

# Flavor Physics for Non-Experts: (a Theory) Overview

*Guido Martinelli  
Dipartimento di Fisica & INFN Sezione di Roma  
Università La Sapienza*

DIPARTIMENTO DI FISICA



SAPIENZA  
UNIVERSITÀ DI ROMA

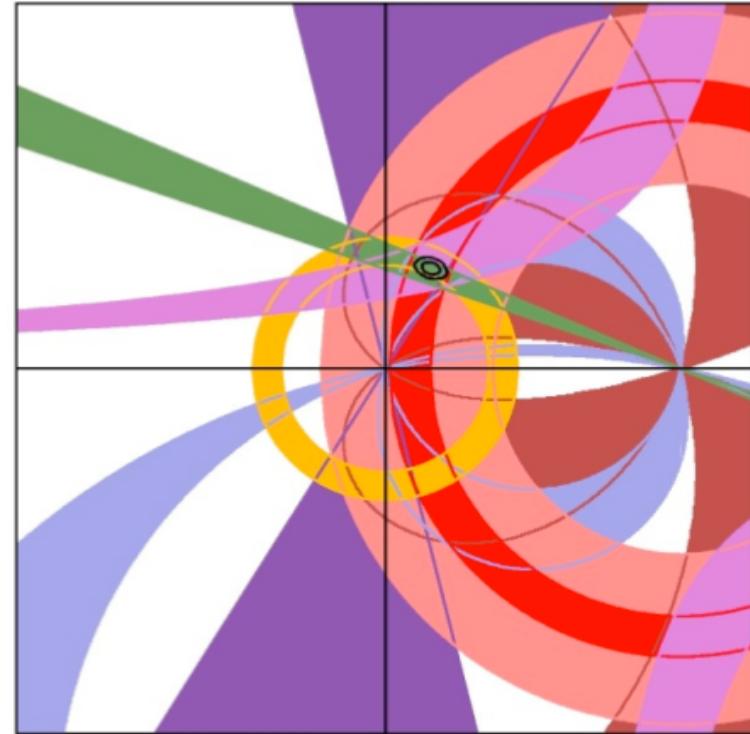


Orsay June 30 2017



# *PLAN OF THE TALK*

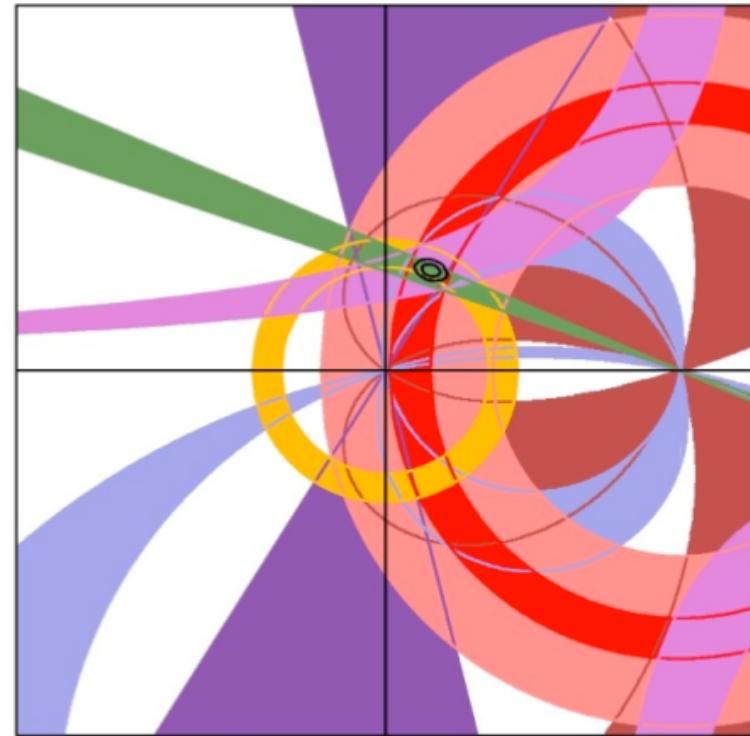
- *General introduction to the Unitary Triangle Fit*
- *SM Analysis*
- *Tensions and unknown*
- *Uncertainties in lattice calculations;*
- *From simple to complicated;*
- *Future directions, new/old ideas*
- *Beyond the SM*
- *Conclusion*



Thanks to  
Bona, Ciuchini, Lubicz,  
Silvestrini, Sachrajda,  
Tantalo, ...

Impossible to cover all recent developments – a selected list of topics –  
apologies for the interesting work that is not reported here

*STANDARD  
MODEL  
UNITARITY  
TRIANGLE  
ANALYSIS  
(Flavor Physics)*



- *Provides the best determination of the CKM parameters;*
- *Tests the consistency of the SM ("direct" vs "indirect" determinations) @ the quantum level;*
- *Provides predictions for SM observables (in the past for example  $\sin 2\beta$  and  $\Delta m_s$ )*
- *It could lead to new discoveries (CP violation, Charm, !?)*

*The fundamental issue is to find signatures of new physics and to unravel the underlying theoretical structure;*

*Precision Flavor physics is a key tool, complementary to the large energy searches at the LHC;*

*If the LHC discovers new elementary particles BSM, then precision flavor physics will be necessary to constrain the underlying framework;*

***The discovery potential of precision flavor physics should not be underestimated.***

*The extraordinary progress of the experimental measurements requires accurate theoretical predictions*

*Precision flavour physics requires the control of hadronic effects for which lattice QCD simulations are essential.*

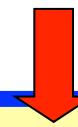
$$Q^{EXP} = V_{CKM} \langle F | \hat{O} | I \rangle$$



$$Q^{EXP} = \sum_i C_{SM}^i(M_W, m_t, \alpha_s) \langle F | \hat{O}_i | I \rangle + \sum_{i'} C_{Beyond}^{i'}(\tilde{m}_\beta, \alpha_s) \langle F | \hat{O}_{i'} | I \rangle$$

# Flavor physics in the Standard Model

In the SM, the quark mass matrix, from which the CKM matrix and  $\mathcal{CP}$  violation originate, is determined by the coupling of the Higgs boson to fermions.



$$\mathcal{L}_{\text{quarks}} = \mathcal{L}^{\text{kinetic}} + \mathcal{L}^{\text{gauge}} + \mathcal{L}^{\text{Yukawa}}$$

$\mathcal{CP}$  invariant

$\mathcal{CP}$  and symmetry breaking  
are strictly correlated

$$\mathcal{L}(\Lambda_{\text{Fermi}}) = \mathcal{L}(\Lambda, H, H^\dagger) + \mathcal{L}^{\text{kin}} + \mathcal{L}_{\text{SM}}^{\text{gauge}} + \mathcal{L}_{\text{SM}}^{\text{Yukawa}} + \frac{\mathcal{L}_5}{\Lambda} + \frac{\mathcal{L}_6}{\Lambda^2} + \dots$$

EWSB

has many accidental  
symmetries

may violate  
accidental  
symmetries

*Absence of FCNC at tree level (& GIM suppression of FCNC @loop level)*

*Almost no CP violation at tree level*

*Flavour Physics is extremely sensitive  
to New Physics (NP)*

*In competition with Electroweak  
Precision Measurements*

# WHY RARE DECAYS ?

Rare decays are a manifestation of broken (accidental) symmetries e.g. of physics beyond the Standard Model

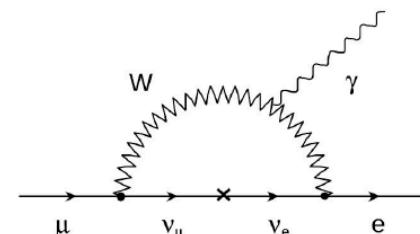
Proton decay

baryon and lepton number conservation

$$\mu \rightarrow e + \gamma$$

lepton flavor number

$$\nu_i \rightarrow \nu_k \text{ found !}$$



$$\mathcal{B}(\mu \rightarrow e\gamma) \sim \alpha \frac{m_\nu^4}{m_W^4} \sim 10^{-52}$$

# RARE DECAYS WHICH ARE ALLOWED IN THE STANDARD MODEL

FCNC:

$$q_i \rightarrow q_k + \nu \bar{\nu}$$

$$q_i \rightarrow q_k + l^+ l^-$$

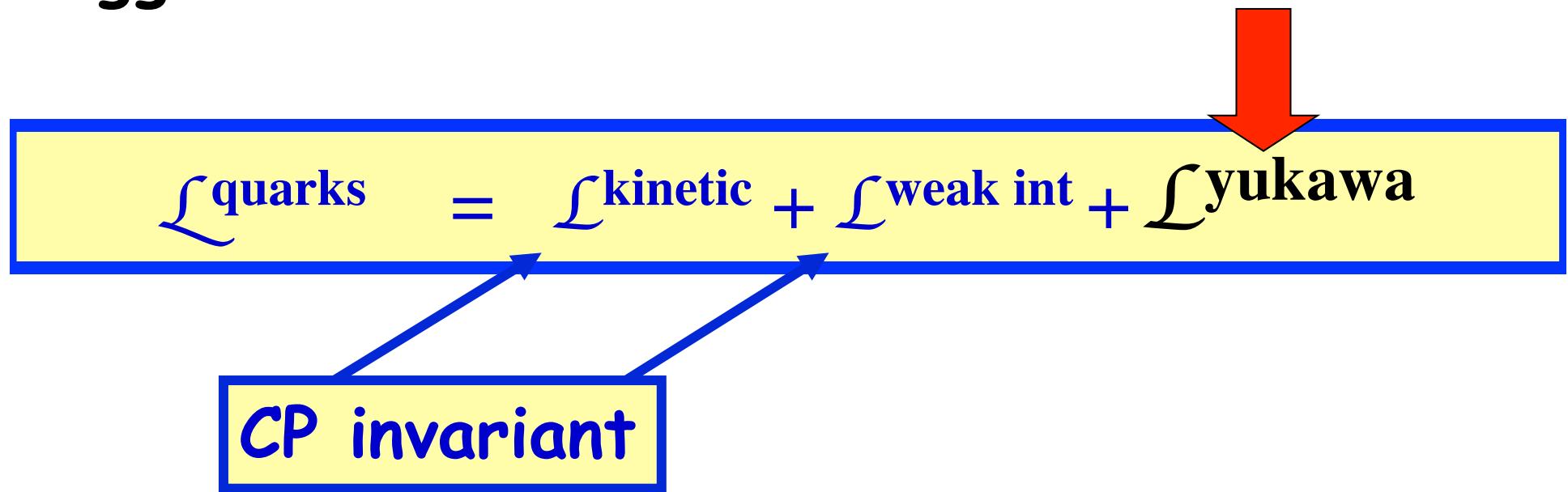
$$q_i \rightarrow q_k + \gamma$$

these decays occur only via loops because of GIM and are suppressed by CKM

THUS THEY ARE SENSITIVE TO  
**NEW PHYSICS**

# **CP Violation in the Standard Model**

In the Standard Model the quark mass matrix, from which the CKM Matrix and  $\mathcal{CP}$  originate, is determined by the Yukawa Lagrangian which couples fermions and Higgs



$\mathcal{CP}$  and symmetry breaking are closely related !

*QUARK MASSES ARE GENERATED  
BY DYNAMICAL SYMMETRY  
BREAKING*

$$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad H^C = i\pi_2 H^*$$

$$\phi^+ \rightarrow 0 \quad \phi^0 \rightarrow \frac{V}{\sqrt{2}}$$

**Charge +2/3**

<i>Elementary Particles</i>			
Quarks	Leptons	Force Carriers	
<i>u</i>	<i>d</i>	<i>t</i>	$\gamma$
<i>c</i>	<i>s</i>	<i>b</i>	<i>g</i>
<i>v<sub>e</sub></i>	<i>v<sub>μ</sub></i>	<i>v<sub>τ</sub></i>	<i>Z</i>
	<i>e</i>	<i>μ</i>	<i>W</i>

*Three Generations of Matter*

$$\mathcal{L}_{\text{yukawa}} \equiv \sum_{i,k=1,N} [ Y_{i,k} (q_L^i H^C) U_R^k + X_{i,k} (q_L^i H) D_R^k + \text{h.c.} ]$$

**Charge -1/3**

$$\sum_{i,k=1,N} [ m_{i,k}^u (\bar{u}_L^i u_R^k) + m_{i,k}^d (\bar{d}_L^i d_R^k) + \text{h.c.} ]$$

## Diagonalization of the Mass Matrix

Up to singular cases, the mass matrix can always be diagonalized by 2 unitary transformations

$$u_L^i \rightarrow U_L^{ik} u_L^k \quad u_R^i \rightarrow U_R^{ik} u_R^k$$
$$M' = U_L^\dagger M U_R \quad (M')^\dagger = U_R^\dagger (M)^\dagger U_L$$

$$\mathcal{L}^{\text{mass}} \equiv m_{\text{up}} (\bar{u}_L u_R + \bar{u}_R u_L) + m_{\text{ch}} (\bar{c}_L c_R + \bar{c}_R c_L)$$
$$+ m_{\text{top}} (\bar{t}_L t_R + \bar{t}_R t_L)$$

$$L_{CC}^{\text{weak int}} = \frac{g_W}{\sqrt{2}} (J_\mu^- W_\mu^+ + h.c.)$$
$$\rightarrow \frac{g_W}{\sqrt{2}} (\bar{u}_L \mathbf{V}^{\text{CKM}} \gamma_\mu d_L W_\mu^+ + \dots)$$

$N(N-1)/2$  angles and  $(N-1)(N-2)/2$  phases

$N=3$       3 angles + 1 phase      KM  
 the phase generates complex couplings i.e. CP  
violation;

**6 masses +3 angles +1 phase = 10 parameters**

$V_{ud}$	$V_{us}$	$V_{ub}$
$V_{cd}$	$V_{cs}$	$V_{cb}$
$V_{tb}$	$V_{ts}$	$V_{tb}$

$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}$$

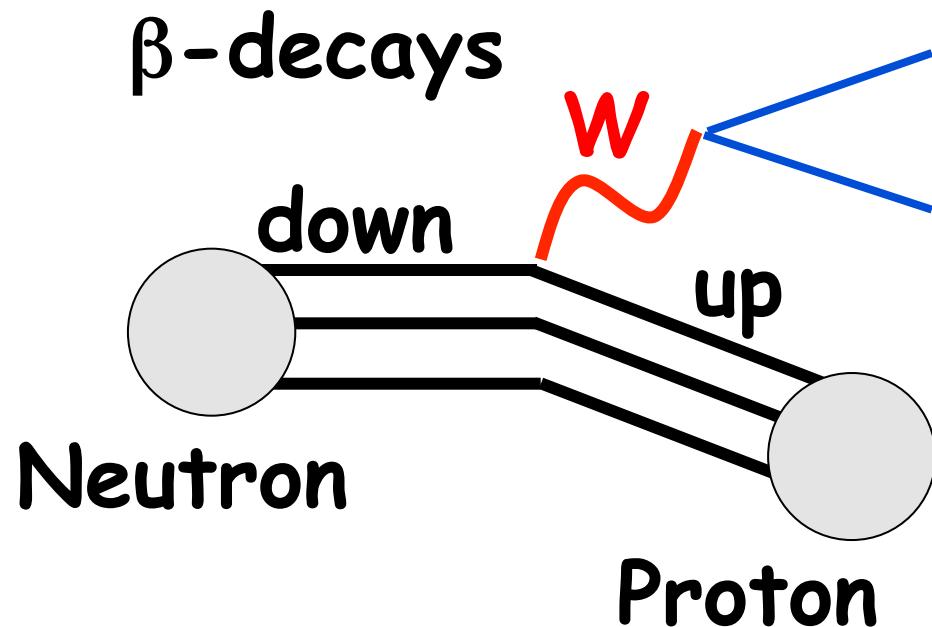
**NO Flavour Changing Neutral Currents (FCNC)  
at Tree Level**  
**(FCNC processes are good candidates for observing  
NEW PHYSICS)**

**CP Violation is natural with three quark  
generations (Kobayashi-Maskawa)**

**With three generations all CP  
phenomena are related to the same  
unique parameter (  $\delta$  )**

$V_{ud}$	$V_{us}$	$V_{ub}$
$V_{cd}$	$V_{cs}$	$V_{cb}$
$V_{td}$	$V_{ts}$	$V_{tb}$

## Quark masses & Generation Mixing



$$|V_{ud}|$$

*updated values later*

$$e^- - \bar{\nu}_e$$

$ V_{ud}  = 0.9735(8)$
$ V_{us}  = 0.2196(23)$
$ V_{cd}  = 0.224(16)$
$ V_{cs}  = 0.970(9)(70)$
$ V_{cb}  = 0.0406(8)$
$ V_{ub}  = 0.00409(25)$
$ V_{tb}  = 0.99(29) (0.999)$

## Textures

There is a clear correlation  
between mixings and masses

$$m_u \sim 4 \text{ MeV} \quad m_c \sim 1200 \text{ MeV} \quad m_t \sim 170 \text{ GeV}$$

$$m_d \sim 8 \text{ MeV} \quad m_s \sim 110 \text{ MeV} \quad m_b \sim 4.3 \text{ GeV}$$

Orizontal  $U(2)$  :  $\psi_L \quad \psi_L^c$

$$\mathcal{L}_{Higgs} = Y H \left[ (\psi_L^a)(\psi_L^b)^c S^{ab} + (\psi_L^a)(\psi_L^b)^c A^{ab} \right]$$

Symmetric  
tensor

Antisymmetric  
tensor

$$M^d = M \begin{pmatrix} 0 & -\sqrt{x} \\ \sqrt{x} & 1+x \end{pmatrix}$$

$\sin \theta_c \sim \sqrt{m_d / m_s}$

R.Gatto '70

$$\text{diag}(M) = M(x, 1) \quad x = m_d / m_s$$

$$V_1 = \begin{pmatrix} 1 \\ \sqrt{x} \end{pmatrix} \quad \lambda_1 = M x$$

$$V_2 = \begin{pmatrix} -\sqrt{x} \\ 1 \end{pmatrix} \quad \lambda_2 = M$$

Masses &  
Mixings  
(including the  
CP phases )  
are related !!

# The Wolfenstein Parametrization

$1 - \frac{1}{2} \lambda^2$	$\lambda$	$A \lambda^3(\rho - i \eta)$
$-\lambda$	$1 - \frac{1}{2} \lambda^2$	$A \lambda^2$
$A \lambda^3 \times (1 - \rho - i \eta)$	$-A \lambda^2$	1

$V_{ub}$

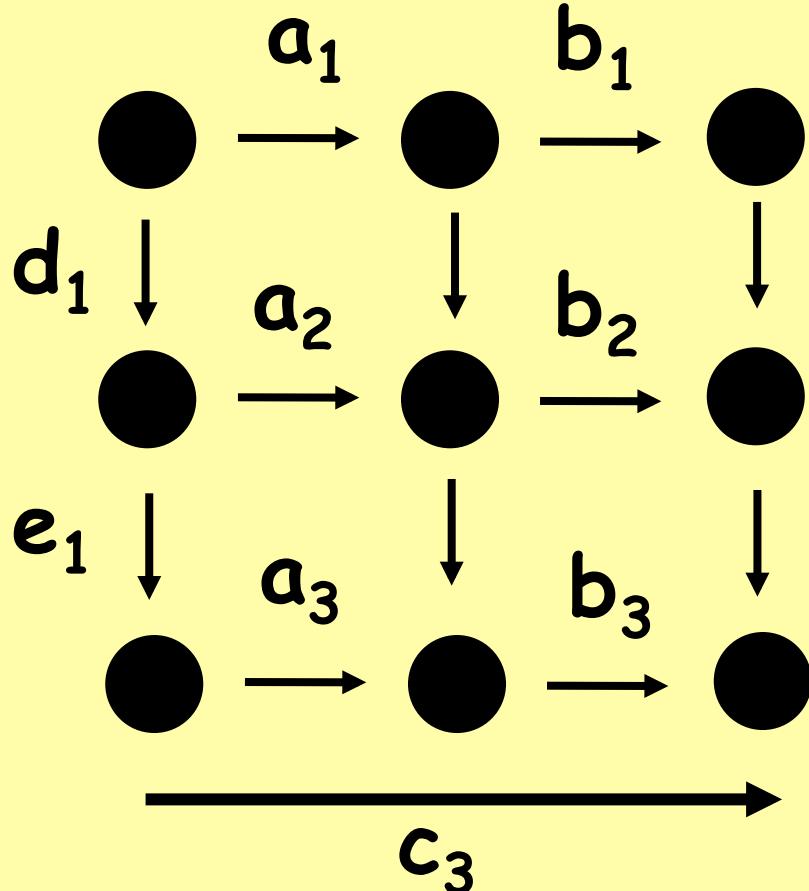
$+ O(\lambda^4)$

$V_{td}$

$$\begin{array}{ll} \lambda \sim 0.2 & A \sim 0.8 \\ \eta \sim 0.2 & \rho \sim 0.3 \end{array}$$

$\sin \theta_{12} = \lambda$
$\sin \theta_{23} = A \lambda^2$
$\sin \theta_{13} = A \lambda^3(\rho - i \eta)$

# The Bjorken-Jarlskog Unitarity Triangle

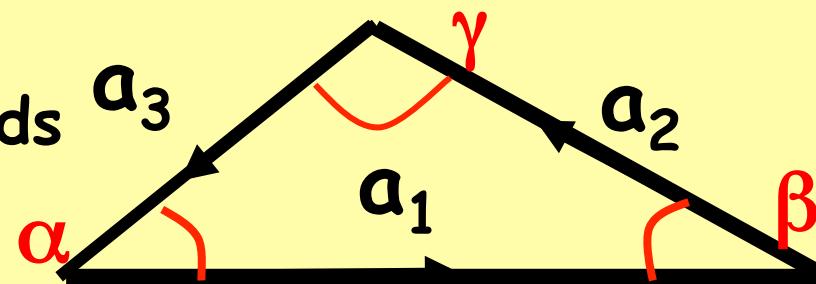


$|V_{ij}|$  is invariant under phase rotations

$$\begin{aligned} a_1 &= V_{11} V_{12}^* = V_{ud} V_{us}^* \\ a_2 &= V_{21} V_{22}^* \quad a_3 = V_{31} V_{32}^* \end{aligned}$$

$$\begin{aligned} a_1 + a_2 + a_3 &= 0 \\ (b_1 + b_2 + b_3 = 0 \text{ etc.}) \end{aligned}$$

Only the orientation depends  
on the phase convention



# STRONG CP VIOLATION

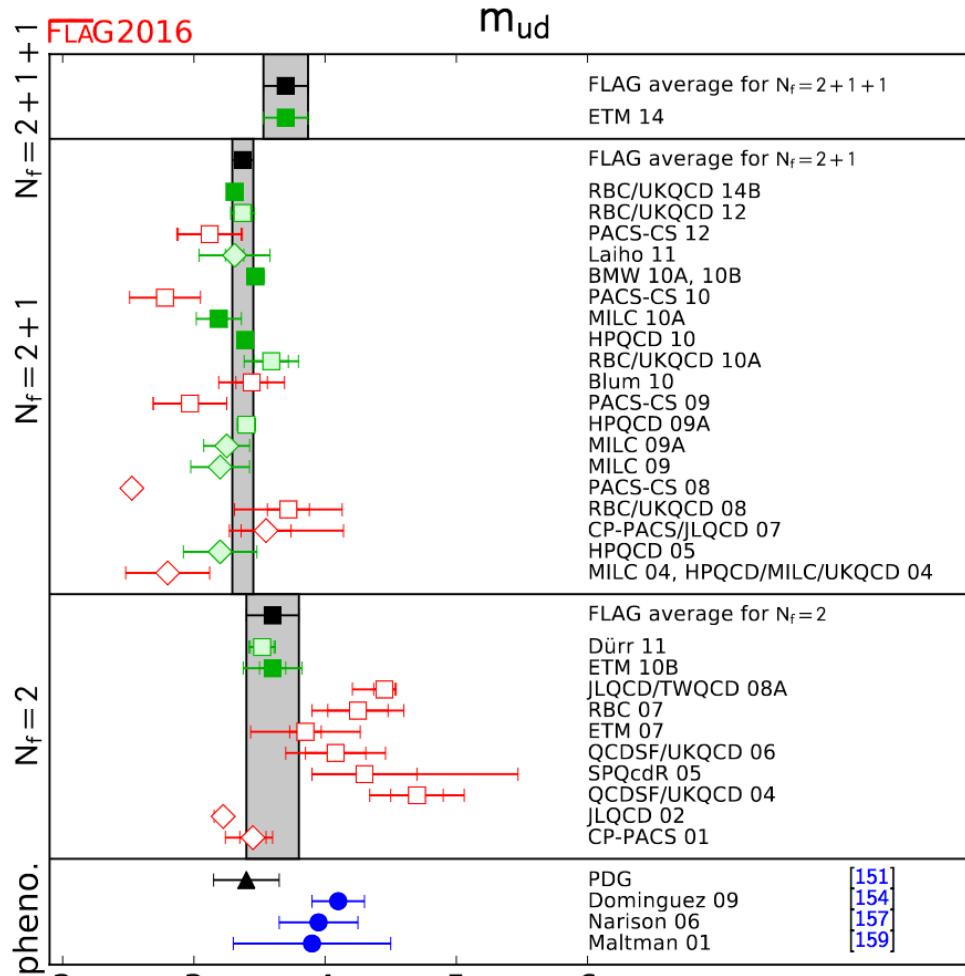
$$\mathcal{L}_\theta = \theta \tilde{G}^{\mu\nu a} G^a_{\mu\nu} \quad \tilde{G}^a_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} G^a_{\rho\sigma}$$

$$\mathcal{L}_\theta \sim \theta \vec{E}^a \cdot \vec{B}^a$$

This term violates CP and gives a contribution to the electric dipole moment of the neutron

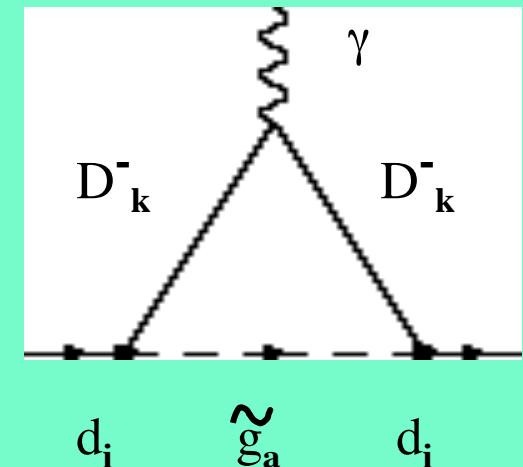
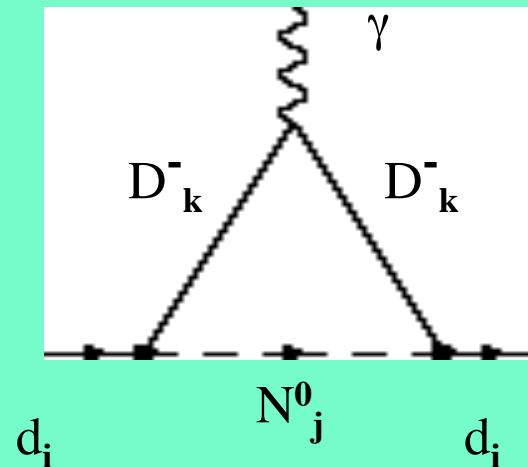
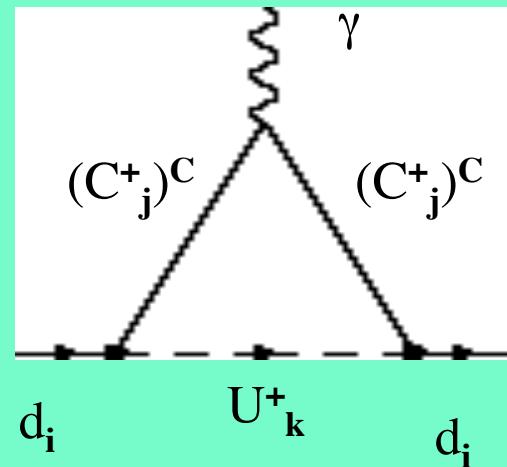
$$e_n < 3 \cdot 10^{-26} \text{ e cm}$$

$\theta < 10^{-10}$  which is quite unnatural !!



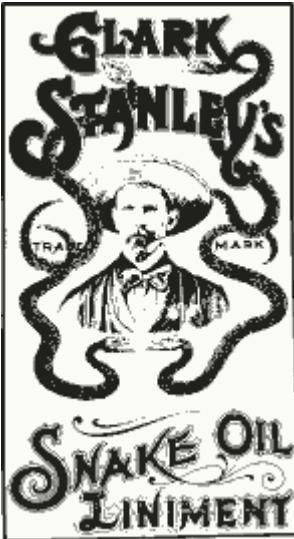
$N_f$	$m_u$	$m_d$	$m_u/m_d$	$R$	$Q$
2+1+1	2.36(24)	5.03(26)	0.470(56)	35.6(5.1)	22.2 (1.6)
2+1	2.16(9)(7)	4.68(14)(7)	0.46(2)(2)	35.0(1.9)(1.8)	22.5(6)(6)
2	2.40(23)	4.80(23)	0.50(4)	40.7(3.7)(2.2)	24.3(1.4)(0.6)

# Neutron electric dipole moment in SuperSymmetry

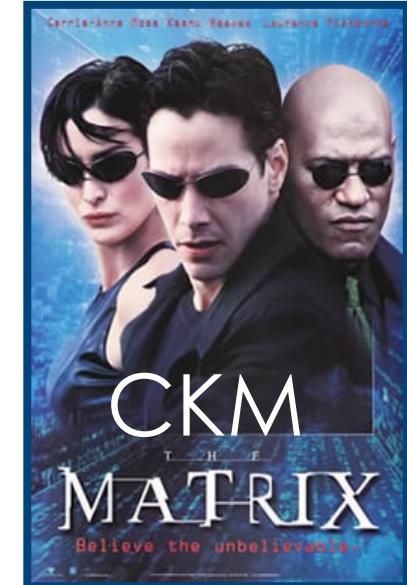


$$\begin{aligned} \mathcal{L}^{\Delta F=0} = & -i/2 C_e \bar{\psi} \sigma_{\mu\nu} \gamma_5 \psi F^{\mu\nu} \\ & -i/2 C_C \bar{\psi} \sigma_{\mu\nu} \gamma_5 t^a \psi G^{\mu\nu a} \\ & -1/6 C_g f_{abc} G^a_{\mu\rho} G^{b\rho}_{\nu} G^c_{\lambda\sigma} \epsilon^{\mu\nu\lambda\sigma} \end{aligned}$$

$C_{e,C,g}$  can be computed perturbatively



[www.utfit.org](http://www.utfit.org)



C. Alpigiani, A. Bevan, M.B., M. Ciuchini,  
D. Derkach, E. Franco, V. Lubicz, G. Martinelli,  
F. Parodi, M. Pierini, C. Schiavi, L. Silvestrini,  
A. Stocchi, V. Sordini, C. Tarantino and V. Vagnoni

Other UT analyses exist, by:

CKMfitter (<http://ckmfitter.in2p3.fr/>),

Laiho&Lunghi&Van de Water (<http://latticeaverages.org/>)

Lunghi&Soni (1010.6069)

Measure	$V_{CKM}$	Other NP parameters
$\Gamma(b \rightarrow u)/\Gamma(b \rightarrow c)$	$\bar{\rho}^2 + \bar{\eta}^2$	$\bar{\Lambda}, \lambda_1, F(1), \dots$
$\epsilon_K$	$\eta [(1 - \bar{\rho}) + \dots]$	$B_K$
$\Delta m_d$	$(1 - \bar{\rho})^2 + \bar{\eta}^2$	$f_{B_d}^2 B_{B_d}$
$\Delta m_d/\Delta m_1$	$(1 - \bar{\rho})^2 + \bar{\eta}^2$	$\xi$
$A_{CP}(B_d \rightarrow J/\psi K_s)$	$\sin 2\beta$	—

$$Q^{EXP} = V_{CKM} \times \langle H_F | \hat{O} | H_I \rangle$$

For details see:  
UTfit Collaboration

<http://www.utfit.org>

*classical UT analysis*

# DIFFERENT LEVELS OF THEORETICAL UNCERTAINTIES (STRONG INTERACTIONS)

1) First class quantities, with reduced or negligible theor. uncertainties

$$A_{CP}(B \rightarrow J/\psi K_s) \quad \gamma \quad \text{from } B \rightarrow DK \\ K^0 \rightarrow \pi^0 \nu \bar{\nu}$$

2) Second class quantities, with theoretical errors of O(10%) or less that can be reliably estimated

$$\varepsilon_K \quad \Delta M_{d,s} \\ \Gamma(B \rightarrow c, u), \quad K^+ \rightarrow \pi^+ \nu \bar{\nu}$$

3) Third class quantities, for which theoretical predictions are model dependent (BBNS, charming, etc.)

In case of discrepancies we cannot tell whether is *new physics or we must blame the model*

$$B \rightarrow K \pi \quad B \rightarrow \pi^0 \pi^0 \\ B \rightarrow \phi K_s$$

# Quantities used in the Standard UT Analysis

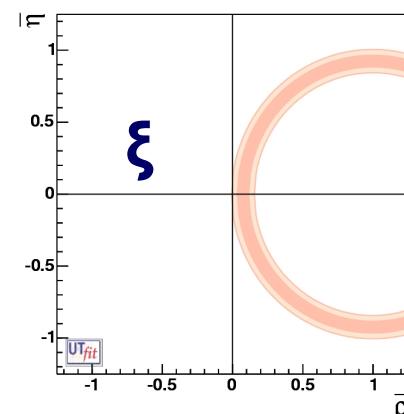
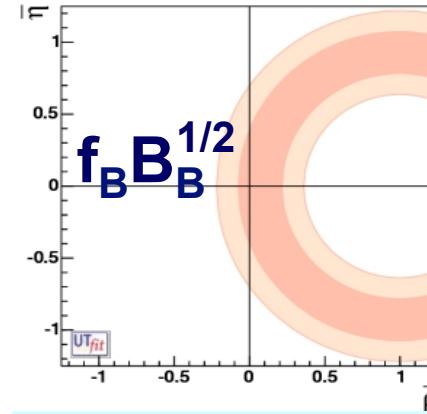
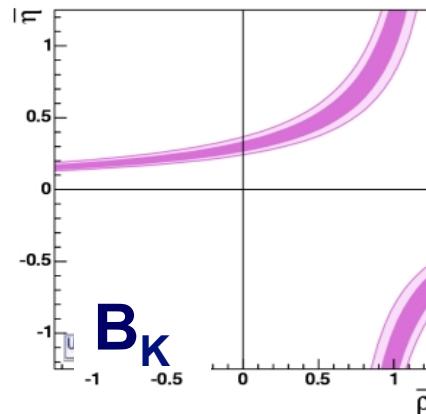
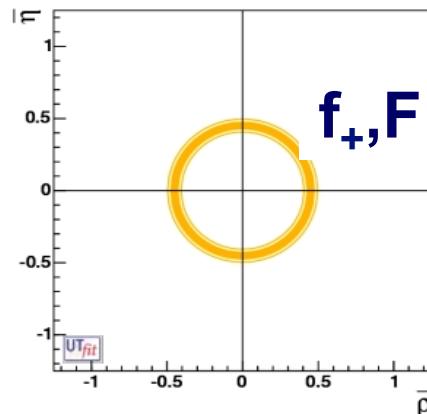
levels @  
68% (95%) CL

$V_{ub}/V_{cb}$

$\varepsilon_K$

$\Delta m_d$

$\Delta m_d/\Delta m_s$



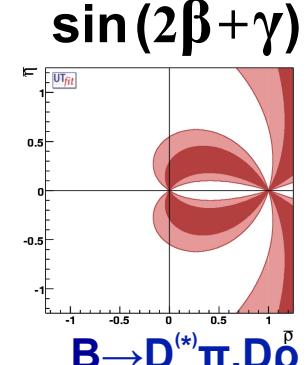
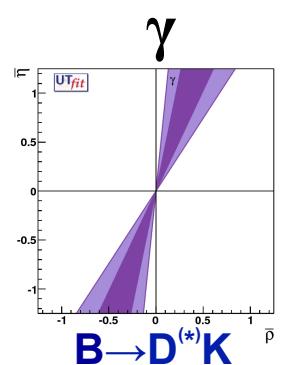
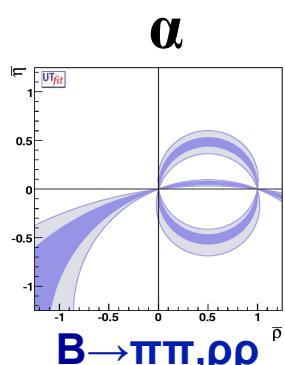
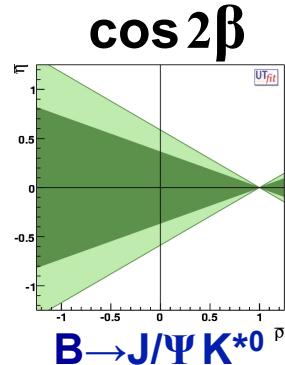
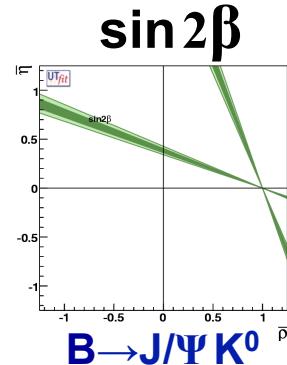
Inclusive vs Exclusive  
Opportunity for lattice  
QCD

**UT-LATTICE**

# Other Quantities used in the UT Analysis

## UT-ANGLES

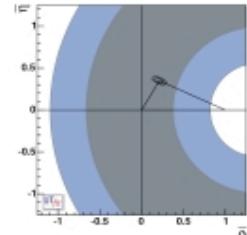
Several new determinations of UT angles are now available, thanks to the results coming from the B-Factory experiments



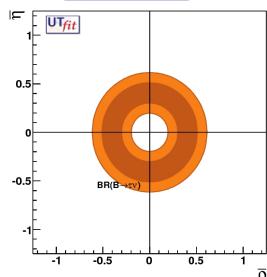
New Constraints from B and K rare decays  
(not used yet)

New bounds are available from rare B and K decays. They do not still have a strong impact on the global fit and they are not used at present.

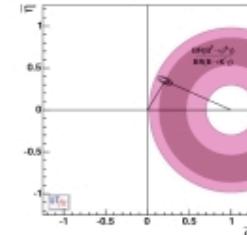
$K \rightarrow \pi \nu \bar{\nu}$



$B \rightarrow \tau \nu$

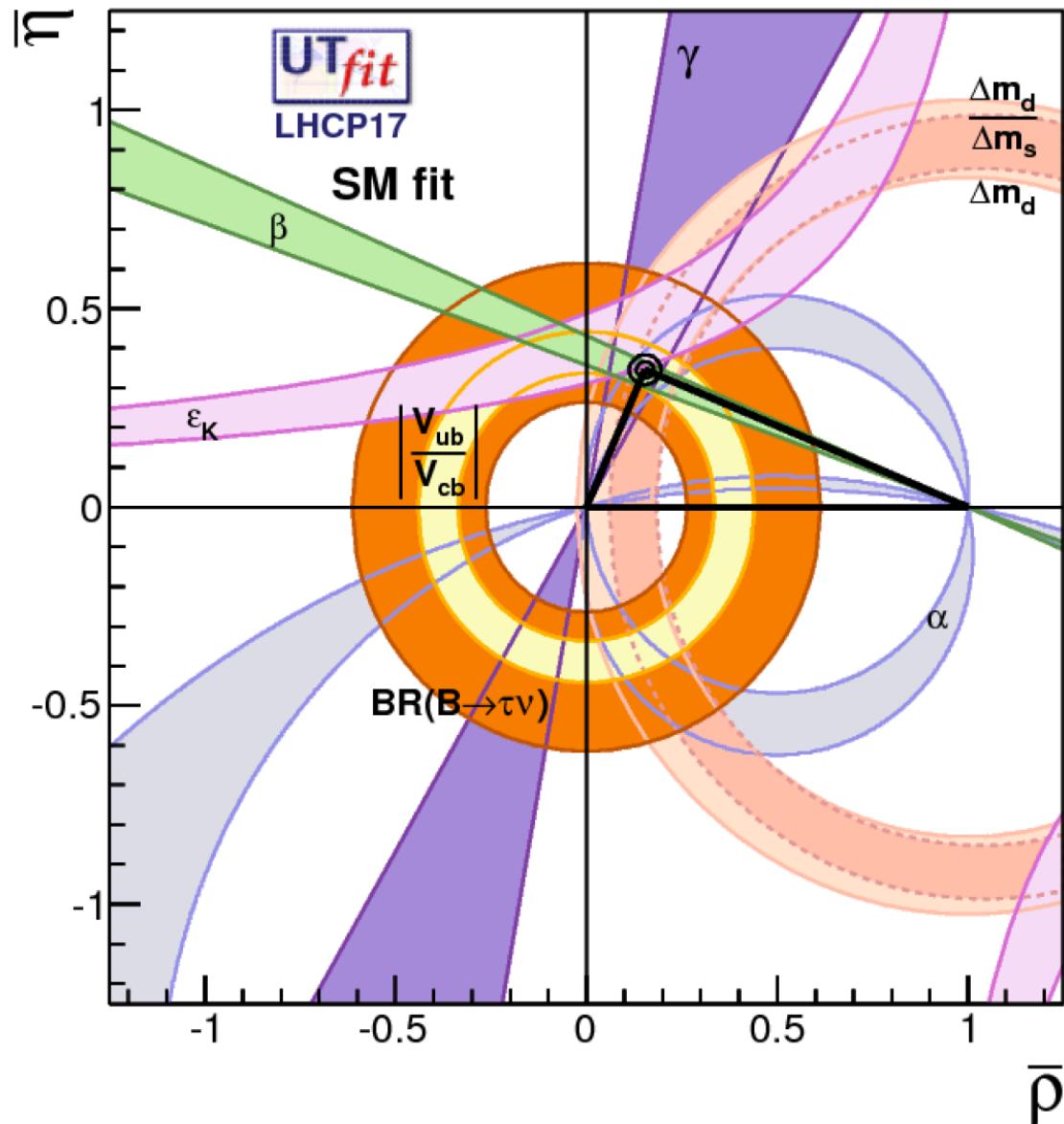


$(B \rightarrow \rho/\omega \gamma)/(B \rightarrow K^* \gamma)$

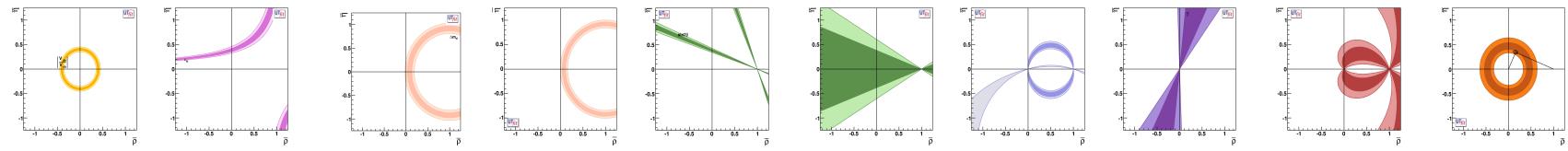




# Unitarity Triangle analysis in the SM:



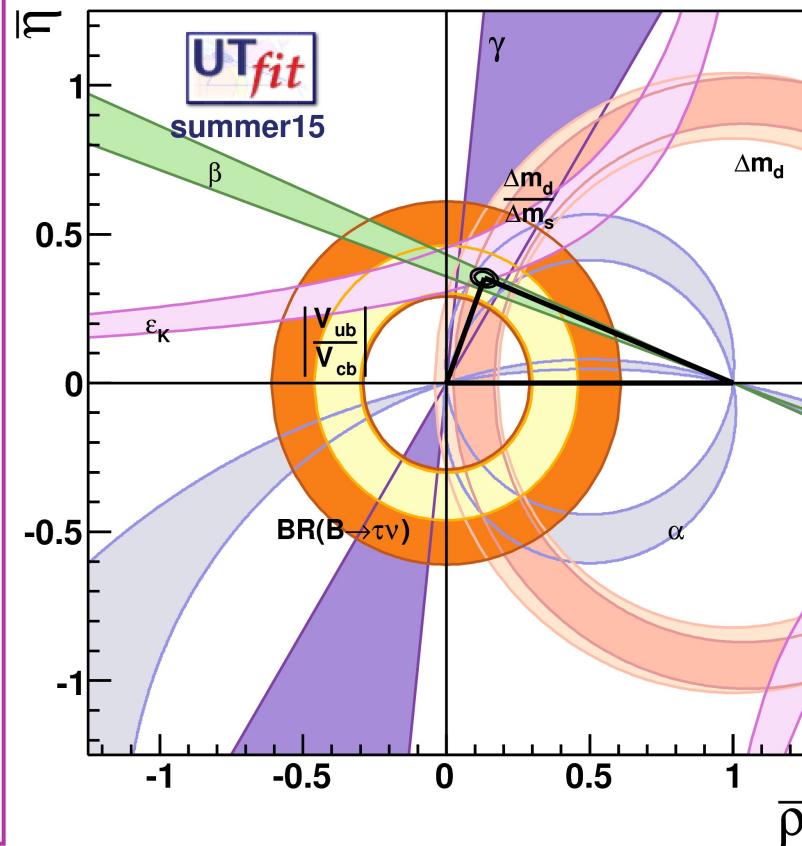
A small off-set for  $\epsilon_K$  in the figure should be corrected



# 2016 results

$$\bar{\rho} = 0.153 \pm 0.013 \quad \bar{\eta} = 0.343 \pm 0.011$$

In the hadronic sector, the SM CKM pattern represents the principal part of the flavor structure and of CP violation



$$\begin{aligned}
 \alpha &= (92.0 \pm 2.0)^0 \\
 \sin 2\beta &= 0.696 \pm 0.018 \\
 \beta &= (21.82 \pm 0.72)^0 \\
 \gamma &= (65.8 \pm 1.9)^0 \\
 A &= 0.833 \pm 0.012 \\
 \lambda &= 0.22497 \pm 0.00069
 \end{aligned}$$

Consistency on an over constrained fit of the CKM parameters

CKM matrix is the dominant source of flavour mixing and CP violation

# CKM Matrix in the SM 2016

CKM matrix thus looks like

$$V_{CKM} = \begin{pmatrix} (0.97431 \pm 0.00015) & (0.22512 \pm 0.00067) & (0.00365 \pm 0.00012)e^{i(-65.88 \pm 1.88)} \\ (-0.22497 \pm 0.00067)e^{i(0.0352 \pm 0.0010)} & (0.97344 \pm 0.00015)e^{i(-0.001877 \pm 0.000055)} & (0.04255 \pm 0.00069) \\ (0.00869 \pm 0.00014)e^{i(-22.00 \pm 0.73)} & (-0.04156 \pm 0.00056)e^{i(1.040 \pm 0.035)} & (0.999097 \pm 0.000024) \end{pmatrix}$$

## Standard Parametrization (PDG)

$$\sin \theta_{12} = 0.22497 \pm 0.00069$$

$$\sin \theta_{23} = 0.04229 \pm 0.00057$$

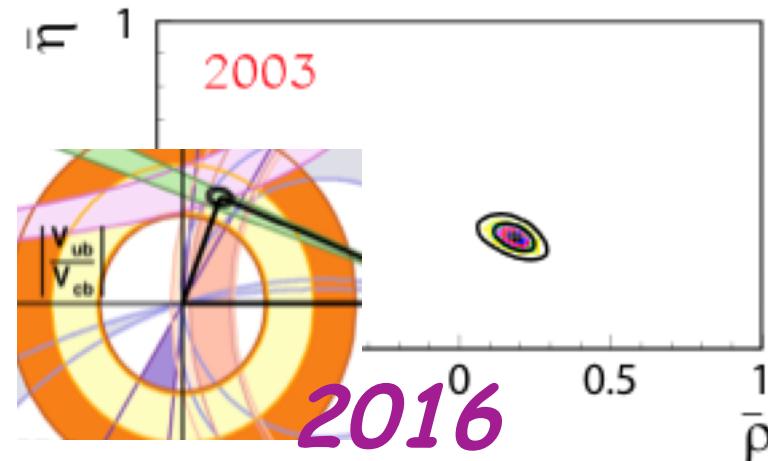
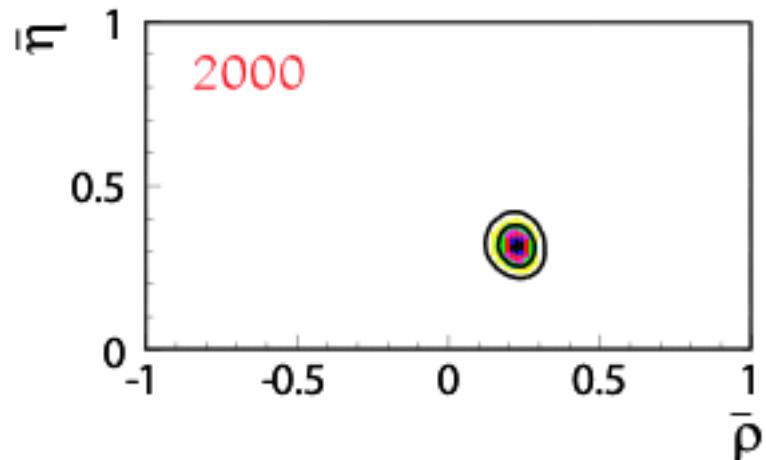
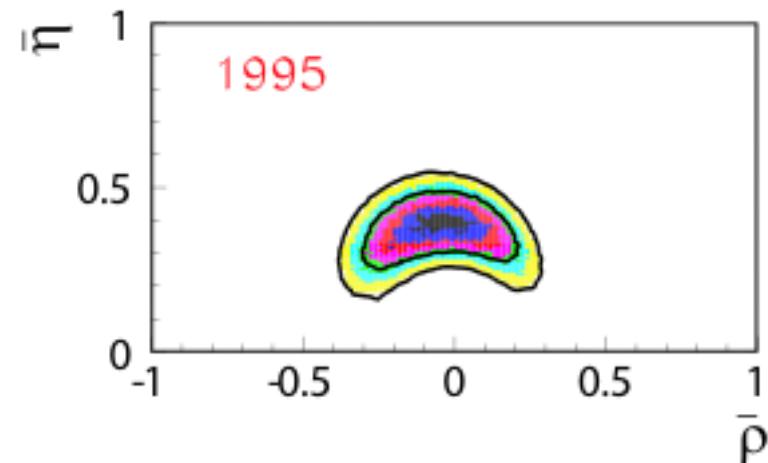
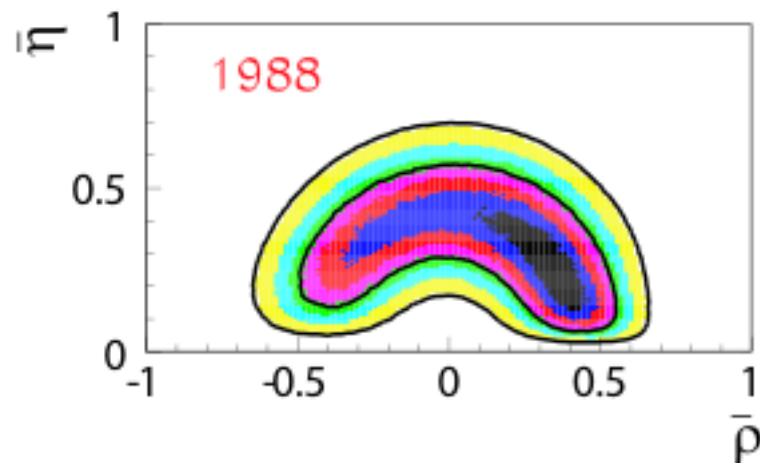
$$\sin \theta_{13} = 0.00368 \pm 0.00002 \quad \delta = 65.9 \pm 2.0$$

## Wolfenstein Parametrization (PDG)

$$\lambda = 0.22497 \pm 0.00069 \quad A = 0.833 \pm 0.012$$

# PROGRESS SINCE 1988

*Experimental progress so impressive that we can fit the hadronic matrix elements (in the SM)*



# *Experimental progress so impressive that we can fit the hadronic matrix elements (in the SM)*

obtained excluding  
the given constraint  
from the fit

Observables	Measurement	Prediction	Pull (# $\sigma$ )
$B_K$	$0.740 \pm 0.029$	$0.81 \pm 0.07$	< 1
$f_{B_s}$	$0.226 \pm 0.005$	$0.220 \pm 0.007$	< 1
$f_{B_s}/f_{B_d}$	$1.203 \pm 0.013$	$1.210 \pm 0.030$	< 1
$B_{B_s}/B_{B_d}$	$1.032 \pm 0.036$	$1.07 \pm 0.05$	< 1
$B_{B_s}$	$1.35 \pm 0.08$	$1.30 \pm 0.07$	< 1

in general: average the Nf=2+1+1 and Nf=2+1 FLAG averages,  
through eq.(28) in arXiv:1403.4504

for  $B_K$ ,  $f_{B_s}$ ,  $f_{B_s}/f_{B_d}$ :

FLAG Nf=2+1+1 (single result) and Nf=2+1 average

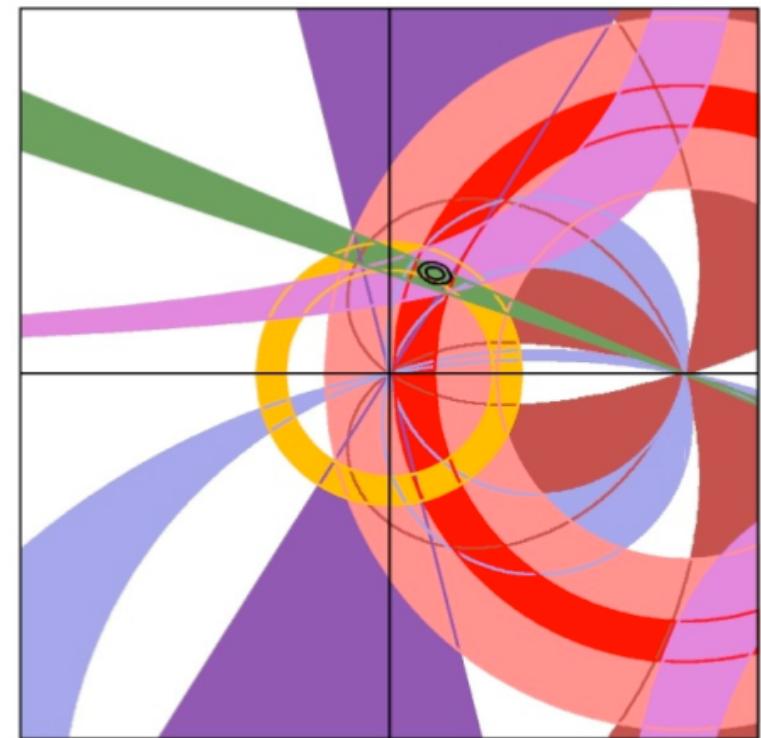
for  $B_{B_s}$ ,  $B_{B_s}/B_{B_d}$ :

update w.r.t. the Nf=2+1 FLAG average (no Nf=2+1+1 results yet)  
updating the FNAL/MILC result to FNAL/MILC 2016 (1602.03560)

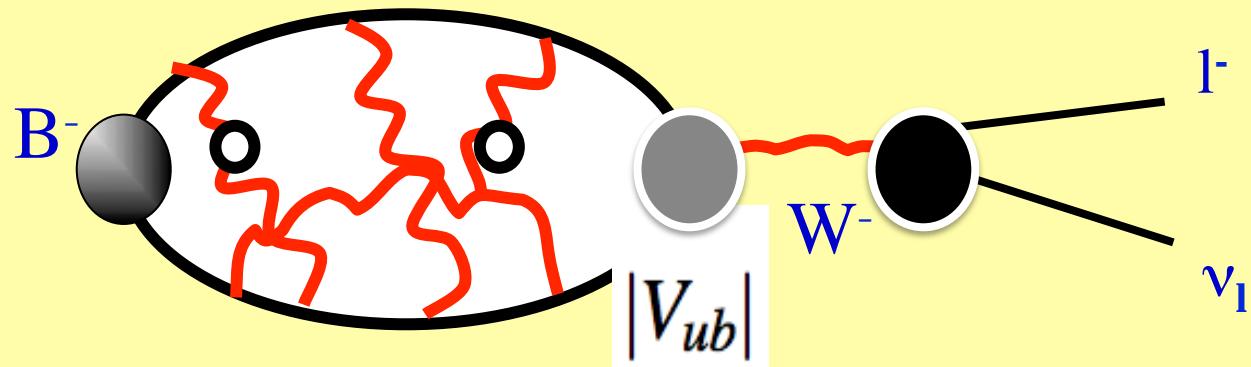
# Do we still care? Tensions and Unknowns

- 1) A ``classical'' example  $B \rightarrow \tau\nu$
- 2)  $|V_{ub}|$  and  $|V_{cb}|$  inclusive vs exclusive
- 3)  $|V_{cb}|$ , B mixing and  $\varepsilon_K$
- 4) D-mixing
- 5)  $R(D)$  and  $R(D^*)$
- 6)  $B \rightarrow K^* ll$
- 7) Physics BSM ?

- What can be computed and what cannot be computed



## The Simplest Example

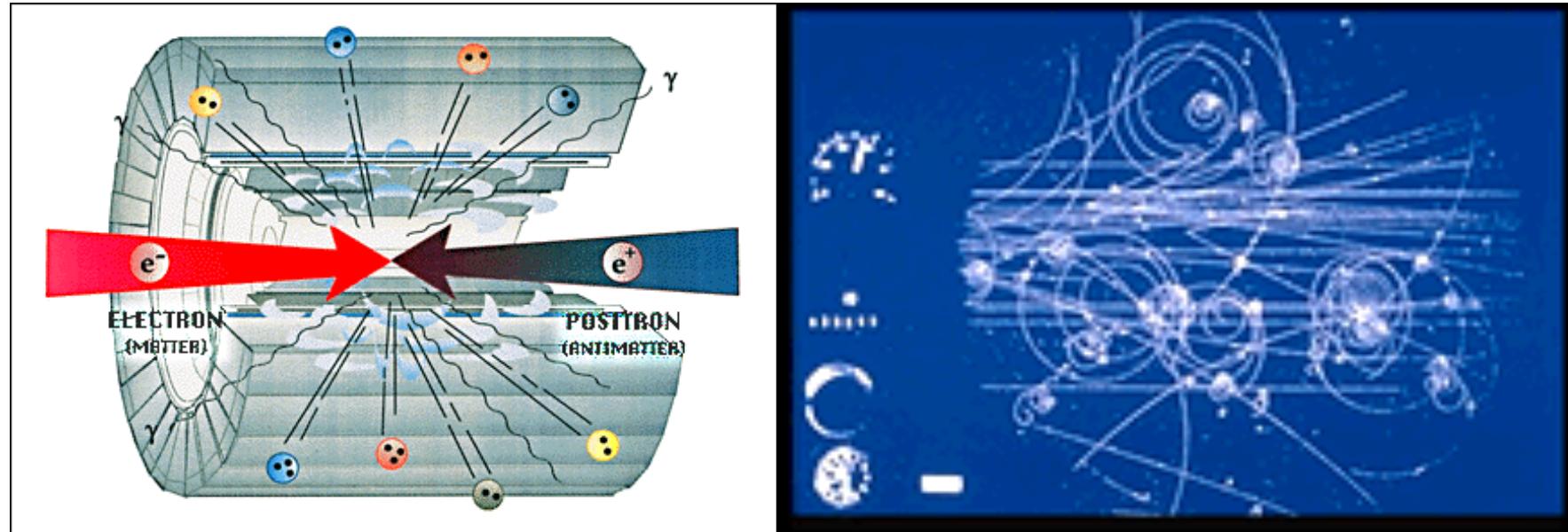


$$BR(B^- \rightarrow \tau^- \bar{\nu}_\tau) = f_B^2 |V_{ub}|^2 \frac{G_F^2 m_B m_\tau^2}{8\pi} \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \tau_B$$

$f_B^2 |V_{ub}|^2$

$$\langle 0 | \bar{b} \gamma_\mu \gamma_5 d | B^0(p) \rangle = i f_B p_\mu$$

COULD WE COMPUTE THIS PROCESS WITH  
SUFFICIENT COMPUTER POWER ?



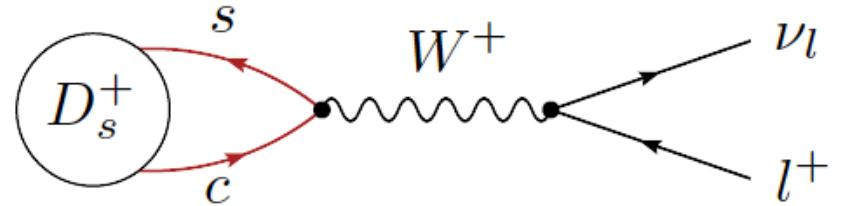
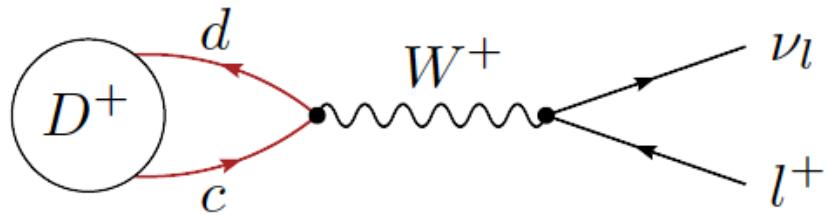
THE ANSWER IS: NO

IT IS NOT ONLY A QUESTION OF COMPUTER POWER  
BECAUSE THERE ARE COMPLICATED  
FIELD THEORETICAL PROBLEMS

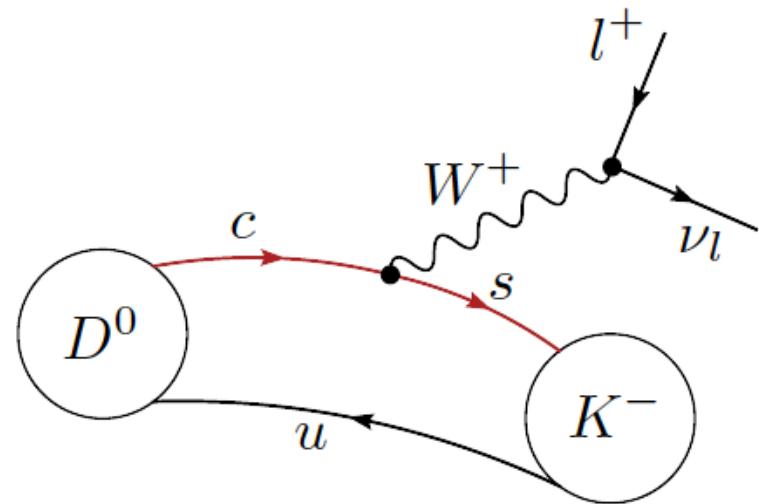
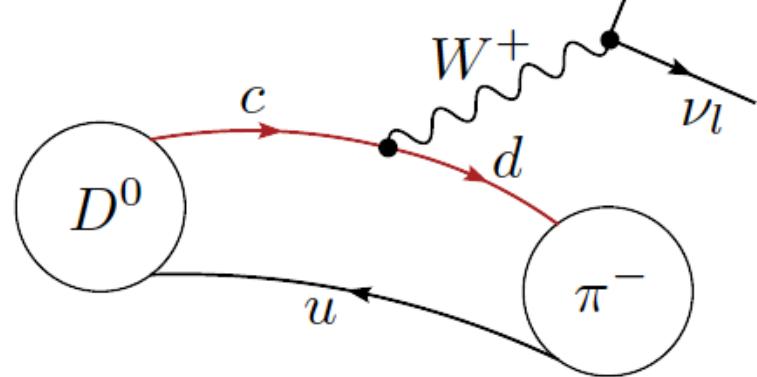
Euclidean vs Minkowski



## Leptonic ( $\pi, K, D, B$ )

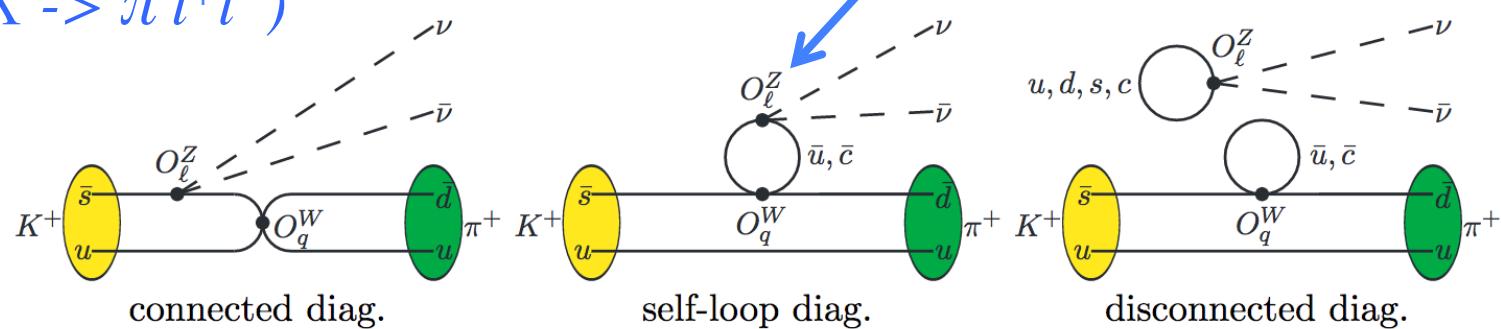


## Semileptonic ( $K, D, B$ ) $l^+$



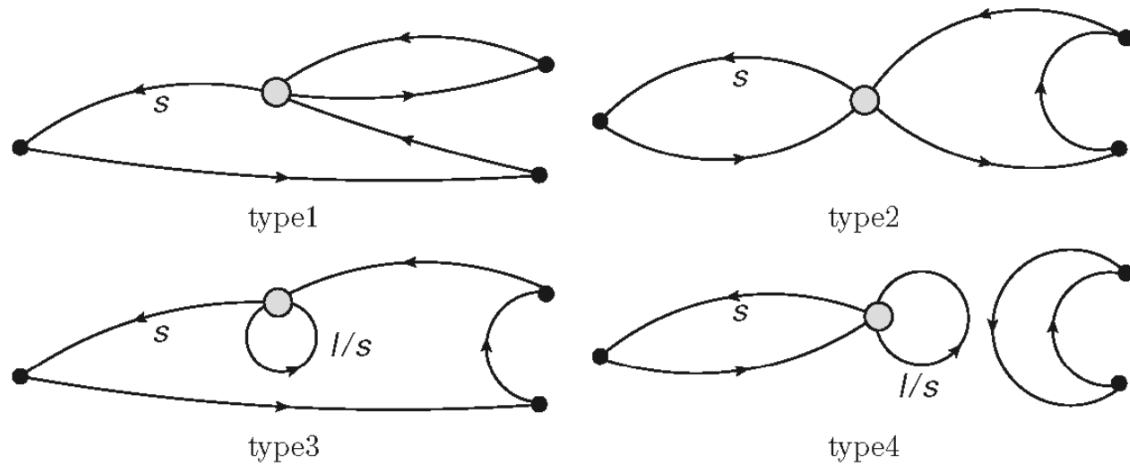
(some) Radiative and Rare  
(also  $K \rightarrow \pi l^+ l^-$ )

long distance effects

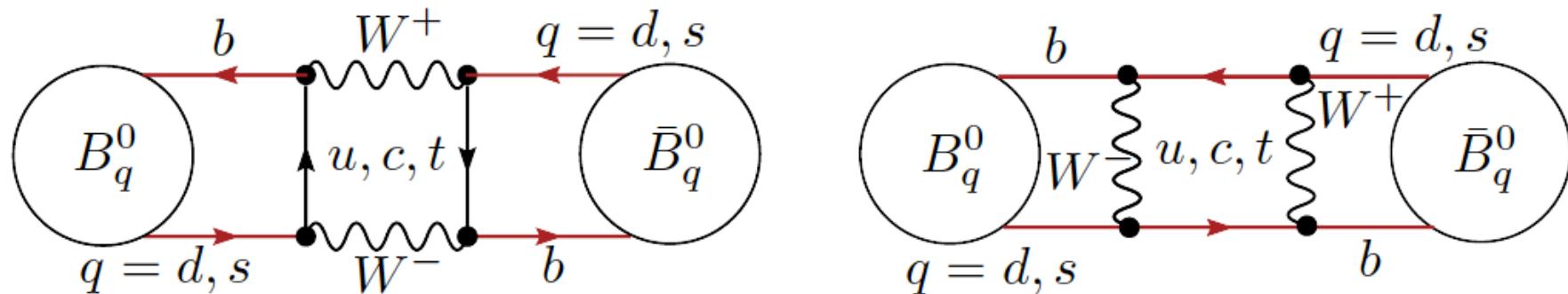


*Non-leptonic  
but only below the  
inelastic threshold  
(may be also  
3 body decays)*

$B \rightarrow \pi\pi, K\pi, \text{etc. No !}$



*Neutral meson mixing (local)*



+ some long distance contributions to  $K$  and  $D$  neutral meson mixing + short distance contributions to  $B \rightarrow K^{(*)} l^+ l^-$

## *Radiative corrections to weak amplitudes*

important for hadron masses, leptonic and semileptonic decays,  $|V_{us}|$ , but also for D and B decays

13

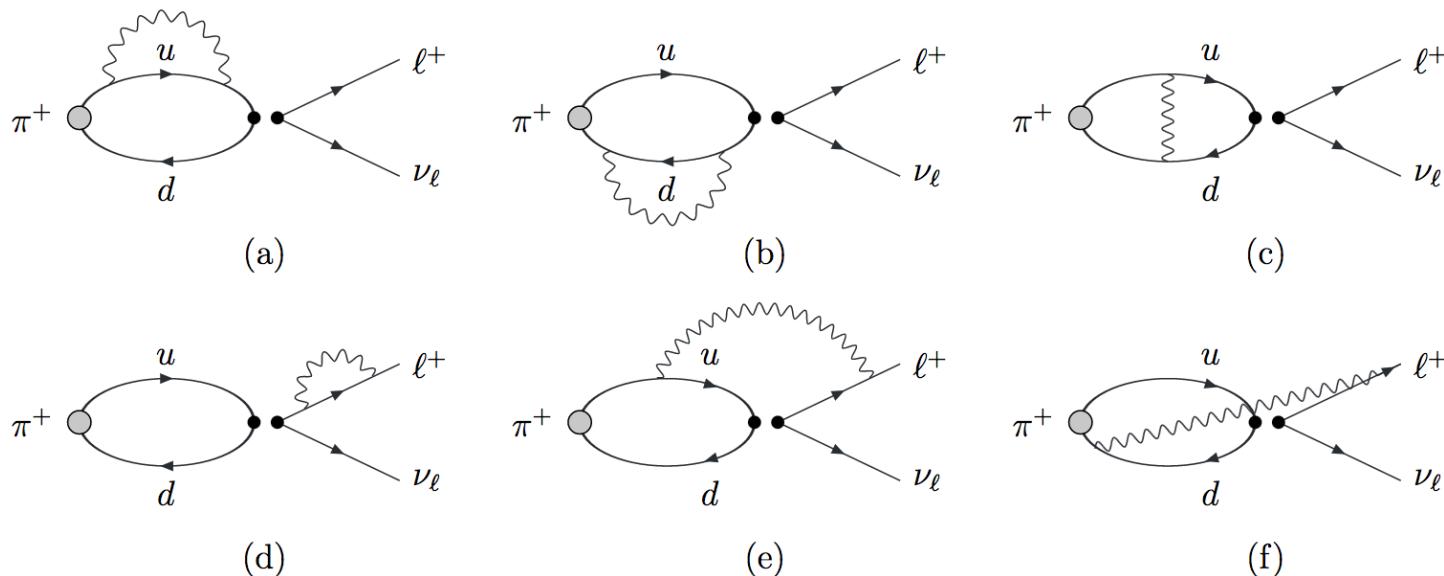


FIG. 5: Connected diagrams contributing at  $O(\alpha)$  contribution to the amplitude for the decay  $\pi^+ \rightarrow \ell^+ \nu_\ell$ .

The accuracy of lattice calculations of the hadron spectrum (and hence of the quark masses) and of the decay constants and form factors is such that isospin breaking and em effects cannot be neglected anymore:

FLAG Collaboration, arXiv:1607.00299

$$N_f = 2+1 \quad m_{ud} = 3.37(8) \text{ MeV} \quad m_s = 92.0(2.1) \text{ MeV}$$

$$m_s/m_{ud} = 27.43(31) \quad \varepsilon = 3\%-6\%$$

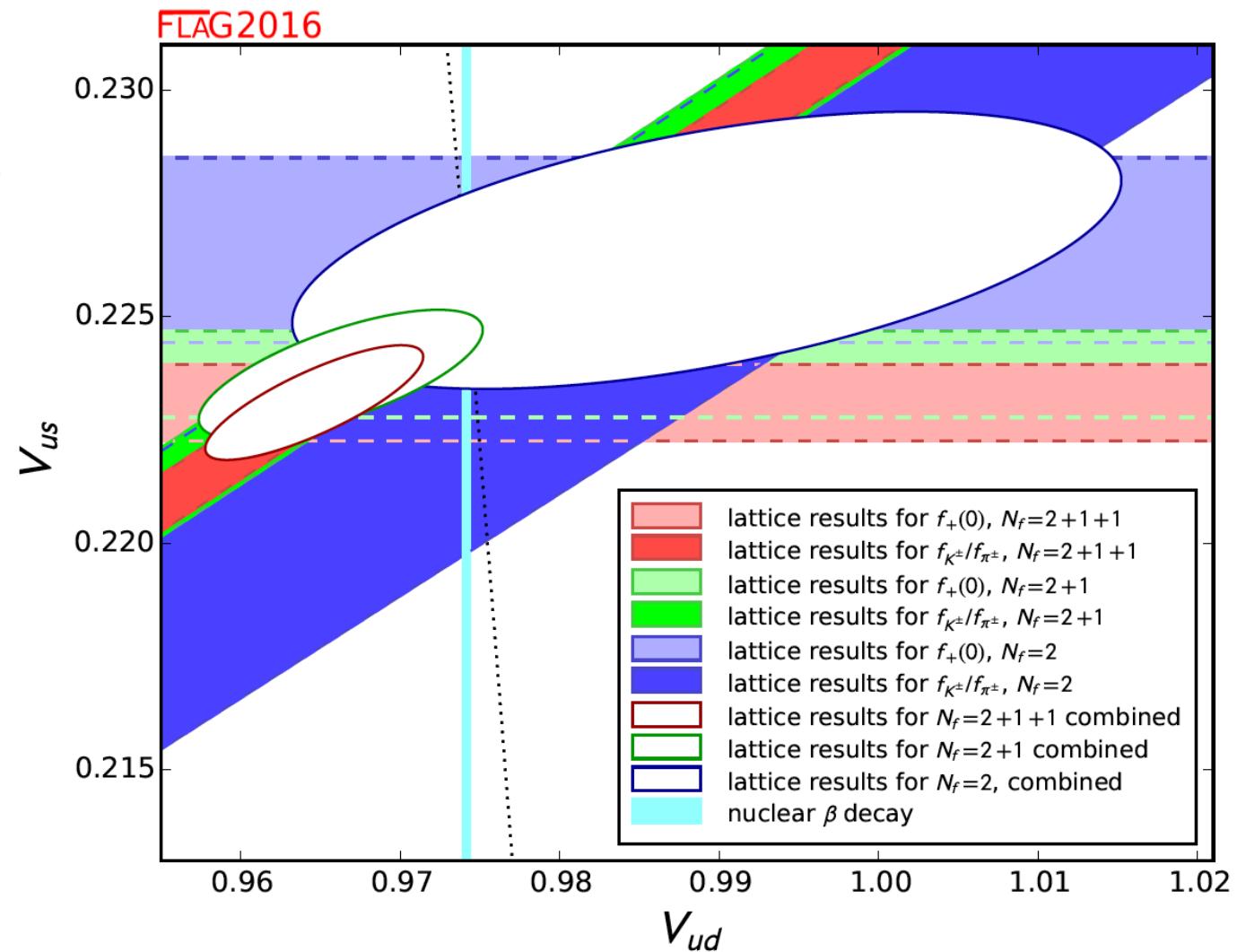
$$N_f = 2+1+1$$

$$m_{ud} = 3.70(17) \text{ MeV} \quad m_s = 93.9(1.1) \text{ MeV}$$
$$m_s/m_{ud} = 27.30(34)$$

$$f_\pi = 130.2(1.4) \text{ MeV} \quad f_K = 155.36(0.4) \text{ MeV} \quad \varepsilon = 0.26\%$$

$$f_K/f_\pi = 1.1933(29) \quad \varepsilon = 0.24\% \quad F^{K\pi}(0) = 0.9704(32) \quad \varepsilon = 0.34\%$$

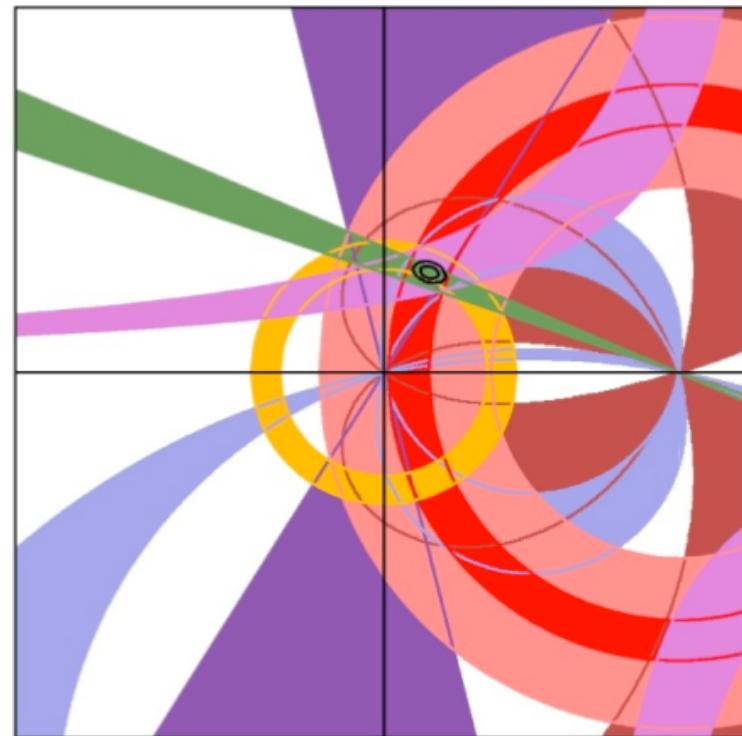
# STANDARD MODEL UNITARITY TRIANGLE ANALYSIS (FLAG)



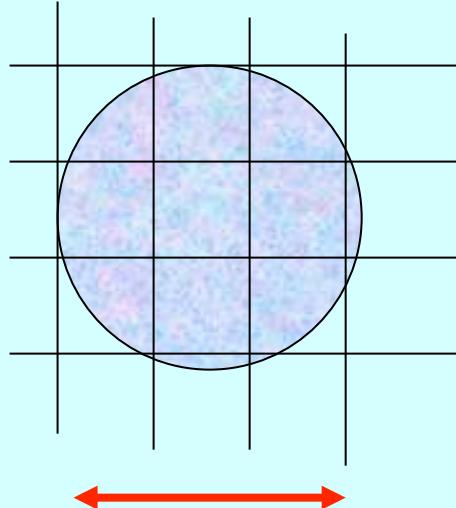
- $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9998(5)$  or  $0.9999(6)$  from semileptonic and leptonic respectively

*Relevant also in D and B meson decays*

- Uncertainties in
- lattice QCD calculations

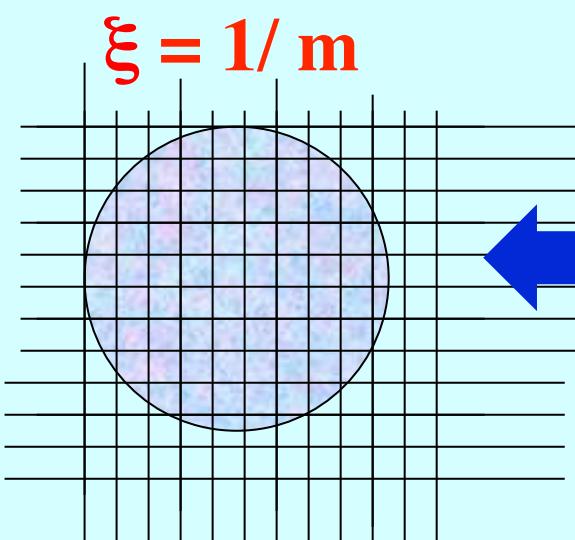


# Continuum limit, discretization and finite volume errors



$a$  Formal  $\lim_{a \rightarrow 0} S_{\text{Lattice}}(\phi) \rightarrow S_{\text{Continuum}}(\phi)$

$a/\xi = m$   $a \sim 1$  The size of the object is comparable to the lattice spacing



$a/\xi \ll 1$  i.e.  $m a \rightarrow 0$  The size of the object is much larger than the lattice spacing

Similar to  $a \sum_n \rightarrow \int dx$

# *Physics Reach (Mainly Heavy Flavor Physics)*

*many slides from Lattice Conferences*

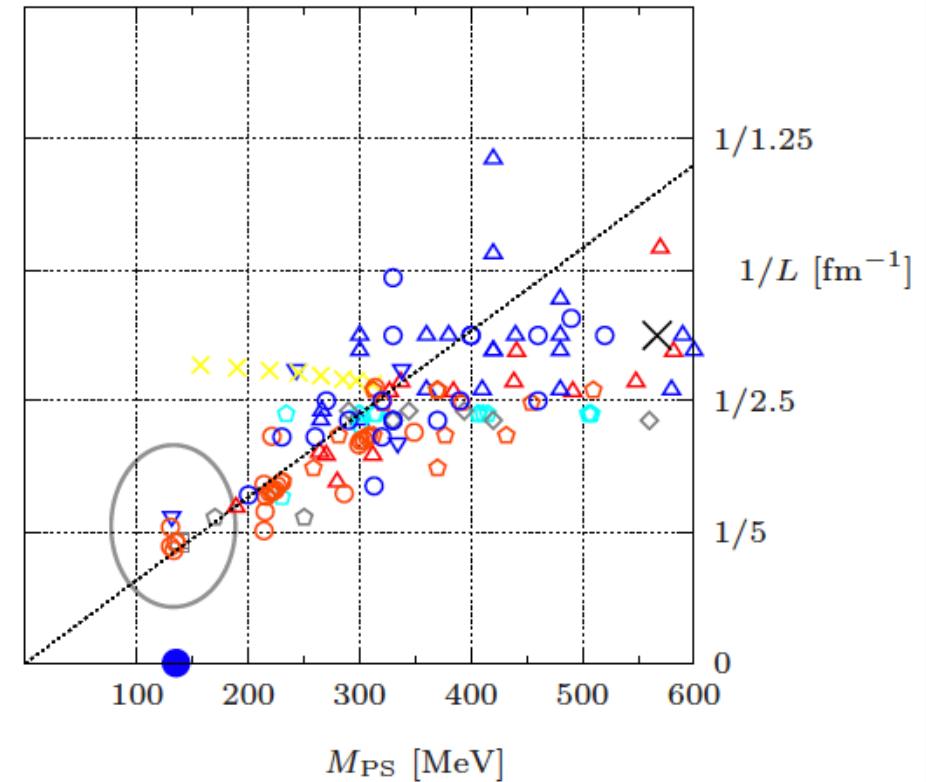
- charm physics directly accessible for some time now
- fraction of available ensembles used for HQ physics still limited

CLS	$N_f = 2$	▲
ETMC	$N_f = 2$	△
(clover) ETMC	$N_f = 2$	▼
(Iwa) TWQCD	$N_f = 2$	×
(Möbius) JLQCD	$N_f = 2 + 1$	○
RBC-UKQCD	$N_f = 2 + 1$	◊
(DSDR) RBC-UKQCD	$N_f = 2 + 1$	◊
(Möbius) RBC-UKQCD	$N_f = 2 + 1$	◻
MILC	$N_f = 2 + 1$	◊
MILC	$N_f = 2 + 1 + 1$	○
ETMC	$N_f = 2 + 1 + 1$	○
JLQCD/CP-PACS (2001)	$N_f = 2$	×
	$M_\pi$ (experiment)	●

—  $\Lambda_{UV} \sim 1/a$

—  $m_Q$

—  $\Lambda_{QCD}$

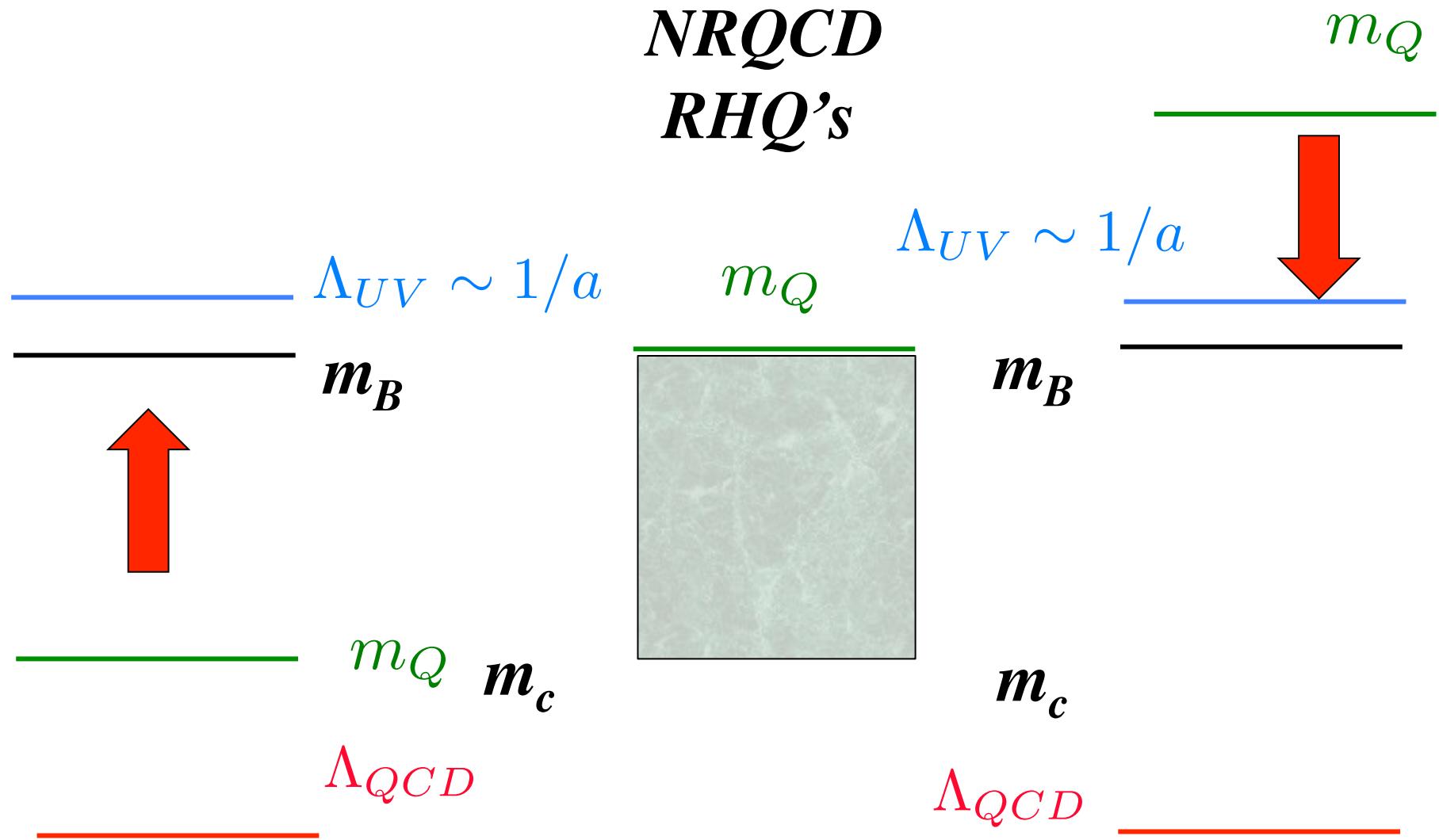


[G Herdoíza]

*a crosscheck of different approaches is fundamental*

*Extrapolation in  $1/m_Q$*   
*Ratio Method*

*npHQET*



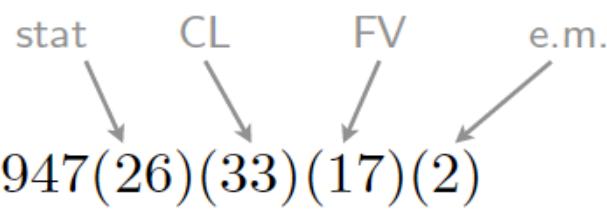
# *ATTENTION TO THE QUOTED ERRORS*

significant differences in estimates of fit and systematic uncertainties in otherwise very similar computations

well-known example from light-quark physics (both computations use MILC ensembles, relatively minor differences)

MILC 13       $f_{K^\pm}/f_{\pi^\pm}|_{N_f=2+1+1} = 1.1947(26)(33)(17)(2)$

stat      CL      FV      e.m.



This diagram shows the error budget for the MILC 13 result. Four arrows point from the right towards the error terms in the equation above. The top arrow is labeled 'e.m.' (electromagnetic). The bottom row of arrows is labeled 'stat' (statistical), 'CL' (constrained least squares), and 'FV' (fit variance). The rightmost arrow is labeled '(misc)' (miscellaneous).

HPQCD 13       $f_{K^\pm}/f_{\pi^\pm}|_{N_f=2+1+1} = 1.1916(15)(12)(1)(10)$

**+ perturbative renormalization**  
courtesy of C. Pena



---

$\Lambda_b \rightarrow p \ell^- \bar{\nu}_\ell$  and  $\Lambda_b \rightarrow \Lambda_c \ell^- \bar{\nu}_\ell$  form factors from lattice QCD  
with relativistic heavy quarks

William Detmold,<sup>1</sup> Christoph Lehner,<sup>2</sup> and Stefan Meinel<sup>3, 4, \*</sup>

<sup>1</sup>*Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA*

<sup>2</sup>*Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA*

<sup>3</sup>*Department of Physics, University of Arizona, Tucson, AZ 85721, USA*

<sup>4</sup>*RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA*

*Very nice paper – interesting for LHCb*

Parameter	coarse	fine
$am_Q^{(b)}$	8.45	3.99
$\xi^{(b)}$	3.1	1.93
$c_{E, B}^{(b)}$	5.8	3.57
$am_Q^{(c)}$	0.1214	-0.0045
$\xi^{(c)}$	1.2362	1.1281
$c_E^{(c)}$	1.6650	1.5311
$c_B^{(c)}$	1.8409	1.6232

TABLE II. Parameters of the bottom and charm quark actions [51, 52].

the parameters  $\nu$ ,  $c_E$ ,  $c_B$  as functions of  $am_Q$ , heavy-quark discretization errors proportional to powers of  $am_Q$  can be removed to all orders. The remaining discretization errors are of order  $a^2|p|^2$ , where  $|p|$  is the typical magnitude of the spatial momentum of the heavy quark inside the hadron. As the continuum limit  $a \rightarrow 0$  is approached, the

# FLAG-2 on $B$ mixing

$BB_s = 1.32(5)$   $N_f=2$ , ETMC

$BB_s = 1.33(6)$   $N_f=2+1$

HPQCD

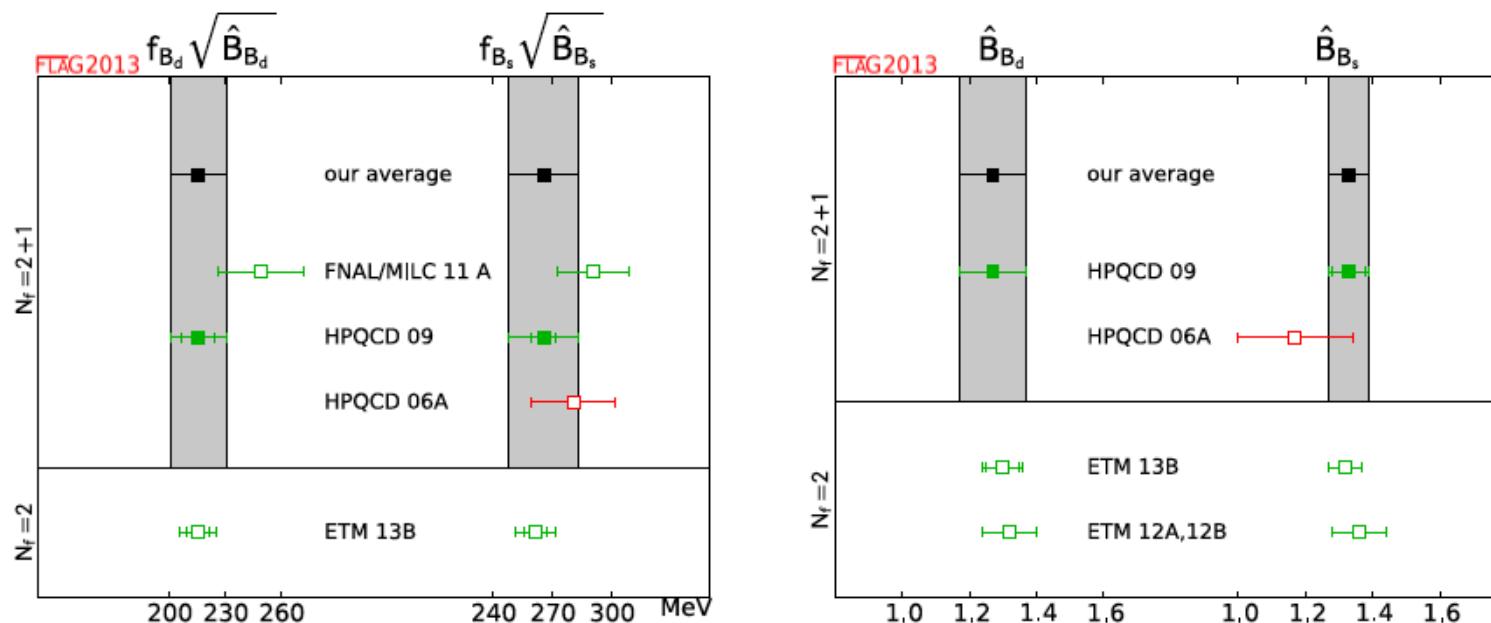
$BB_s = 1.492(92)$   $N_f=2+1$ , NEW

FNAL/MILC

UTFIT AV.       $BB_s = 1.38(11)$

now published

Collaboration	Ref.	$N_f$	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization/matching	$f_{B_d} \sqrt{\hat{B}_{B_d}}$	$f_{B_s} \sqrt{\hat{B}_{B_s}}$	$\hat{B}_{B_d}$	$\hat{B}_{B_s}$
FNAL/MILC 11A	[411]	2+1	C	★ ○	★ ○	✓	250(23) <sup>†</sup>	291(18) <sup>†</sup>	—	—	—
HPQCD 09	[402]	2+1	A	○ ○ <sup>▼</sup>	★ ○	✓	216(15)*	266(18)*	1.27(10)*	1.33(6)*	—
HPQCD 06A	[412]	2+1	A	■ ■	★ ○	✓	—	281(21)	—	—	1.17(17)
ETM 13B	[334]	2	P	★ ○	★ ★	✓	216(6)(8)	262(6)(8)	1.30(5)(3)	1.32(5)(2)	—
ETM 12A, 12B	[392, 413]	2	C	★ ○	★ ★	✓	—	—	1.32(8) <sup>°</sup>	1.36(8) <sup>°</sup>	—

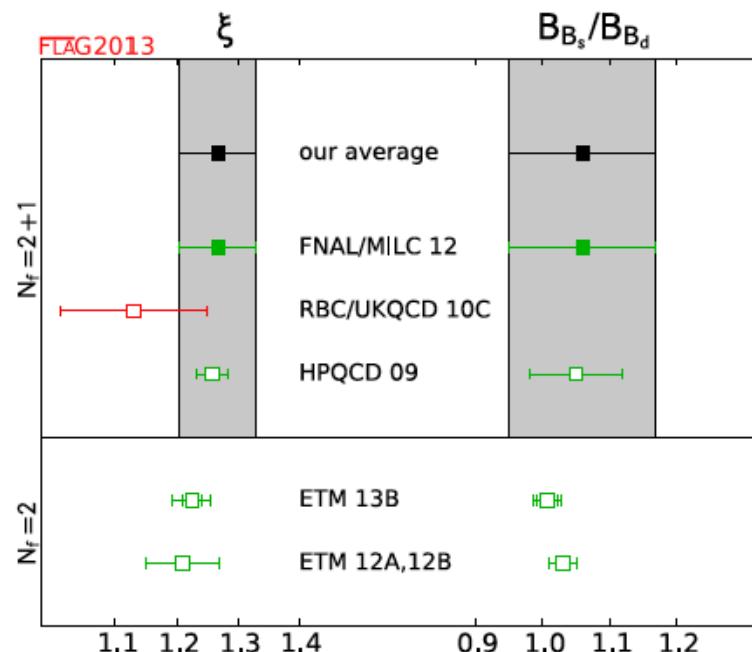


# *FLAG-2 on $B$ mixing*

**$FLAG2 \, BBs/BBd = 1.06(11)$**   
 **$UTFIT \, BBs/BBd = 1.012(27)$**

now published

Collaboration	Ref.	$N_f$	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization/matching	$\xi$	$B_{B_s}/B_{B_d}$
FNAL/MILC 12	[414]	2+1	A	○	○	★	○	✓	1.268(63)
RBC/UKQCD 10C	[405]	2+1	A	■	■	★	○	✓	1.13(12)
HPQCD 09	[402]	2+1	A	○	○ <sup>▼</sup>	★	○	✓	1.258(33)
<hr/>									
ETM 13B	[334]	2	P	★	○	★	★	✓	1.225(16)(14)(22)
ETM 12A, 12B	[392, 413]	2	C	★	○	★	★	✓	1.21(6) 1.03(2)
<hr/>									



# Do we still care? Tensions and Unknowns

- 1) A ``classical'' example  $B \rightarrow \tau\nu$
- 2)  $|V_{ub}|$  and  $|V_{cb}|$  inclusive vs exclusive
- 3)  $|V_{cb}|$ , B mixing and  $\varepsilon_K$
- 4) D-mixing
- 5)  $R(D)$  and  $R(D^*)$
- 6)  $B \rightarrow K^* ll$
- 7) Physics BSM ?

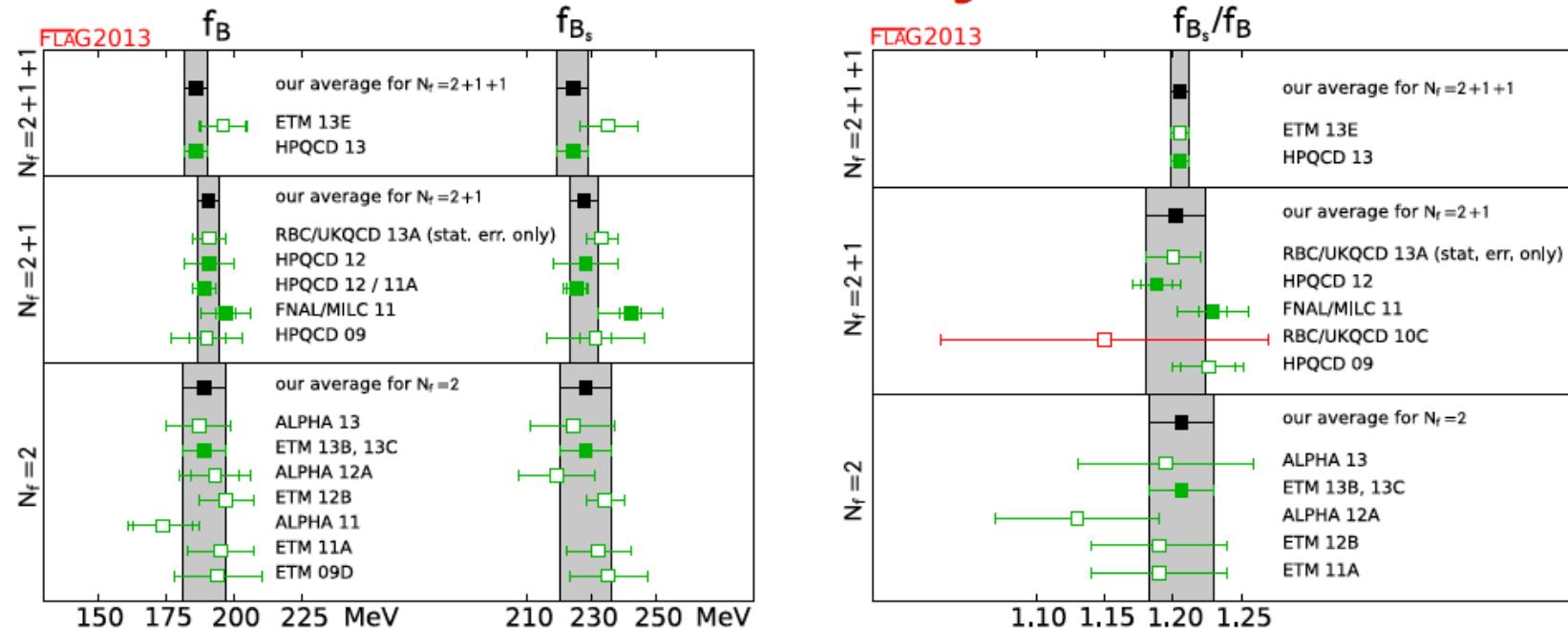
# CKM-TRIANGLE ANALYSIS

## State of The Art 2015

	Measurement	Fit	Prediction	Pull
$\alpha$	$(92.7 \pm 6.2)^\circ$ 6.7 %	$(90.1 \pm 2.7)^\circ$ 2.9 %	$(88.3 \pm 3.4)^\circ$ 3.8 %	0.6
$\sin 2\beta$	$0.680 \pm 0.024$ 3.5 %	$0.696 \pm 0.022$ 2.6 %	$0.747 \pm 0.039$ 5.2 %	1.8
$\gamma$	$(71.4 \pm 6.5)^\circ$ 9.1 %	$(67.4 \pm 2.8)^\circ$ 4.2 %	$(66.7 \pm 3.0)^\circ$ 4.5 %	0.7
$ V_{ub}  \times 10^3$	$3.81 \pm 0.40$ 10 %	$3.66 \pm 0.12$ 3.3 %	$3.64 \pm 0.12$ 3.3 %	0.5
$ V_{cb}  \times 10^2$	$4.09 \pm 0.11$ 2.6 %	$4.206 \pm 0.053$ 1.2 %	$4.240 \pm 0.062$ 1.4 %	0.9
$\varepsilon_K \times 10^3$	$2.228 \pm 0.011$ 0.5 %	$2.227 \pm 0.011$ 0.5 %	$2.08 \pm 0.18$ 8.7 %	0.8
$\Delta m_s \text{ (ps}^{-1})$	$17.761 \pm 0.022$ 0.1 %	$17.755 \pm 0.022$ 0.1 %	$17.3 \pm 1.0$ 5.7 %	0.2
$BR(B \rightarrow \tau\nu) \times 10^4$	$1.06 \pm 0.20$ 18.9 %	$0.83 \pm 0.07$ 7.9 %	$0.81 \pm 0.7$ 8.2 %	1.3
$BR(B_s \rightarrow \mu\mu) \times 10^9$	<del><math>2.9 \pm 0.7</math></del> 24.1 %	<del><math>3.00 \pm 0.15</math></del> 3.8 %	<del><math>3.04 \pm 0.16</math></del> 4.0 %	ew corrections not included
$BR(B_d \rightarrow \mu\mu) \times 10^9$	$0.39 \pm 0.15$ 38.5 %	$0.1098 \pm 0.0057$ 5.2 %	$0.1103 \pm 0.0058$ 5.2 %	ew corrections not included
$\beta_s$	$(0.97 \pm 0.95)^\circ$ 98 %	$(1.056 \pm 0.039)^\circ$ 4.4 %	$(1.056 \pm 0.039)^\circ$ 4.1 %	0.1 not included in the fit

$$B(B \rightarrow \tau\nu)_{\text{Old}} = (1.67 \pm 0.30) 10^{-4}$$

# FLAG-2 on B decay constants



$N_f$	$f_B$ [MeV]	$f_{B_s}$ [MeV]	$f_{B_s}/f_B$
2	189(8)	228(8)	1.206(24)
2+1	190.5(4.2)	227.7(4.5)	1.202(22)
2+1+1	186(4)	224(5)	1.205(7)

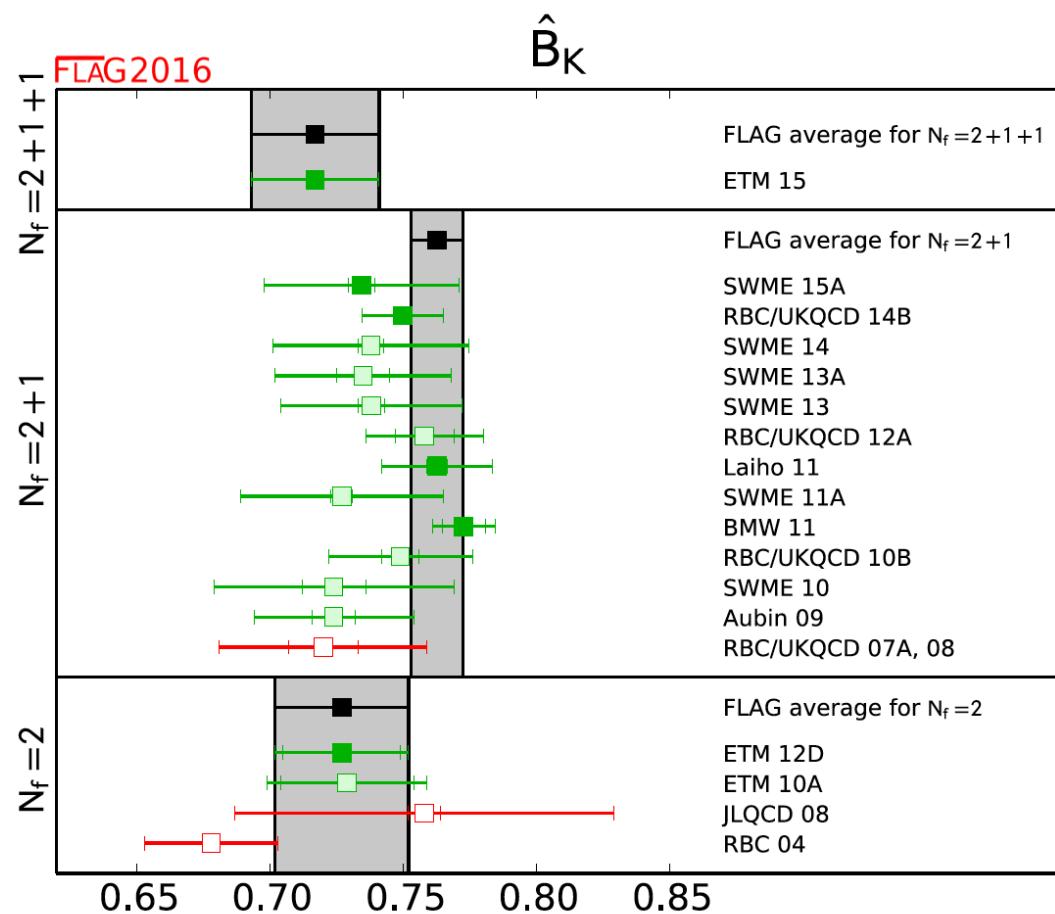
[ FLAG 2013, Eur J Phys C74 (2014) 2890, arXiv:1310.8555v2 ]

(+ HPQCD results for  $f_{B_c}$ , not covered by FLAG) [ PRD 86 (2012) 074503 ]

# FLAG average for Standard Model $B_K$

- $B_K$  in NDR- $\overline{\text{MS}}$  scheme:  $B_K(\mu) = \frac{\langle \bar{K}^0 | Q^{\Delta S=2}(\mu) | K^0 \rangle}{\frac{8}{3} f_K^2 m_K^2}$
- Renormalization group independent  $B$  parameter  $\hat{B}_K$ :  

$$\hat{B}_K = \left( \frac{\bar{g}(\mu)^2}{4\pi} \right)^{-\gamma_0/(2\beta_0)} \exp \left\{ \int_0^{\bar{g}(\mu)} dg \left( \frac{\gamma(g)}{\beta(g)} + \frac{\gamma_0}{\beta_0 g} \right) \right\} B_K(\mu)$$



- $N_f = 2 + 1 + 1$ :  
 $\hat{B}_K = 0.717(24)$
- $N_f = 2 + 1$ :  
 $\hat{B}_K = 0.763(10)$
- $N_f = 2$ :  
 $\hat{B}_K = 0.727(25)$

## LATTICE PARAMETERS (2017)

*It does not make sense to improve the precision  
on  $B_K$  if we do not control long distance effects;  
Similarly for  $f_\pi$  or  $f_K$  without radiative corrections*

Observables	Measurement	Prediction	Pull (# $\sigma$ )
$B_K$	$0.740 \pm 0.029$	$0.81 \pm 0.07$	< 1
$f_{B_s}$	$0.226 \pm 0.005$	$0.220 \pm 0.007$	< 1
$f_{B_s}/f_{B_d}$	$1.203 \pm 0.013$	$1.210 \pm 0.030$	< 1
$B_{B_s}/B_{B_d}$	$1.032 \pm 0.036$	$1.07 \pm 0.05$	< 1
$B_{B_s}$	$1.35 \pm 0.08$	$1.30 \pm 0.07$	< 1

# Long Distance Effects in Neutral Meson Mixing

N.H.Christ, T.Izubuchi, CTS, A.Soni & J.Yu (RBC-UKQCD), arXiv:1212.5931

Z.Bai, N.H.Christ, T.Izubuchi, CTS, A.Soni & J.Yu (RBC-UKQCD), arXiv:1406.0916

Z.Bai (RBC-UKQCD), arXiv:1411.3210

$$\exp \quad \Delta m_K \equiv m_{K_L} - m_{K_S} = 3.483(6) \times 10^{-12} \text{ MeV.}$$

*3.19(41)(96)  
lattice unphysical  
masses*

- Historically led to the prediction of the energy scale of the charm quark.  
Mohapatra, Rao & Marshak (1968); GIM (1970); Gaillard & Lee (1974)
- Tiny quantity  $\Rightarrow$  places strong constraints on BSM Physics.
- Within the standard model,  $\Delta m_K$  arises from  $K^0 - \bar{K}^0$  mixing at second order in the weak interactions:

$$\Delta M_K = 2\mathcal{P} \sum_{\alpha} \frac{\langle \bar{K}^0 | H_W | \alpha \rangle \langle \alpha | H_W | K^0 \rangle}{m_K - E_{\alpha}},$$

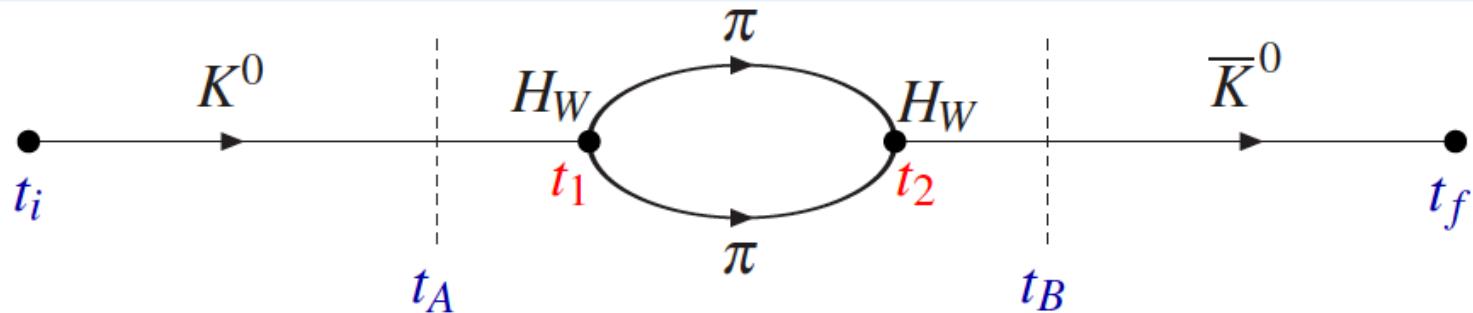
**New project:**  $64^3 \times 128$ ,  $a^{-1} = 2.36$  GeV,  $m_c = 1.2$  GeV,  $m_{\pi} = 136$  MeV

- Based on 59 configurations:  $\Delta M_K = 5.5(1.7) \times 10^{-12}$  MeV

Lattice 2017

[C. Sachrajda's talk, Wednesday 12:30@Seminarios 6+7]

# Long Distance Effects in Neutral Meson Mixing



- $\Delta m_K$  is given by

$$\Delta m_K \equiv m_{K_L} - m_{K_S} = 2\mathcal{P} \sum_{\alpha} \frac{\langle \bar{K}^0 | \mathcal{H}_W | \alpha \rangle \langle \alpha | \mathcal{H}_W | K^0 \rangle}{m_K - E_{\alpha}} = 3.483(6) \times 10^{-12} \text{ MeV}.$$

- The above correlation function gives ( $T = t_B - t_A + 1$ )

$$C_4(t_A, t_B; t_i, t_f) = |Z_K|^2 e^{-m_K(t_f - t_i)} \sum_n \frac{\langle \bar{K}^0 | \mathcal{H}_W | n \rangle \langle n | \mathcal{H}_W | K^0 \rangle}{(m_K - E_n)^2} \times \\ \left\{ e^{(M_K - E_n)T} - (m_K - E_n)T - 1 \right\}.$$

- From the coefficient of  $T$  we can therefore obtain

$$\Delta m_K^{\text{FV}} \equiv 2 \sum_n \frac{\langle \bar{K}^0 | \mathcal{H}_W | n \rangle \langle n | \mathcal{H}_W | K^0 \rangle}{(m_K - E_n)}.$$

# Long Distance Effects in Neutral Meson Mixing

- The general formula can be written:  
N.H.Christ, G.Martinelli & CTS, arXiv:1401.1362  
N.H.Christ, X.Feng, G.Martinelli & CTS, arXiv:1504.01170

$$\Delta m_K = \Delta m_K^{\text{FV}} - 2\pi v \langle \bar{K}^0 | H | n_0 \rangle_V v \langle n_0 | H | K^0 \rangle_V \left[ \cot \pi h \frac{dh}{dE} \right]_{m_K},$$

where  $h(E, L)\pi \equiv \phi(q) + \delta(k)$ .

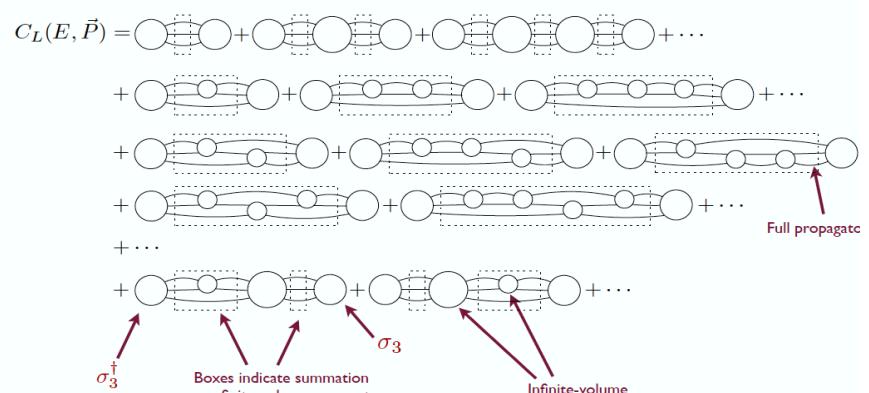
- This formula reproduces the result for the special case when the volume is such that there is a two-pion state with energy  $= m_K$ .  
N.H.Christ, arXiv:1012.6034
- Increasing the volumes keeping  $h = n/2$  and thus avoiding the power corrections is an intriguing possibility.

*Within reasonable approximations  
can be extended to D meson mixing*

M. Ciuchini, V. Lubicz, L. Silvestrini, S. Simula  
(progresses made by M. T. Hansen & S.  
Sharpe, 1204.0826v4, 1409.7012v, 1504.04248v1)

Also CPV in  $D \rightarrow \pi\pi$  or  $KK$

3-particle correlator



# D MIXING

- D mixing is described by:
  - Dispersive  $D \rightarrow \bar{D}$  amplitude  $M_{12}$ 
    - SM: long-distance dominated, not calculable
    - NP: short distance, calculable w. lattice
  - Absorptive  $D \rightarrow \bar{D}$  amplitude  $\Gamma_{12}$ 
    - SM: long-distance, not calculable
    - NP: negligible
  - Observables:  $|M_{12}|$ ,  $|\Gamma_{12}|$ ,  $\Phi_{12} = \arg(\Gamma_{12}/M_{12})$

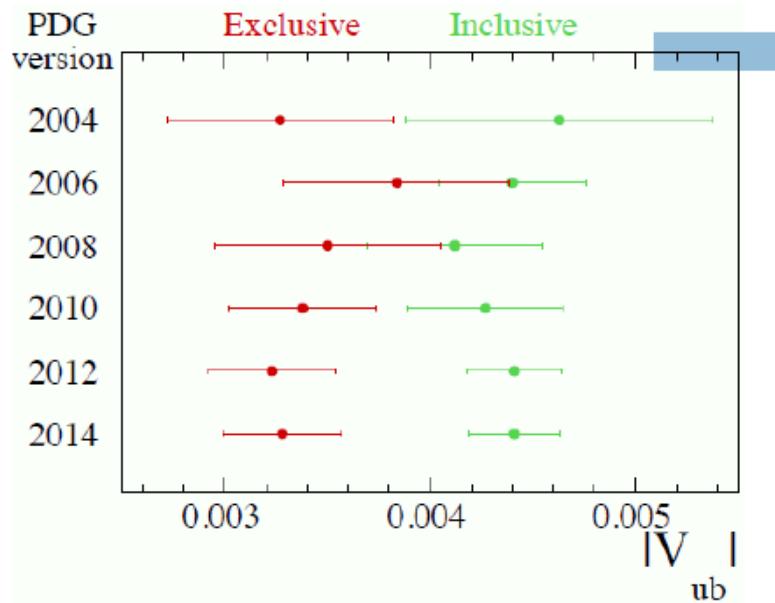
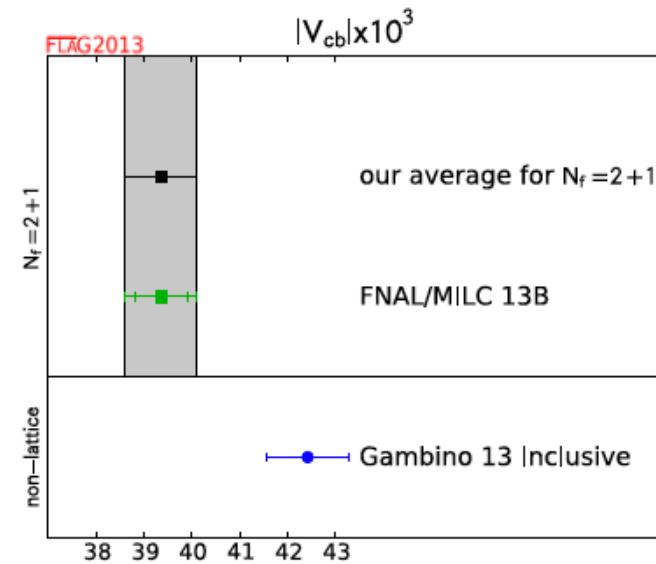
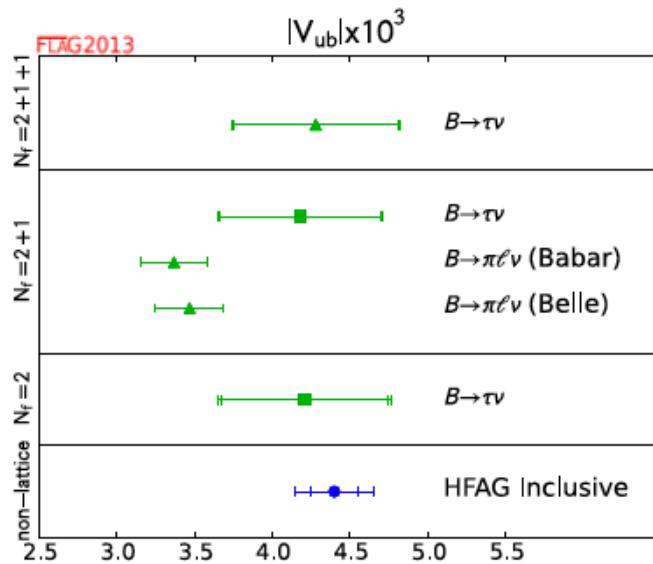
*Let us assume that the Standard Model contributions to  $M_{12}$  and  $\Gamma_{12}$  are real*

# Do we still care? Tensions and Unknowns

- 1) A ``classical'' example  $B \rightarrow \tau\nu$
- 2)  $|V_{ub}|$  and  $|V_{cb}|$  inclusive vs exclusive
- 3)  $|V_{cb}|$ , B mixing and  $\varepsilon_K$
- 4) D-mixing
- 5)  $R(D)$  and  $R(D^*)$
- 6)  $B \rightarrow K^* ll$
- 7) Physics BSM ?

2016

# $|V_{ub}|, |V_{cb}|$



$V_{ub}$  Exclusive =  $0.00369 \pm 0.00015$   
 $V_{cb}$  Exclusive =  $0.0392 \pm 0.0007$   
 $V_{ub}/V_{cb}$  Exclusive =  $0.083 \pm 0.006$   
 $V_{ub}$  Inclusive =  $0.00441 \pm 0.00022$   
 $V_{cb}$  Inclusive =  $0.0422 \pm 0.0007$   
 $Belle$  =  $0.04247 \pm 0.00100$



## $V_{cb}$ and $V_{ub}$

Updates from UTfit

New HFAG (HFLAV) @CKM16

$$|V_{cb}| \text{ (excl)} = (38.88 \pm 0.60) 10^{-3}$$

$$|V_{cb}| \text{ (incl)} = (42.19 \pm 0.78) 10^{-3}$$

New HFAG @CKM16

$\sim 3.3\sigma$  discrepancy

New HFAG @CKM16

$$|V_{ub}| \text{ (excl)} = (3.65 \pm 0.14) 10^{-3}$$

$$|V_{ub}| \text{ (incl)} = (4.50 \pm 0.20) 10^{-3}$$

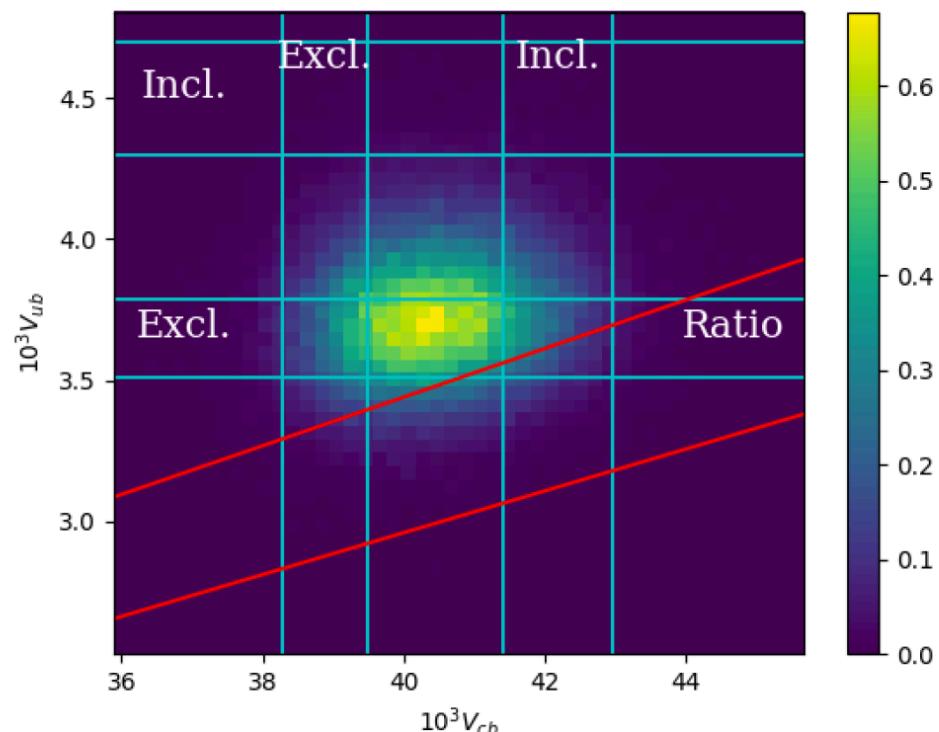
New HFAG @CKM16

$\sim 3.4\sigma$  discrepancy

$$|V_{ub} / V_{cb}| \text{ (LHCb)} = (8.0 \pm 0.6) 10^{-2}$$

Updated value

updated for LHCP17





## $V_{cb}$ and $V_{ub}$

Updates from UTfit

2D average inspired by  
D'Agostini skeptical procedure  
(hep-ex/9910036) with  $\sigma=1$ .  
Very similar results obtained  
from a 2D a la PDG procedure.

$$|V_{cb}| = (40.5 \pm 1.1) 10^{-3}$$

uncertainty  $\sim 2.4\%$

$$|V_{ub}| = (3.74 \pm 0.23) 10^{-3}$$

uncertainty  $\sim 5.6\%$

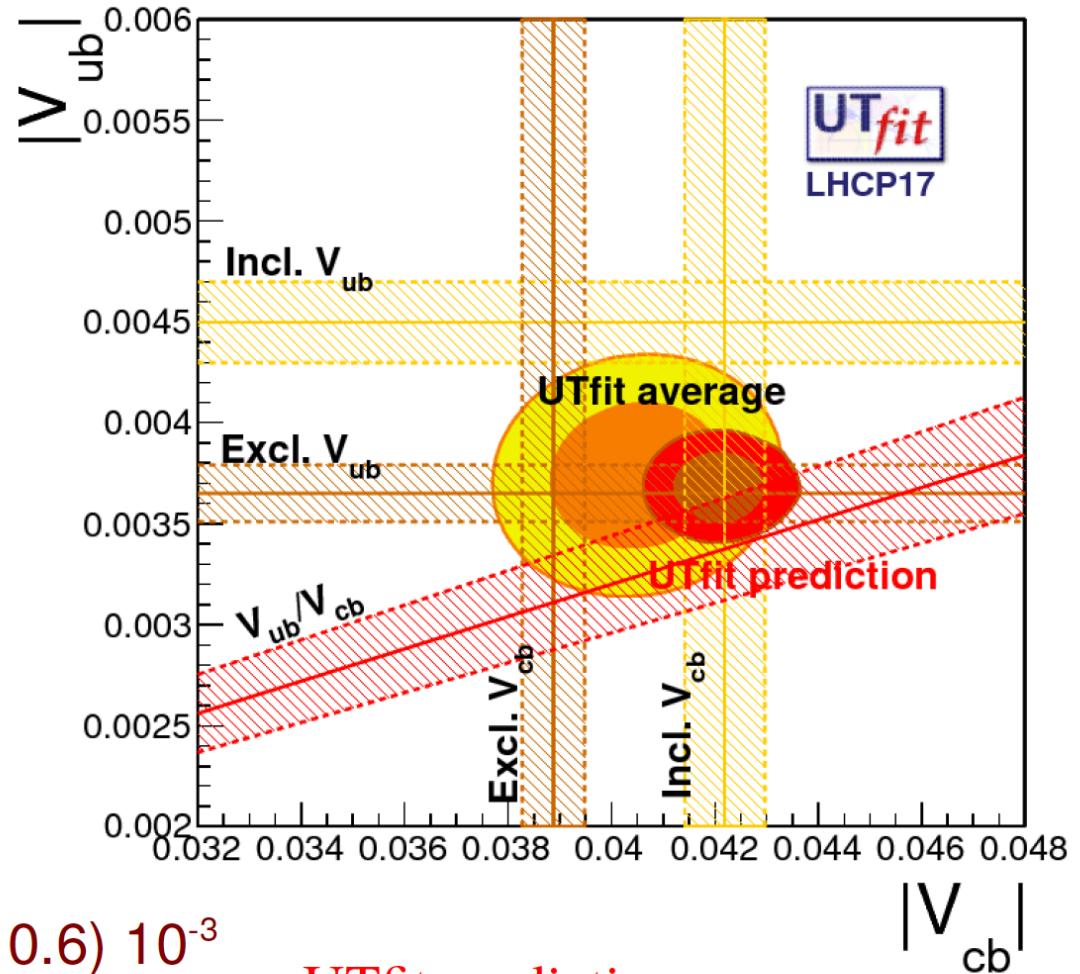
**Incl**

$$|V_{cb}| = (42.1 \pm 0.6) 10^{-3}$$

**Excl**  
Marcella Bona

$$|V_{ub}| = (3.68 \pm 0.11) 10^{-3}$$

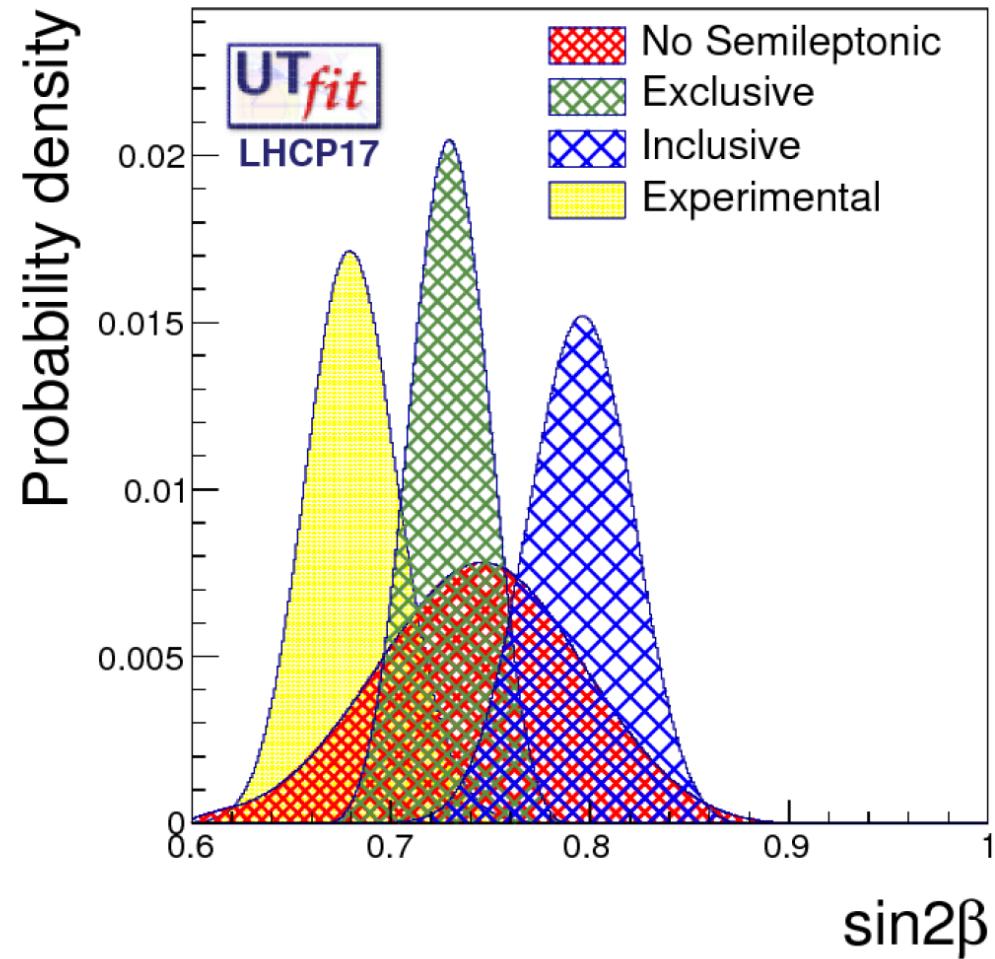
updated for LHPC17



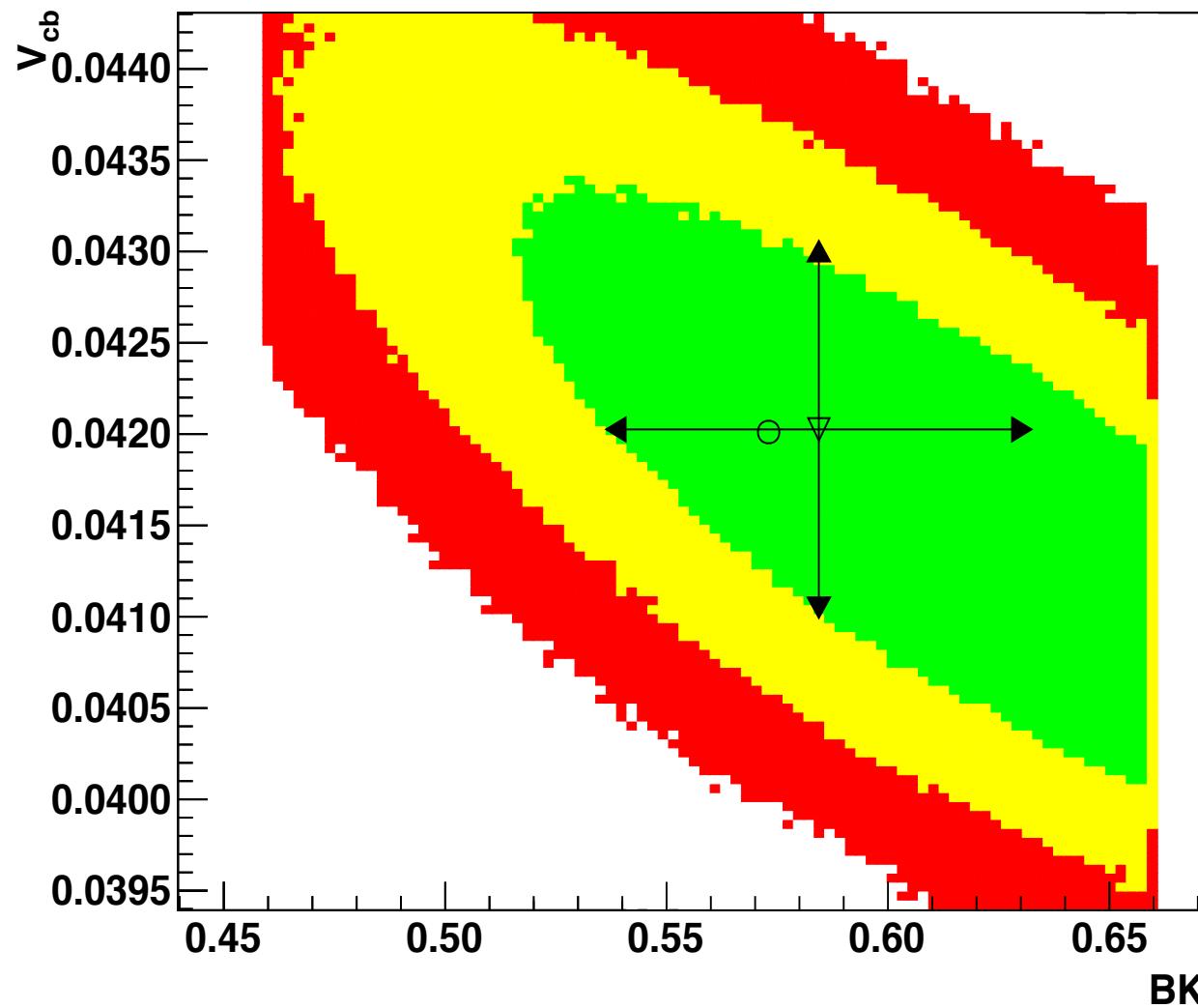
UTfit predictions



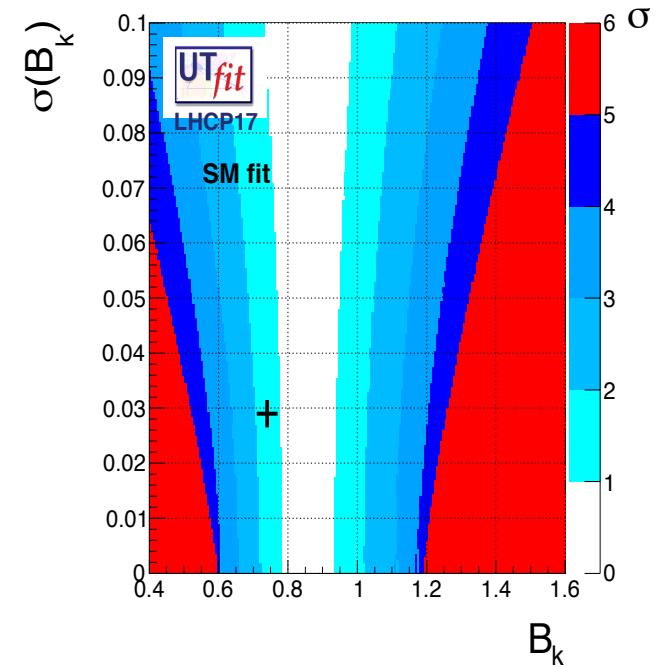
## exclusives vs inclusives



# UT-fit Preliminary



- $\varepsilon_K$  large  $V_{cb}$
- $B$  mixing with large lattice matrix elements smaller  $V_{cb}$



## Model-Independent Extraction of $|V_{cb}|$ from $\bar{B} \rightarrow D^* \ell \bar{\nu}$ , cont'd

- New Belle analysis released:

**Abdesselam et al (Belle) 1702.01521**

- ▶ Unfolded data, full correlation matrix
- ▶ Large dataset, energy and angular distributions
- ▶ CLN:  $|V_{cb}| = (37.4 \pm 1.3) \times 10^{-3}$

**Exclusive**

- Two independent analyses using BGL:

- ▶ Very consistent fits:

$$|V_{cb}| = (41.7^{+2.0}_{-2.1}) \times 10^{-3}$$

**Bigi, Gambino & Schacht, 1703.06124**

$$|V_{cb}| = (41.9^{+2.0}_{-1.9}) \times 10^{-3}$$

**BG & Kobach, 1703.08170**

- ▶ Robust: different numerical inputs
- ▶ Likely culprit: independent form factors (no HQET symmetry)

$$\begin{aligned} \langle D^*(\varepsilon, p') | \bar{c} \gamma^\mu b | \bar{B}(p) \rangle &= i g \epsilon^{\mu\nu\alpha\beta} \varepsilon_\nu^* p_\alpha p'_\beta, \\ \langle D^*(\varepsilon, p') | \bar{c} \gamma^\mu \gamma^5 b | \bar{B}(p) \rangle &= f \varepsilon^{*\mu} + (\varepsilon^* \cdot p) [a_+(p + p')^\mu + a_-(p - p')^\mu], \end{aligned}$$

Recall: BGL introduced  $z$ -parametrization, eg,

$$g(z) = \frac{1}{P_g(z)\phi_g(z)} \sum_{n=0}^N a_n z^n \quad \text{with} \quad \sum_n a_n^2 \leq 1 \quad \text{and} \quad 0 \leq z \leq z_{\max} = 0.056$$

with calculable outer function  $\phi$  and Blaschke factor  $P$

- ▶ CLN uses BGL technique, but imposes HQET conditions

Work ahead:

- Experiments: release unfolded data
- Experiments' next best alternative: do BGL fits
- Global analysts: do BGL fits, others (e.g., polynomial in  $q^2$ )?
- Theorists:  $\Lambda/m_c$  effects?
- Theorists: Is BGL better than polynomial for independent form factors?
- Can this affect  $B \rightarrow D^{(*)}\tau\nu$
- **LATTICE !!**

If I may be so bold: *problem solved*

- Retrospect: What went wrong?
  - ▶ The problem was sociological!

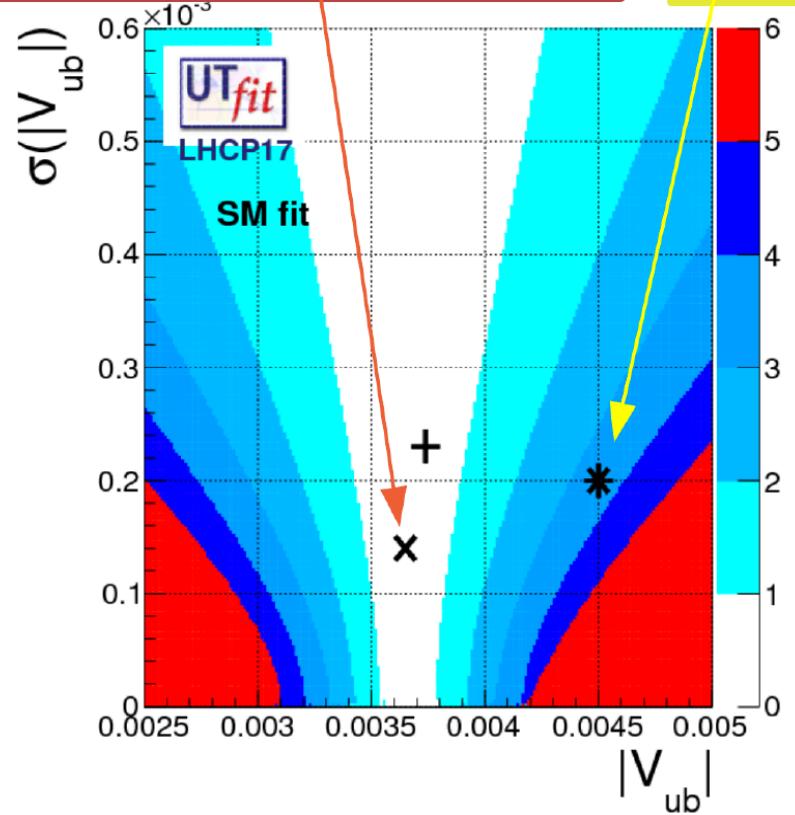
Also: FF calculations  
only on MILC configurations  
⇒ need confirmation with  
different methods



*tensions? not really.. still that  $V_{ub}$  inclusive*

$$V_{ub} (\text{excl}) = (3.65 \pm 0.14) \cdot 10^{-3}$$

$$V_{ub} (\text{incl}) = (4.50 \pm 0.20) \cdot 10^{-3}$$



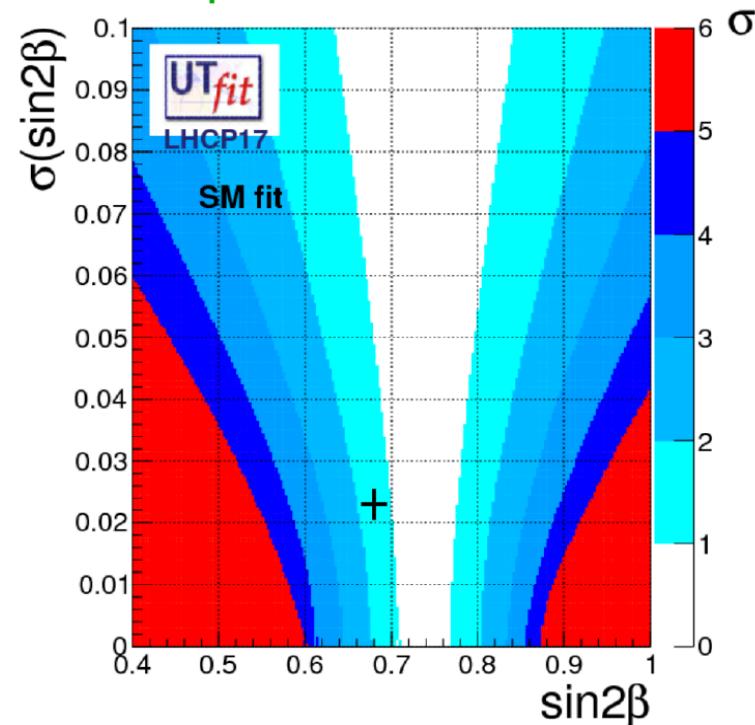
$$V_{ub\text{exp}} = (3.74 \pm 0.23) \cdot 10^{-3}$$

$$V_{ub\text{UTfit}} = (3.66 \pm 0.13) \cdot 10^{-3}$$

$\sim 1.4\sigma$

$$\sin 2\beta_{\text{exp}} = 0.680 \pm 0.023$$

$$\sin 2\beta_{\text{UTfit}} = 0.737 \pm 0.031$$



# Universal Unitarity Triangle 2016 and the Tension Between $\Delta M_{s,d}$ and $\varepsilon_K$ in CMFV Models

Monika Blanke<sup>a,b</sup> and Andrzej J. Buras<sup>c</sup>

<sup>a</sup> Institut fur Kernphysik, Karlsruhe Institute of Technology,  
Hermann-von-Helmholtz-Platz 1, D-76344 Eggenstein-Leopoldshafen, Germany

<sup>b</sup> Institut fur Theoretische Teilchenphysik, Karlsruhe Institute of Technology,  
Engesserstraße 7, D-76128 Karlsruhe, Germany

<sup>c</sup> TUM-IAS, Lichtenbergstr. 2a, D-85748 Garching, Germany  
Physik Department, TUM, D-85748 Garching, Germany

## Abstract

Motivated by the recently improved results from the Fermilab Lattice and MILC Collaborations on the hadronic matrix elements entering  $\Delta M_{s,d}$  in  $B_{s,d}^0 - \bar{B}_{s,d}^0$  mixing, we determine the Universal Unitarity Triangle (UUT) in models with Constrained Minimal Flavour Vi-

$$F_{B_s} \sqrt{\hat{B}_{B_s}}, \quad F_{B_d} \sqrt{\hat{B}_{B_d}}, \quad \hat{B}_K . \quad (2)$$

Fortunately, during the last years these uncertainties decreased significantly. In particular, concerning  $F_{B_s} \sqrt{\hat{B}_{B_s}}$  and  $F_{B_d} \sqrt{\hat{B}_{B_d}}$ , an impressive progress has recently been made by the Fermilab Lattice and MILC Collaborations (Fermilab-MILC) that find [3]

$$F_{B_s} \sqrt{\hat{B}_{B_s}} = (274.6 \pm 8.8) \text{ MeV}, \quad F_{B_d} \sqrt{\hat{B}_{B_d}} = (227.7 \pm 9.8) \text{ MeV} , \quad (3)$$

with uncertainties of 3% and 4%, respectively. An even higher precision is achieved for the ratio

$$\xi = \frac{F_{B_s} \sqrt{\hat{B}_{B_s}}}{F_{B_d} \sqrt{\hat{B}_{B_d}}} = 1.206 \pm 0.019 . \quad (4)$$

## CKM Uncertainties

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (8.39 \pm 0.30) \cdot 10^{-11} \left[ \frac{|V_{cb}|}{0.0407} \right]^{2.8} \left[ \frac{\gamma}{73.2^\circ} \right]^{0.71}$$
$$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (3.36 \pm 0.09) \cdot 10^{-11} \left[ \frac{|V_{ub}|}{3.88 \cdot 10^{-3}} \right]^2 \left[ \frac{|V_{cb}|}{0.0407} \right]^2 \left[ \frac{\sin \gamma}{\sin(73.2)} \right]^2$$

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (65.3 \pm 3.1) [\bar{\text{Br}}(B_s \rightarrow \mu^+ \mu^-)]^{1.4} \left[ \frac{\gamma}{70^\circ} \right]^{0.71} \left[ \frac{227 \text{ MeV}}{F_{B_s}} \right]^{2.8}$$

A. Buras , Buttazzo,  
Girrbach-Noe,  
Knegjens  
1503.02693

For  $B_s \rightarrow \mu^+ \mu^-$  we use the formula from [56], slightly modified in [2]

$$\overline{\mathcal{B}}(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}} = (3.65 \pm 0.06) \cdot 10^{-9} \left[ \frac{m_t(m_t)}{163.5 \text{ GeV}} \right]^{3.02} \left[ \frac{\alpha_s(M_Z)}{0.1184} \right]^{0.032} R_s$$

where

$$R_s = \left[ \frac{F_{B_s}}{227.7 \text{ MeV}} \right]^2 \left[ \frac{\tau_{B_s}}{1.516 \text{ ps}} \right] \left[ \frac{0.938}{r(y_s)} \right] \left[ \frac{|V_{ts}|}{41.5 \cdot 10^{-3}} \right]^2.$$

Now,

$$|V_{td}| = |V_{us}| |V_{cb}| R_t, \quad |V_{ts}| = \eta_R |V_{cb}|$$

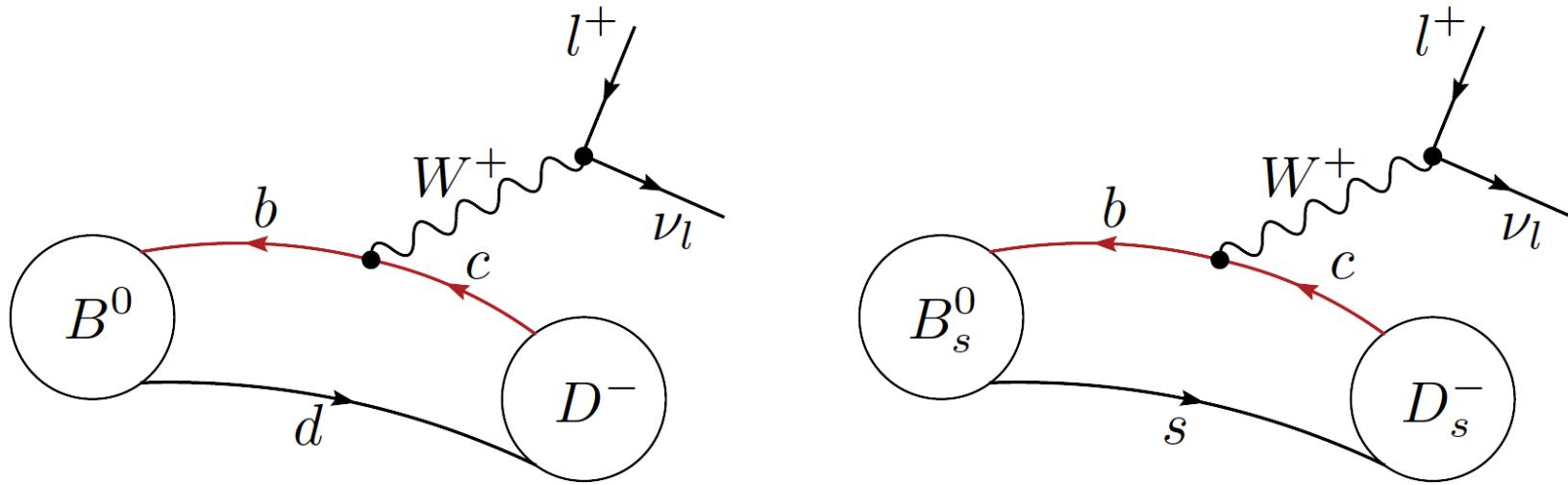
with  $R_t$  being one of the sides of the unitarity triangle (see Fig. 1) and

$$\eta_R = 1 - |V_{us}| \xi \sqrt{\frac{\Delta M_d}{\Delta M_s}} \sqrt{\frac{m_{B_s}}{m_{B_d}}} \cos \beta + \frac{\lambda^2}{2} + \mathcal{O}(\lambda^4) = 0.9825,$$

# Do we still care? Tensions and Unknowns

- 1) A ``classical'' example  $B \rightarrow \tau\nu$
- 2)  $|V_{ub}|$  and  $|V_{cb}|$  inclusive vs exclusive
- 3)  $|V_{cb}|$ , B mixing and  $\varepsilon_K$
- 4) D-mixing (already discussed)
- 5)  $R(D)$  and  $R(D^*)$  (and  $V_{cb}$  of course)
- 6)  $B \rightarrow K^{(*)} ll$
- 7) Physics BSM ?

# B semileptonic decay: $|V_{cb}|$

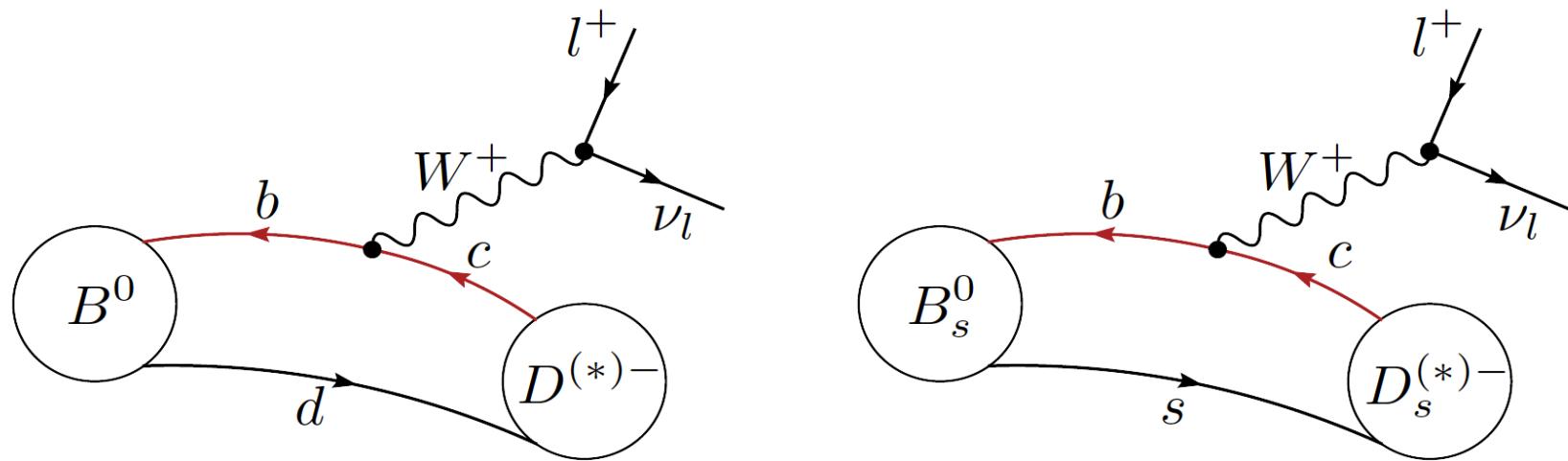


$$\frac{d\Gamma(B_{(s)} \rightarrow Pl\nu)}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{24\pi^3} \frac{(q^2 - m_l^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_{B_{(s)}}^2} \left[ \left(1 + \frac{m_l^2}{2q^2}\right) m_{B_{(s)}}^2 (E_P^2 - m_P^2) |f_+(q^2)|^2 + \frac{3m_l^2}{8q^2} (m_{B_{(s)}}^2 - m_P^2)^2 |f_0(q^2)|^2 \right]$$

*e, μ suppressed* ←

uncertainties from kinematical factors / neglected h.o. OPE at the permille level

# B semileptonic decay: $|V_{cb}|$



$$\frac{d\Gamma(B \rightarrow D l \nu_l)}{dw} = \frac{G_F^2}{48\pi^3} (m_B + m_D)^2 (w^2 - 1)^{3/2} |\eta_{EW}|^2 |V_{cb}|^2 |\mathcal{G}(w)|^2 + \mathcal{O}\left(\frac{m_l^2}{q^2}\right)$$

$$\frac{d\Gamma(B \rightarrow D^* l \nu_l)}{dw} = \frac{G_F^2}{4\pi^3} (m_B - m_{D^*})^2 (w^2 - 1)^{1/2} |\eta_{EW}|^2 \chi(w) |V_{cb}|^2 |\mathcal{F}(w)|^2 + \mathcal{O}\left(\frac{m_l^2}{q^2}\right)$$

$$w = \frac{p_B \cdot p_{D^{(*)}}}{m_B m_{D^{(*)}}}$$

$$\mathcal{G}(w) = \frac{4 \frac{m_D}{m_B}}{1 + \frac{m_D}{m_B}} f_+(q^2) \quad \text{etc}$$

*Low recoil region ( $w=1$ ) accessible to lattice calculations*

# $B \rightarrow D-D^*$

same lattice configurations used  $m_b a \approx 1.1$  in the best case

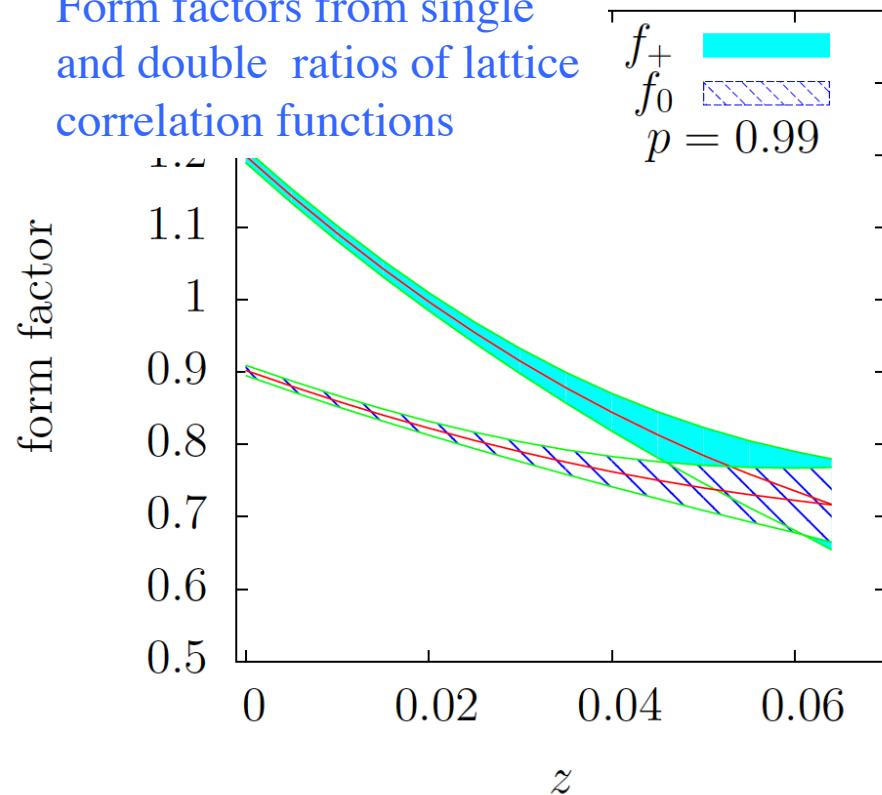
	FNAL/MILC*	FNAL/MILC	HPQCD
process	$B \rightarrow D^* \ell \nu$	$B \rightarrow D \ell \nu$	$B \rightarrow D \ell \nu$
kinematics	$w = 1$	$w \geq 1$	$w \geq 1$
ensembles	MILC	MILC	MILC
$N_f$	2+1	2+1	2+1
$a$ (fm)	5/0.045 – 0.15	4/0.045 – 0.12	2/0.09, 0.12
$M_\pi^{\min}$ [MeV]	260	220	260
$M_\pi^{\min} L$	3.8	3.8	3.8
$l$ quarks	asqtad	asqtad	asqtad
$c$ quark	RHQ (Fermilab)	RHQ (Fermilab)	HISQ
$b$ quark	RHQ (Fermilab)	RHQ (Fermilab)	NRQCD
reference	[ 1403.0635 ]	[ 1503.07237 ]	[ 1505.03925 ]

(\* full publication of  $B \rightarrow D^*$  results, no changes wrt proceedings value quoted in FLAG)

# new results for $B \rightarrow Dl\nu$

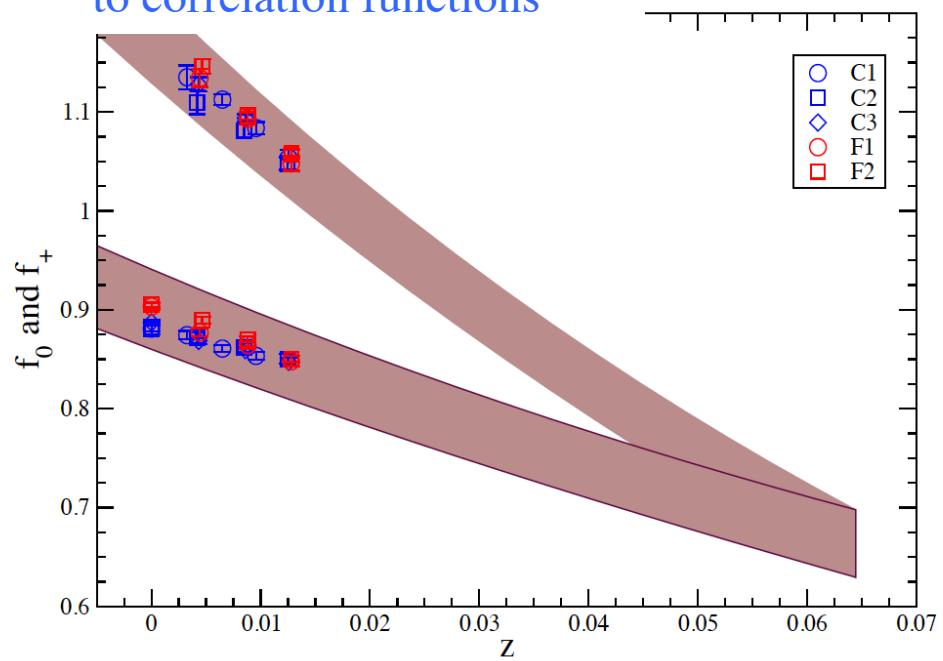
[ FNAL/MILC ]

Form factors from single  
and double ratios of lattice  
correlation functions



[ HPQCD ]

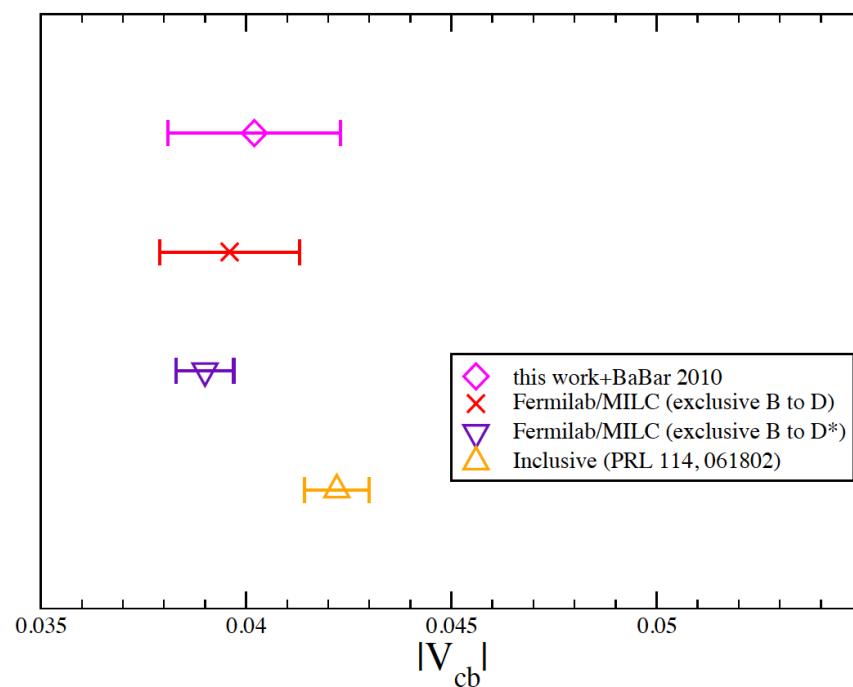
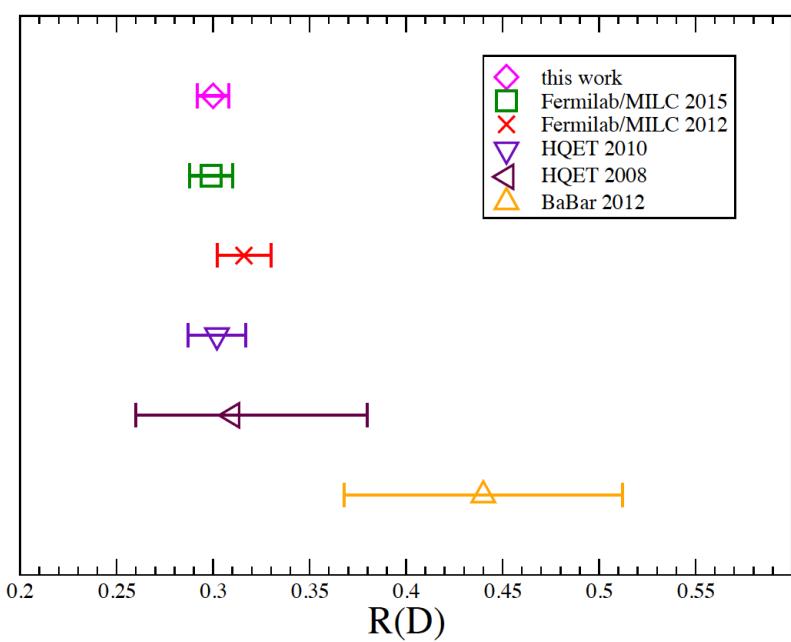
Form factors from direct fit  
to correlation functions

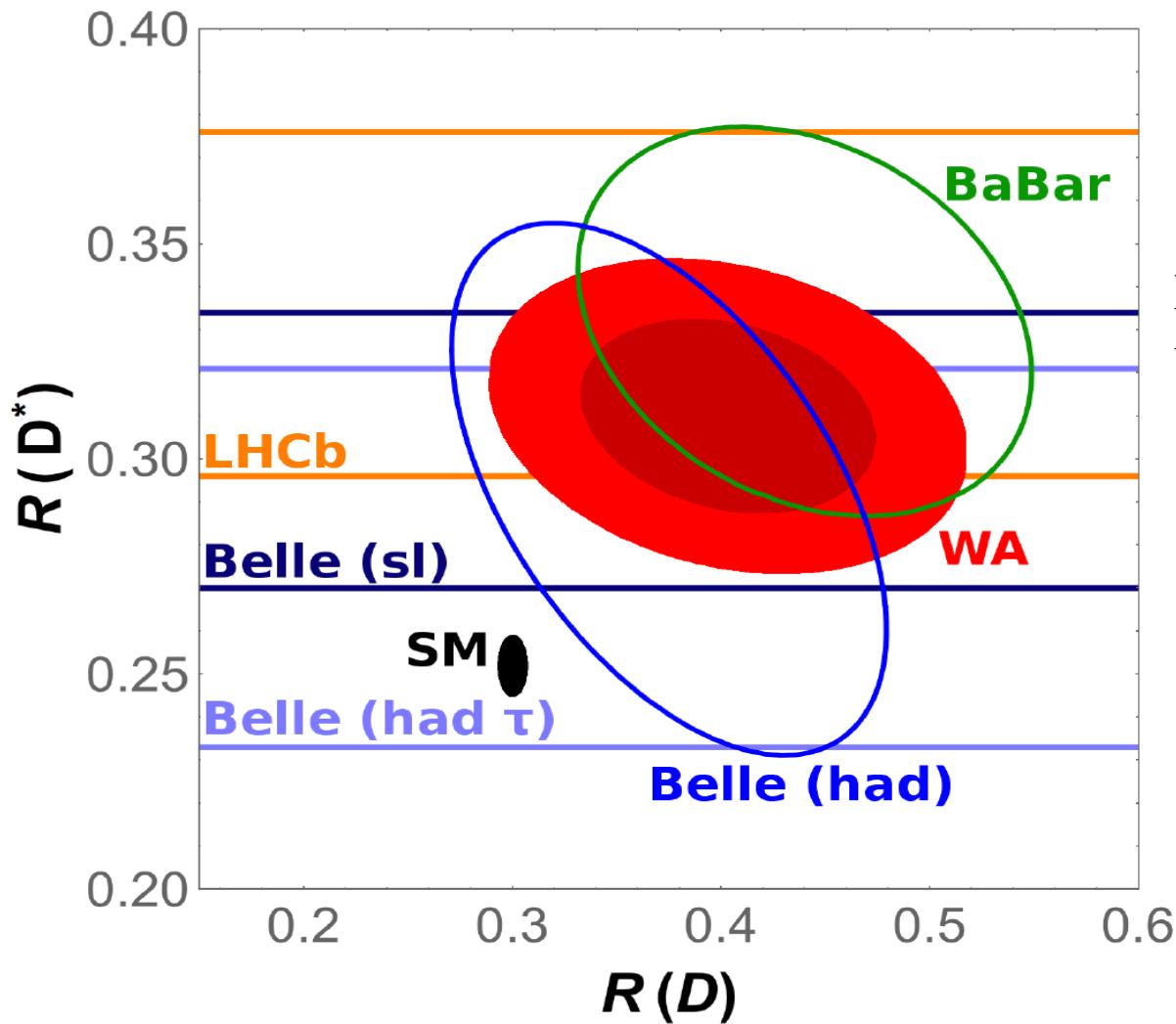


$$R(D) = \frac{\mathcal{B}(B \rightarrow D\tau\nu)}{\mathcal{B}(B \rightarrow D\ell\nu)} = 0.299(11)$$

$$0.300(8)$$

# HPQCD June 13 2016





A.Celis, M. Jung, X.  
Li, A. Pich  
arXiv:1612.07757v2

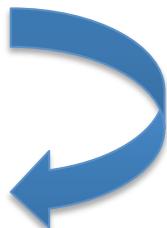
FIG. 1. Average of  $R(D^{(*)})$  measurements, displayed as red filled ellipses (68% CL and 95% CL). The SM prediction is shown as a black ellipse (95% CL), and the individual measurements as continuous contours (68% CL): Belle (blue ellipse and horizontal bands), BaBar (green ellipse), and LHCb (horizontal orange band).

## $|V_{ub}|$ & $|V_{cb}|$ inclusive vs exclusive and all that

- 1) On the long run exclusive decays based on non-perturbative (lattice) determination of the relevant form factors will win;
- 2) The precision of the theoretical predictions for inclusive decays cannot be improved (are the present quoted errors reliable?);
- 3) Still (much) more work is needed, and different lattice approaches to the physical B should be used and compared;
- 4) R(D) and R(D\*) is an open problem; more lattice collaborations should work on these calculations. A comparison with Bs and Bc decays fundamental;
- 5) Theoretical calculations and experimental analyses should not be biased by the HQFT - after all  $\Lambda_{\text{QCD}}/m_c \approx O(1)$ ;
- 6) I hope to be wrong, but the possibility of new physics in tree level  $b \rightarrow c$  decays looks to me quite remote.

# Do we still care? Tensions and Unknowns

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- 2)  $|V_{ub}|$  and  $|V_{cb}|$  inclusive vs exclusive
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- 4) D-mixing (already discussed)
- 5)  $R(D)$  and  $R(D^*)$  (and  $V_{cb}$  of course)
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- 7) Physics BSM ?



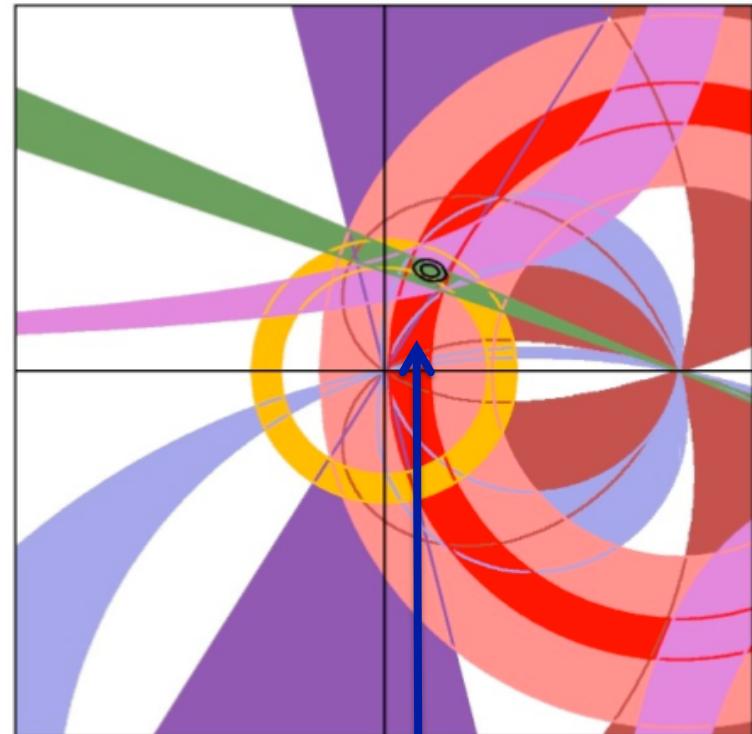
## Is the present picture showing a Model Standardissimo ?

An evidence, an evidence, my kingdom for an evidence

From Shakespeare's *Richard III*  
and A. Stocchi

- 1) Fit of NP- $\Delta F=2$  parameters in a Model “independent” way
- 2) “Scale” analysis in  $\Delta F=2$

# *BSM*



***VERY GOOD CONSISTENCY WITHIN THE SM !***

# .... beyond the Standard Model

- New Physics in Kaon decays
- New Physics in  $B \rightarrow K^{(*)} l^+ l^-$
- New Physics in Mixing

$$\varepsilon'/\varepsilon = (1.4 \pm 7.0) \cdot 10^{-4}$$

$$\left( \frac{\text{Re } A_0}{\text{Re } A_2} \right) = 31.0 \pm 6.6$$

$$(\varepsilon'/\varepsilon)_{\text{exp}} = (16.6 \pm 2.3) \cdot 10^{-4}$$

$$\left( \frac{\text{Re } A_0}{\text{Re } A_2} \right)_{\text{exp}} = 22.4$$

*Courtesy by A. Buras*

# Results for $\text{Re}[A_0]$ , $\text{Im}[A_0]$ and $\text{Re}[\epsilon'/\epsilon]$

Xu Feng Lattice 2017

[RBC-UKQCD, PRL115 (2015) 212001]

- Determine the  $K \rightarrow \pi\pi (I=0)$  amplitude  $A_0$ 
  - ▶ Lattice results

$$\text{Re}[A_0] = 4.66(1.00)_{\text{stat}}(1.26)_{\text{syst}} \times 10^{-7} \text{ GeV}$$

$$\text{Im}[A_0] = -1.90(1.23)_{\text{stat}}(1.08)_{\text{syst}} \times 10^{-11} \text{ GeV}$$

- ▶ Experimental measurement

$$\text{Re}[A_0] = 3.3201(18) \times 10^{-7} \text{ GeV}$$

$\text{Im}[A_0]$  is unknown

- Determine the direct  $CP$  violation  $\text{Re}[\epsilon'/\epsilon]$

$$\text{Re}[\epsilon'/\epsilon] = 0.14(52)_{\text{stat}}(46)_{\text{syst}} \times 10^{-3} \quad \text{Lattice}$$

$$\text{Re}[\epsilon'/\epsilon] = 1.66(23) \times 10^{-3} \quad \text{Experiment}$$

2.1  $\sigma$  deviation  $\Rightarrow$  require more accurate lattice results

# Four dominant contributions to $\varepsilon'/\varepsilon$ in the SM

AJB, Jamin, Lautenbacher (1993); AJB, Gorbahn, Jäger, Jamin (2015)

$$\text{Re}(\varepsilon'/\varepsilon) = \left[ \frac{\text{Im}(V_{td} V_{ts}^*)}{1.4 \cdot 10^{-4}} \right] 10^{-4} \left[ -3.7 + 21.2 \cdot B_6^{(1/2)} + 1.1 - 9.6 \cdot B_8^{(3/2)} \right]$$

From  $\text{Re}A_0$

From  $\text{Re}A_2$

(Q<sub>4</sub>)  $(V-A) \otimes (V-A)$   
QCD Penguins

$(V-A) \otimes (V+A)$   
QCD Penguins

$(V-A) \otimes (V-A)$   
EW Penguins

$(V-A) \otimes (V+A)$   
EW Penguins

```
graph TD; A["From ReA0"] --> B["-3.7 + 21.2 · B6^(1/2)"]; A --> C["1.1 - 9.6 · B8^(3/2)"]; D["From ReA2"] --> B; D --> C; B --> Q4["(Q4) (V-A) ⊗ (V-A)  
QCD Penguins"]; B --> VA1["(V-A) ⊗ (V+A)  
QCD Penguins"]; C --> VA2["(V-A) ⊗ (V-A)  
EW Penguins"]; C --> VA3["(V-A) ⊗ (V+A)  
EW Penguins"]
```

Assumes that  $\text{Re}A_0$  and  $\text{Re}A_2$  ( $\Delta l=1/2$  Rule) fully described by SM  
(includes isospin breaking corrections)

$\varepsilon'/\varepsilon$  from RBC-UKQCD

Calculate all contributions directly  
(no isospin breaking corrections)

$$[-(6.5 \pm 3.2) + 25.3 \cdot B_6^{(1/2)} + (1.2 \pm 0.8) - 10.2 \cdot B_8^{(3/2)}]$$

# $\varepsilon'/\varepsilon$ from RBC-UKQCD

Anatomy: AJB, Gorbahn, Jäger, Jamin (2015)

$$\text{Re}(\varepsilon'/\varepsilon) = \left[ \frac{\text{Im}(V_{td} V_{ts}^*)}{1.4 \cdot 10^{-4}} \right] 10^{-4} [-6.5 + 25.3 \cdot B_6^{(1/2)} + 1.2 - 10.2 \cdot B_8^{(3/2)}]$$

Calculate all contributions directly

$(Q_4)$	$(V-A) \otimes (V-A)$ QCD Penguins	$(V-A) \otimes (V+A)$ QCD Penguins	$(V-A) \otimes (V-A)$ EW Penguins
			$(V-A) \otimes (V+A)$ EW Penguins

Extracted from

RBC-UKQCD

$$B_6^{(1/2)} = B_8^{(3/2)} = 1 \text{ in the large N limit}$$

$$B_6^{(1/2)} = 0.57 \pm 0.15$$

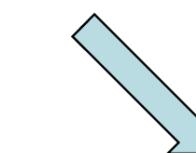
$$B_8^{(3/2)} = 0.76 \pm 0.05$$

EW penguins in full agreement  
with BGJJ but

+ third term  
very similar to BGJJ  
 $(\text{Re}A_2)_{\text{Lattice}} \approx (\text{Re}A_2)_{\text{exp}}$

$$\left[ \frac{(\text{Re}A_0)}{(\text{Re}A_0)_{\text{exp}}} \approx 1.4 \right]$$

The negative  
contribution of  
 $Q_4$  overestimated



$$\left( \frac{\varepsilon'}{\varepsilon} \right)_{\text{Lattice}} = (1.4 \pm 7.0) \cdot 10^{-4}$$

---

# Anatomy of $\epsilon'/\epsilon$ – A new flavour anomaly?

AJB, Gorbahn, Jäger, Jamin,, 1507.xxxx

**RBC-UKQCD**

$$\epsilon'/\epsilon = (1.4 \pm 7.0) \cdot 10^{-4}$$

(3.2 $\sigma$ )  $\epsilon'/\epsilon = (2.2 \pm 3.8) \cdot 10^{-4}$

$$\epsilon'/\epsilon = (6.3 \pm 2.5) \cdot 10^{-4}$$

$$\epsilon'/\epsilon = (9.1 \pm 3.3) \cdot 10^{-4}$$

exp:  $\epsilon'/\epsilon = (16.6 \pm 3.3) \cdot 10^{-4}$

RBC-QCD values

$$B_6^{(1/2)} = 0.57 \pm 0.15$$
$$B_8^{(3/2)} = 0.76 \pm 0.05$$

large  $N$  bounds (AJB, Gérard)

$$B_6^{(1/2)} = B_8^{(3/2)} = 0.76$$

large  $N$  bounds (AJB, Gérard)

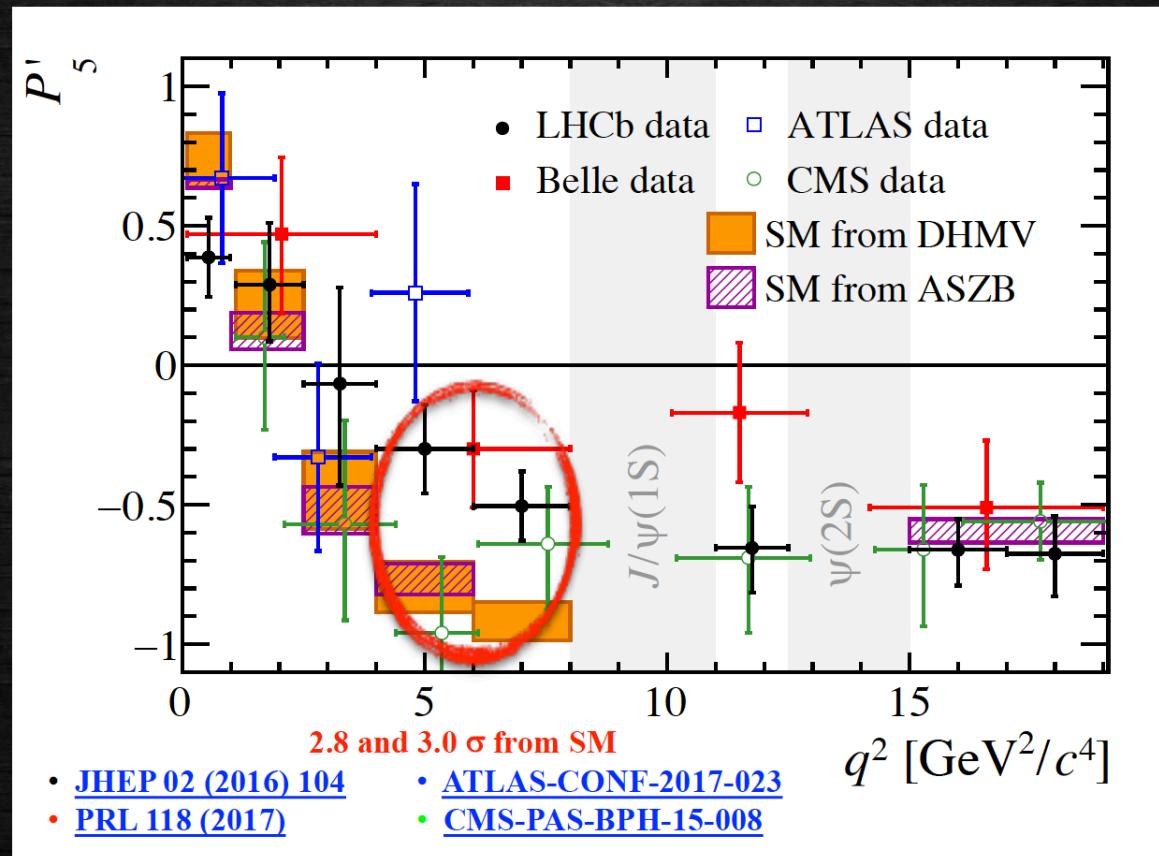
$$B_6^{(1/2)} = B_8^{(3/2)} = 1.0$$



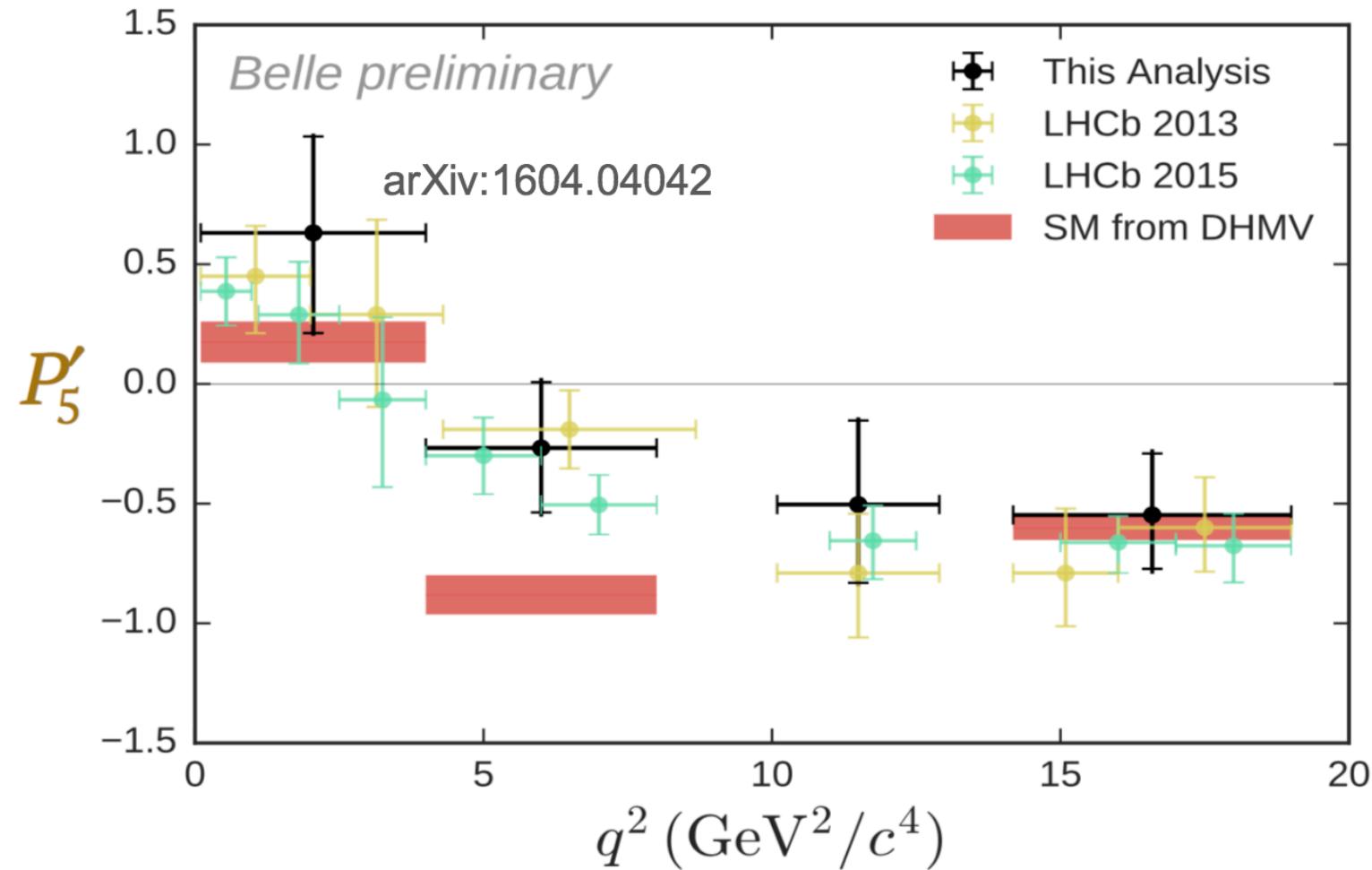
# Angular Analyses



- › First **full angular analysis** of  $B^0 \rightarrow K^{*0} \mu \mu$ : measured all CP-averaged angular terms and CP-asymmetries
- › Can construct **less form-factor dependent ratios of observables**



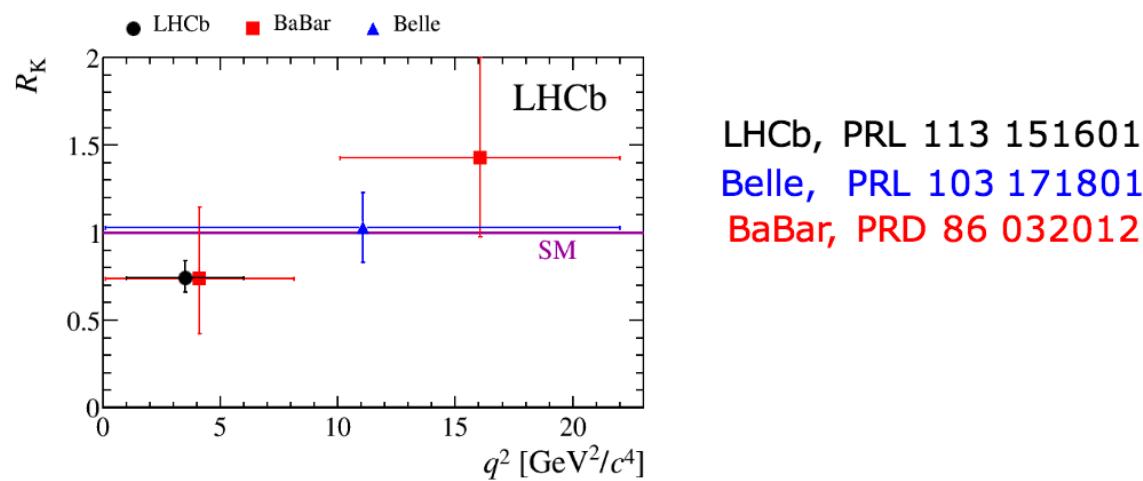
# New analysis from Belle



# Reminder:

## $R_K = \mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-) / \mathcal{B}(B^+ \rightarrow K^+ e^+ e^-)$

- Test of lepton universality :  $R_K \sim 1$  in SM, with negligible theoretical uncertainties



$$R_K(1 < q^2 < 6 \text{ GeV}^2) = 0.745^{+0.090}_{-0.074} (\text{stat}) \pm 0.036 (\text{syst})$$

- Compatible with SM at  $2.6\sigma$
- Experimentally challenging
  - lower trigger efficiency for electrons, resolution deteriorated by bremsstrahlung
- Other modes suitable for same test:  
 $B^0 \rightarrow K^{*0} l^+ l^-$ ,  $B_s \rightarrow \phi l^+ l^-$ ,  $\Lambda_B \rightarrow \Lambda l^+ l^-$

## **AND NOW:**

The hint that the loop induced decays  $b \rightarrow s\ell\ell$  can break lepton flavor universality (1) was corroborated by the most recent LHCb results [4],

$$R_{K^*}^{\text{low}} = \frac{\mathcal{B}(B \rightarrow K^* \mu\mu)_{q^2 \in [0.045, 1.1] \text{GeV}^2}}{\mathcal{B}(B \rightarrow K^* ee)_{q^2 \in [0.045, 1.1] \text{GeV}^2}} = 0.660 \pm^{0.110}_{0.070} \pm 0.024,$$
$$R_{K^*}^{\text{central}} = \frac{\mathcal{B}(B \rightarrow K^* \mu\mu)_{q^2 \in [1.1, 6] \text{GeV}^2}}{\mathcal{B}(B \rightarrow K^* ee)_{q^2 \in [1.1, 6] \text{GeV}^2}} = 0.685 \pm^{0.113}_{0.069} \pm 0.047, \quad (2)$$

***VERY DIFFICULT TO EXPLAIN WITH  
HADRONIC UNCERTAINIES!!***

# Heavy to light semileptonic

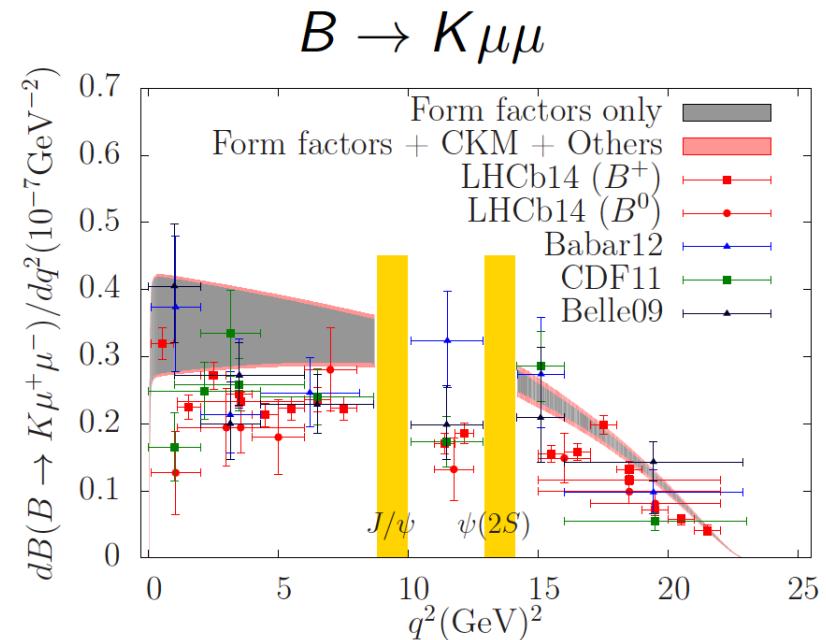
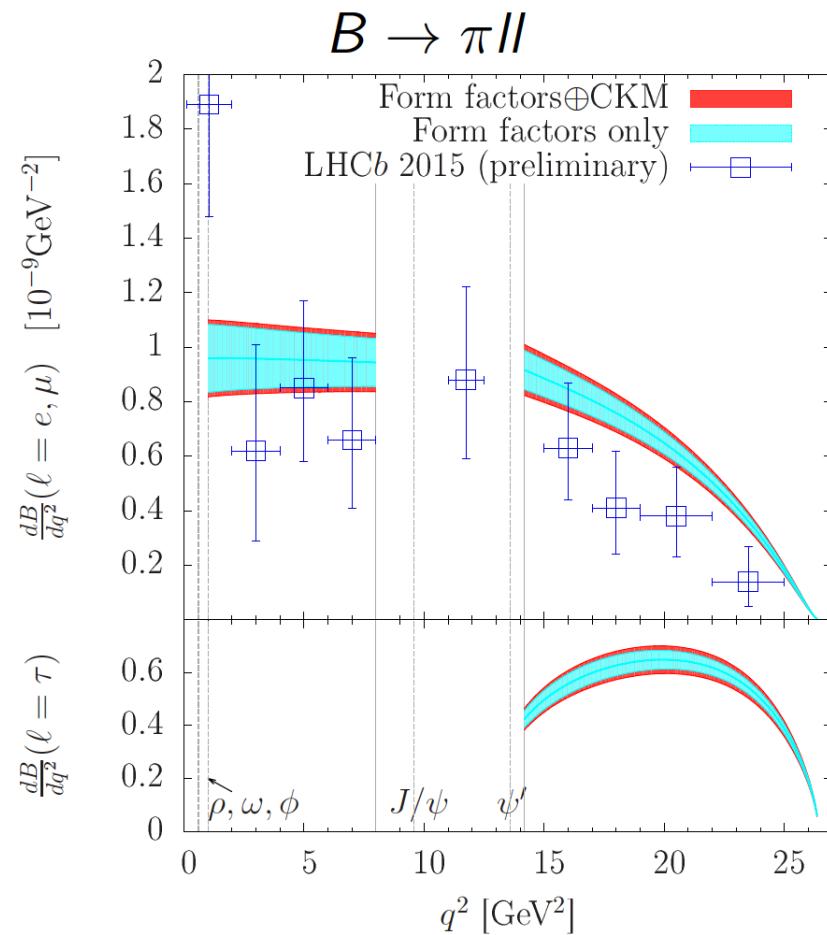
New results after Lattice 2015:

***Local operator contribution only***

	Fermilab/MILC	Fermilab/MILC	Detmold and Meinel
process	$B \rightarrow K^{*+}$ , full $q^2$	$B \rightarrow \pi^{*+}$ , full $q^2$	$\Lambda_b \rightarrow \Lambda$ full $q^2$
kinematics	MILC asqtad	MILC asqtad	RBC/UKQCD DWF
ensembles			
$N_f$	2+1	2+1	2+1
$a$	4/0.045-0.12	4/0.045-0.12	2/0.09-0.12
$M_\pi^{\min}$	260	260	227
light quark	asqtad	asqtad	DWF
$b$ quark	Fermilab	Fermilab	RHQ
Ref.	PRD.93.025026	PRL.115.152002	PRD.93.074501

- PRD.93.034005 (Fermilab/MILC,  $B$  rare decay pheno)
- PRD.94.013007 (Meinel and van Dyk,  $\Lambda_b$  rare decay pheno)
- PRD.88.054509, PRL.111.162002 (HPQCD,  $B \rightarrow K^{*+}$  ff and pheno),  
PRD.89.094501, PRL.112.212003 ( $B \rightarrow K^{*+}$  ff and pheno)

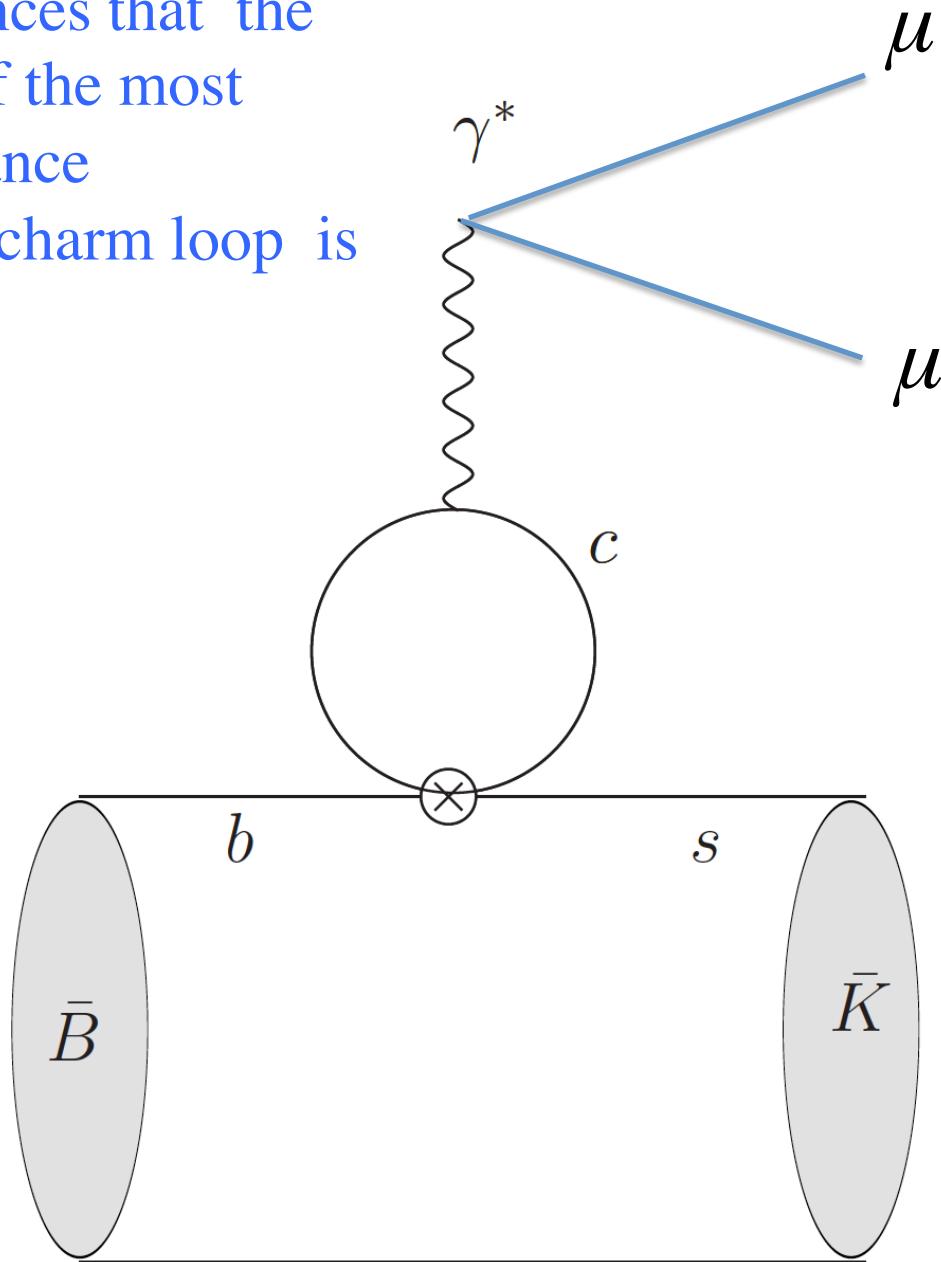
# Standard Model predictions of $B$ rare decays



- Standard-Model predictions of the differential decay rate in  $B \rightarrow \pi//$  and  $B \rightarrow K//$  process (PRL.115.152002, PRD.93.034005).

There are good chances that the lattice calculation of the most important long distance contributions via a charm loop is possible

M. Ciuchini,  
V. Lubicz, G.M.,  
L. Silvestrini,  
S. Simula



# RADIATIVE/RARE KAON DECAYS

*G. Isidori, G. M., and P. Turchetti, Phys.Lett. B633, 75 (2006),  
arXiv:hep-lat/0506026*

N.H. Christ X. Feng A. Portelli and C.T. Sachrajda *Phys.Rev. D92 (2015) no.9, 094512* [10.1103/PhysRevD.92.094512](https://doi.org/10.1103/PhysRevD.92.094512) \*

$$K \rightarrow \pi l^+ l^- \qquad \qquad K \rightarrow \pi \nu \bar{\nu}$$

*Conserved currents and GIM important*

## 2.1 $K \rightarrow \pi \ell^+ \ell^-$

G. Isidori, G. M., and P. Turchetti

The main non-perturbative correlators relevant for these decays are those with the electromagnetic current. In particular, the relevant  $T$ -product in Minkowski space is [7, 8]

$$(\mathcal{T}_i^j)_{\text{em}}^\mu(q^2) = -i \int d^4x e^{-iq \cdot x} \langle \pi^j(p) | T \{ J_{\text{em}}^\mu(x) [Q_i^u(0) - Q_i^c(0)] \} | K^j(k) \rangle , \quad (11)$$

$$J_{\text{em}}^\mu = \frac{2}{3} \sum_{q=u,c} \bar{q} \gamma^\mu q - \frac{1}{3} \sum_{q=d,s} \bar{q} \gamma^\mu q \quad (12)$$

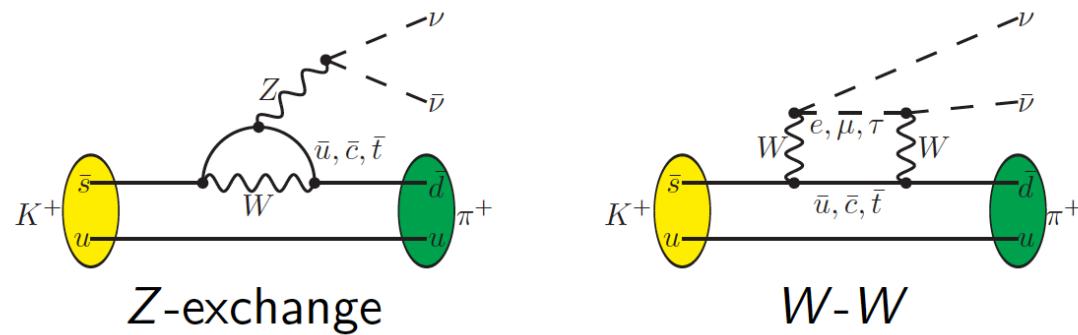
for  $i = 1, 2$  and  $j = +, 0$ . Thanks to gauge invariance we can write

$$(\mathcal{T}_i^j)_{\text{em}}^\mu(q^2) = \frac{w_i^j(q^2)}{(4\pi)^2} [q^2(k+p)^\mu - (m_k^2 - m_\pi^2)q^\mu] . \quad (13)$$

The normalization of (13) is such that the  $O(1)$  scale-independent low-energy couplings  $a_{+,0}$  defined in [8] can be expressed as

$$a_j = \frac{1}{\sqrt{2}} V_{us}^* V_{ud} \left[ C_1 w_1^j(0) + C_2 w_2^j(0) + \frac{2N_j}{\sin^2 \theta_W} f_+(0) C_{7V} \right] . \quad (14)$$

# $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ : Experiment vs Standard model



$K^+ \rightarrow \pi^+ \nu \bar{\nu}$ : largest contribution from top quark loop, thus theoretically clean

$$\mathcal{H}_{\text{eff}} \sim \frac{G_F}{\sqrt{2}} \cdot \underbrace{\frac{\alpha_{\text{EM}}}{2\pi \sin^2 \theta_W} \lambda_t X_t(x_t) \cdot (\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}}_{\mathcal{N} \sim 2 \times 10^{-5}}$$

Probe the new physics at scales of  $\mathcal{N}^{-\frac{1}{2}} M_W = O(10 \text{ TeV})$

Past experimental measurement is 2 times larger than SM prediction

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{exp}} = 1.73^{+1.15}_{-1.05} \times 10^{-10} \quad [\text{BNL E949, '08}]$$

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}} = 9.11 \pm 0.72 \times 10^{-11} \quad [\text{Buras et. al., '15}]$$

but still consistent with > 60% exp. error

## Results for charm quark contribution

### Charm quark contribution $P_c$

$$P_c = P_c^{\text{SD}} + \delta P_{c,u}$$

NNLO QCD [Buras, Gorbahn, Haisch, Nierste, '06]:

$$P_c^{\text{SD}} = 0.365(12)$$

Phenomenological ansatz [Isidori, Mescia, Smith, '05]

$$\delta P_{c,u} = 0.040(20)$$

Lattice results @ $m_\pi = 420$  MeV,  $m_c = 860$  MeV

[RBC-UKQCD, arXiv:1701.02858]

$$P_c = 0.2529(\pm 13)_{\text{stat}} (\pm 32)_{\text{scale}} (-45)_{\text{FV}}$$

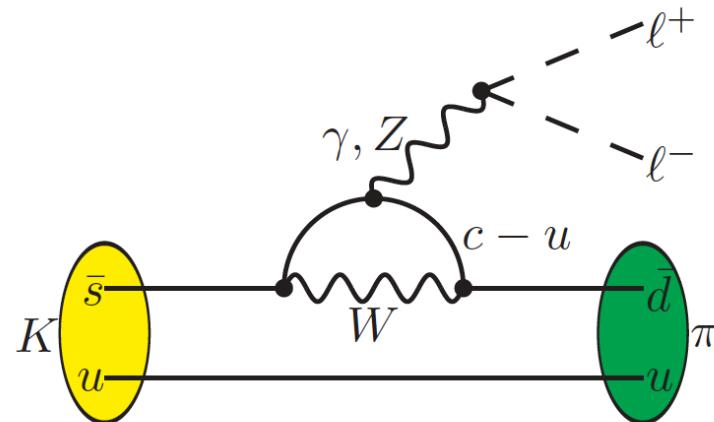
$$P_c - P_c^{\text{SD}} = 0.0040(\pm 13)_{\text{stat}} (\pm 32)_{\text{scale}} (-45)_{\text{FV}}$$

- As a smaller  $m_c$  is used,  $P_c$  is also smaller
- Cancellation in  $W$ - $W$  and  $Z$ -exchange diag. leads to small  $P_c - P_c^{\text{SD}}$
- Important to perform the calculation at physical  $m_\pi$  and  $m_c$

## $K \rightarrow \pi \ell^+ \ell^-$ : CP conserving channel

CP conserving decay:  $K^+ \rightarrow \pi^+ \ell^+ \ell^-$  and  $K_S \rightarrow \pi^0 \ell^+ \ell^-$

- Involve both  $\gamma$ - and  $Z$ -exchange diagram, but  $\gamma$ -exchange is much larger



- Unlike  $Z$ -exchange, the  $\gamma$ -exchange diagram is LD dominated
  - ▶ By power counting, loop integral is quadratically UV divergent
  - ▶ EM gauge invariance reduces divergence to logarithmic
  - ▶  $c - u$  GIM cancellation further reduces log divergence to be UV finite

# First exploratory calculation on $K^+ \rightarrow \pi^+ \ell^+ \ell^-$

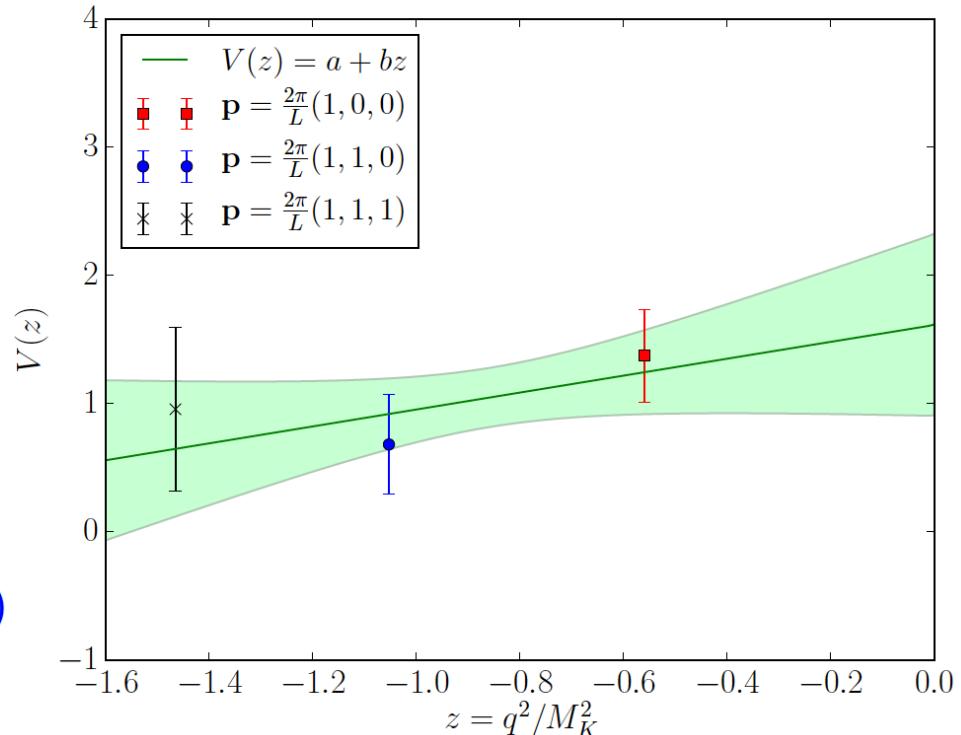
Use  $24^3 \times 64$  ensemble,  $N_{\text{conf}} = 128$   
 [RBC-UKQCD, PRD94 (2016) 114516]

$$a^{-1} = 1.78 \text{ GeV}, m_\pi = 430 \text{ MeV}$$

$$m_K = 625 \text{ MeV}, m_c = 530 \text{ MeV}$$

Momentum dependence of  $V_+(z)$

$$V_+(z) = a_+ + b_+ z \\ \Rightarrow a_+ = 1.6(7), b_+ = 0.7(8)$$



$K^+ \rightarrow \pi^+ e^+ e^-$  data + phenomenological analysis:  $a_+ = -0.58(2), b_+ = -0.78(7)$   
 [Cirigliano, et. al., Rev. Mod. Phys. 84 (2012) 399]

$$V_j(z) = a_j + b_j z + \underbrace{\frac{\alpha_j r_\pi^2 + \beta_j(z - z_0)}{G_F M_K^2 r_\pi^4}}_{K \rightarrow \pi\pi\pi} \underbrace{\left[1 + \frac{z}{r_V^2}\right]}_{F_V(z)} \underbrace{\left[\phi(z/r_\pi^2) + \frac{1}{6}\right]}_{\text{loop}}, \quad j = +, S$$

- Experimental data only provide  $\frac{d\Gamma}{dz} \Rightarrow$  square of form factor  $|V_+(z)|^2$
- Need phenomenological knowledge to determine the sign for  $a_+, b_+$

# TESTING THE NEW PHYSICS SCALE

## Effective Theory Analysis $\Delta F=2$

Effective Hamiltonian in the mixing amplitudes

$$H_{\text{eff}}^{\Delta B=2} = \sum_{i=1}^5 C_i(\mu) Q_i(\mu) + \sum_{i=1}^3 \tilde{C}_i(\mu) \tilde{Q}_i(\mu)$$

$$Q_1 = \bar{q}_L^\alpha \gamma_\mu b_L^\alpha \bar{q}_L^\beta \gamma^\mu b_L^\beta \quad (\text{SM/MFV})$$

$$Q_2 = \bar{q}_R^\alpha b_L^\alpha \bar{q}_R^\beta b_L^\beta$$

$$Q_3 = \bar{q}_R^\alpha b_L^\beta \bar{q}_R^\beta b_L^\beta$$

$$Q_4 = \bar{q}_R^\alpha b_L^\alpha \bar{q}_L^\beta b_R^\beta$$

$$Q_5 = \bar{q}_R^\alpha b_L^\beta \bar{q}_L^\beta b_R^\beta$$

$$\tilde{Q}_1 = \bar{q}_R^\alpha \gamma_\mu b_R^\alpha \bar{q}_R^\beta \gamma^\mu b_R^\beta$$

$$\tilde{Q}_2 = \bar{q}_L^\alpha b_R^\alpha \bar{q}_L^\beta b_R^\beta$$

$$\tilde{Q}_3 = \bar{q}_L^\alpha b_R^\beta \bar{q}_L^\beta b_R^\beta$$

$$\begin{aligned} F_1 &= F_{\text{SM}} = (V_{tq} V_{tb}^*)^2 \\ F_{j=1} &= 0 \end{aligned}$$

$$\begin{aligned} |F_j| &= F_{\text{SM}} \\ \text{arbitrary phases} \end{aligned}$$

$$\begin{aligned} |F_j| &= 1 \\ \text{arbitrary phases} \end{aligned}$$

$$C_j(\Lambda) = \frac{LF_j}{\Lambda^2} \Rightarrow \Lambda = \sqrt{\frac{LF_j}{C_j(\Lambda)}}$$

$C(\Lambda)$  coefficients are extracted from data

L is loop factor and should be :

L=1 tree/strong int. NP

L= $\alpha_s^2$  or  $\alpha_W^2$  for strong/weak perturb. NP

MFV

NMFV

*LATTICE  
CALCULATIONS  
ESSENTIAL IN  
THIS CASE !!*

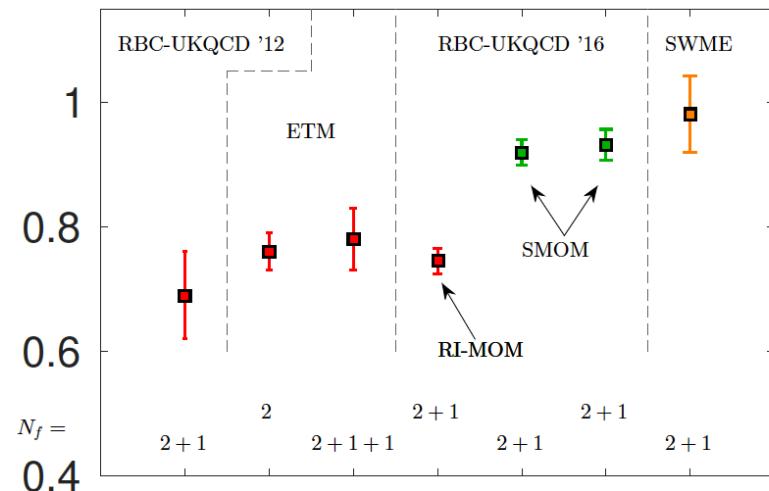
Flavour generic

# Resolution of the discrepancy for $B_4$ , $B_5$

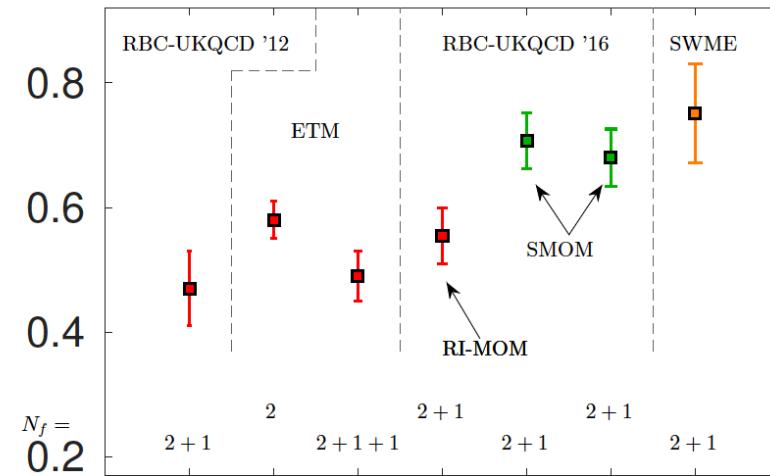
$N_f = 2 + 1$  DWF,  $a = 0.08, 0.11$  fm,  $m_\pi = 300$  MeV [RBC-UKQCD, JHEP11(2016)001]

open question

$B_4$



$B_5$



Plot, courtesy of N. Garron

- Use both RI/MOM and SMOM  $\Rightarrow$  the former is significantly smaller
- Use two RI/SMOM schemes,  $(\not{q}, \not{q})$  and  $(\gamma_\mu, \gamma_\mu)$   $\Rightarrow$  consistent results
- RI/(S)MOM result compatible with previous RI/(S)MOM calculation

Study suggests RI/MOM suffers from large IR artifacts  $\Rightarrow$  discrepancy

On-going project: [J. Kettle's talk, Wednesday 11:30@Seminarios 6+7]

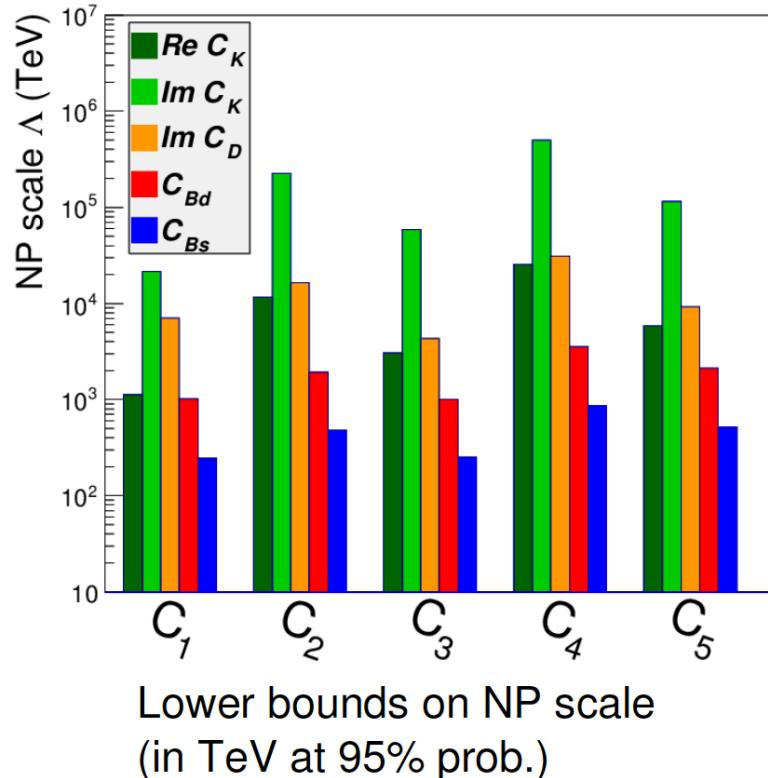
- $64^3$  and  $48^3$  ensembles with physical  $m_\pi$  and finer lattice spacing



## results from the Wilson coefficients

**Generic:**  $C(\Lambda) = \alpha/\Lambda^2$ ,  $F_i \sim 1$ , arbitrary phase

$\alpha \sim 1$  for strongly coupled NP



Non-perturbative NP  
 $\Lambda > 5.0 \times 10^5$  TeV

To obtain the lower bound for loop-mediated contributions, one simply multiplies the bounds by  $\alpha_s (\sim 0.1)$  or by  $\alpha_w (\sim 0.03)$ .

$\alpha \sim \alpha_w$  in case of loop coupling through weak interactions

NP in  $\alpha_w$  loops  
 $\Lambda > 1.5 \times 10^4$  TeV

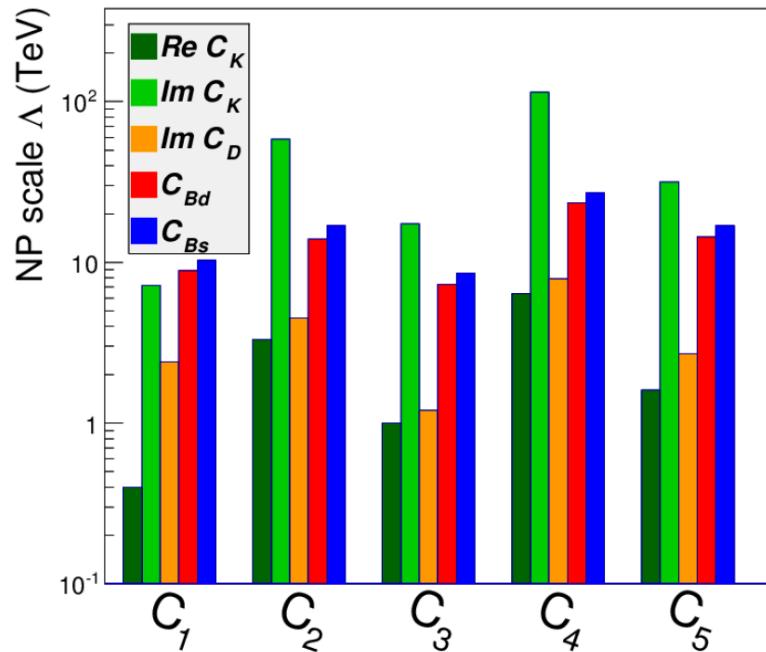
Best bound from  $\varepsilon_K$   
dominated by CKM error  
CPV in charm mixing follows,  
exp error dominant  
Best CP conserving from  $\Delta m_K$ ,  
dominated by long distance  
 $B_d$  and  $B_s$  behind,  
errors from both CKM  
and B-parameters



## results from the Wilson coefficients

NMFV:  $C(\Lambda) = \alpha \times |F_{\text{SM}}|/\Lambda^2$ ,  $F_i \sim |F_{\text{SM}}|$ , arbitrary phase

$\alpha \sim 1$  for strongly coupled NP



Lower bounds on NP scale  
(in TeV at 95% prob.)

Non-perturbative NP  
 $\Lambda > 114$  TeV

To obtain the lower bound for loop-mediated contributions, one simply multiplies the bounds by  $\alpha_s$  ( $\sim 0.1$ ) or by  $\alpha_w$  ( $\sim 0.03$ ).

$\alpha \sim \alpha_w$  in case of loop coupling through weak interactions

NP in  $\alpha_w$  loops  
 $\Lambda > 3.4$  TeV

If new chiral structures present,  
 $\epsilon_K$  still leading  
 $B_{(s)}$  mixing provides very stringent constraints, especially if no new chiral structures are present  
Constraining power of the various sectors depends on unknown NP flavour structure.



absence says more than presence

FRANK HERBERT  
(Dune)

THANKS FOR YOUR ATTENTION

