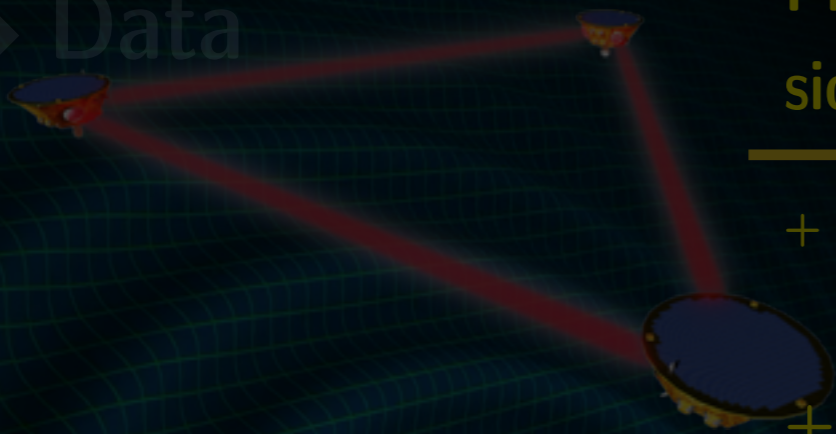
L3

Catalogs of GWs sources
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Gravitational wave sources
emitting between 0.02mHz
and 100 mHz

LISA data

► Data



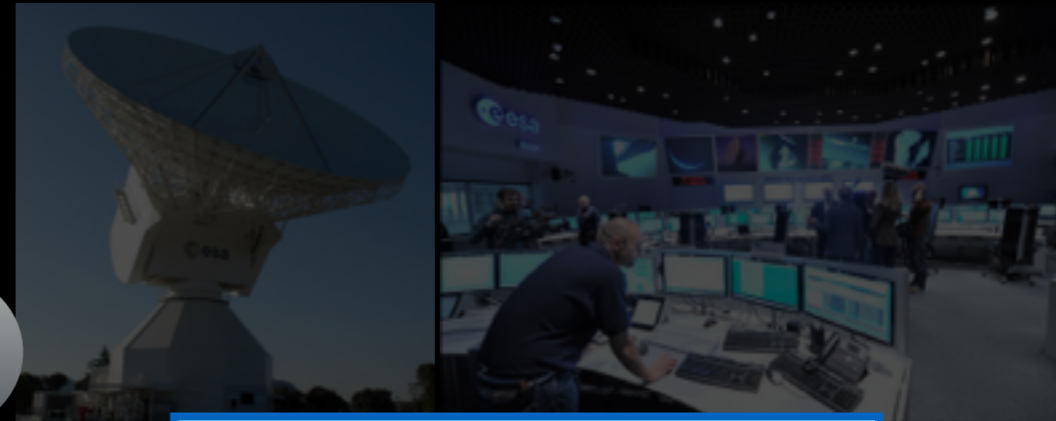
'Survey' type observatory

Phasemeters (carrier, sidebands, distance)

+ Gravitational Reference Sensor

+ Auxiliary channels

L0



Calibrations corrections

Resynchronisation (clock)

Time-Delay Interferometry
reduction of laser noise

L1

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L2

Data Analysis of GWs

L3

Catalogs of GWs sources
with their waveform

Gravitational wave sources
emitting between 0.02mHz
and 100 mHz

From L0 to L1

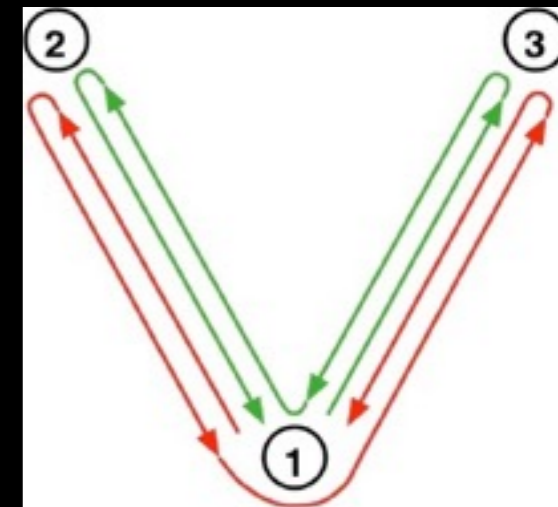
- ▶ **Consolidate** the data
- ▶ **Check data quality**
- ▶ **Calibrations and correction of data (amplitude & time):**
=> convert data in usable measurements
- ▶ **Correct** the main measurements by subtracting various effects measured using other channels (a la LISAPathfinder):
 - ex: subtract cross-talk effects
- ▶ **Synchronise time references (clock)** between the 3 spacecrafts

From L0 to L1: TDI

► Time Delay Interferometry:

- Combine delayed measurements to reduce laser noises, optical bench noises, clock noises, ... ?
- Algebraic development : many combinations (generators)

$$\begin{aligned}
 X &= -s_1 - D_3 s'_2 - D_3 D_{3'} s'_1 - D_3 D_{3'} D_{2'} s_3 \\
 &\quad + s'_1 + D_{2'} s_3 - D_{2'} D_2 s_1 - D_{2'} D_2 D_3 s_3 \\
 &\simeq 0
 \end{aligned}$$

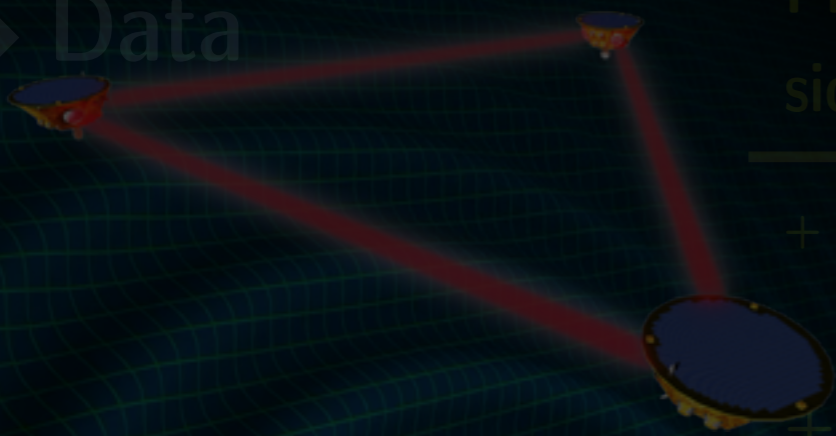


- Different precisions level

- 1st generation : rigid formation of LISA : $D_{i'} s = D_i s$,
- 1.5 generation : Sagnac effect : $D_{i'} s \neq D_i s$ but $D_j D_i s = D_i D_j s$,
- 2nd generation : flexing and Sagnac effect : $D_j D_i s \neq D_i D_j s$

LISA data

► Data



'Survey' type observatory

Phasemeters (carrier, sidebands, distance)

+ Gravitational Reference Sensor

+ Auxiliary channels

L0



Calibrations corrections

Resynchronisation (clock)

Time-Delay Interferometry
reduction of laser noise

L1

2 data channels TDI non-correlated

L2

Data Analysis of GWs

L3

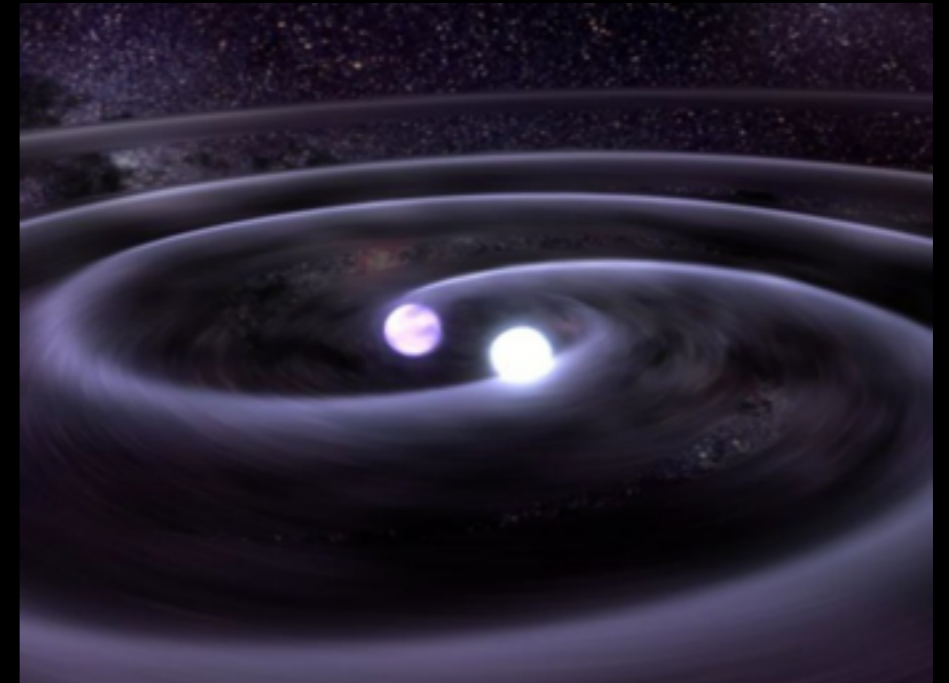
Catalogs of GWs sources
with their waveform

GW sources

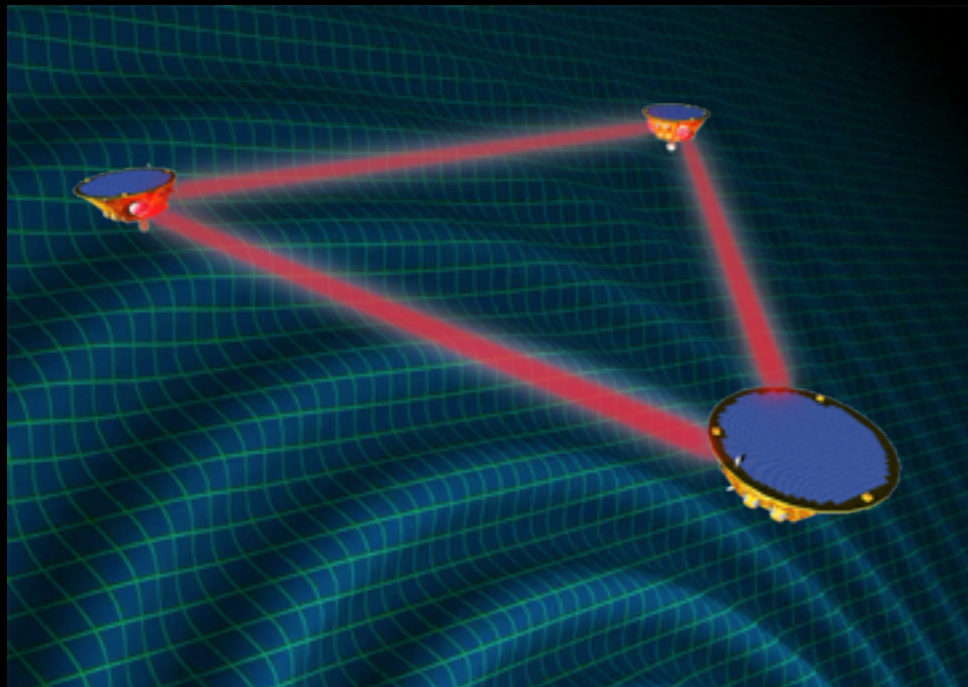
- 6×10^7 galactic binaries
- 10-100/year SMBHBs
- 10-1000/year EMRIs
- large number of Stellar Origin BH binaries (LIGO/Virgo)
- Cosmological backgrounds
- Unknown sources

Galactic binaries

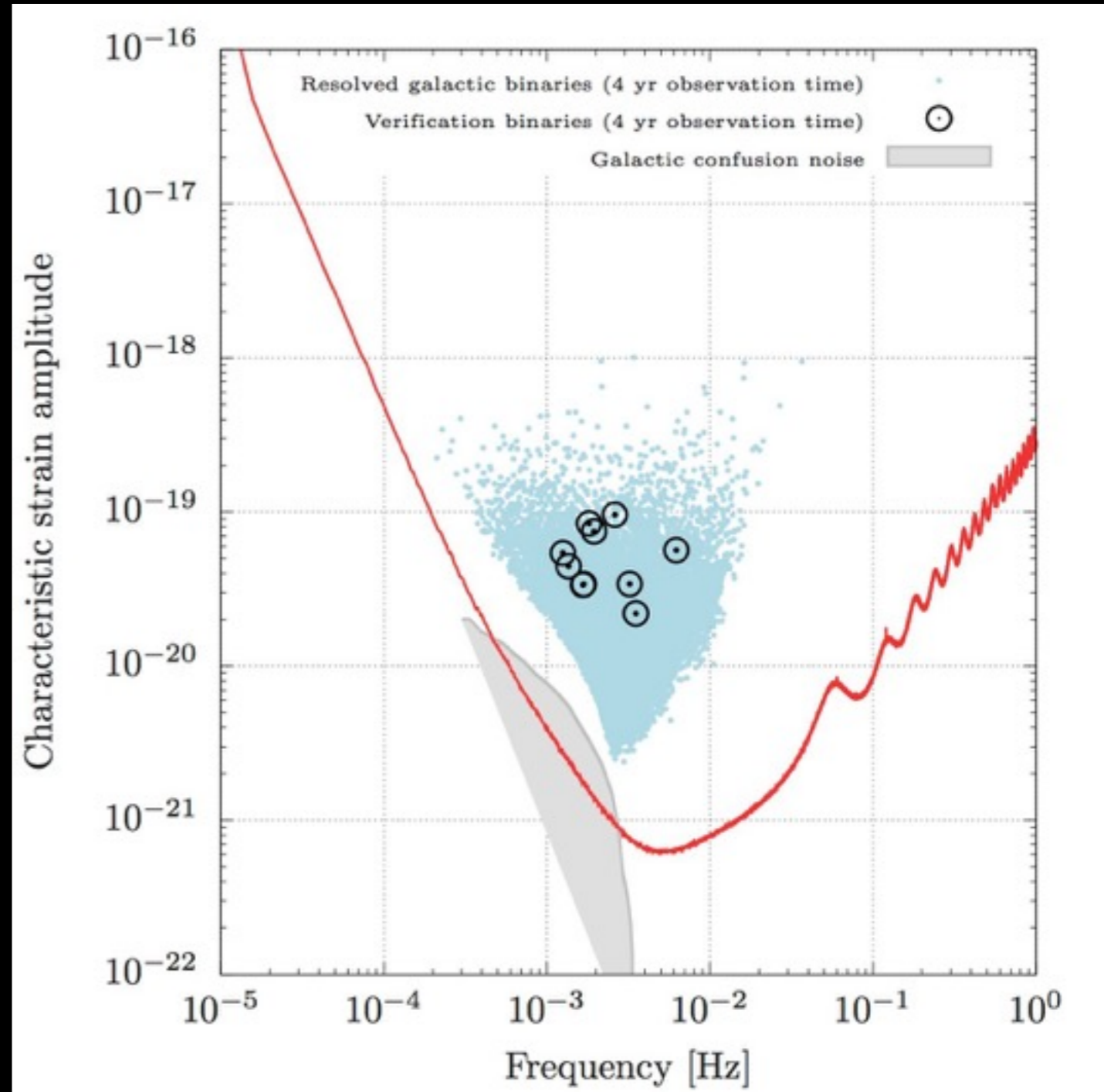
- ▶ **Gravitational wave:**
 - quasi monochromatic
- ▶ **Duration: permanent**
- ▶ **Signal to noise ratio:**
 - detected sources: 7 - 1000
 - confusion noise from non-detected sources
- ▶ **Event rate:**
 - 25 000 detected sources
 - more than 10 guaranteed sources (verification binaries)



Galactic binaries



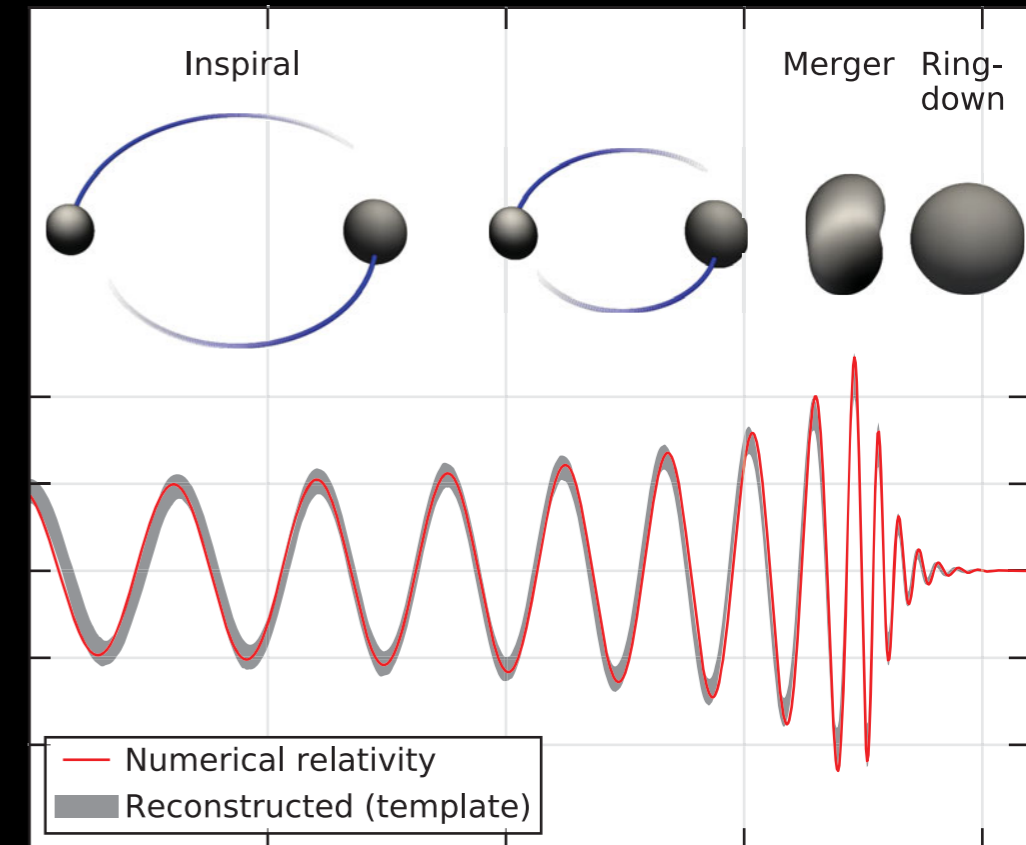
GW sources
- 6×10^7 galactic binaries



Super Massive Black Hole Binaries

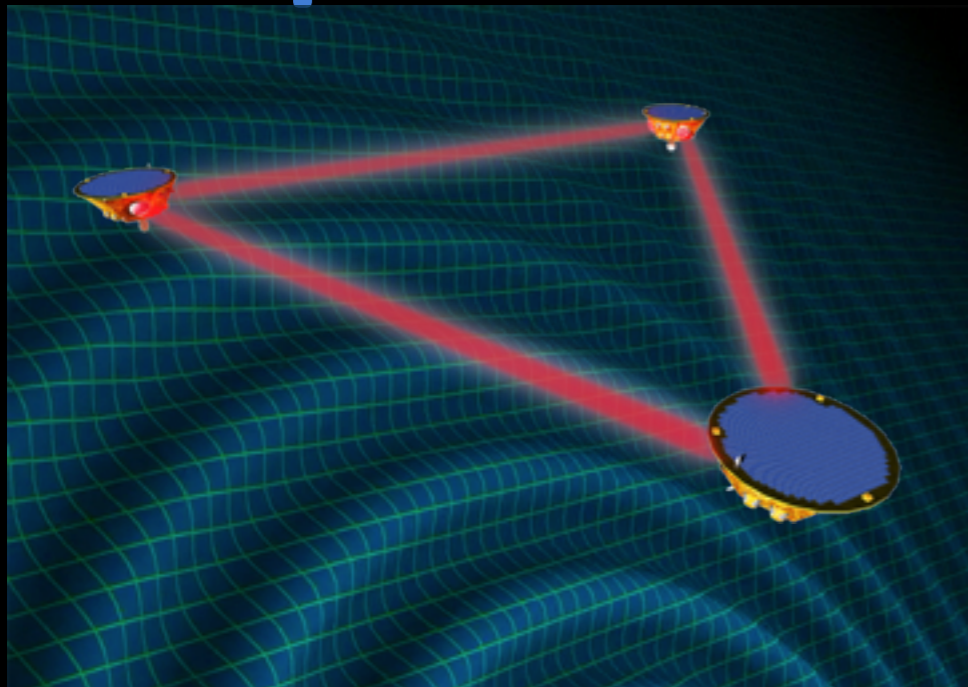
► Gravitational wave:

- Inspiral: Post-Newtonian,
- Merger: Numerical relativity,
- Ringdown: Oscillation of the resulting MBH.



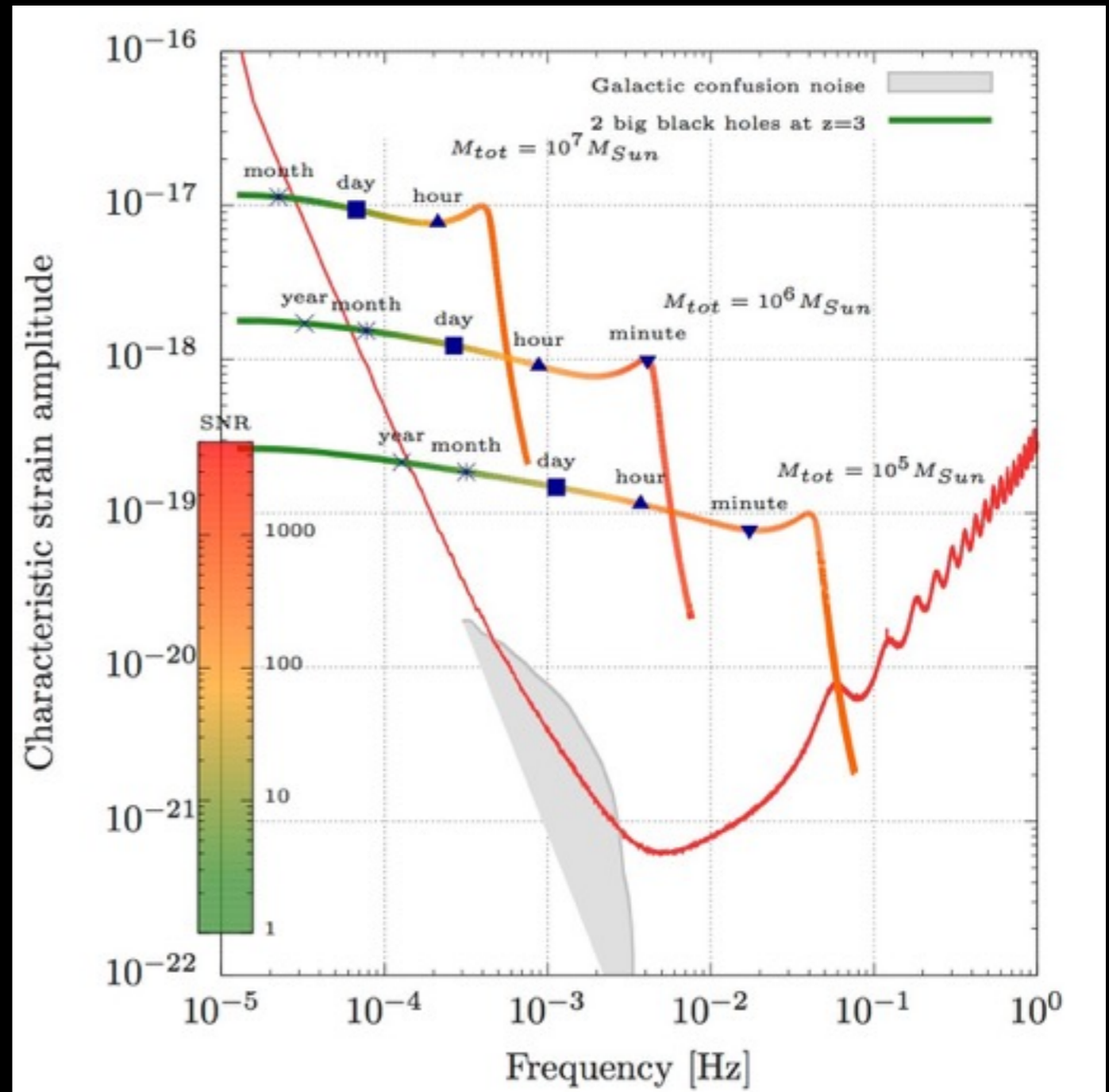
- Duration: between few hours and several months
- Signal to noise ratio: until few thousands
- Event rate: 10-100/year

Super Massive Black Hole Binaries



OG sources

- 6×10^7 galactic binaries
- 10-100/year SMBHBs



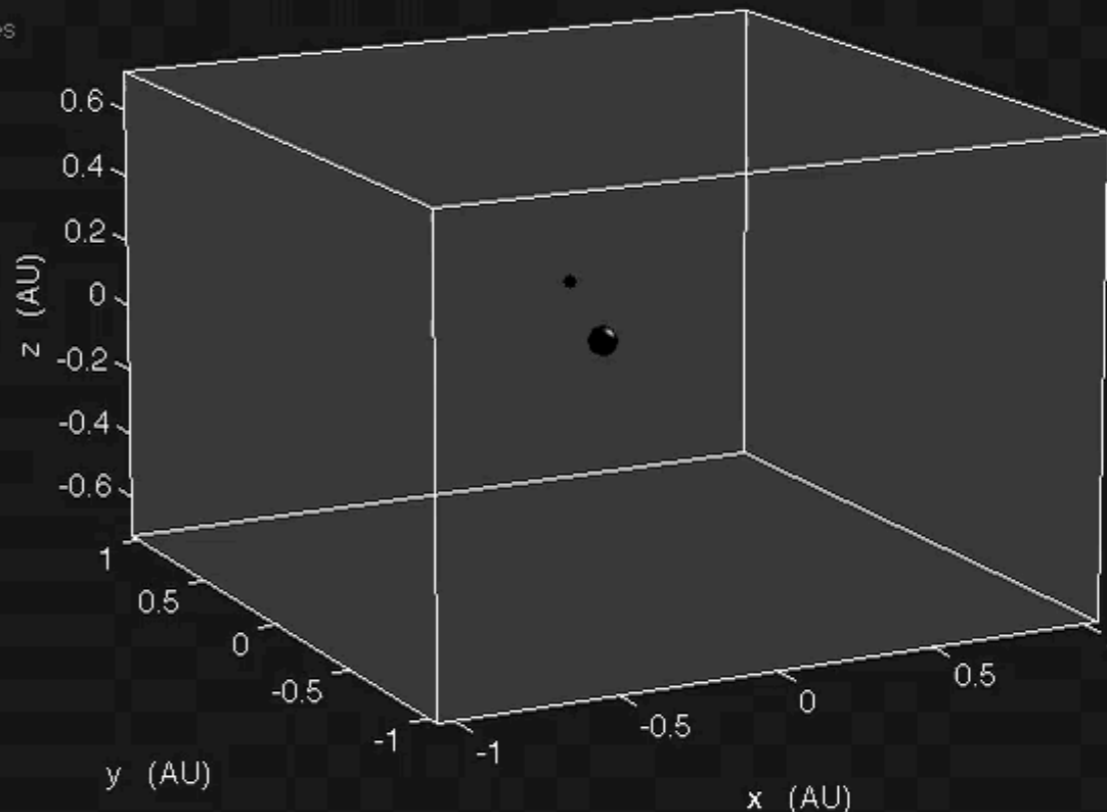
EMRIs

- ▶ **Gravitational wave:**
 - very complex waveform
 - No precise simulation at the moment
- ▶ **Duration: about 1 year**
- ▶ **Signal to Noise Ratio: from tens to few hundreds**
- ▶ **Event rate:**
from few events per year to few hundreds

Large black hole:
shown to scale
3,000,000 solar masses
90% maximal spin

Small black hole:
shown enlarged
270 solar masses
negligible spin

Trace duration:
1 day



Steve Drasco
Max Planck Institute
for Gravitational Physics
(Albert Einstein Institute)
sdrasco@aei.mpg.de

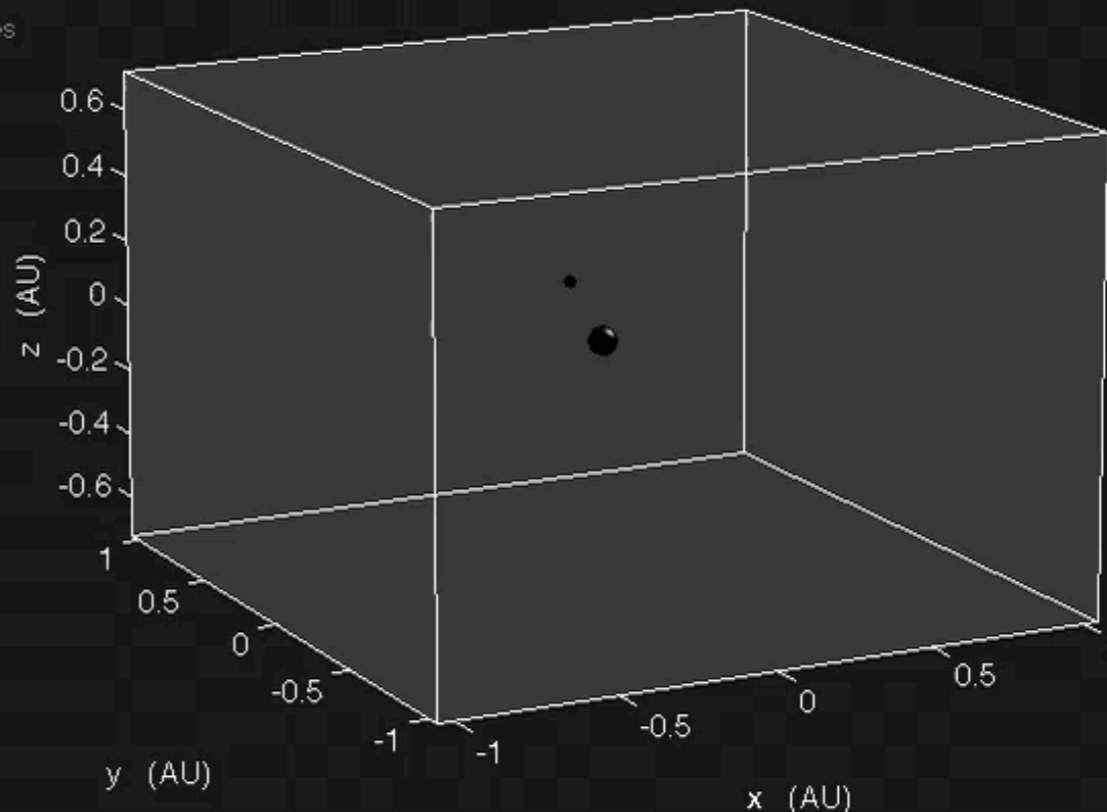
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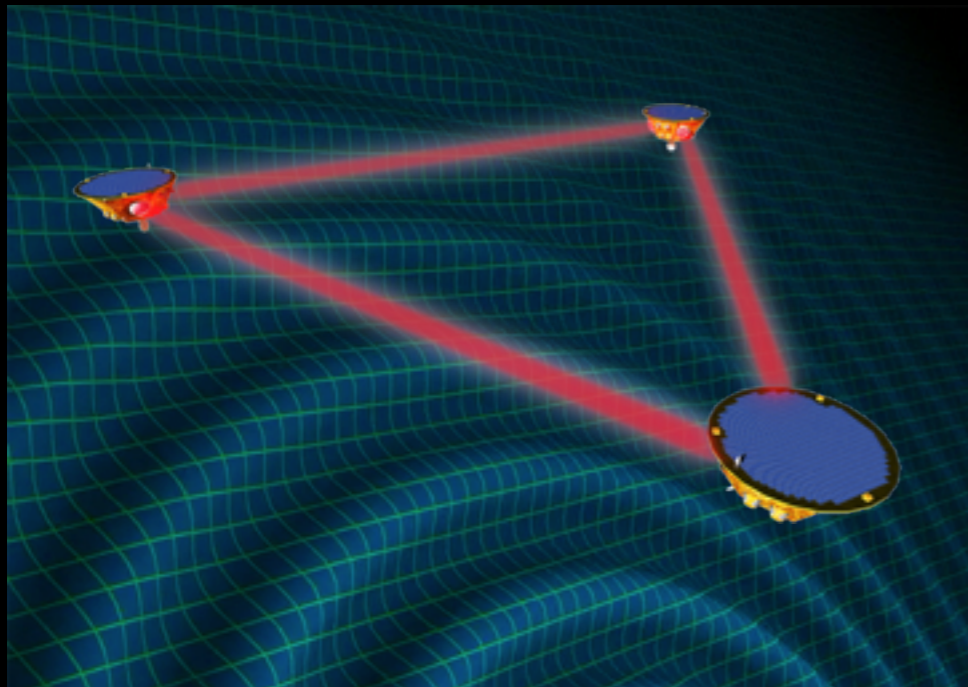
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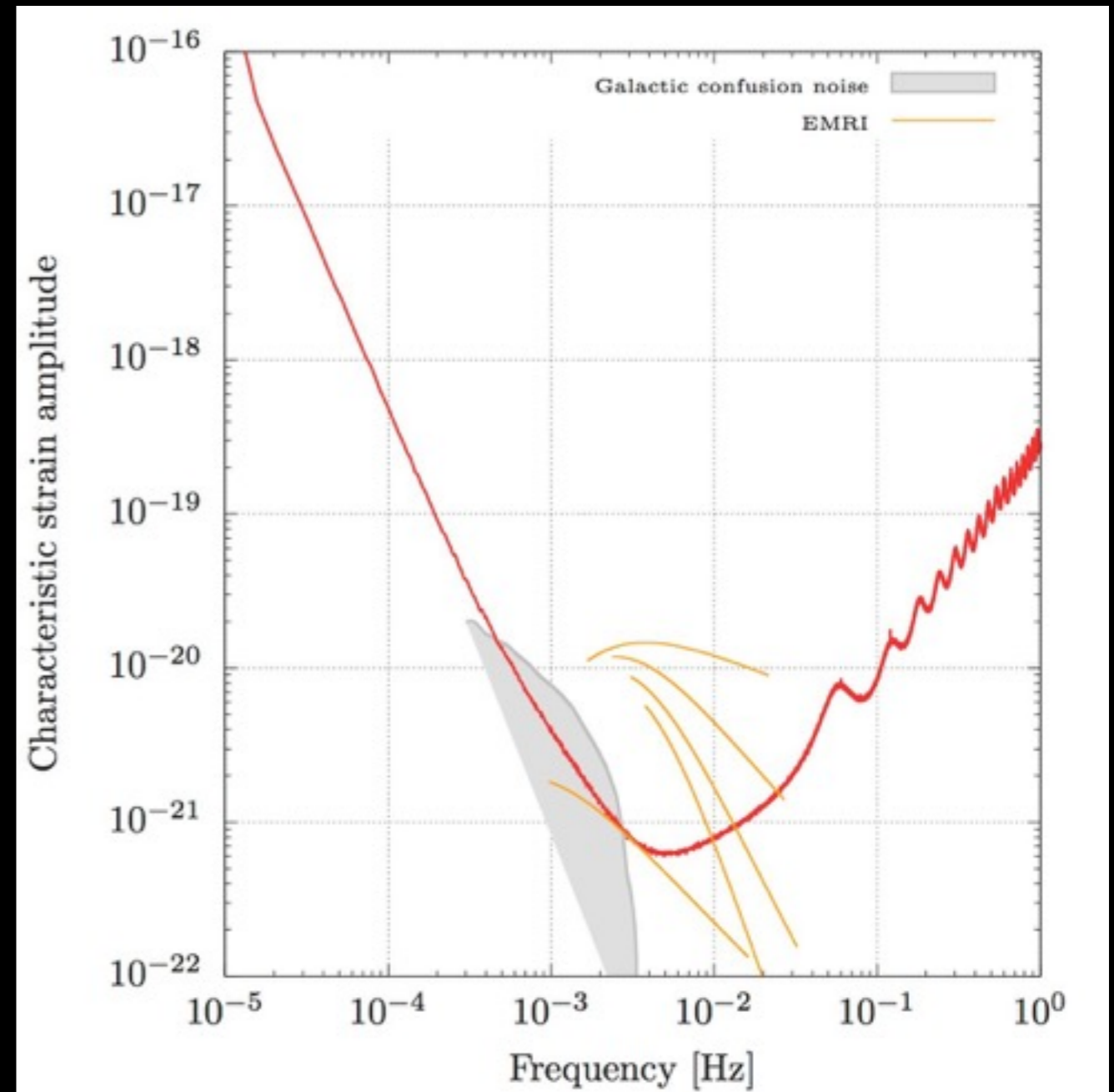
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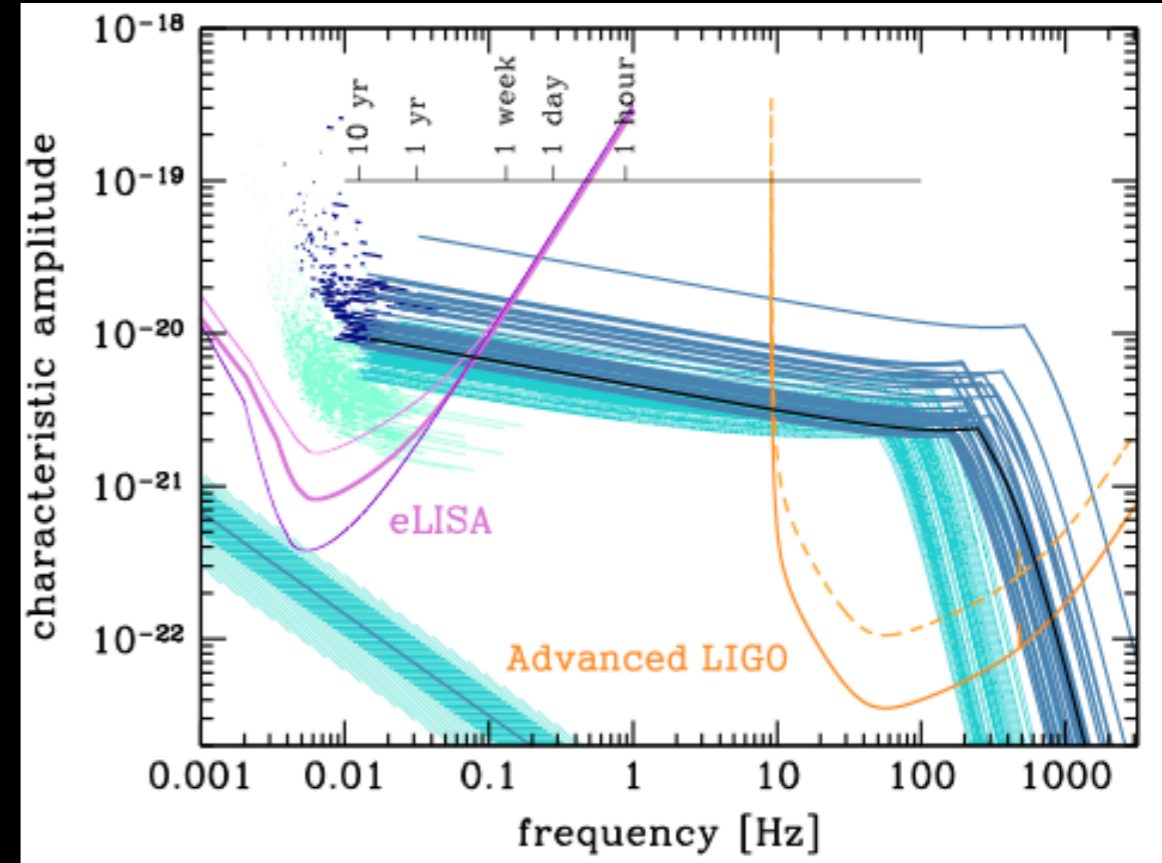
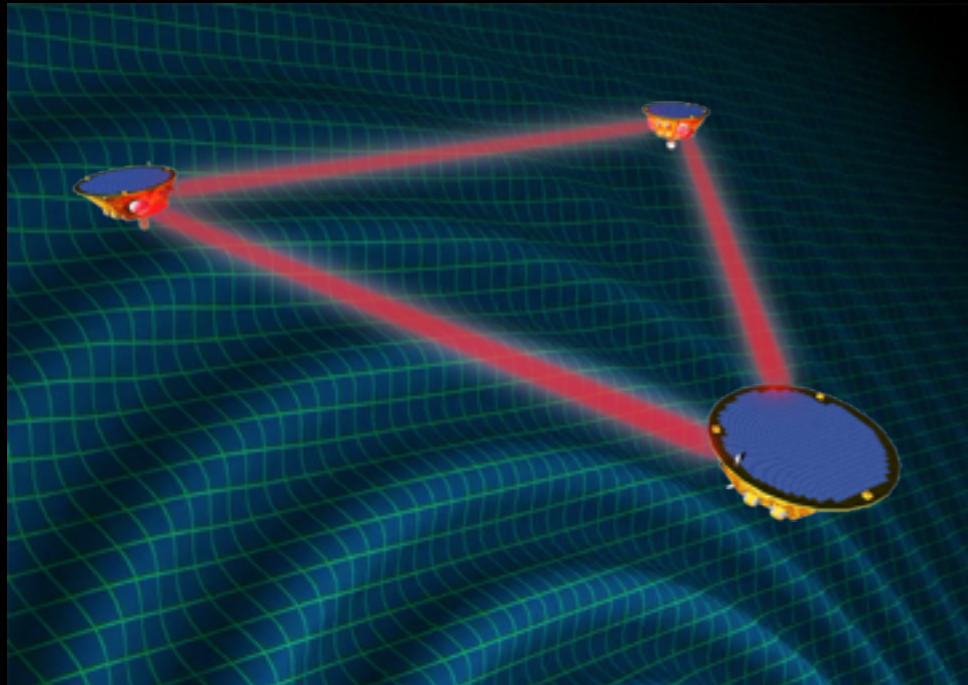


OG sources

- 6×10^7 galactic binaries
- 10-100/year SMBHBs
- 10-1000/years EMRIs

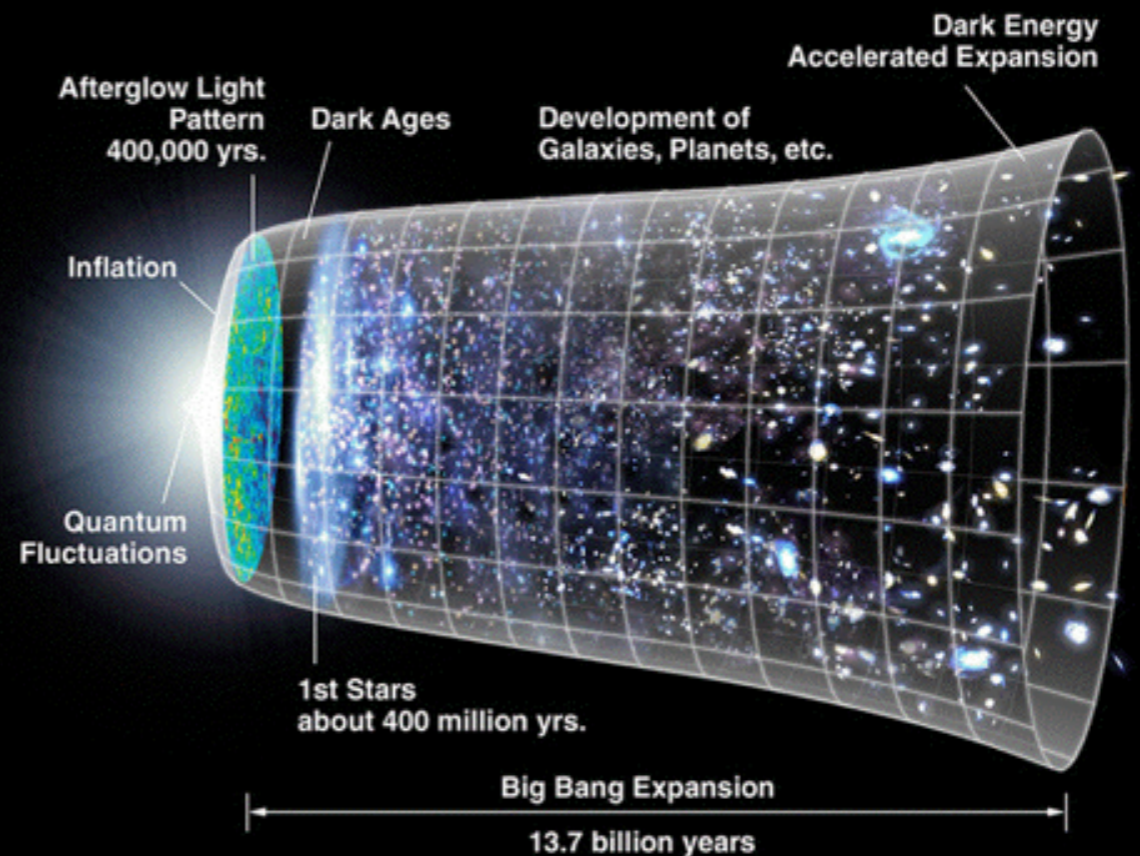


Others sources

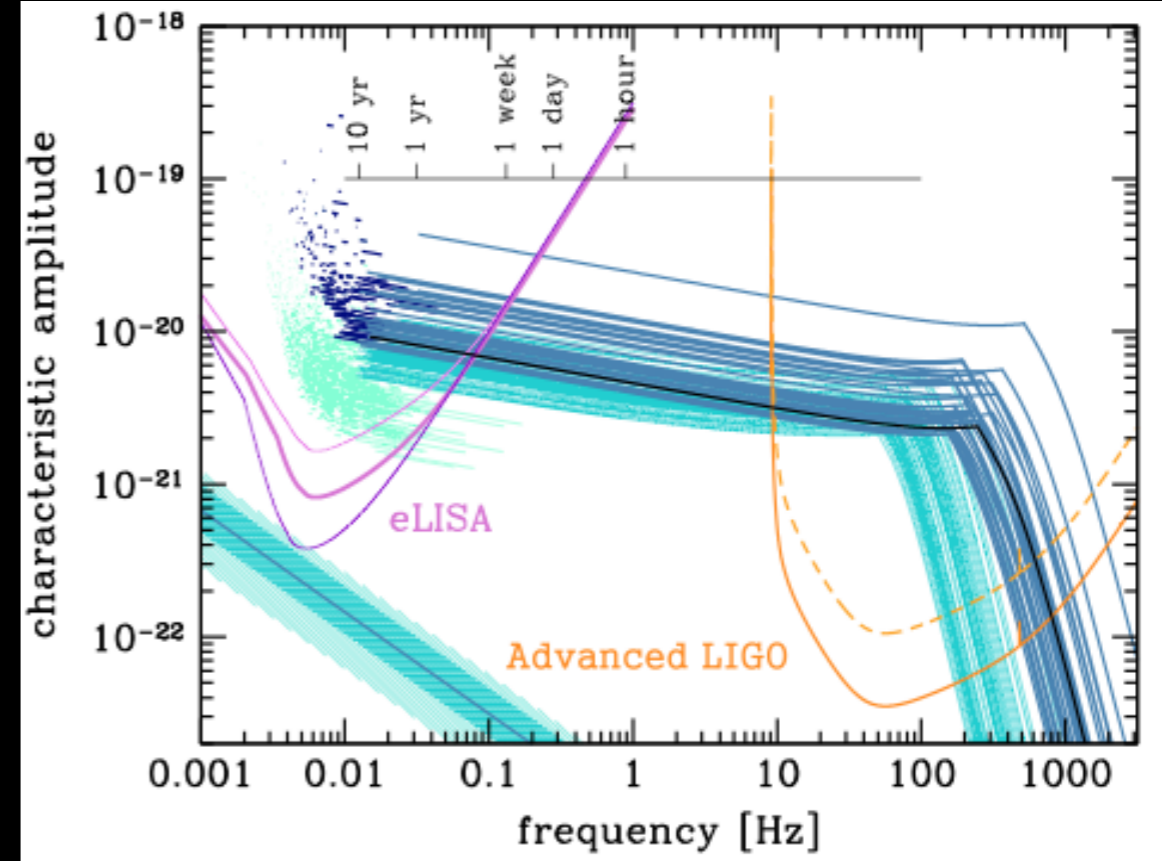
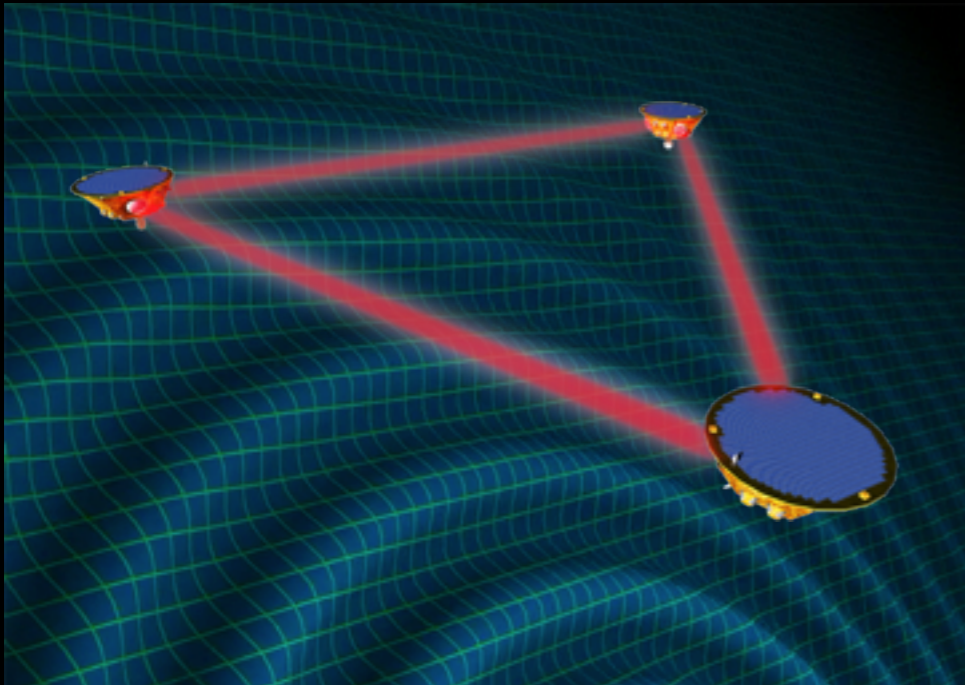


GW sources

- 6×10^7 galactic binaries
- 10-100/year SMBHBs
- 10-1000/year EMRIs
- large number of Stellar Origin BH binaries (LIGO/Virgo)
- Cosmological backgrounds
- Unknown sources

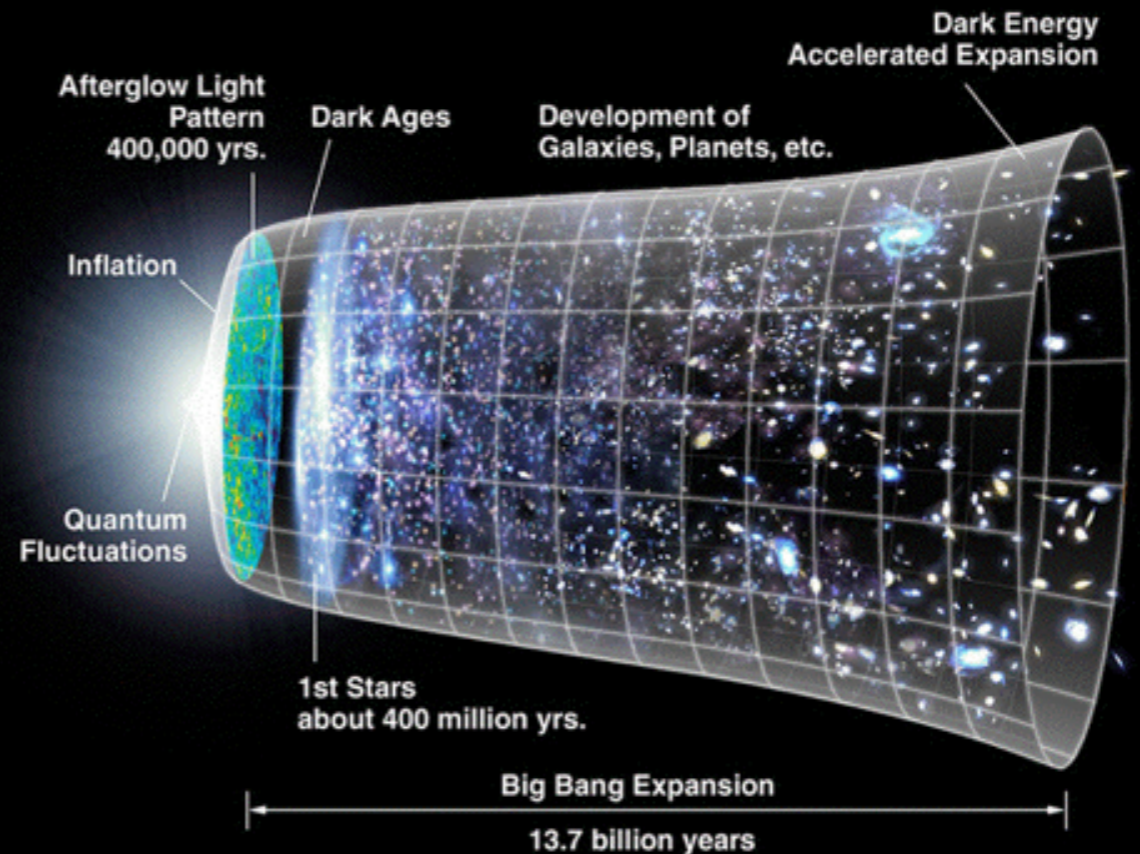


Others sources



GW sources

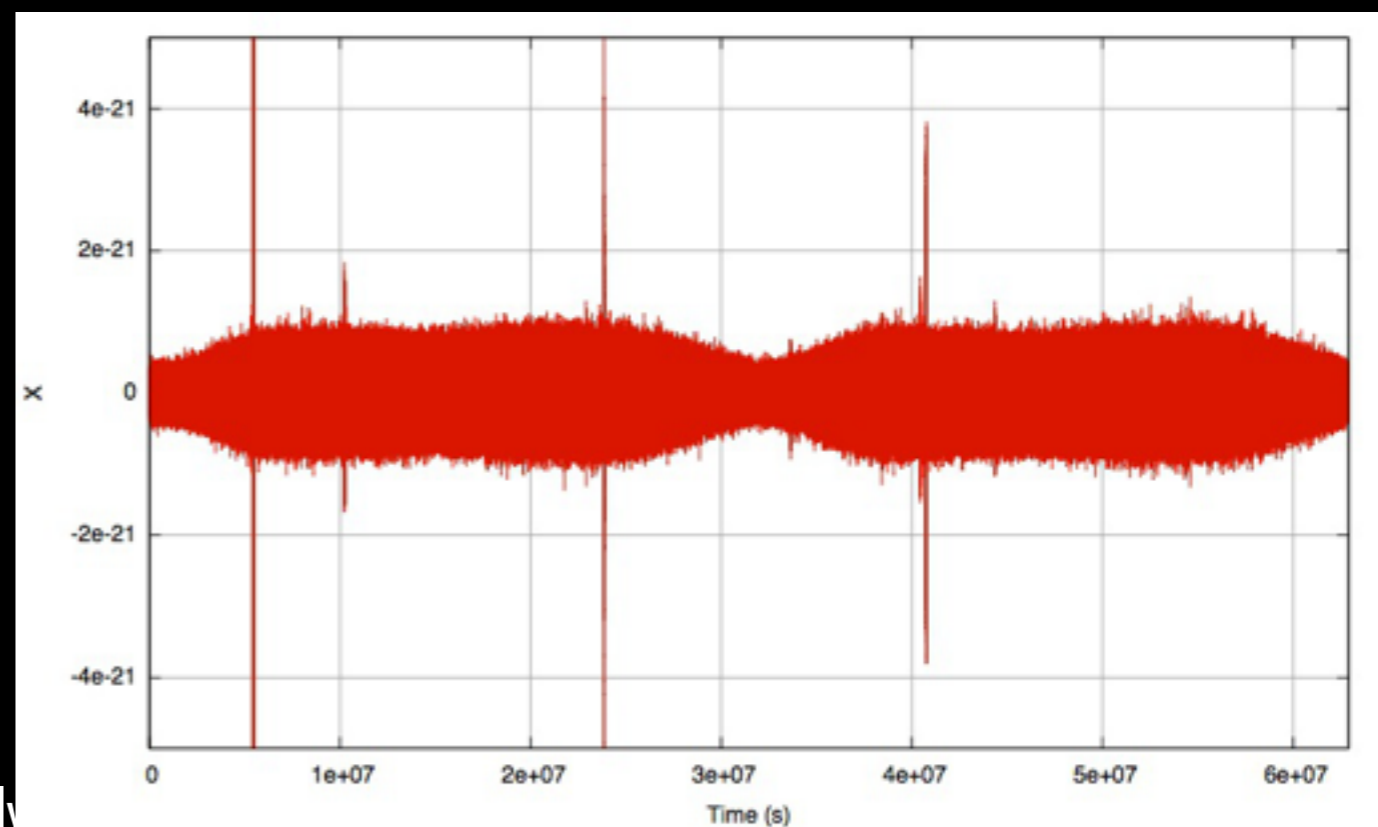
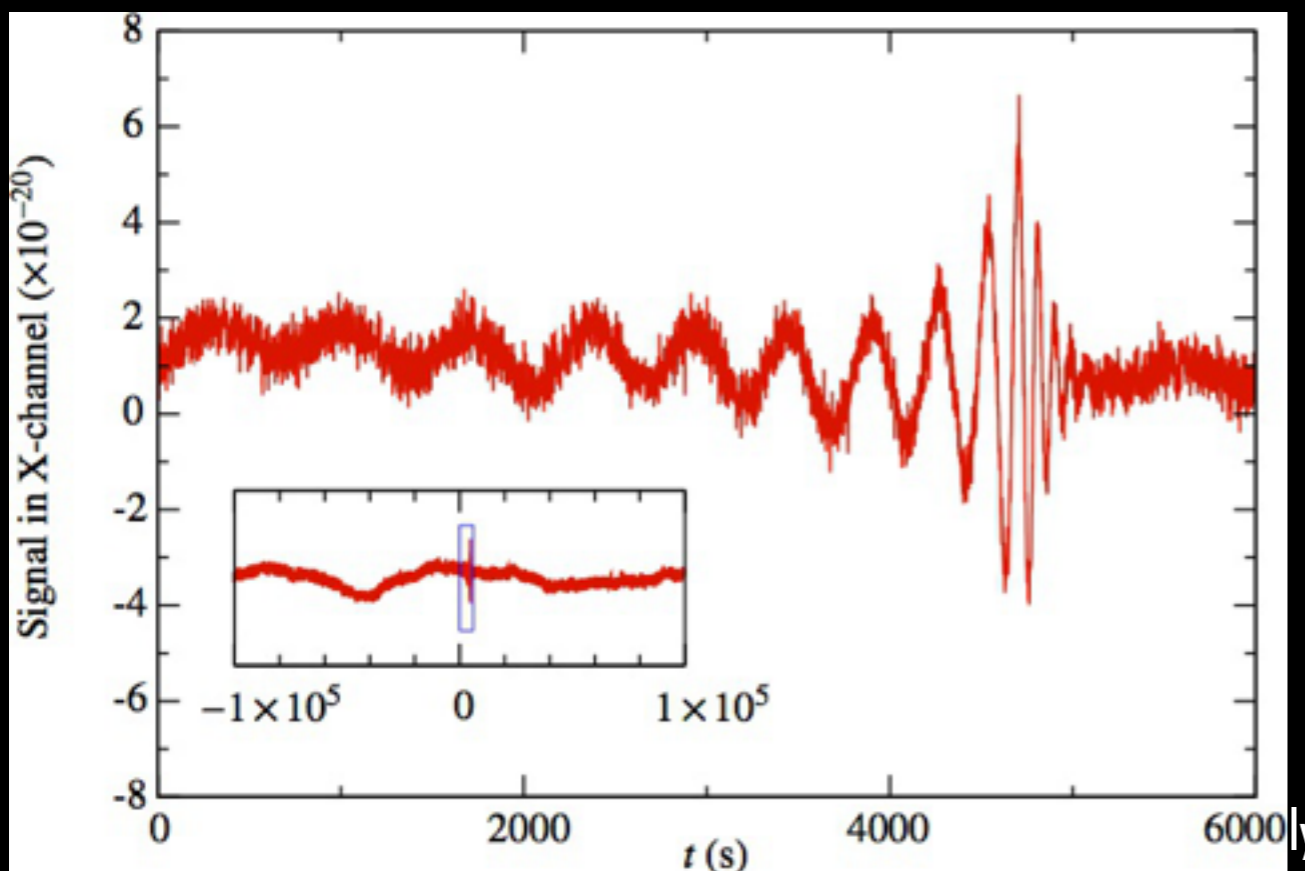
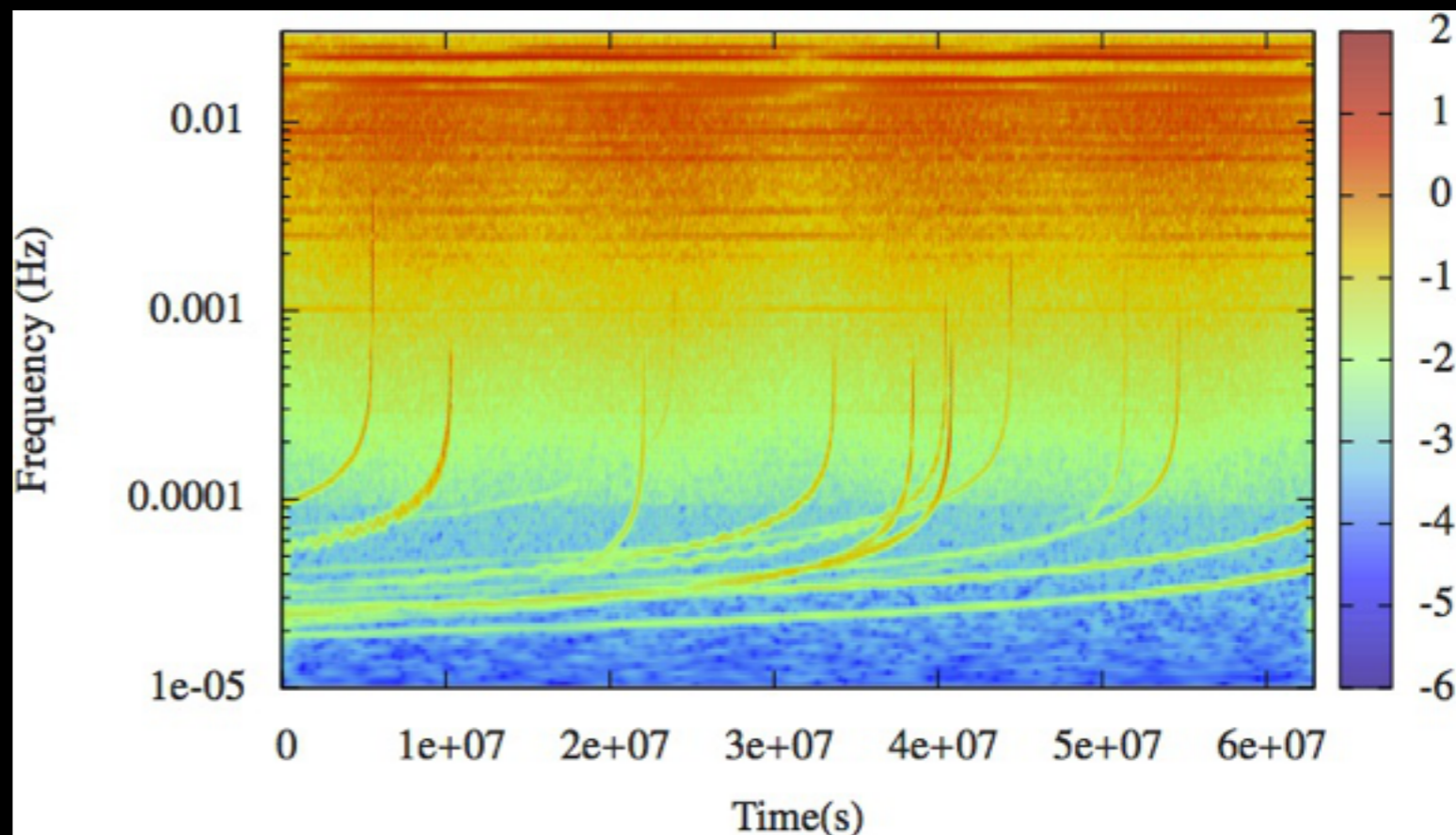
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- Cosmological backgrounds
- Unknown sources



GWs in LISA data

▶ Example of simulated data (LISACode):

- about 100 SMBHs,
- Galactic binaries



Global analysis

- ▶ How many parameters in a global model ?
 - Full binary system of black holes: 17 parameters
 - 11 internal param.: masses (2), phase(1), spins(6), eccentricity(2)
 - 6 external param.: position(3), orientation(2), reference time(1)
 - Galactic binaries: quasi-monochromatic binary \Rightarrow 9 parameters
 - f , df/dt , d^2f/dt^2 , phase, t_{ref} , sky position(2), orientation(2)
 - SMBHB & EMRIs \Rightarrow 17 parameters
 - SOBH: spin parameters can be neglected \Rightarrow 11 parameters

Global analysis

- ▶ How many parameters in a global model ?
 - Cosmic string cusps: 5 parameters:
 - sky position(2), polarisation(1), amplitude(1), reference time(1)
 - Stochastic background: few 10 parameters
- ▶ Total number of parameters:
 - 25000 GBs x 9
 - + 4x(50 SMBHBs + 100 EMRIs) x 17
 - + 100 SOBHBs x 11
 - + 40 cosmic string x 5
 - + 10 (stochastic background)
 - = about 240 000 parameters to estimate !

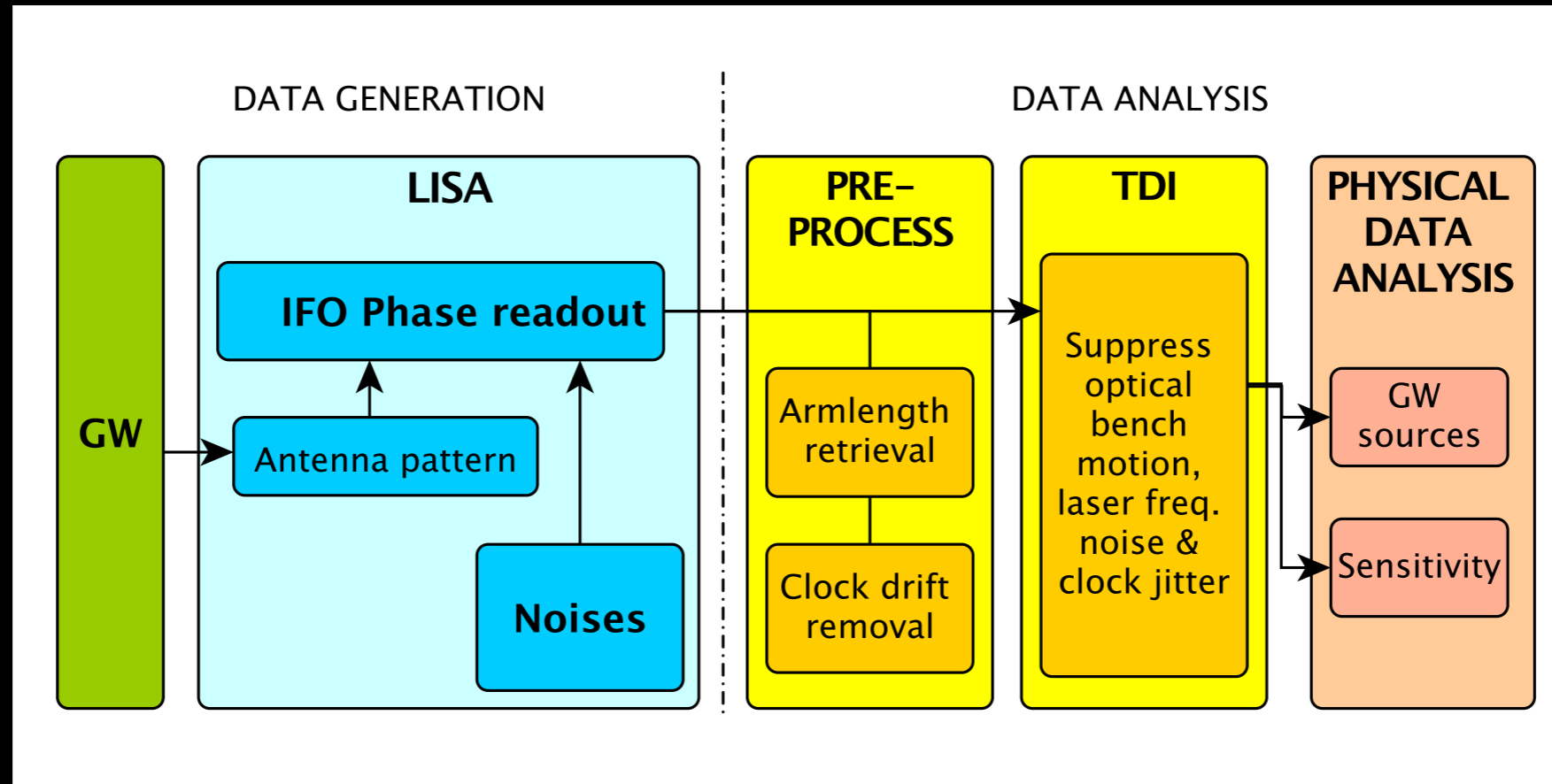
Global analysis

- ▶ Global analysis seems impossible !
- ▶ Iterative process, step by step
 - identify most powerful sources
 - subtract them from the data
 - search for the next ones
 - ...

=> Could be quite challenging.

- Other ideas are welcome

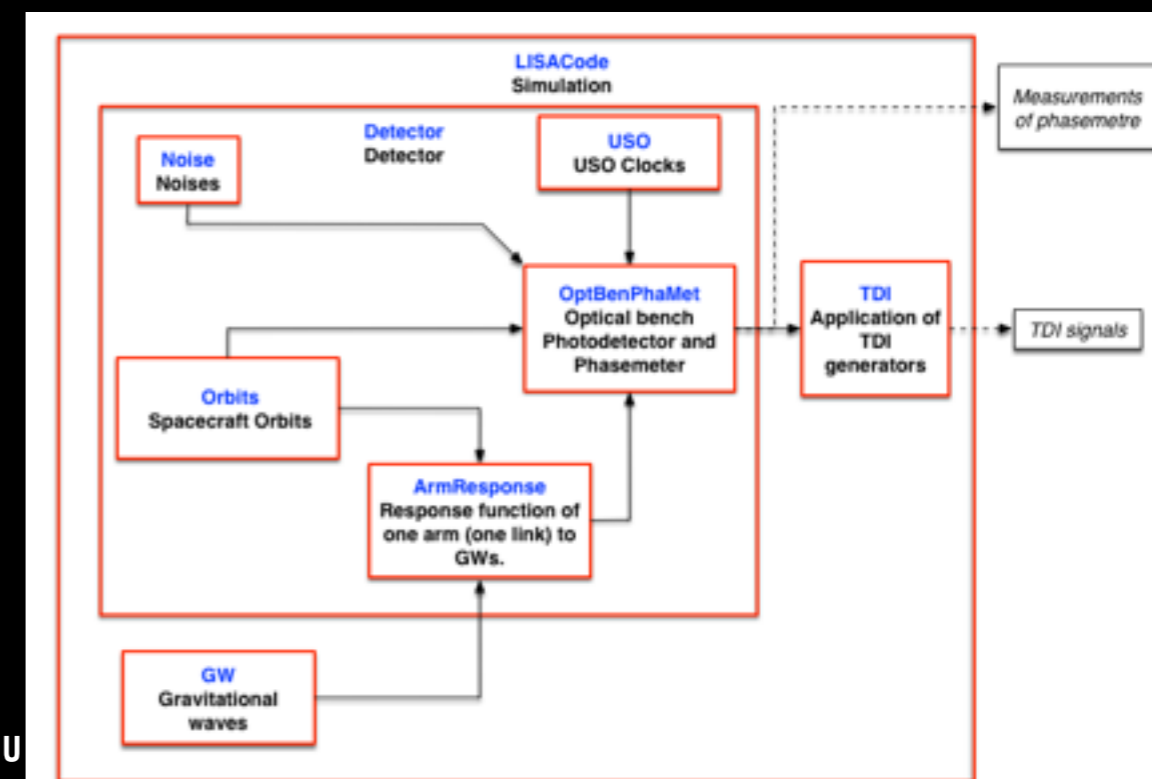
Simulation



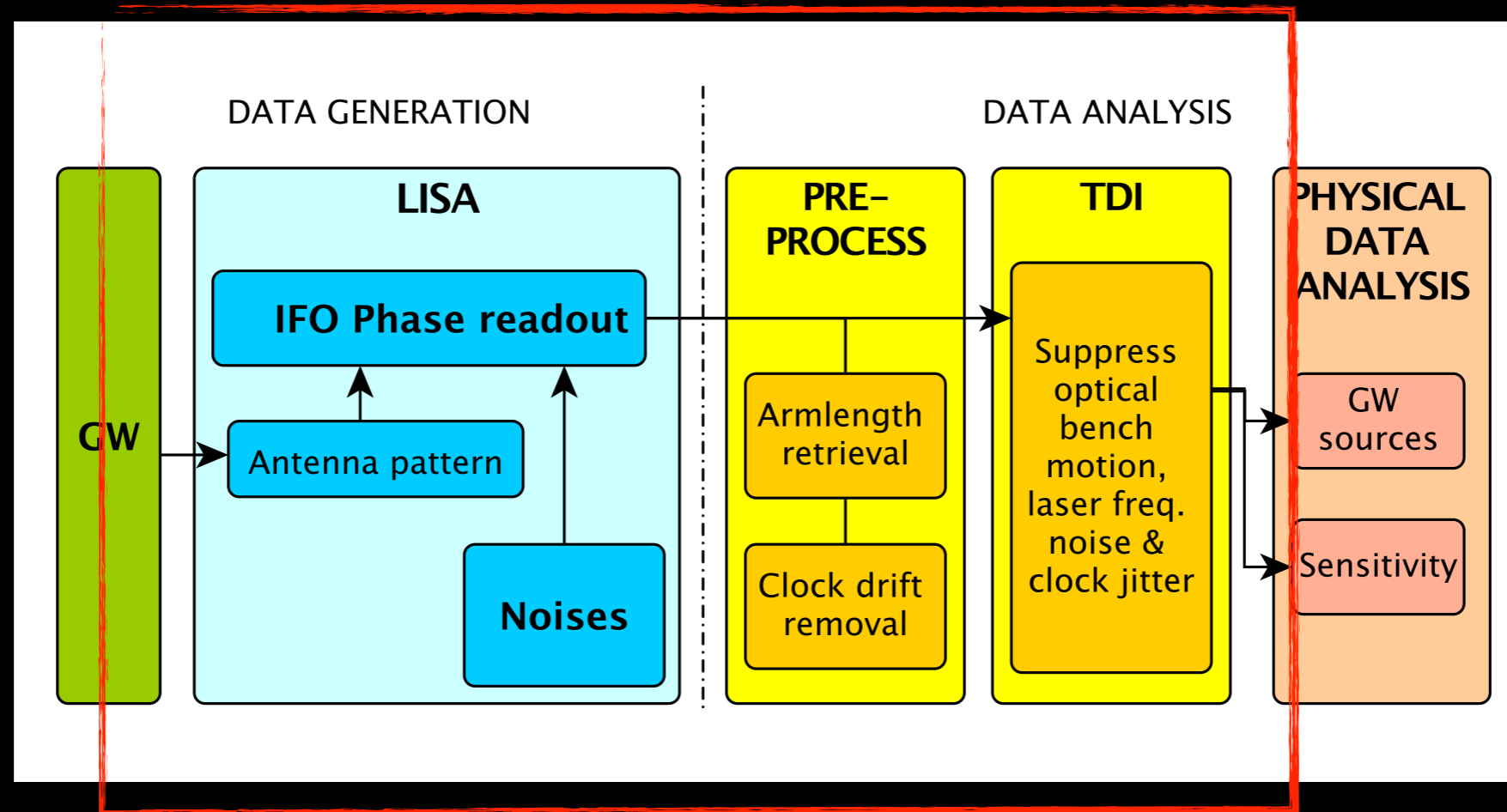
▶ LISACode (new modular version in development)

▶ 2 complementary simulators:

- TDISim (check TDI)
- LISADyn (3D dynamic)



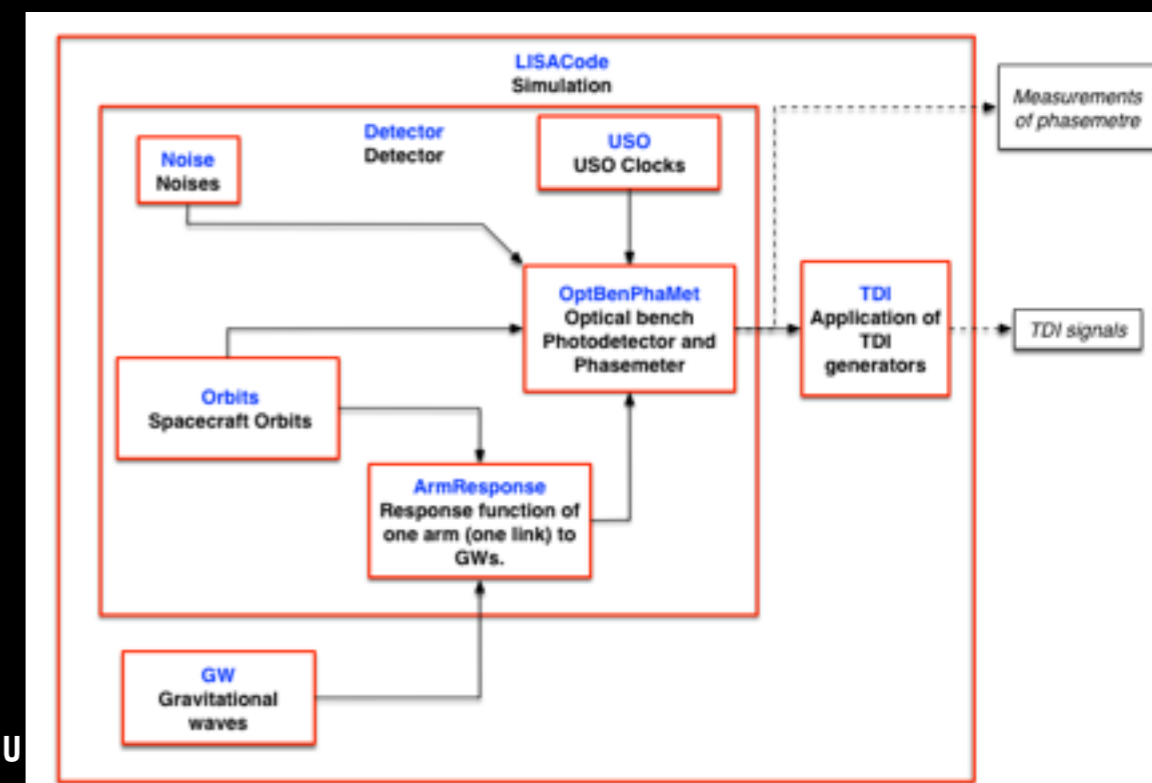
Simulation



▶ LISACode (new modular version in development)

▶ 2 complementary simulators:

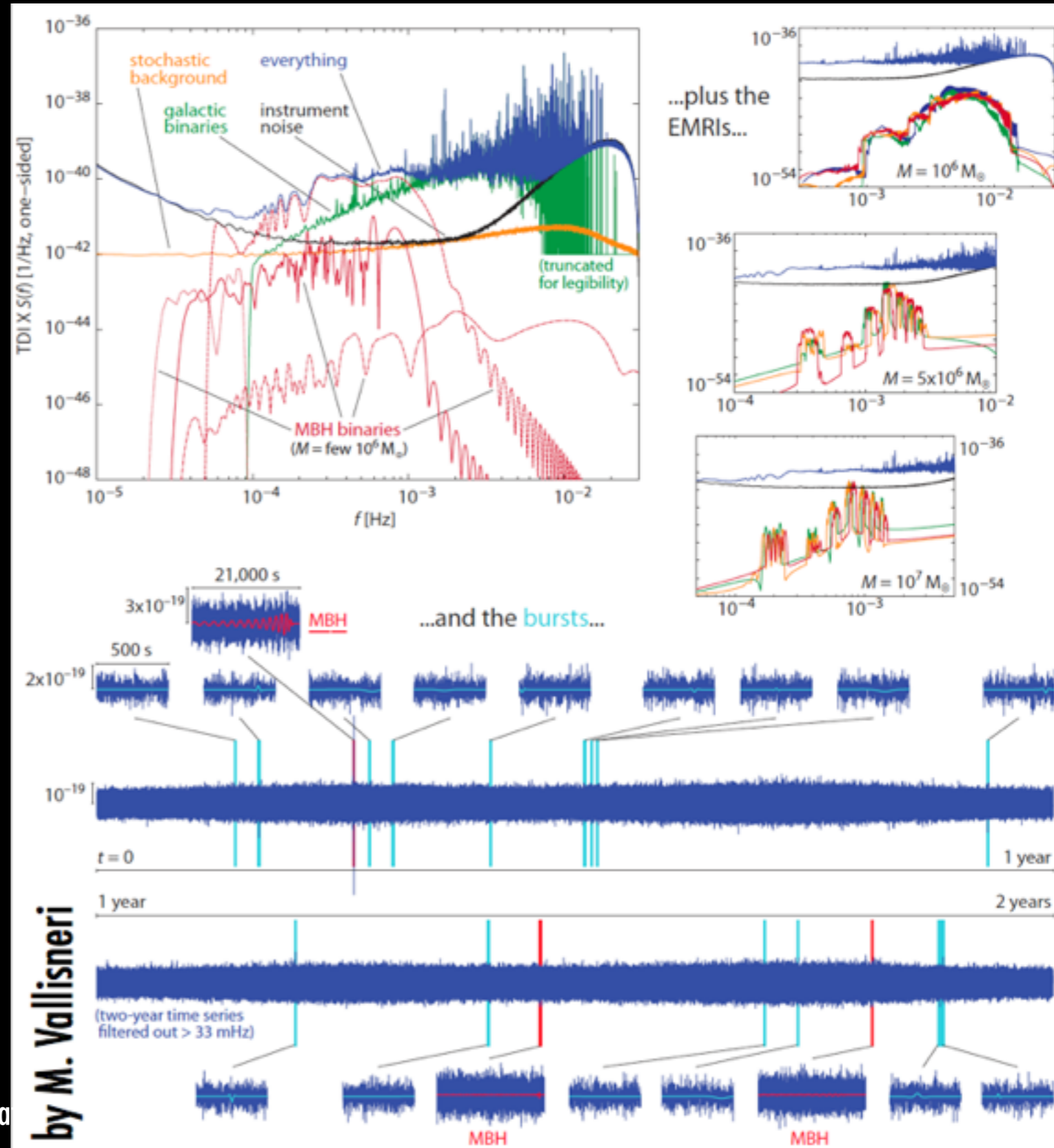
- TDISim (check TDI)
- LISADyn (3D dynamic)



LISA Data Challenges

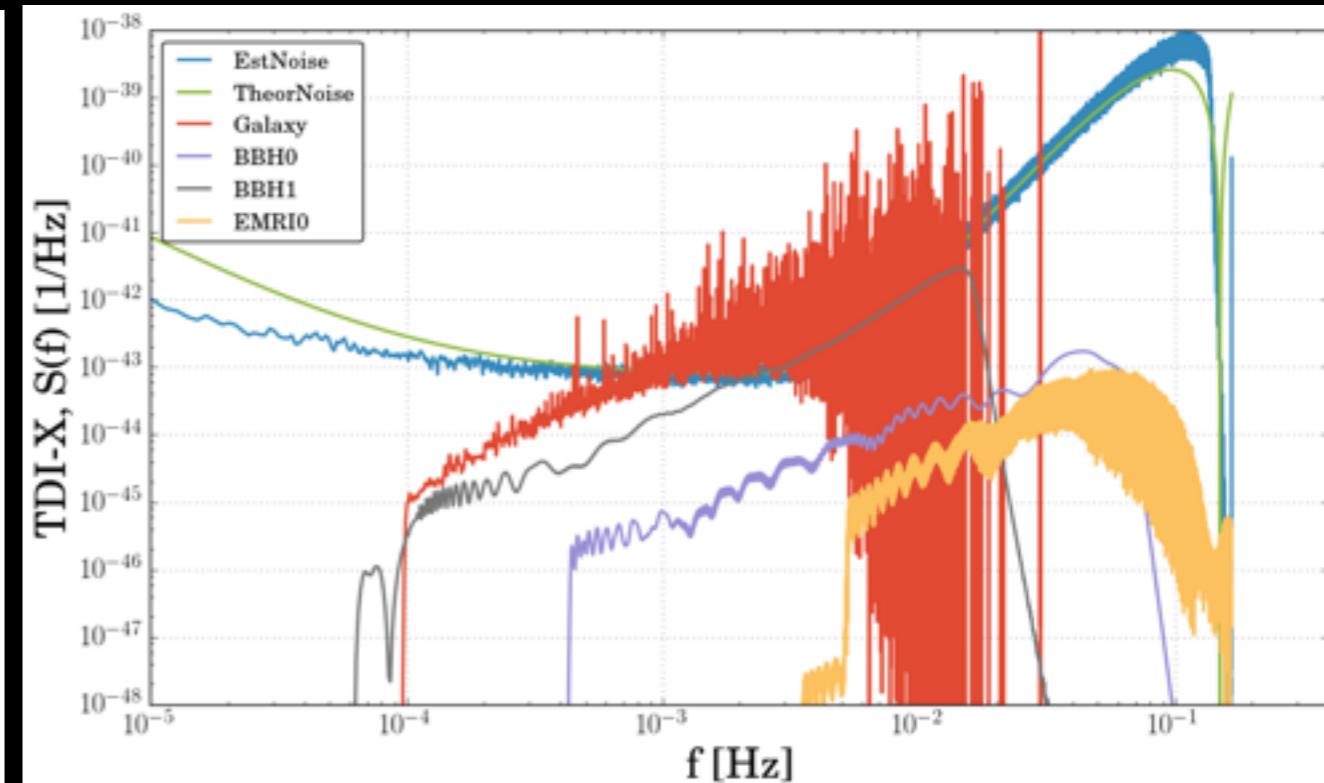
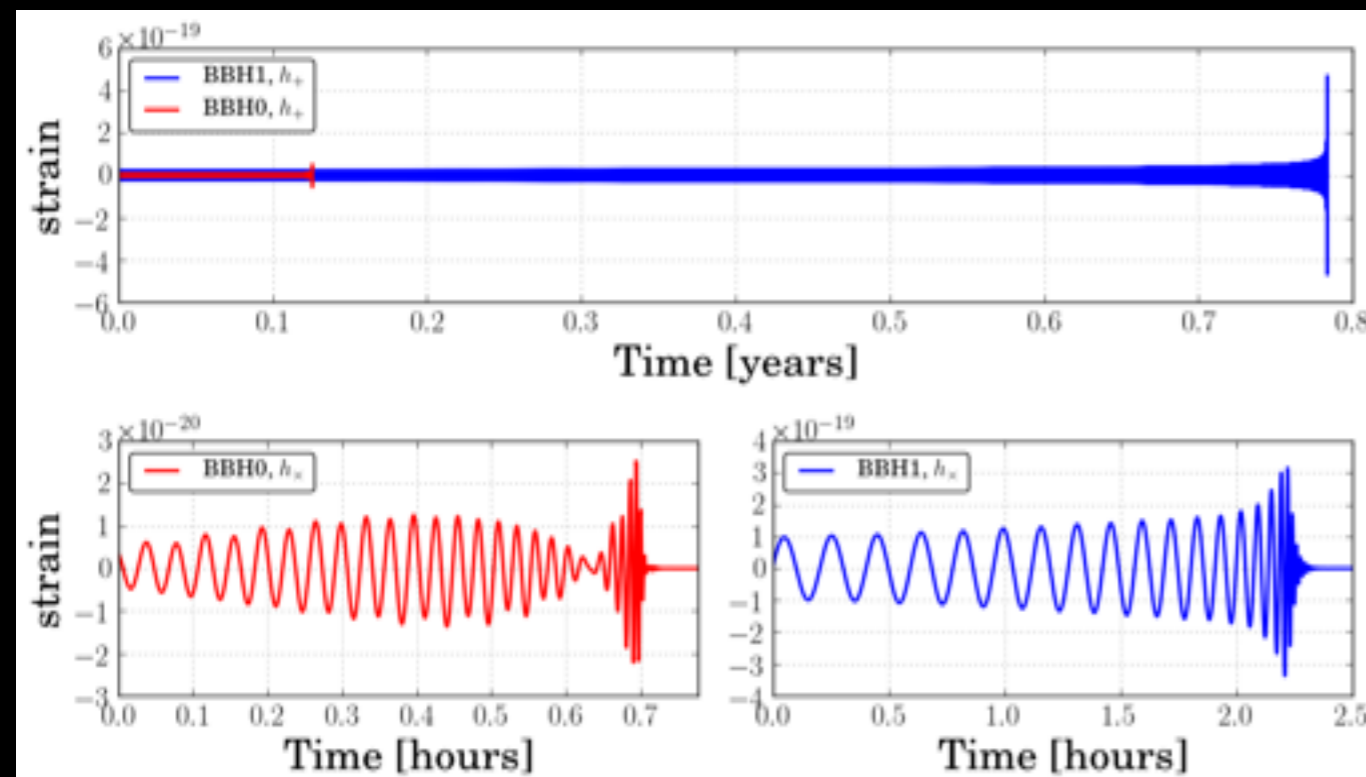
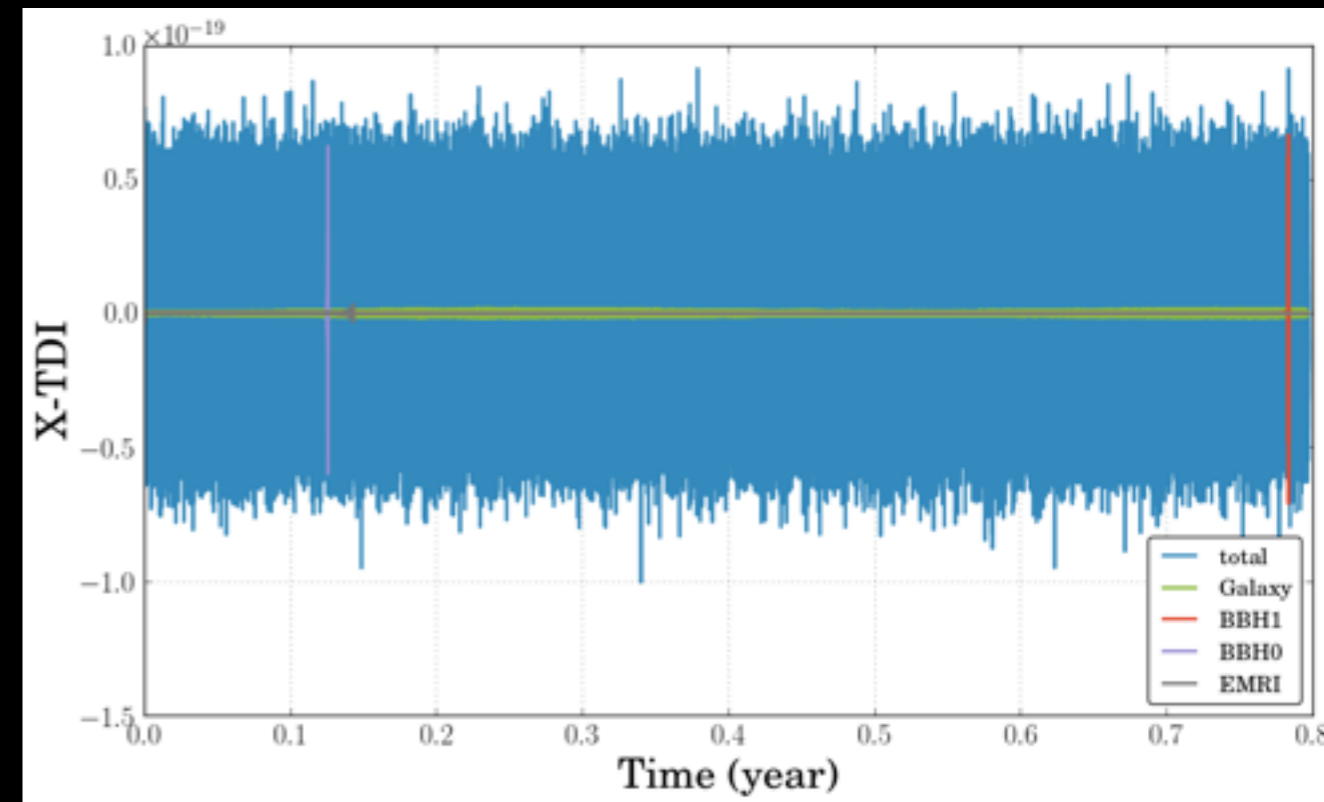
- ▶ Mock LDC: 2005→2011
- ▶ Data: few sources + simplified noises
- ▶ Challenges of increasing complexities
- ▶ Goals of the MLDC:
 - Check the feasibility of LISA data analysis
 - Develop data analysis
- ▶ now (2017): start of the LDC

MLDC 4



LISA Data Challenges

- ▶ 2017: start of the LDC
- ▶ Additional goal:
 - Design the pipelines of the mission
- ▶ Example of the potential data for LDC1 (from S. Babak)



Data, signal and noise

▶ The time data $d(t)$ contains:

- $h(t)$: signals that can be characterized by a sets of parameters
 - deterministic / stochastic
 - resolvable or not
- $n(t)$: noises from
 - instrument itself
 - other sources

▶ Assumption 1: GW and noise are linearly independent:

$$d(t) = h_{real}(t) + n(t)$$

- $h(t)$: GW perturbation $h_{ab}(t, \vec{x})$ convolved with instrument

Likelihood

- ▶ **Goal:** find the $h_{model} = h_{real}$
- ▶ **Likelihood:** found by demanding residual compatible with noise distribution $p_n(x)$:
 - The likelihood of observing $d \equiv \{d_1, d_2, \dots, d_N\}$ where $d_i = d(t_i)$, is given by:

$$p(d(t)/h_{real}(t)) = p_n(r(t)) = p_n(d(t) - h_{real}(t))$$

- ▶ **So if $p(d(t)/h_{model}(t))$ is compatible with the noise distribution:** $h_{model}(t) = h_{real}(t)$

Likelihood

- ▶ Usual case: noise is a multi-variate gaussian distribution:

$$p(d|h) = p_n(r) = \frac{1}{\sqrt{\det(2\pi C_n)}} e^{-\frac{1}{2} \sum_{i,j} r_i (C_n^{-1})_{ij} r_j}$$

where the correlation matrix is : $C_n = \langle n_i n_j \rangle - \langle n_i \rangle \langle n_j \rangle$

- ▶ Generalization for a network of detectors:

$$p(d|h) = \frac{1}{\sqrt{\det(2\pi C_n)}} e^{-\frac{1}{2} \sum_{Ii, Jj} r_{Ii} (C_n^{-1})_{Ii, Jj} r_{Jj}}$$

where I, J labels the detector and i, j the discrete time or frequency sample

Likelihood

► Inner product:

$$\langle x|y \rangle = \sum_{Ii, Jj} x_{Ii} (C_n^{-1})_{Ii, Jj} y_{Jj}$$

► Likelihood:

$$\mathcal{L} = p(d|h) = \frac{1}{\sqrt{\det(2\pi C_n)}} e^{-\frac{1}{2} \langle d-h|d-h \rangle}$$

► If C_n^{-1} is diagonal with $1/\sigma_i^2$ the inner product is similar to

$$\chi^2 = \sum_i \left(\frac{d_i - h_i}{\sigma_i} \right)^2 \Rightarrow \mathcal{L} = C e^{-\frac{1}{2} \langle d-h|d-h \rangle} = C e^{-\frac{1}{2} \chi^2}$$

Likelihood

► If stationary noise

→ C_n only depend to $|t_i - t_j|$

→ $C_n \sim$ diagonal in the Fourier domain (Discrete Fourier Transform) with on the diagonal $S_{n,k} T/2$

→ Inner product: $\langle \tilde{x} | \tilde{y} \rangle = 2 \sum_{j=0}^{N/2-1} \Delta f \frac{\tilde{x}_j^* \tilde{h}_j + \tilde{x}_j \tilde{h}_j^*}{S_{n,j}}$

→ Continuous limit: $\langle \tilde{x} | \tilde{y} \rangle = 2 \int_0^\infty df \frac{\tilde{x}^*(f) \tilde{h}(f) + \tilde{x}(f) \tilde{h}^*(f)}{S_n(f)}$

$$= 4 \Re \int_0^\infty df \frac{\tilde{x}^*(f) \tilde{h}(f)}{S_n(f)}$$

Likelihood

- ▶ If noise C_n is known (i.e. known parameters) and stationary, the factor in front is neglected and we only consider the logarithm of likelihood:

$$\begin{aligned}\log \mathcal{L} &= -\frac{1}{2} \langle d - h | d - h \rangle \\ &= \langle d | h \rangle - \frac{1}{2} \langle h | h \rangle - \frac{1}{2} \langle d | d \rangle\end{aligned}$$

- ▶ $\langle d | d \rangle$ is fixed so the relevant term that is usually used is the reduced likelihood:

$$\log \mathcal{L}' = \langle d | h \rangle - \frac{1}{2} \langle h | h \rangle$$

Data analysis for deterministic sources

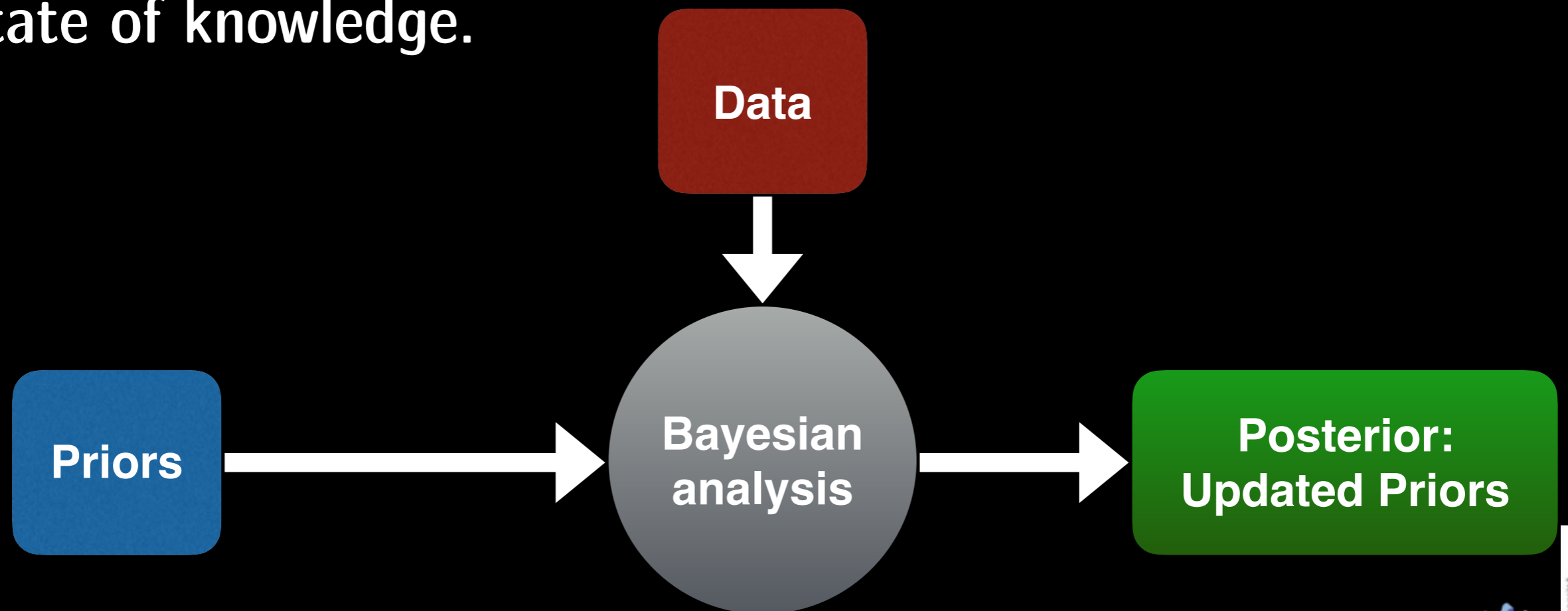
- ▶ Deterministic sources: binaries, cusps
- ▶ Bayesian or frequentist analysis
- ▶ Most part of the methods are based on **match-filtering**
 - Core: computation of likelihood:

$$\log \mathcal{L}' = \langle d|h \rangle - \frac{1}{2} \langle h|h \rangle$$
$$\langle \tilde{x}|\tilde{y} \rangle = 2 \int_0^\infty df \frac{\tilde{x}^*(f)\tilde{h}(f) + \tilde{x}(f)\tilde{h}^*(f)}{S_n(f)}$$
$$= 4 \Re \int_0^\infty df \frac{\tilde{x}^*(f)\tilde{h}(f)}{S_n(f)}$$

- Need for h , the model of the signal in TDI outputs

Bayesian inference

- ▶ Data are given
- ▶ The uncertainties are on the model / parameters
- ▶ Our prior knowledge is updated by what we learn from the data, as measured by the likelihood to give our posterior state of knowledge.



Bayesian inference

- ▶ Bayes theorem:

$$p(a|d) = \frac{p(a) p(d|a)}{p(d)}$$

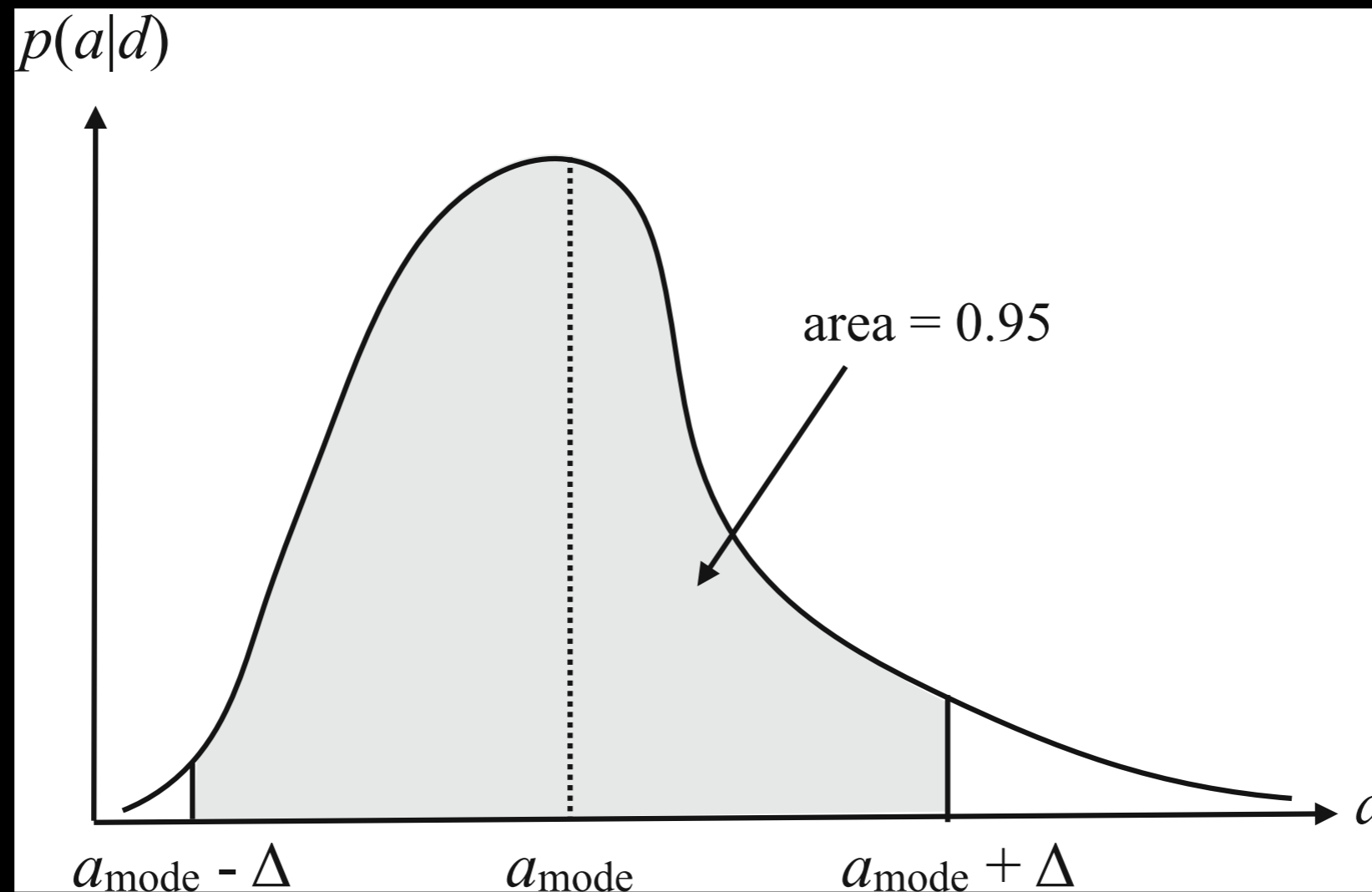
Diagram illustrating Bayes' theorem with labels and arrows:

- prior** (points to $p(a)$)
- likelihood** (points to $p(d|a)$)
- evidence** (points to $p(d)$)
- posterior distribution** (points to $p(a|d)$)

- ▶ “Everything” about the parameters is in the posterior distribution

Bayesian inference

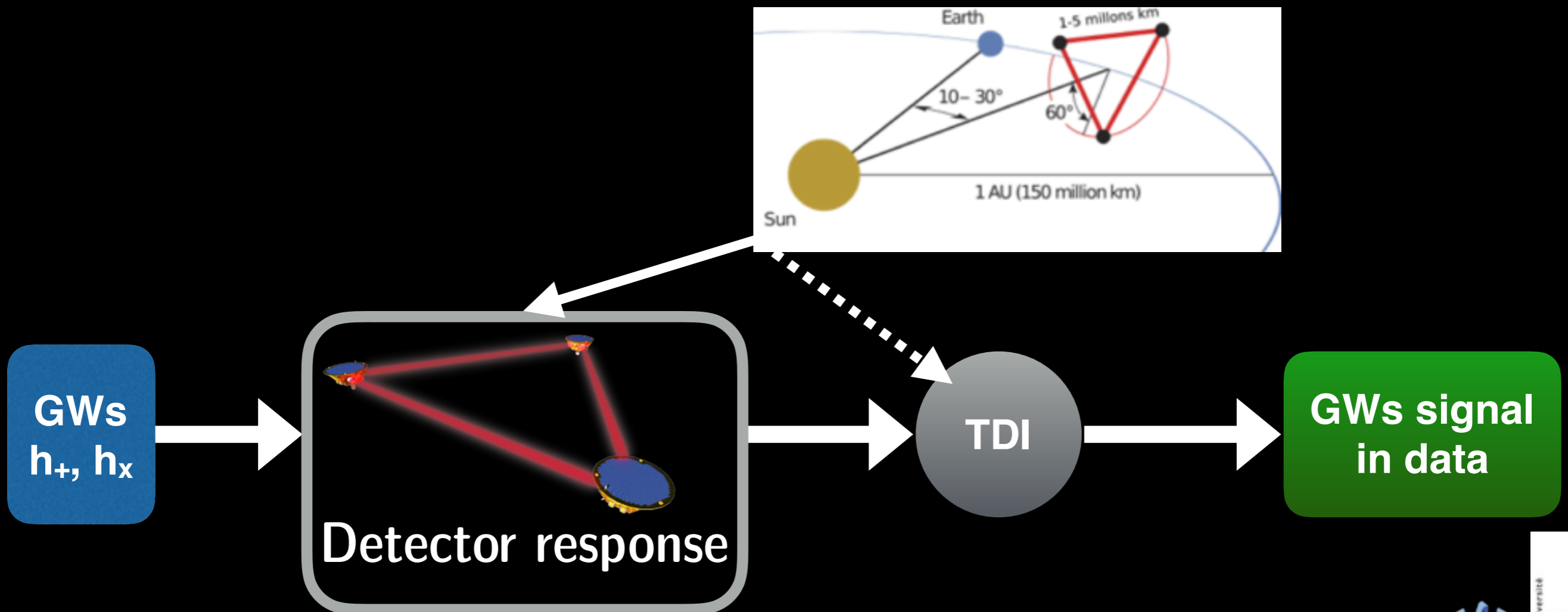
- ▶ Confidence interval = credible interval (degree of belief): area under the posterior between one parameter value and another



Romano & Cornish, LRR 2017

Model of the signal

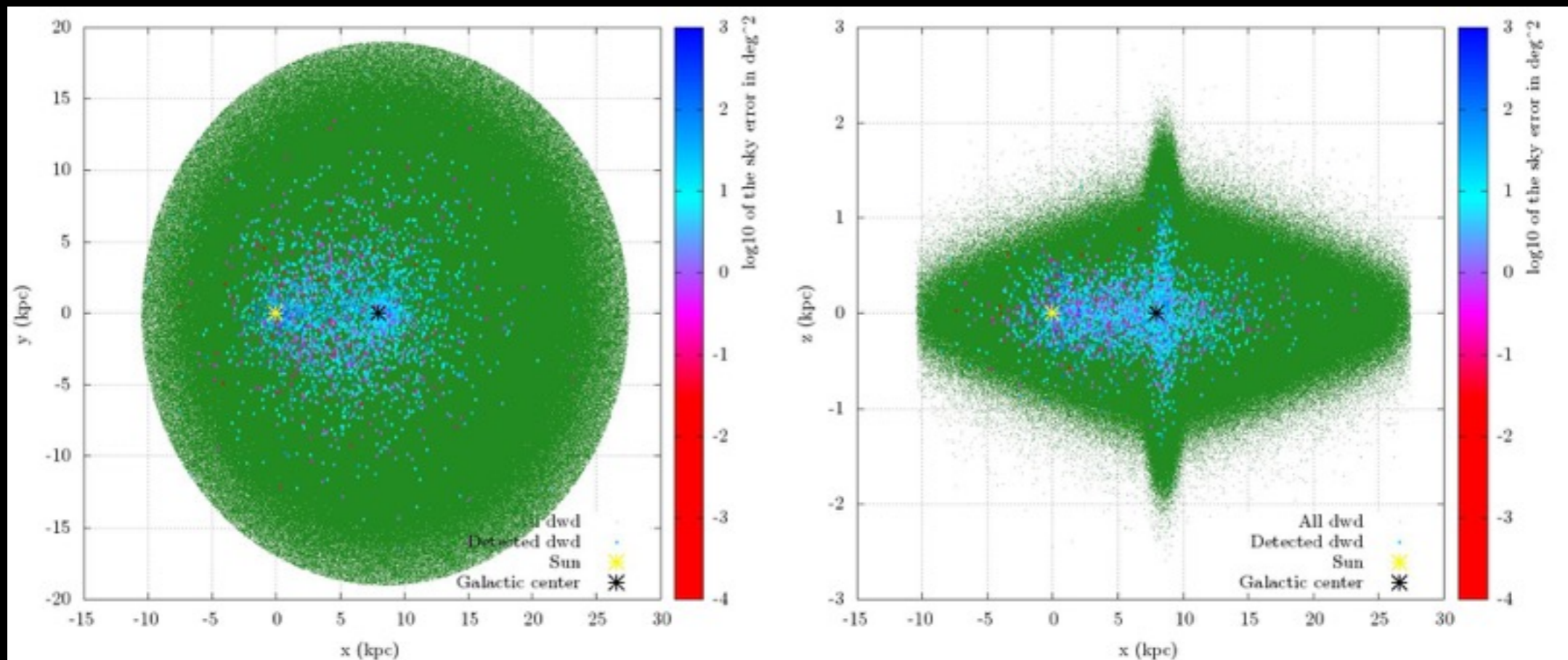
- ▶ In a model of the GW signal in the output data (TDI), 2 modifications have to be considered:
 - Response of the detector to GW: arm response
 - Time Delay Interferometry



Analysis of galactic binaries

► Characteristics of the problem

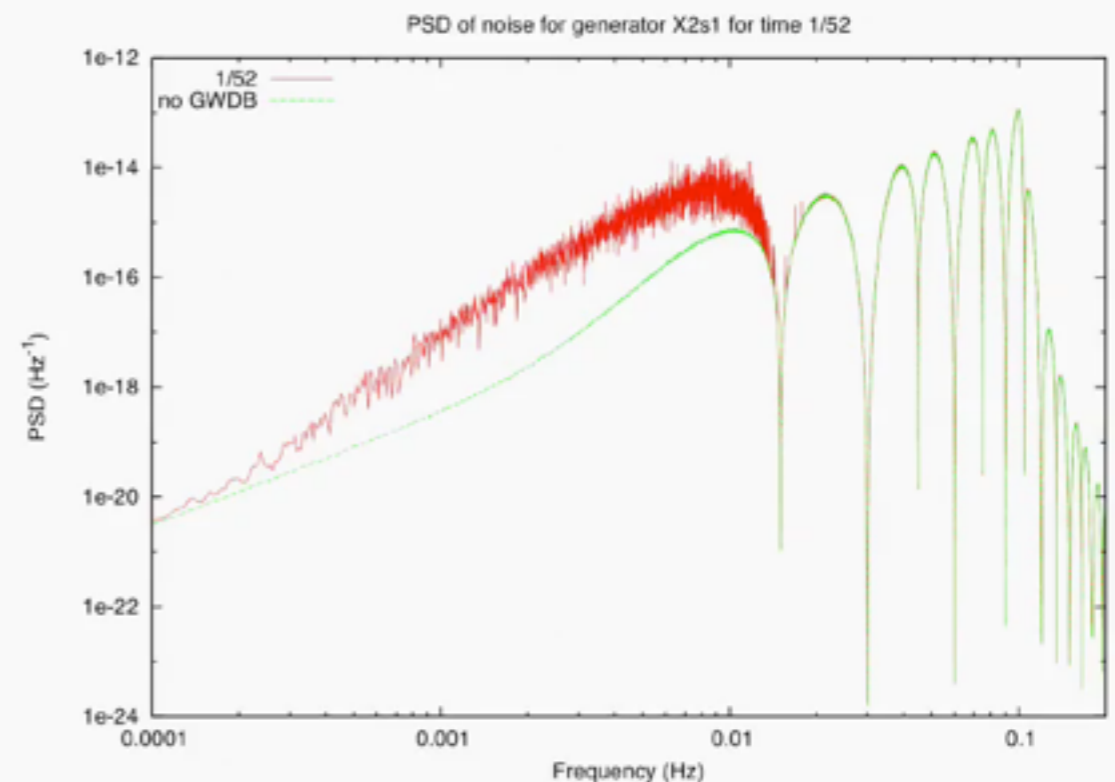
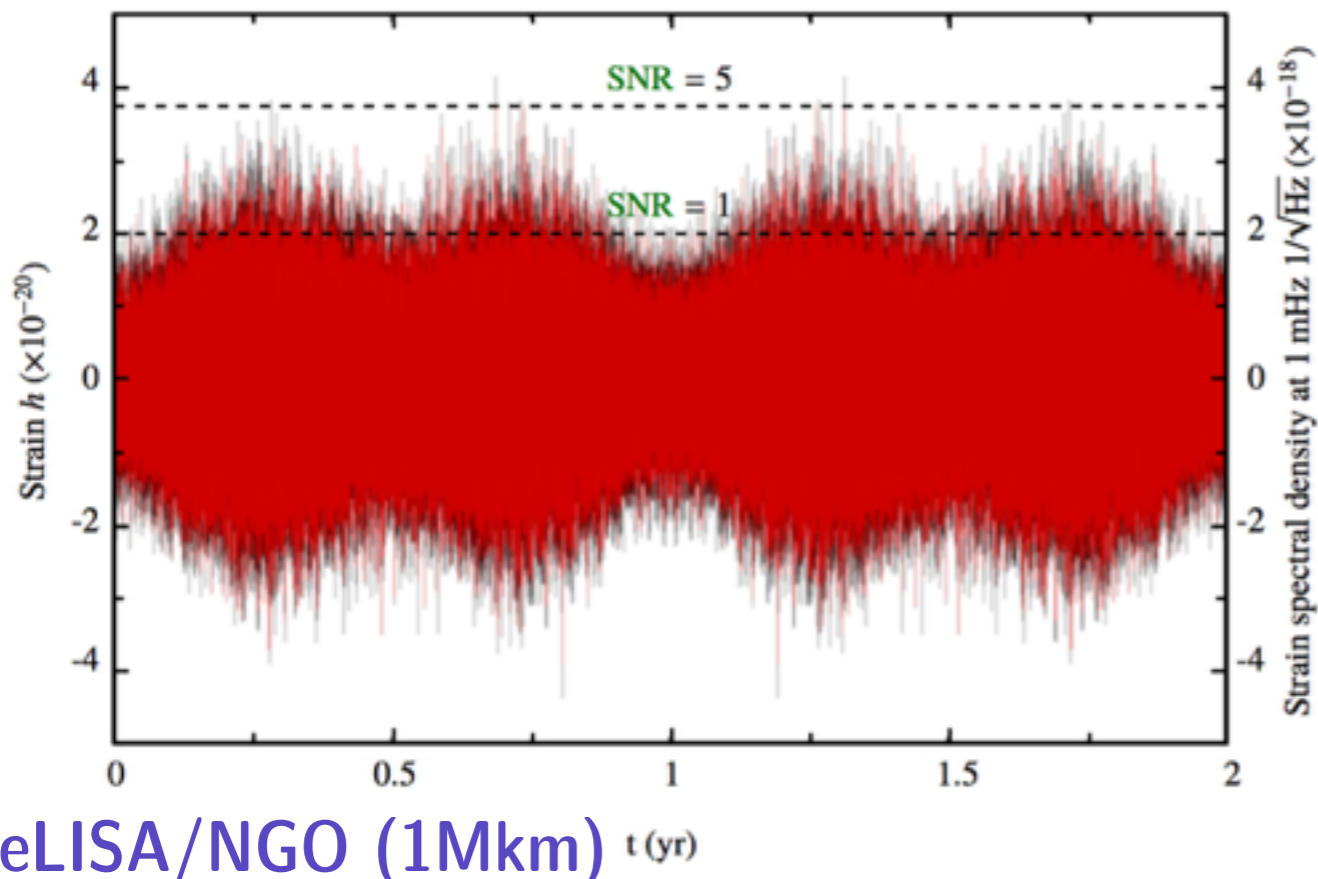
- Simple model: quasi-monochromatic
- Large number of sources
- Sources at low frequency ($f < 10\text{mHz}$)
- Distributed in the galaxy \Rightarrow more around the Galactic Center



Analysis of galactic binaries

► Characteristics of the problem

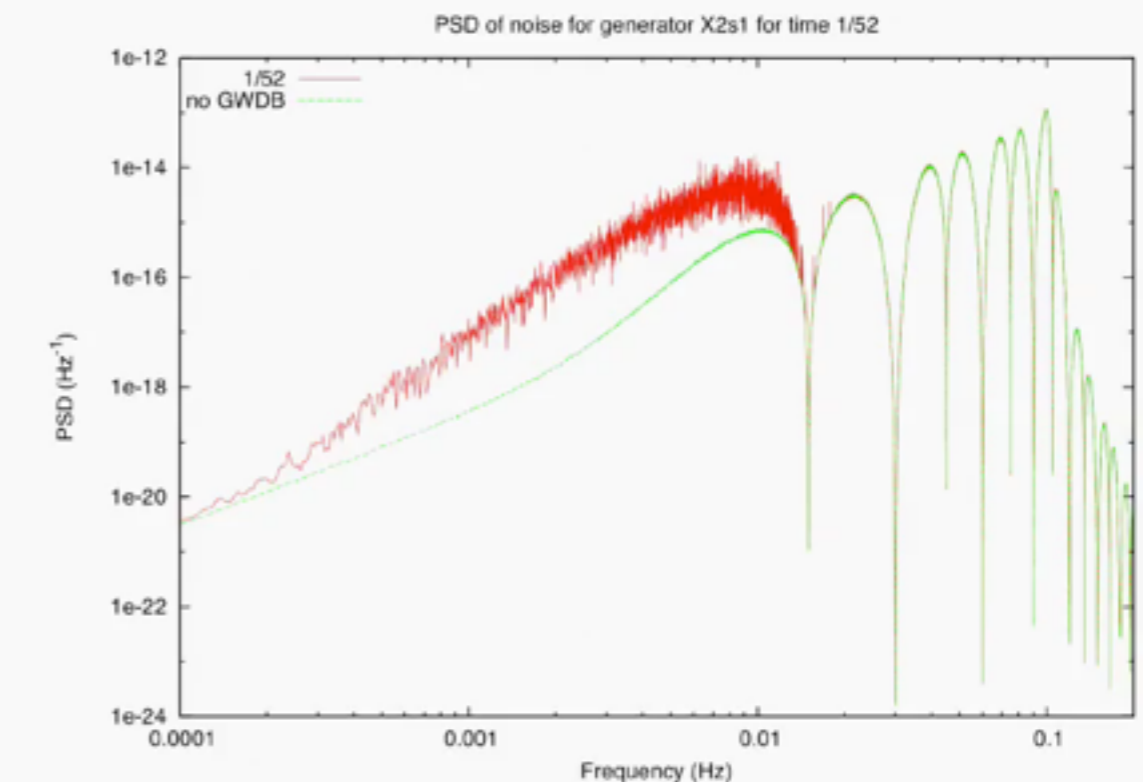
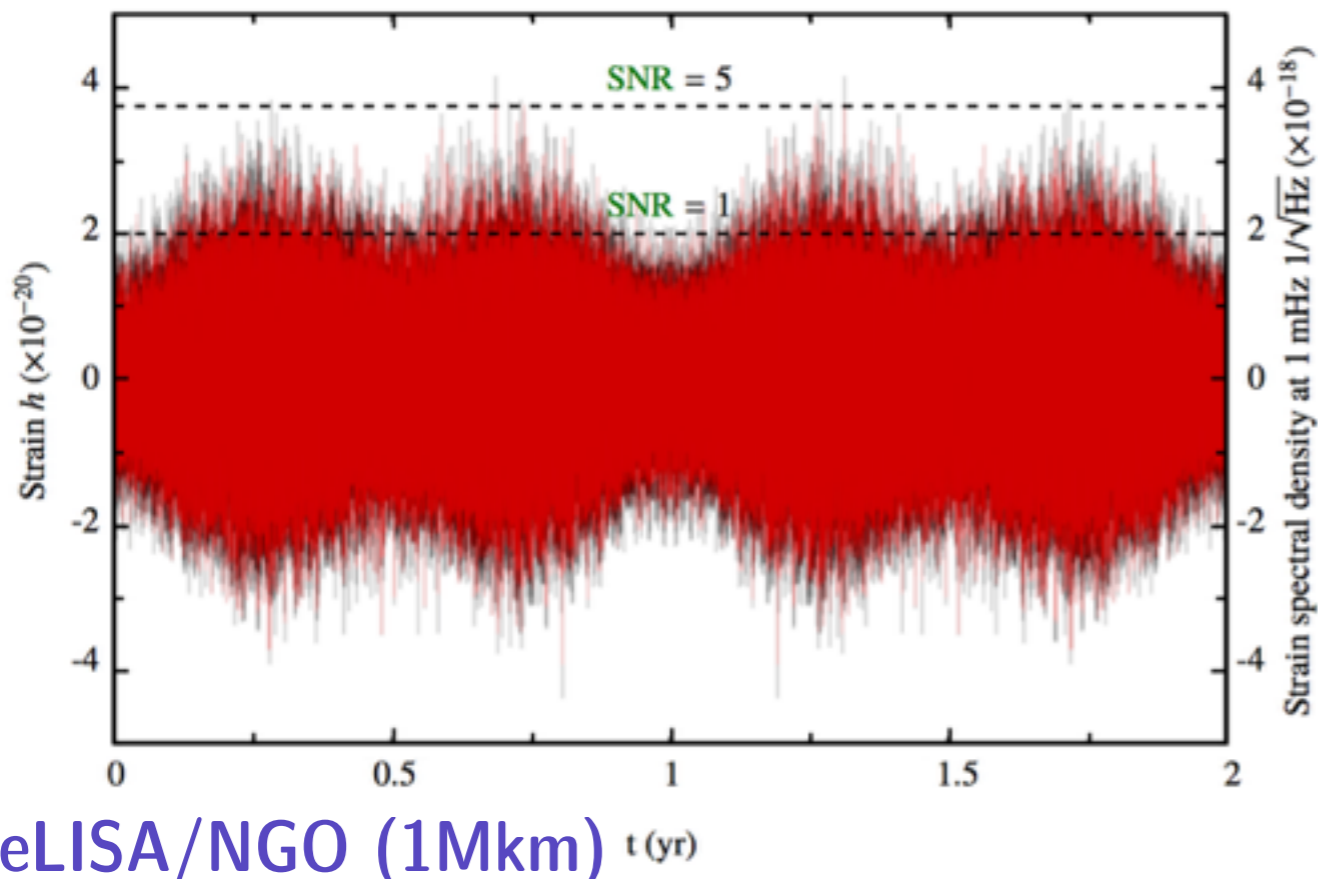
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Analysis of galactic binaries

► Characteristics of the problem

- Simple model: quasi-monochromatic
- Large number of sources
- Sources at low frequency ($f < 10\text{mHz}$)
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Analysis of galactic binaries

▶ Example of technics (developed for MLDC):

- Metropolis–Hastings Monte Carlo (MHMC) [Cornish & Crowder]:

- separate runs for overlapping frequency bands
- different hypothesized numbers of sources;
- model comparison to determine the most probable number of sources in each band.

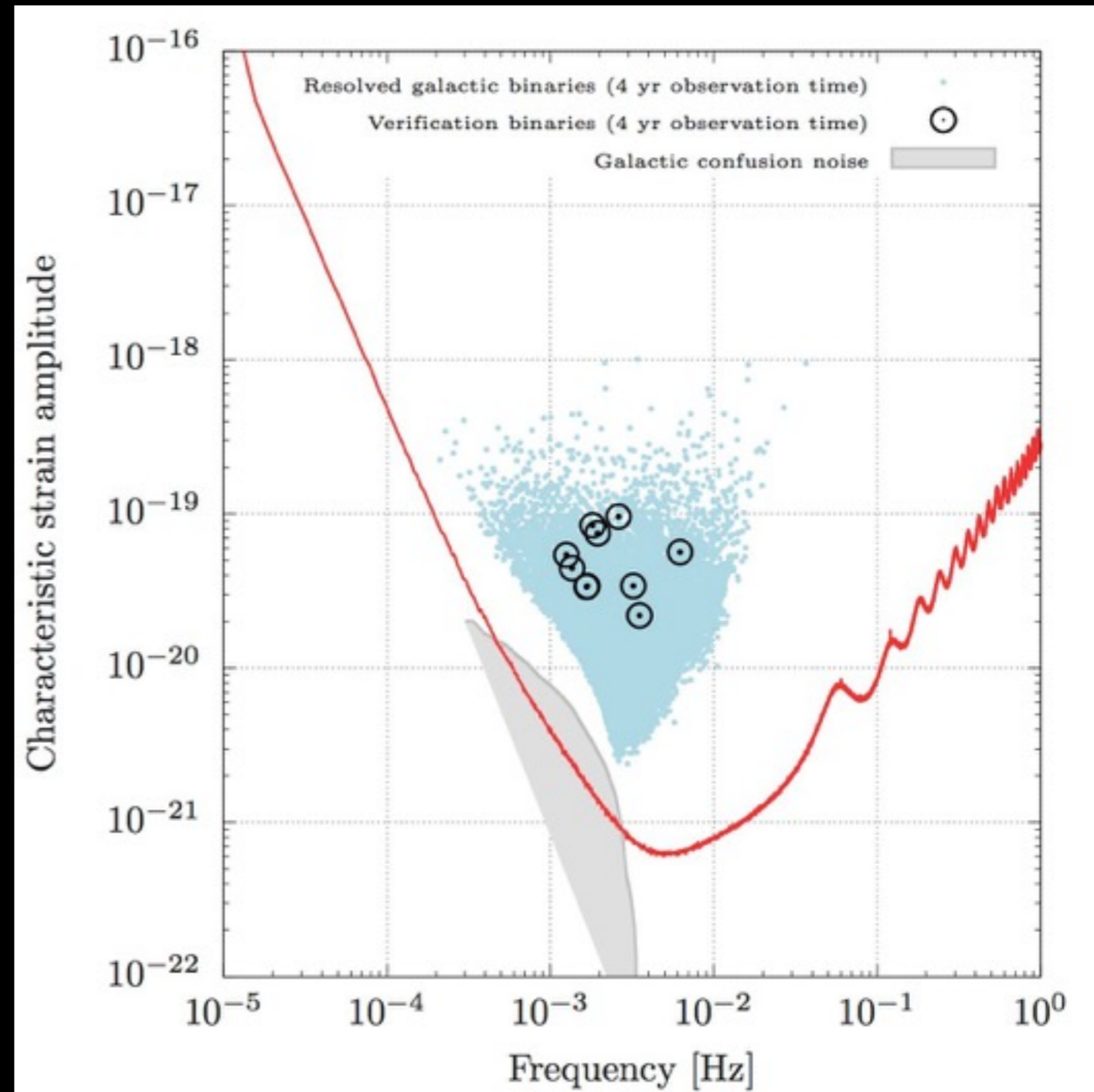
⇒ 19,324 sources identified !

- Template-bank–based matched-filtering [Prix&Whelan, Krolak et al.]

- Fstatistic: analytical maximization of the likelihood over some par
- template bank
- frequency bands

Analysis of galactic binaries

- ▶ Verification binaries:
 - Check the instrument
 - Analysis
 - prior on the parameters that are already measure
 - better estimation of the others parameters
- ▶ The non-resolved GBs will form a foreground for the other sources

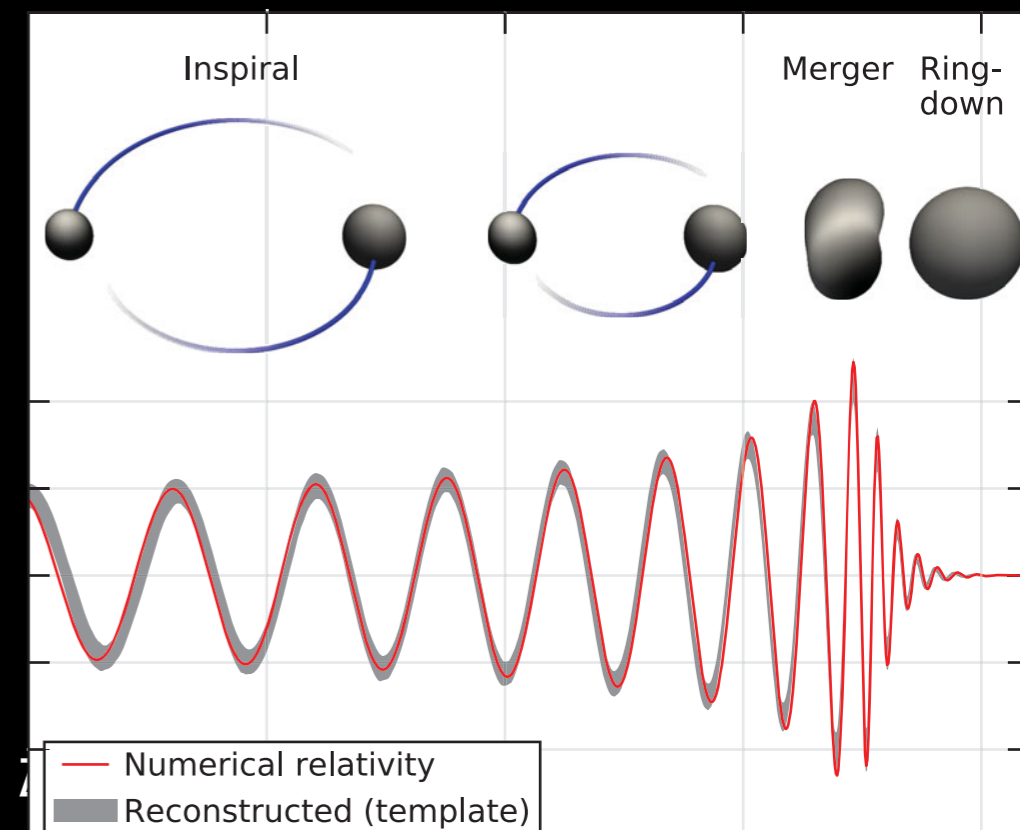
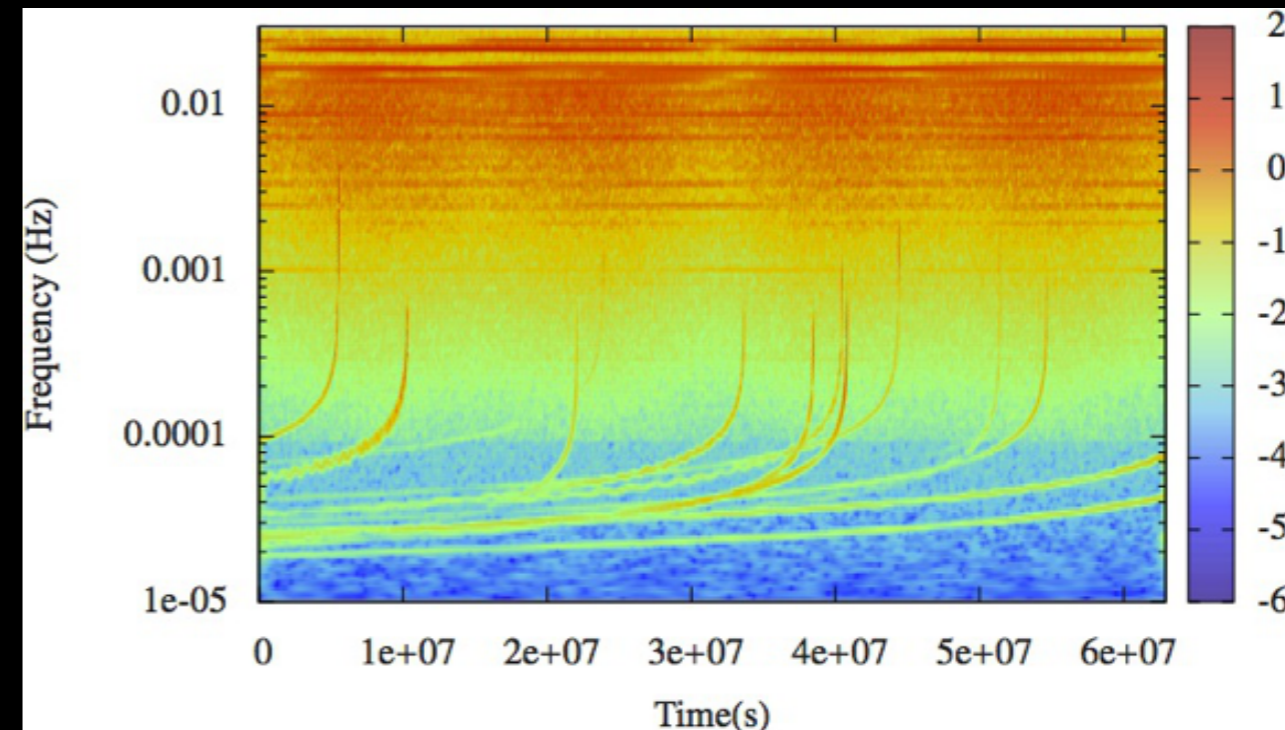


=> Add a component in the noise S_n .

SMBHBs

► Characteristics:

- 17 parameters
- waveforms:
 - inspiral: semi-analytic (PN)
 - merger: numerical relativity
 - ring-down: analytic
- duration: day to month
- model: semi-analytic approximation:
 - Spinning Effective One Body
 - PhemomC, D, P
- 10-100 events/years



SMBHBs analysis

- ▶ Examples of technics (developed for MLDC3-2010, inspiral only but spinning \Rightarrow precessing):
 - Multimodal genetic algorithm with A-statistic [Petiteau et al.]
 - MultiNest with A-statistic [Bridges et al.]
 - Tempered Metropolis-Hastings MCMC algorithm [Arnaud et al.]
 - Two stages search using a non-spinning MBH search & MultiNest [Brown et al.]
 - Parallel tempered MHMC algorithm using thermostated/frequency annealing [Cornish et al.]

Spinning MBHB: MLDC 3.2

Results :

Coalescence after the observation : $t_c > T_{obs}$ Coalescence during the observation : $t_c < T_{obs}$

source (SNR _{true})	group	$\Delta M_c / M_c$ $\times 10^{-5}$	$\Delta \eta / \eta$ $\times 10^{-4}$	Δt_c (sec)	Δsky (deg)	Δa_1 $\times 10^{-3}$	Δa_2 $\times 10^{-3}$	$\Delta D / D$ $\times 10^{-2}$	SNR	FF _A	FF _E
MBH-1 (1670.58)	AEI	2.4	6.1	62.9	11.6	7.6	47.4	8.0	1657.71	0.9936	0.9914
	CambAEI	3.4	40.7	24.8	2.0	8.5	79.6	0.7	1657.19	0.9925	0.9917
	MTAPC	24.8	41.2	619.2	171.0	13.3	28.7	4.0	1669.97	0.9996	0.9997
	JPL	40.5	186.6	23.0	26.9	39.4	66.1	6.9	1664.87	0.9972	0.9981
	GSFC	1904.0	593.2	183.9	82.5	5.7	124.3	94.9	267.04	0.1827	0.1426
MBH-3 (847.61)	AEI	9.0	5.2	100.8	175.9	6.2	18.6	2.7	846.96	0.9995	0.9989
	CambAEI	13.5	57.4	138.9	179.0	21.3	7.2	1.5	847.04	0.9993	0.9993
	MTAPC	333.0	234.1	615.7	80.2	71.6	177.2	16.1	842.96	0.9943	0.9945
	JPL	153.0	51.4	356.8	11.2	187.7	414.9	2.7	835.73	0.9826	0.9898
	GSFC	8168.4	2489.9	3276.9	77.9	316.3	69.9	95.6	218.05	0.2815	0.2314
MBH-4 (160.05)	AEI	4.5	75.2	31.4	0.1	47.1	173.6	9.1	160.05	0.9989	0.9994
	CambAEI	3.2	171.9	30.7	0.2	52.9	346.1	21.6	160.02	0.9991	0.9992
	MTAPC	48.6	2861.0	5.8	7.3	33.1	321.1	33.0	149.98	0.8766	0.9352
	JPL	302.6	262.0	289.3	4.0	47.6	184.5	28.3	158.34	0.8895	0.9925
	GSFC	831.3	1589.2	1597.6	94.4	59.8	566.7	95.4	-45.53	-0.1725	-0.2937
MBH-2 (18.95)	AEI	1114.1	952.2	38160.8	171.1	331.7	409.0	15.3	20.54	0.9399	0.9469
	CambAEI	88.7	386.6	6139.7	172.4	210.8	130.7	24.4	20.36	0.9592	0.9697
	MTAPC	128.6	45.8	16612.0	8.9	321.4	242.4	13.1	20.27	0.9228	0.9260
	JPL	287.0	597.7	11015.7	11.8	375.3	146.3	9.9	18.69	0.9661	0.9709
MBH-6 (12.82)	AEI	1042.3	1235.6	82343.2	2.1	258.2	191.6	26.0	13.69	0.9288	0.9293
	CambAEI	5253.2	1598.8	953108.0	158.3	350.8	215.4	29.4	10.17	0.4018	0.4399
	MTAPC	56608.7	296.7	180458.8	119.7	369.2	297.6	25.1	11.34	-0.0004	0.0016



Conclusion

- ▶ LISA will observe gravitational wave sources in the frequency band 0.2 - 100 mHz: large number of sources: binaries, backgrounds, ...
- ▶ Complex instrument with very **high precision metrology**: free-fall, long arm interferometry, ...
- ▶ Good technology readiness: key aspects validated by LISAPathfinder
- ▶ **Pre-processing** of data to reduce noises => **2 scientific data channels** containing all the GW informations to be extracted
- ▶ LISA Data Analysis is **challenging but tractable**
- ▶ LISA has been accepted and is detailed definition phase.
- ▶ Increasing activities on instrument side and data analysis



Thank you





Thank you

