Physique au LHC: théorie

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- **Le Modèle Standard**

1. Le Modèle Standard de la physique des particules

2. Statut du MS

3. Contraintes sur M_H dans le MS

4. Problèmes et insuffisances du MS

- **Tests du MS au LHC**
- **Le Higgs au LHC**
- **SUSY au LHC**
- **Nouvelle physique au LHC**

1. The Standard Model

The SM is based on a local gauge symmetry: invariance under

$$\mathrm{G_{SM} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y}$$

- **The group $\mathrm{SU(3)_C}$ describes the strong force:**

– interaction between q, q , q which are SU(3) triplets

– mediated by 8 gluons, G_μ^a corresponding to 8 generators of $\mathbf{SU(3)_C}$

Gell-Man 3×3 matrices: $[\mathbf{T^a}, \mathbf{T^b}] = \mathbf{if^{abc}T_c}$ with $\mathrm{Tr}[\mathbf{T^aT^b}] = \frac{1}{2}\delta_{\mathbf{ab}}$

– asymptotic freedom: interaction "weak" at high energy, $\alpha_{\mathbf{s}} = \frac{\mathbf{g_s^2}}{\mathbf{4}\pi} \ll \mathbf{1}$

The Lagrangian of the theory is given by:

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4}\mathbf{G^a_{\mu\nu}G^{\mu\nu}_a} + \mathbf{i}\sum_\mathbf{i} \mathbf{\bar{q}_i}(\partial_\mu - \mathbf{ig_sT_aG^a_\mu})\gamma^\mu \mathbf{q_i}(-\sum_\mathbf{i} \mathbf{m_i\bar{q}_iq_i})$$

with $\mathbf{G^a_{\mu\nu}} = \partial_\mu \mathbf{G^a_\nu} - \partial_\nu \mathbf{G^a_\mu} + \mathbf{g_s\, f^{abc}G^b_\mu G^c_\nu}$

– fermion gauge boson couplings : $-g_i\overline{\psi}V_\mu\gamma^\mu\psi$

– triple gauge boson couplings : $ig_i\,\mathrm{Tr}(\partial_\nu V_\mu - \partial_\mu V_\nu)[V_\mu, V_\nu]$

– quartic gauge boson couplings : $\frac{1}{2}g_i^2\,\mathrm{Tr}[V_\mu, V_\nu]^2$

1. The SM: brief introduction

• $\mathrm{SU(2)_L \times U(1)_Y}$ **describes the electroweak interaction:**

– between the three familles of quarks and leptons

$$\mathbf{I_f^{3L,3R}} = \pm\tfrac{1}{2}, 0 \quad \Rightarrow \mathbf{L} = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_{\mathbf{L}}, \ \mathbf{R} = \mathbf{e_R^-}, \ \mathbf{Q} = \begin{pmatrix} u \\ d \end{pmatrix}_{\mathbf{L}}, \ \mathbf{u_R}, \ \mathbf{d_R}$$

$$\mathbf{Y_f = 2Q_f - 2I_f^3 \Rightarrow Y_L = -1, Y_R = -2, Y_Q = \tfrac{1}{3}, Y_{u_R} = \tfrac{4}{3}, Y_{d_R} = -\tfrac{2}{3}}$$

Same holds for the two other generations: $\mu, \nu_\mu, \mathrm{c}, \mathrm{s}; \ \tau, \nu_\tau, \mathrm{t}, \mathrm{b}$.

There is no $\nu_{\mathbf{R}}$ **(and neutrinos are and stay exactly massless)!**

– mediated by the $\tilde{\mathrm{W}}_\mu$ **(isospin) and** B_μ **(hypercharge) gauge bosons**

the gauge bosons, corresp. to generators, are exactly massless

$$\mathbf{T^a} = \tfrac{1}{2}\tau^{\mathbf{a}}\,; \quad [\mathbf{T^a}, \mathbf{T^b}] = \mathbf{i}\epsilon^{\mathbf{abc}}\mathbf{T_c} \ \text{ and } \ [\mathbf{Y}, \mathbf{Y}] = \mathbf{0}$$

Lagrangian simple: with fields strenghs and covariant derivatives

$$\mathbf{W^a_{\mu\nu}} = \partial_\mu \mathbf{W^a_\nu} - \partial_\nu \mathbf{W^a_\mu} + \mathbf{g_2}\epsilon^{\mathbf{abc}}\mathbf{W^b_\mu W^c_\nu}, \ \mathbf{B}_{\mu\nu} = \partial_\mu \mathbf{B}_\nu - \partial_\nu \mathbf{B}_\mu$$

$$\mathbf{D}_\mu\psi = \left(\partial_\mu - \mathbf{igT_aW^a_\mu} - \mathbf{ig'}\tfrac{\mathbf{Y}}{\mathbf{2}}\mathbf{B}_\mu\right)\psi\,, \quad \mathbf{T^a} = \tfrac{1}{2}\tau^{\mathbf{a}}$$

$$\mathcal{L}_{\mathrm{SM}} = -\tfrac{1}{4}\mathbf{W^a_{\mu\nu}W^{\mu\nu}_a} - \tfrac{1}{4}\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu} + \bar{\mathbf{F}}_{\mathbf{Li}}\,\mathbf{iD}_\mu\gamma^\mu\,\mathbf{F_{Li}} + \bar{\mathbf{f}}_{\mathbf{Ri}}\,\mathbf{iD}_\mu\gamma^\mu\,\mathbf{f_{R_i}}$$

1. The Higgs in the SM: the potential

But if gauge boson and fermion masses are put by hand in $\mathcal{L}_{\rm SM}$

$\frac{1}{2}{\rm M_V^2 V^\mu V_\mu}$ **and/or** ${\rm m_f \bar{f}_L f_R}$ **terms: breaking of gauge symmetry.**

We need a less “brutal” way to generate particle masses in the SM.

In the SM, for the mechanism of spontaneous EW symmetry breaking,

$\Rightarrow$ **introduce a doublet of complex scalar fields**

$$\mathbf{\Phi} = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \text{with} \quad \mathbf{Y_\Phi = +1}$$

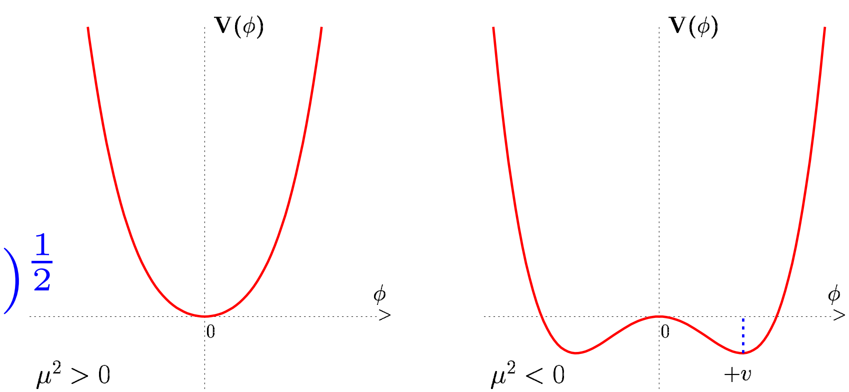
with a Lagrangian that is invariant under $\mathbf{SU(2)_L \times U(1)_Y}$

$$\mathcal{L}_{\mathbf{S}} = (\mathbf{D}^\mu \mathbf{\Phi})^\dagger (\mathbf{D}_\mu \mathbf{\Phi}) - \mu^2 \mathbf{\Phi}^\dagger \mathbf{\Phi} - \lambda (\mathbf{\Phi}^\dagger \mathbf{\Phi})^2$$

$\mu^2 > 0$**: 4 scalar particles.**

$\mu^2 < 0$**:** Φ **develops a vev:**

$$\langle 0 | \Phi | 0 \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \quad v = \left(-\frac{\mu^2}{\lambda}\right)^{\frac{1}{2}}$$



1. The Higgs in the SM: the physical fields

To obtain the physical states, write $\mathcal{L}_S$ with the true vacuum:

- **Write Φ in terms of four fields $\theta_{1,2,3}(x)$ and H(x) at 1st order:**

$$\Phi(x) = e^{i\theta_a(x)\tau^a(x)/v} \frac{1}{\sqrt{2}}\binom{0}{v+H(x)} \simeq \frac{1}{\sqrt{2}}\binom{\theta_2+i\theta_1}{v+H-i\theta_3}$$

- **Make a gauge transformation on Φ to go to the unitary gauge:**

$$\Phi(x) \to e^{-i\theta_a(x)\tau^a(x)}\,\Phi(x) = \frac{1}{\sqrt{2}}\binom{0}{v+H(x)}$$

- **Then fully develop the term $|D_\mu\Phi)|^2$ of the Lagrangian $\mathcal{L}_S$:**

$$|D_\mu\Phi)|^2 = \left|\left(\partial_\mu - ig_1\frac{\tau_a}{2}W_\mu^a - i\frac{g_2}{2}B_\mu\right)\Phi\right|^2$$

$$= \frac{1}{2}\left|\begin{pmatrix} \partial_\mu - \frac{i}{2}(g_2W_\mu^3+g_1B_\mu) & -\frac{ig_2}{2}(W_\mu^1-iW_\mu^2) \\ -\frac{ig_2}{2}(W_\mu^1+iW_\mu^2) & \partial_\mu+\frac{i}{2}(g_2W_\mu^3-g_1B_\mu)\end{pmatrix}\binom{0}{v+H}\right|^2$$

$$= \frac{1}{2}(\partial_\mu H)^2 + \frac{1}{8}g_2^2(v+H)^2|W_\mu^1+iW_\mu^2|^2 + \frac{1}{8}(v+H)^2|g_2W_\mu^3-g_1B_\mu|^2$$

- **Define the new fields $W_\mu^\pm$ and Z_μ [A_μ is the orthogonal of Z_μ]:**

$$W^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp W_\mu^2)\,,\; Z_\mu = \frac{g_2W_\mu^3-g_1B_\mu}{\sqrt{g_2^2+g_1^2}}\,,\; A_\mu = \frac{g_2W_\mu^3+g_1B_\mu}{\sqrt{g_2^2+g_1^2}}$$

$$\sin^2\theta_W \equiv g_2/\sqrt{g_2^2+g_1^2} = e/g_2$$

1. The Higgs in the SM: the masses

- And pick up the terms which are bilinear in the fields $\mathbf{W^\pm, Z, A}$:

$$\mathbf{M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu + \frac{1}{2} M_A^2 A_\mu A^\mu}$$

$\Rightarrow$ 3 degrees of freedom for $\mathbf{W_L^\pm, Z_L}$ and thus $\mathbf{M_{W^\pm}, M_Z}$:

$$\mathbf{M_W = \frac{1}{2} v g_2\ ,\ M_Z = \frac{1}{2} v \sqrt{g_2^2 + g_1^2}\ ,\ M_A = 0\ ,}$$

with the value of the vev given by: $\mathbf{v = 1/(\sqrt{2} G_F)^{1/2} \sim 246\ \mathrm{GeV}}$.

$\Rightarrow$ The photon stays massless, $\mathrm{U(1)_{QED}}$ is preserved.

- For fermion masses, use <u>same</u> doublet field $\mathbf{\Phi}$ and its <u>conjugate</u> field $\mathbf{\tilde{\Phi} = i\tau_2 \Phi^*}$ and introduce $\mathcal{L}_{\rm Yuk}$ which is invariant under SU(2)xU(1):

$$\mathcal{L}_{\rm Yuk} \mathbf{= - f_e (\bar{e}, \bar{\nu})_L \Phi e_R - f_d (\bar{u}, \bar{d})_L \Phi d_R - f_u (\bar{u}, \bar{d})_L \tilde{\Phi} u_R + \cdots}$$

$$\mathbf{= - \frac{1}{\sqrt{2}} f_e (\bar{\nu}_e, \bar{e}_L) \binom{0}{v+H} e_R \cdots = - \frac{1}{\sqrt{2}} (v+H) \bar{e}_L e_R \cdots}$$

$$\mathbf{\Rightarrow m_e = \frac{f_e\, v}{\sqrt{2}}\ ,\ m_u = \frac{f_u\, v}{\sqrt{2}}\ ,\ m_d = \frac{f_d\, v}{\sqrt{2}}}$$

With same Φ, we have generated gauge boson and fermion masses, while preserving SU(2)xU(1) gauge symmetry (which is now hidden)!

What about the residual degree of freedom?

1. The Higgs in the SM: the Higgs boson

It will correspond to the physical spin–zero scalar Higgs particle, H.

The kinetic part of H field, $\frac{1}{2}(\partial_\mu \mathrm{H})^2$, comes from $|\mathrm{D}_\mu \Phi)|^2$ term.

Mass and self-interaction part from $\mathrm{V}(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2$:

$$\mathrm{V} = \frac{\mu^2}{2}(0, \mathrm{v}+\mathrm{H})\binom{0}{\mathrm{v}+\mathrm{H}} + \frac{\lambda}{2}|(0, \mathrm{v}+\mathrm{H})\binom{0}{\mathrm{v}+\mathrm{H}}|^2$$

Doing the exercise you find that the Lagrangian containing H is,

$$\mathcal{L}_\mathrm{H} = \frac{1}{2}(\partial_\mu \mathrm{H})(\partial^\mu \mathrm{H}) - \mathrm{V} = \frac{1}{2}(\partial^\mu \mathrm{H})^2 - \lambda \mathrm{v}^2 \mathrm{H}^2 - \lambda \mathrm{v}\, \mathrm{H}^3 - \frac{\lambda}{4} \mathrm{H}^4$$

The Higgs boson mass is given by: $\mathrm{M_H^2} = 2\lambda \mathrm{v}^2 = -2\mu^2$.

The Higgs triple and quartic self–interaction vertices are:

$$\mathrm{g_{H^3}} = 3\mathrm{i}\, \mathrm{M_H^2}/\mathrm{v}\,,\ \mathrm{g_{H^4}} = 3\mathrm{i} \mathrm{M_H^2}/\mathrm{v}^2$$

What about the Higgs boson couplings to gauge bosons and fermions?

They were almost derived previously, when we calculated the masses:

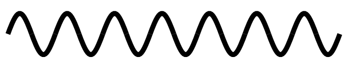
$$\mathcal{L}_{\mathrm{M_V}} \sim \mathrm{M_V^2}(1+\mathrm{H}/\mathrm{v})^2\ ,\ \mathcal{L}_{\mathrm{m_f}} \sim -\mathrm{m_f}(1+\mathrm{H}/\mathrm{v})$$

$$\Rightarrow \mathrm{g_{Hff}} = \mathrm{i}\mathrm{m_f}/\mathrm{v}\,,\ \mathrm{g_{HVV}} = -2\mathrm{i}\mathrm{M_V^2}/\mathrm{v}\,,\ \mathrm{g_{HHVV}} = -2\mathrm{i}\mathrm{M_V^2}/\mathrm{v}^2$$

Since v is known, the only free parameter in the SM is $\mathrm{M_H}$ or λ.

1. SM physics: $W/Z/H$ at high energies

Propagators of gauge and Goldstone bosons in a general ζ gauge:



$\longrightarrow q$

$$\frac{-i}{q^2-M_V^2+i\epsilon}\left[g_{\mu\nu}+(\zeta-1)\frac{q_\mu q_\mu}{q^2-\zeta M_V^2}\right]$$

$\zeta=\infty$: **Landau gauge**

$\zeta=1$: **'t Hooft-Feynman**

$\omega^\pm, \omega^0$:

$\longrightarrow q$

$$\frac{-i}{q^2-\zeta M_V^2+i\epsilon}$$

- **In unitary gauge, Goldstones do not propagate and gauge bosons have usual propagators of massive spin–1 particles (old IVB theory).**
- **At very high energies, $\mathrm{s} \gg \mathrm{M_V^2}$, an approximation is $\mathrm{M_V} \sim 0$. The $\mathrm{V_L}$ components of V can be replaced by the Goldstones, $\mathrm{V_L} \to \omega$.**
- **In fact, the electroweak equivalence theorem tells that at high energies, massive vector bosons are equivalent to Goldstones. In VV scattering e.g.**

$$\mathbf{A(V_L^1 \cdots V_L^n \to V_L^1 \cdots V_L^{n'}) = (i)^n(-i)^{n'} A(w^1 \cdots w^n \to w^1 \cdots w^{n'})}$$

Thus, we simply replace V by w in the scalar potential and use w:

$$\mathbf{V = \frac{M_H^2}{2v}(H^2+w_0^2+2w^+w^-)H+\frac{M_H^2}{8v^2}(H^2+w_0^2+2w^+w^-)^2}$$

2. Status of the SM

The parameters of the SM at tree—level:

In the SM, there are 18 free parameters ($+\theta_{\rm QCD} + \nu$ sector):

- **9 fermions masses, 4 CKM parameters (see below for details).**
- **3 coupling g_s, g_2, g_1 and 2 parameters from scalar potential μ, λ**

More precise inputs, $\alpha_s, \alpha(M_Z^2), G_F, M_Z$ and M_H (unknown)

Weak interactions of fermions with gauge bosons

$$\mathcal{L}_{\rm NC} = e J_\mu^{\rm A} A^\mu + \frac{g_2}{\cos\theta_W} J_\mu^Z Z^\mu \ , \ \ \mathcal{L}_{\rm CC} = \frac{g_2}{\sqrt{2}} (J_\mu^+ W^{+\mu} + J_\mu^- W^{-\mu})$$

$$J_\mu^A = Q_f \bar{f} \gamma_\mu f \ , \ J_\mu^Z = \frac{1}{4} \bar{f} \gamma_\mu [\hat{v}_f - \gamma_5 \hat{a}_f] f \ , \ J_\mu^+ = \frac{1}{2} \bar{f}_u \gamma_\mu (1-\gamma_5) f_d$$

with $v_f = \frac{\hat{v}_f}{4 s_W c_W} = \frac{2I_f^3 - 4 Q_f s_W^2}{4 s_W c_W} \ , \ a_f = \frac{\hat{a}_f}{4 s_W c_W} = \frac{2 I_f^3}{4 s_W c_W}$

3-families: complication in CC as current eigenstates $\neq$ mass eigenstates

connected by a unitary transformation: $(d', s', b') = V_{\rm CKM} (d, s, b)$

$V_{\rm CKM} \equiv 3\times 3$ **unitarity matrix; NC are diagonal in both bases (GIM).**

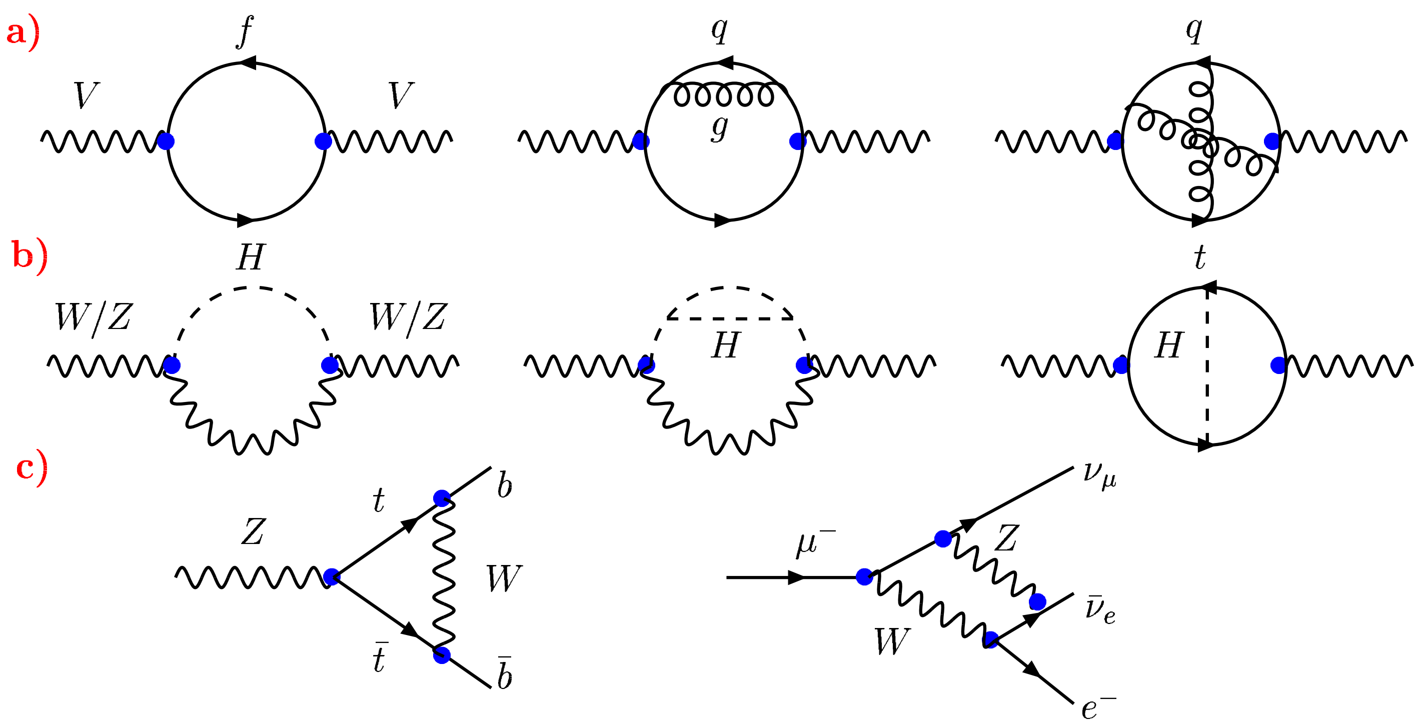
Parametrized by 3 angles and 1 CPV phase: tests at B–factories.

2. Status of the SM: precision tests

M_W **and** $\sin^2\theta_W$ **predicted:** $\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha(M_Z^2)}{2M_W^2(1-M_W^2/M_Z^2)}$; $\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$

In fact, they are related by $\rho = \frac{M_W^2}{c_W^2 M_Z^2} \equiv 1$ **at tree–level in the SM**

To have very precise predictions, include the radiative corrections:



The dominant correction is, besides $\Delta\alpha$**, the one to the** ρ **parameter**

$$\rho = \frac{1}{1-\Delta\rho}\ ,\ \Delta\rho = \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2} = \frac{3G_\mu m_t^2}{8\sqrt{2}\pi^2} - \frac{G_\mu M_W^2}{8\sqrt{2}\pi^2}\log\frac{M_H^2}{M_W^2} + \cdots$$

2. Status of the SM: high–precision data

- **Z boson lineshape parameters at LEP1 ($\sqrt{\rm s} \sim \rm M_Z$):**

$$M_Z, \Gamma_Z, \sigma(e^+e^- \to \text{hadrons})$$

- **Partial decay widths and asymmetries in Z decays at LEP1:**

$$\Gamma(Z \to f\bar{f}) = \frac{2\alpha}{3} N_c M_Z (v_f^2 + a_f^2)\ ,\ A_{FB}^f = \frac{3}{4} \frac{2a_e v_e}{v_e^2+a_e^2} \frac{2a_f v_f}{v_f^2+a_f^2}$$

- **Left–right polarized asymmetries in Z decays at SLC:**

$$A_{LR} = \frac{2a_e v_e}{v_e^2+a_e^2}\ ,\ A_{LR/FB}^f = \frac{3}{4} \frac{2a_f v_f}{v_f^2+a_f^2}$$

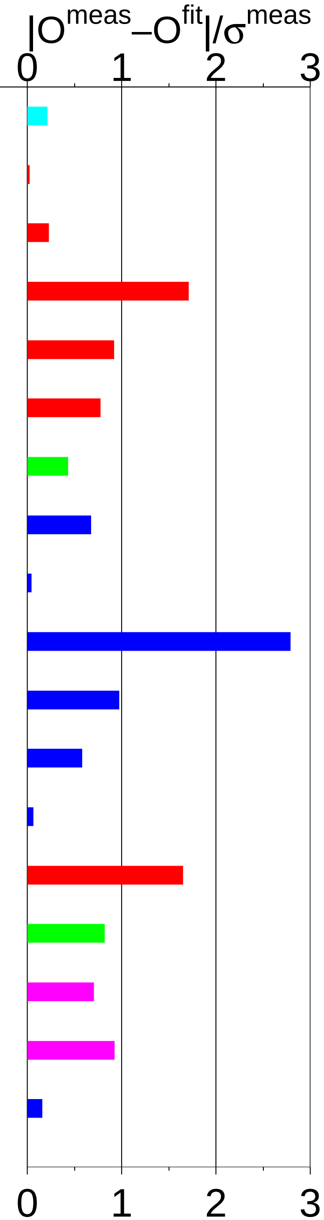
- **W boson parameters: M_W and Γ_W at LEP2 and Tevatron.**
- **Other observables at low–energy: ν_e DIS, PV in Cs and Th ...**
- **Use top quark mass value from Tevatron $m_t = 171 \pm 2$ GeV**
- **Use value of α_s from LEP and elsewhere: $\alpha_s = 0.1172 \pm 0.002$**
- **Use $\alpha(M_Z)$ with $\Delta\alpha = 0.028 \pm 0.00036$ from low–energy data**

$\Rightarrow$ **Very high precision tests of the SM at the quantum level: 1%–0.1%**

SM describes precisely (almost) all available experimental data!

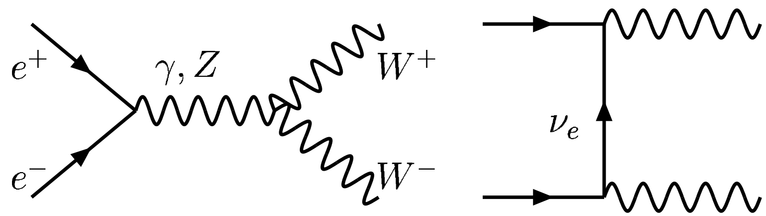
2. Status of the SM: high–precision tests

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02758 ± 0.00035	0.02766
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4957
σ^0_{had} [nb]	41.540 ± 0.037	41.477
R_l	20.767 ± 0.025	20.744
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01640
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1479
R_b	0.21629 ± 0.00066	0.21585
R_c	0.1721 ± 0.0030	0.1722
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1037
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0741
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1479
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.392 ± 0.029	80.371
Γ_W [GeV]	2.147 ± 0.060	2.091
m_t [GeV]	171.4 ± 2.1	171.7



2. Tests of the SM: gauge structure

WW production at at LEP2:



General CPC WWV coupling given by:

$$\mathcal{L}_{\rm eff}^{WWV} \propto g_1^V V^\mu \left(W^-_{\mu\nu} W^{+\nu} - W^+_{\mu\nu} W^{-\nu} \right) + \kappa_V W^+_\mu W^-_\nu V^{\mu\nu} + \frac{\lambda_V}{M_W^2} V^{\mu\nu} W^{+\rho}_\nu W^-_{\rho\mu}$$

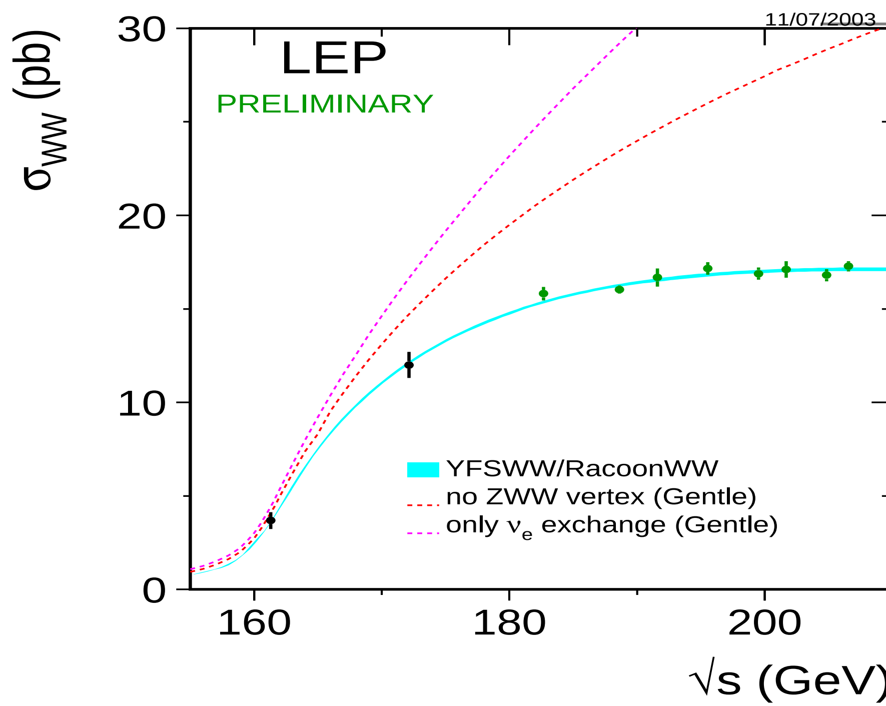
In SM: $g_1^V = 1, \kappa_V = 1, \lambda_V = 0$

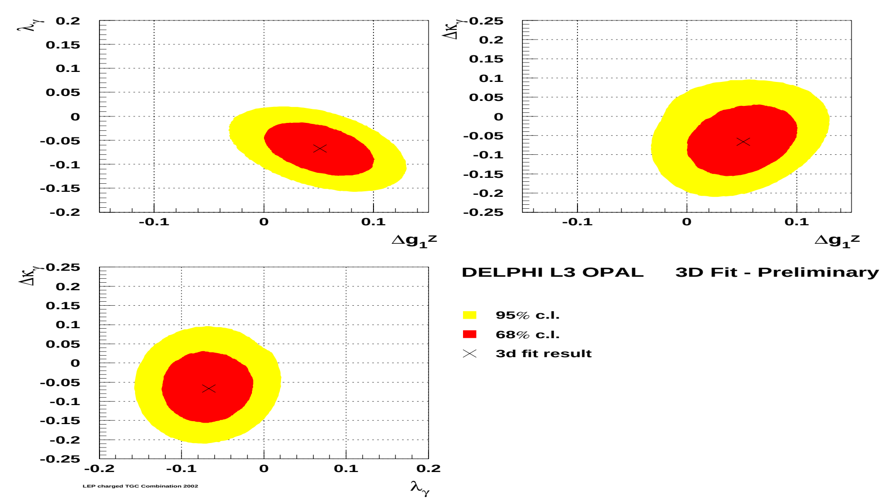
$\mathbf{SU(2)_L \times U(1)_Y}$ gauge structure checked rather precisely at LEP2

Note: QCD also very precisely tested!

– running of α_s from m_τ to LEP2.

– 3 gluon vertex determined at LEP1.

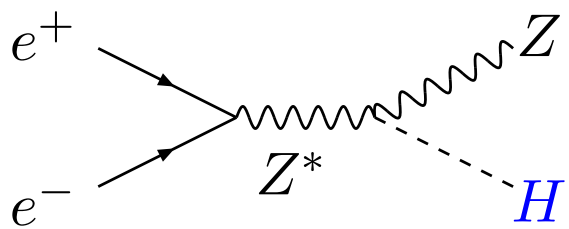




3. Constraints on M_H: experiment

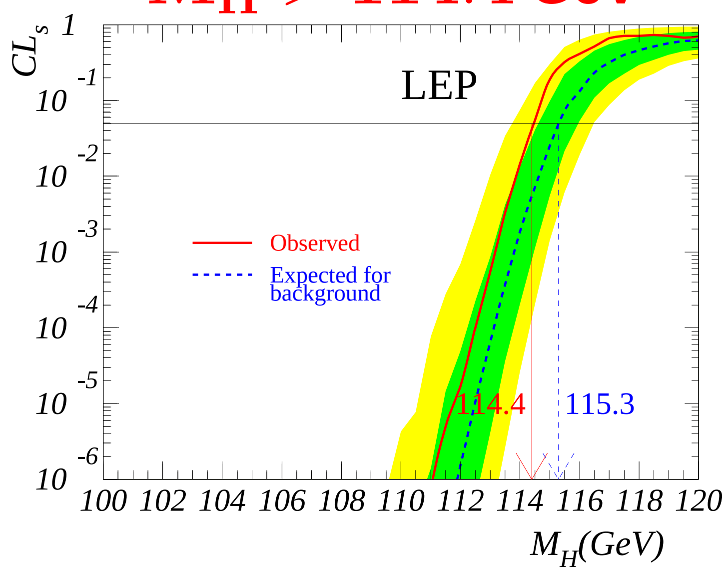
Direct searches at LEP:

H looked for in $\mathrm{e^+e^- \to ZH}$



We have a limit at 95% CL:

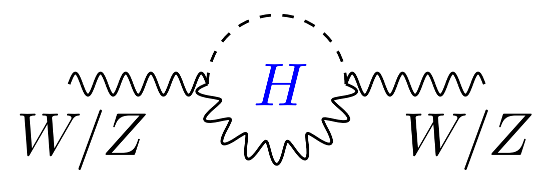
$\mathbf{M_H > 114.4}$ **GeV**



(1.7σ excess at $M_H \sim 116$ GeV)

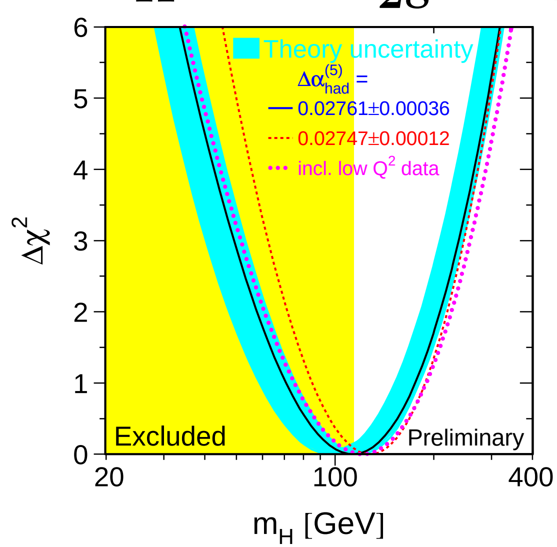
Indirect searches:

H contributes to RC to W/Z masses:



Fit the EW precision measurements:

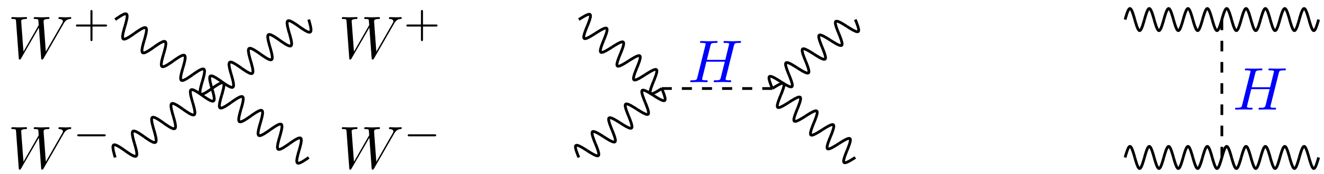
we obtain $\mathbf{M_H = 85^{+39}_{-28}}$ GeV, or



$\mathbf{M_H \lesssim 160}$ **GeV at 95% CL**

3. Constraints on M_H: perturbative unitarity

Scattering of massive gauge bosons $\mathbf{V_L V_L \to V_L V_L}$ at high-energy



Because w interactions increase with energy (q^μ terms in V propagator),

$\mathbf{s \gg M_W^2 \Rightarrow \sigma(w^+w^- \to w^+w^-) \propto s}$: $\Rightarrow$ **unitarity violation possible!**

Decomposition into partial waves and choose J=0 for $\mathbf{s \gg M_W^2}$:

$$\mathbf{a_0 = -\frac{M_H^2}{8\pi v^2}\left[1 + \frac{M_H^2}{s - M_H^2} + \frac{M_H^2}{s}\log\left(1 + \frac{s}{M_H^2}\right)\right]}$$

For unitarity to be fullfiled, we need the condition $\mathbf{|Re(a_0)| < 1/2}$.

At high energies, $\mathbf{s \gg M_H, M_W}$, we have: $\mathbf{a_0 \xrightarrow{s \gg M_H^2} -\frac{M_H^2}{8\pi v^2}}$

$$\mathbf{unitarity \Rightarrow M_H \lesssim 870\ GeV\ \ (M_H \lesssim 710\ GeV)}$$

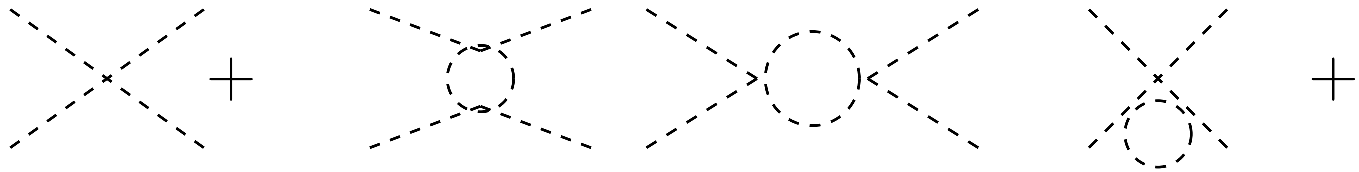
For a very heavy or no Higgs boson, we have: $\mathbf{a_0 \xrightarrow{s \ll M_H^2} -\frac{s}{32\pi v^2}}$

$$\mathbf{unitarity \Rightarrow \sqrt{s} \lesssim 1.7\ TeV\ \ (\sqrt{s} \lesssim 1.2\ TeV)}$$

Otherwise (strong?) New Physics should appear to restore unitarity.

3. Constraints on M_H: triviality

The quartic coupling of the Higgs boson λ ($\propto M_H^2$) increases with energy.



The RGE evolution of λ with Q^2 and its solution are given by:

$$\frac{\mathrm{d}\lambda(Q^2)}{\mathrm{d}Q^2} = \frac{3}{4\pi^2}\lambda^2(Q^2) \Rightarrow \lambda(Q^2) = \lambda(v^2)\left[1 - \frac{3}{4\pi^2}\lambda(v^2)\log\frac{Q^2}{v^2}\right]^{-1}$$

- If $Q^2 \ll v^2$, $\lambda(Q^2) \to 0_+$: **the theory is said to be trivial (no int.).**
- If $Q^2 \gg v^2$, $\lambda(Q^2) \to \infty$: **Landau pole at** $Q = v \exp\left(\frac{4\pi^2 v^2}{M_H^2}\right)$.

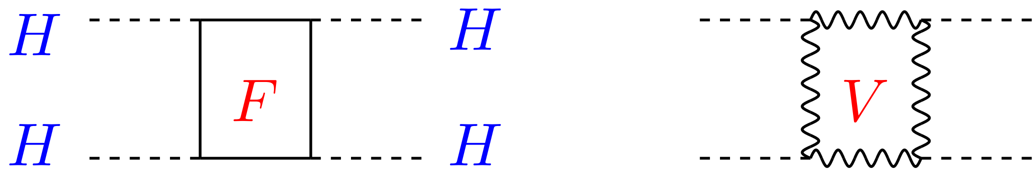
The SM is valid only at scales before λ becomes infinite:

If $\Lambda_C = M_H$, $\lambda \lesssim 4\pi \Rightarrow M_H \lesssim 650$ **GeV**

(Comparable to results obtained with simulations on the lattice!)

3. Constraints on M_H: vacuum stability

The top quark and gauge bosons also contribute to the evolution of λ.



The RGE evolution of the coupling at one–loop is given by

$$\lambda(Q^2) = \lambda(v^2) + \frac{1}{16\pi^2}\left[-12\frac{m_t^4}{v^4} + \frac{3}{16}\left(2g_2^4 + (g_2^2+g_1^2)^2\right)\right]\log\frac{Q^2}{v^2}$$

If λ is small (H is light), top loops might lead to $\lambda(0) < \lambda(v)$:

v is not the minimum of the potentiel and the EW vacuum is instable.

$\Rightarrow$ Impose that the coupling λ stays always positive:

$$\lambda(Q^2) > 0 \Rightarrow M_H^2 > \frac{v^2}{8\pi^2}\left[-12\frac{m_t^4}{v^4} + \frac{3}{16}\left(2g_2^4 + (g_2^2+g_1^2)^2\right)\right]\log\frac{Q^2}{v^2}$$

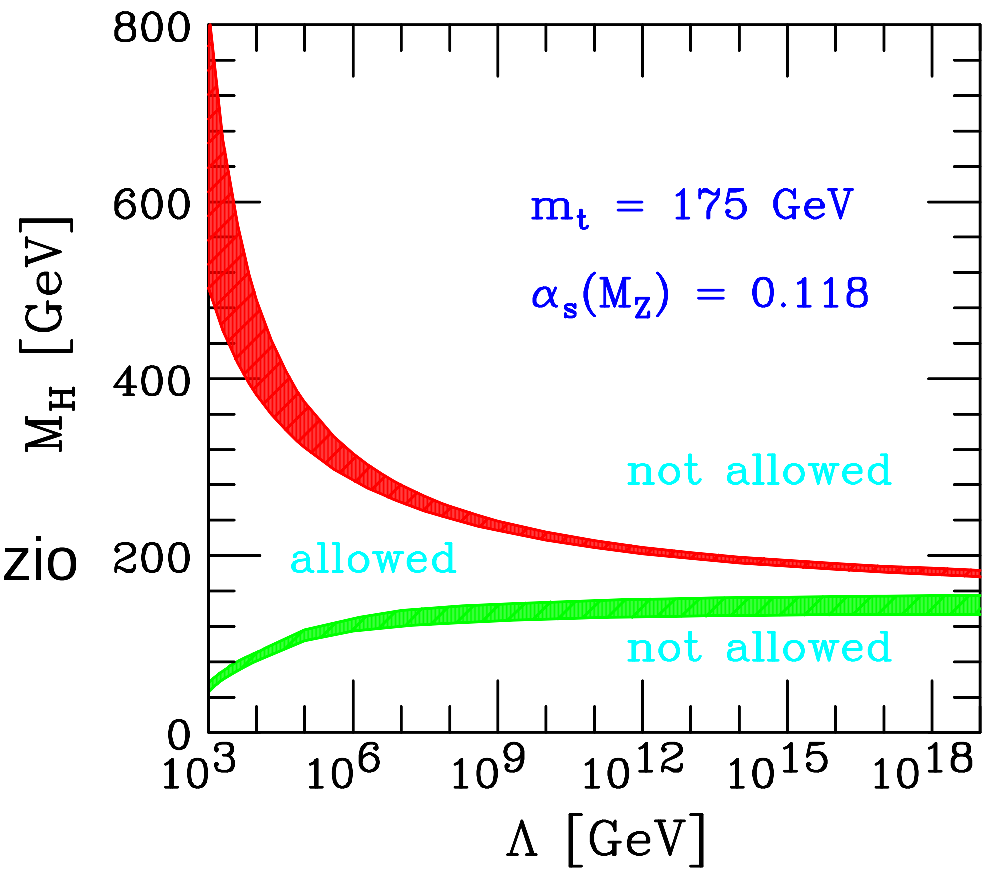
Very strong constraint: $Q = \Lambda_C \sim 1$ TeV $\Rightarrow M_H \gtrsim 70$ GeV

3. Constraints on M_H: triviality+stability

Combine the two constraints and include all possible effects:

- **corrections at two loops**
- **theoretical errors**
- **experimental errors**
- **other refinements** $\cdots$

Cabibbo, Maiani, Parisi, Petronzio
Hambye, Riesselmann



$$\Lambda_C \sim 10^3 \text{ GeV} \quad \Rightarrow \quad 70 \text{ GeV} \lesssim M_H \lesssim 700 \text{ GeV}$$

$$\Lambda_C \sim 10^{16} \text{ GeV} \quad \Rightarrow \quad 130 \text{ GeV} \lesssim M_H \lesssim 180 \text{ GeV}$$

4. Problems and shortcomings of the SM

The SM has many attractive theoretical/experimental features:

- Based on gauge principle, unitary, perturbative, renormalisable $\cdots$
- Once M_H fixed: everything is predictible with great accuracy.
- And has passed all experimental tests up to now.

But the model has too many shortcomings:

- Too many free parameters (19!) in the model, put by hand...
- No satisfactory explanation for $\mu^2 < 0$ (put ad hoc).
- Does not include the fourth fundamental force, gravity, ..
- Does not say anything about the masses of the neutrinos.
- Does not explain the baryon asymmetry in the universe.
- No real unification of the three gauge interactions; fast P decay.
- There is no stable, weak, massive particle for dark matter.

And above all that, there is the hierarchy or naturalness problem.

4. Problems of the SM: unification

In SM, we have 3 different gauge groups with 3 coupling constants:

$\Rightarrow$ **SU(3)xSU(2)xU(1) subgroup of a bigger unifying group.**

Grand Unified Theory (GUT): SU(5), SO(10), E_6 etc....

- **only one coupling constant a the GUT scale $M_{GUT} = M_U$**

Spontaneous breakdown to G_{SM} at M_U (intermdeiate scale?).

- **GUT has fundamental representation including all SM fermions.**

Ex: SO(10) has dim. 16 repr. which incorporates 15 SM fermions.

- **Space left for RH neutrinos: generation of m_ν via see–saw.**
- **Baryon asymmetry of the universe through leptogenesis**
- **Explains charge quantization (ex. in SU(5): e,d in multiplet).**
- **Can relate the masses of fermions at M_U (Yukama coupling unif.)**

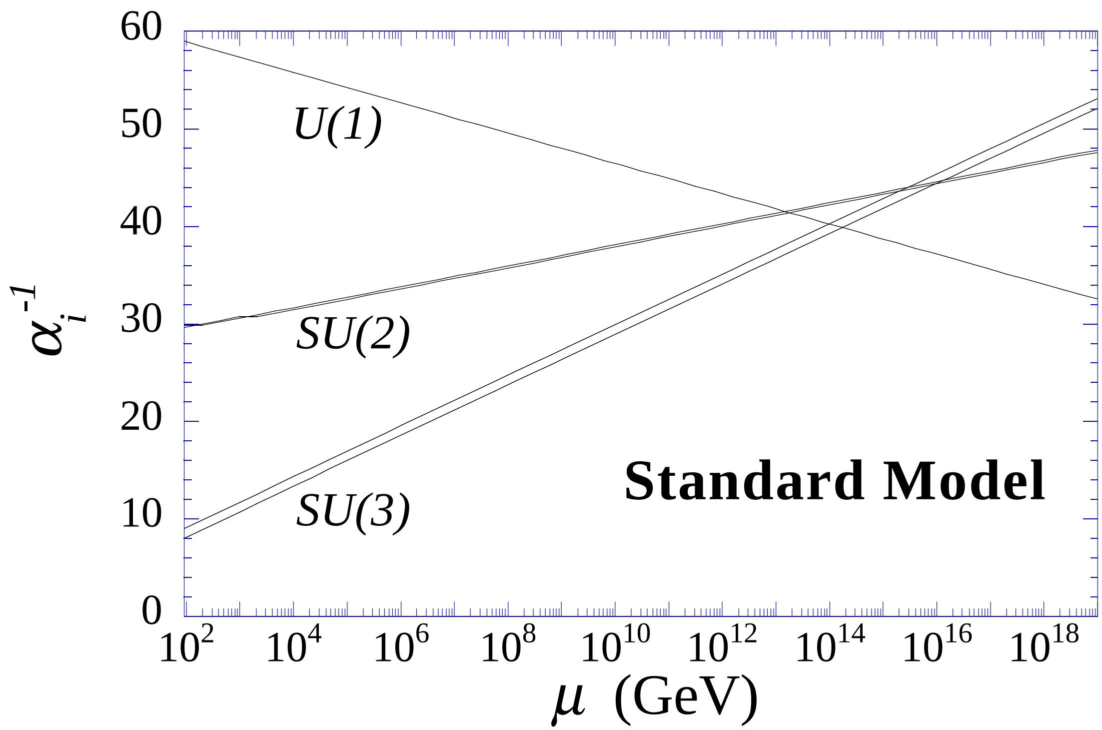
However, there is a problem in non-SUSY GUTS:

the SU(3), SU(2) and U(1) gauge couplings $\alpha_i = g_i^2/(4\pi)$ do not unify:

4. Problems of the SM: unification

The running of the coupling constants: due to radiative corrections to the interaction term in the original Lagrangian ($\gamma f\bar{f}$ in QED); equivalent to ren. of two–point functions; evolution determined by RGE which depends on relevant gauge group and particle content.

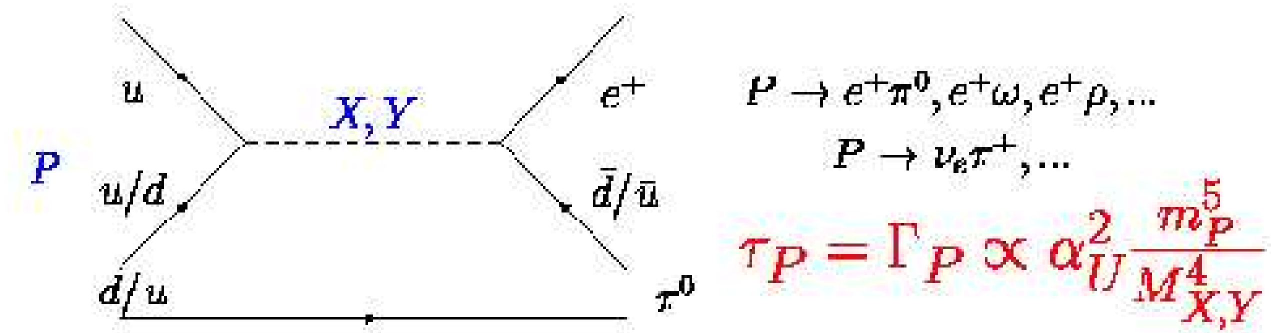
In the context of SU(5), there is no unification with SM particle content



Alternative view: couplings do not meet at a single point near M_{GUT}.

4. Problems of the SM: proton decay

P decay occurs via exchange of the heavy SU(5) gauge bosons X,Y:



– Compute the effective 4–fermion interaction (CKM... dependent).

– Run down vertices from high scale $\mathrm{M_{X,Y}} \sim \mathrm{M_{GUT}}$ to $\mathrm{m_P}$.

– Calculate hadronic ME of the 4–fermion operator (model dep..).

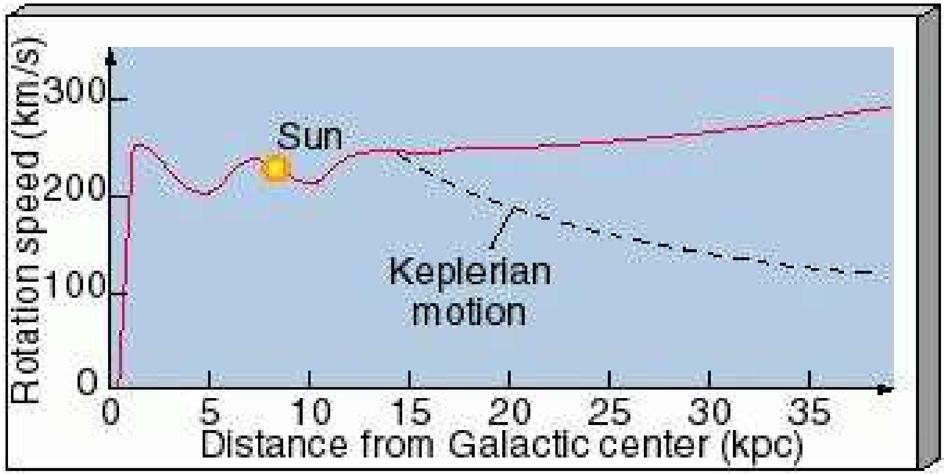
With the input GUT scale from $\mathrm{g_i's}$, $\mathrm{M_{GUT}} \sim 10^{15}$ GeV, one has:

$$\tau_{\mathbf{P}}^{\rm non-SUSY\ GUT} = \mathbf{10^{30\pm1.7}}\ \rm years$$

To be compared to $\tau_{\mathbf{P}}^{\rm exp} \gtrsim 10^{33}$ years: P decay is far too fast!!!

4. Problems of the SM: no cold dark matter

The experimental measurement of the galaxy rotation curve:



shows that some dark matter should be present in universe.

From large structure formation: DM should be cold (non relativistic)

The WMAP satelitte has shown that there is 25% of CDM:

$$\Omega_{\rm DM}\, h^2 \simeq 0.113 \pm 0.009 \Rightarrow 0.09 \leq \Omega_{\rm DM}\, h^2 \leq 0.14 \text{ at } 99\% \text{ CL}$$

Needs a particle that fullfils the following conditions:

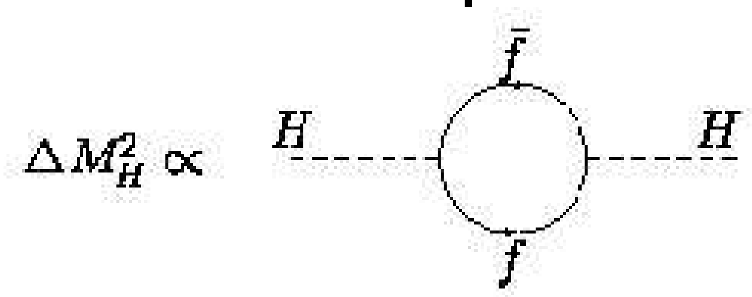
electrically neutral, weakly interacting, rather massive and stable!

There is no such a particle in the SM and also in non–SUSY GUTs!

4. Problems of the SM: the hierarchy problem

Radiative corrections to the Higgs boson mass in the SM

Let us first consider the fermion loop contribution to $\mathbf{M_H^2}$



Using a cut–off Λ (see excercises later) one obtains:

$$\mathbf{\Delta M_H^2 = N_f \frac{\lambda_f^2}{8\pi^2} \left[-\Lambda^2 + 6 m_f^2 \log\frac{\Lambda}{m_f} - 2 m_f^2 \right] + \mathcal{O}(1/\Lambda^2)}$$

We have thus a quadratic divergence, $\mathbf{\Delta M_H^2 \sim \Lambda^2}$.

Divergence is independent of $\mathbf{M_H}$, and does not disappear if $\mathbf{M_H = 0}$:

The choice $\mathbf{M_H = 0}$ does not increase the symmetry of $\mathcal{L}_{\rm SM}$.

If we fix the cut–off Λ to $\mathrm{M_{GUT}}$ or $\mathbf{M_P}$: $\Rightarrow \mathbf{M_H} \sim 10^{14}$ to 10^{17} GeV!

The Higgs boson mass prefers to be close to the very high scale:

This is the hierarchy problem.

4. Problems of the SM: the hierarchy problem

But we want a light Higgs ($M_H \lesssim 1$ TeV) for unitarity etc... reasons.

We need thus to make: $M_H^2|^{\rm Physical} = M_H^2|^0 + \Delta M_H^2$+ counterterm

And adjust this counterterm with a precision of 10^{-30} (30 digits)

This fine-tunning would be very unnatural...

In SM, besides fermion loops, there are also contributions to M_H

from the massive gauge bosons and from the Higgs boson itself:

$$\Rightarrow \Delta M_H^2 \propto [3(M_W^2 + M_Z^2 + M_H^2)/4 - \sum m_f^2](\Lambda^2/M_W^2)$$

We can adjust the unknown M_H so that the quadratic divergence

disappears (would be a prediction for Higgs mass, $M_H \sim 200$ GeV).

However: does not work at two–loop level or at higher orders....

Summary: the problem of the quadratic divergences to M_H is there.

Photon and fermion masses protected by gauge and chiral symmetry,

.... but here is no symmetry which protects M_H in the SM.