Finite Unified Theories:
Predictions for Collider Physics

Sven Heinemeyer, IFCA (Santander)

Orsay, 02/2008

1. The model
2. The constraints
3. The predictions
4. Conclusions

Finite Unified Theories:
Predictions for Collider Physics

Sven Heinemeyer, IFCA (Santander)

Orsay, 02/2008

1. The model ⇒ covered by George

2. The constraints

3. The predictions

4. Conclusions

2. The constraints

How does it work with the constraints?

Input:

– parameters at the GUT scale

– some SM parameters

⇒ application of RGE's

Output:

– all SUSY parameters:
  – SUSY masses and mixings
  – other SUSY parameters: $\tan\beta$, ...

– some SM parameters (e.g. heavy quark masses)

⇒ evaluation of observables possible

→ constraints: can be tested now, reduce allowed parameter space

→ predictions: to be tested at the LHC, ...

The constraints

A) The heavy quark masses

B) The decay BR($b \to s\gamma$)

C) The decay $B_s \to \mu^+\mu^-$

D) The lightest Higgs boson mass M_h

E) Cold Dark Matter (CDM) density

F) The anomalous magnetic moment of the muon

G) . . .

2A) The heavy quark masses

FUT input: m_τ

FUT output/prediction: m_t, m_b

We use as constraints:

$$\begin{aligned} m_t^{\text{pole,exp}} &= 170.9 \pm 1.8 \text{ GeV} \\ m_b(M_Z)^{\text{exp}} &= 2.82 \pm 0.07 \text{ GeV} \end{aligned}$$

Note:

prediction of $m_b(M_Z)$ involves loop corrections to the relation between mass and Yukawa coupling:

$$y_b \sim \frac{m_b}{1 + \Delta_b}$$

with

$$\Delta_b = \frac{2\alpha_s}{3\,\pi} m_{\tilde{g}}\, \mu \tan\beta \times I(m_{\tilde{b}_1}, m_{\tilde{b}_2}, m_{\tilde{g}}) + \frac{\alpha_t}{4\,\pi} A_t\, \mu \tan\beta \times I(m_{\tilde{t}_1}, m_{\tilde{t}_2}, \mu)$$

Checks:

use expansion of denominator as estimate for size of higher-order uncertainties.
Compare:

$$\frac{1}{1 + \Delta_b}, \quad 1 - \Delta_b + \Delta_b^2, \quad 1 - \Delta_b$$

2B) The decay $\mathrm{BR}(b \to s\gamma)$

Experimental result:

[*HFAG '07*]

$$\mathrm{BR}(b \to s\gamma)^{\text{exp}} = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4}$$

Latest SM prediction:

[*M. Misiak et al. '07*]

$$\mathrm{BR}(b \to s\gamma)^{\text{theo,SM}} = (3.15 \pm 0.23) \times 10^{-4}$$

SUSY corrections by code from [*G. Hiller et al.*] ,
crosschecked with code from [*Micromegas '07*]

Additional error from unknown SUSY corrections: $\pm 0.15 \times 10^{-4}$

Conservative approach: add theoretical and experimental error linearly

2C) The decay $\mathrm{BR}(B_s \to \mu^+\mu^-)$

Experimental result:

[*CDF '07, D0 '07*]

$$\mathrm{BR}(B_s \to \mu^+\mu^-)^{\mathrm{exp}} < 5.8 \times 10^{-8} \quad \text{at 95\% C.L.}$$

SM result:

[*Buchalla, Buras, Misiak, Urban '93-'03*]

$$\mathrm{BR}(B_s \to \mu^+\mu^-)^{\mathrm{theo,SM}} = (3.4 \pm 0.5) \times 10^{-9}$$

→ so far negligible

SUSY contributions taken from [*Micromegas '07*] ,
checked with [*Dedes, Dreiner, Nierste '01*]

Error estimate: not available
("error is small, even in SUSY")

2D) The lightest Higgs boson mass M_h

MSSM predicts upper bound on M_h:

tree-level bound: $m_h < M_Z$, excluded by LEP Higgs searches!

Large radiative corrections:

Yukawa couplings: $\frac{\mathrm{e}\, m_t}{2 M_W s_\mathrm{W}}, \frac{\mathrm{e}\, m_t^2}{M_W s_\mathrm{W}}, \ldots$

$\Rightarrow$ Dominant one-loop corrections: $\Delta M_h^2 \sim G_\mu m_t^4 \log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$

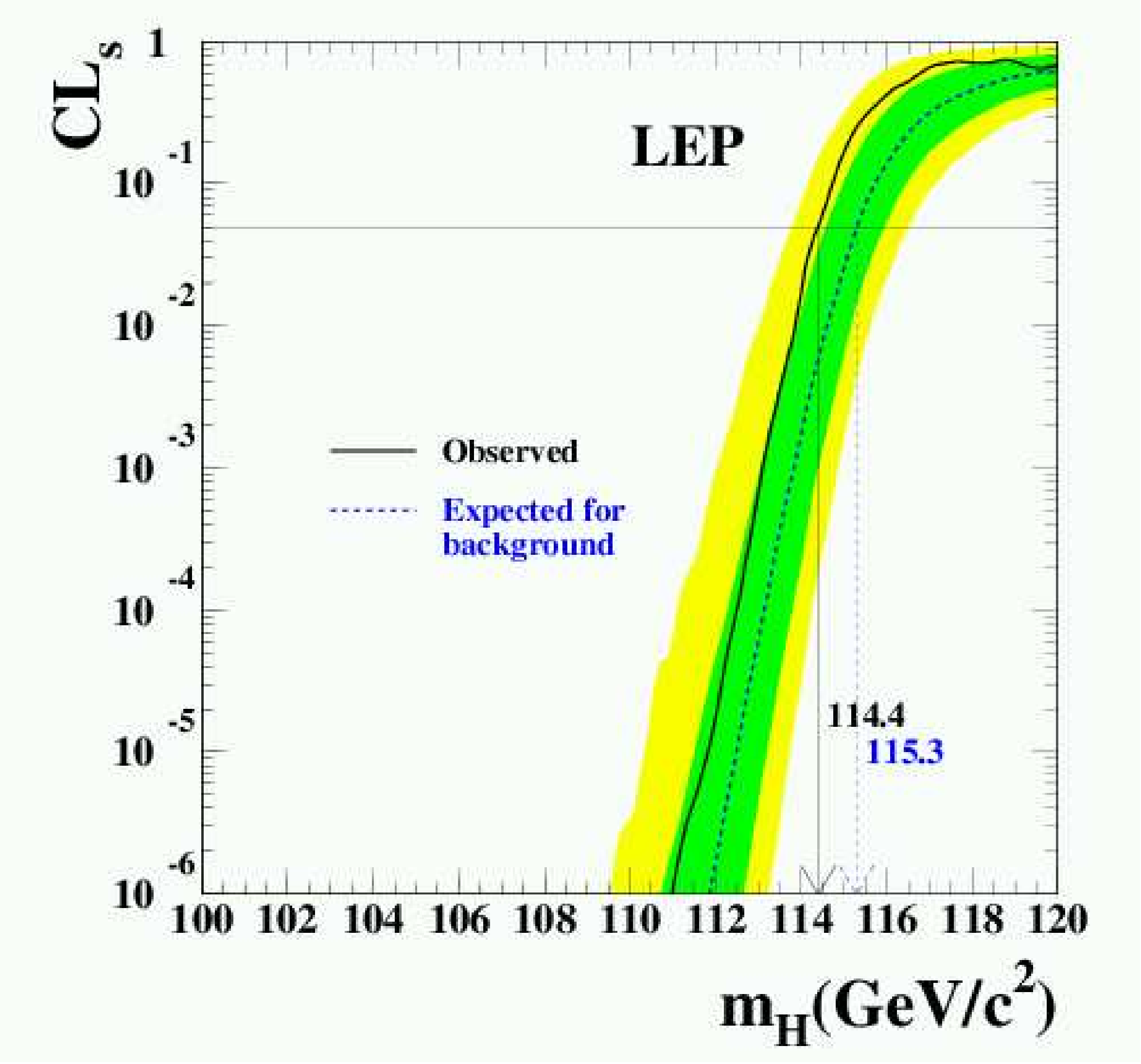
The MSSM Higgs sector is connected to all other sector via loop corrections (especially to the scalar top sector)

Present status of M_h prediction in the MSSM:

Complete one-loop and ‘almost complete’ two-loop result available

In FUT:

SM bound of M_H search can be used [*LEP Higgs Working Group '03*]



$M_h > 114.4$ GeV

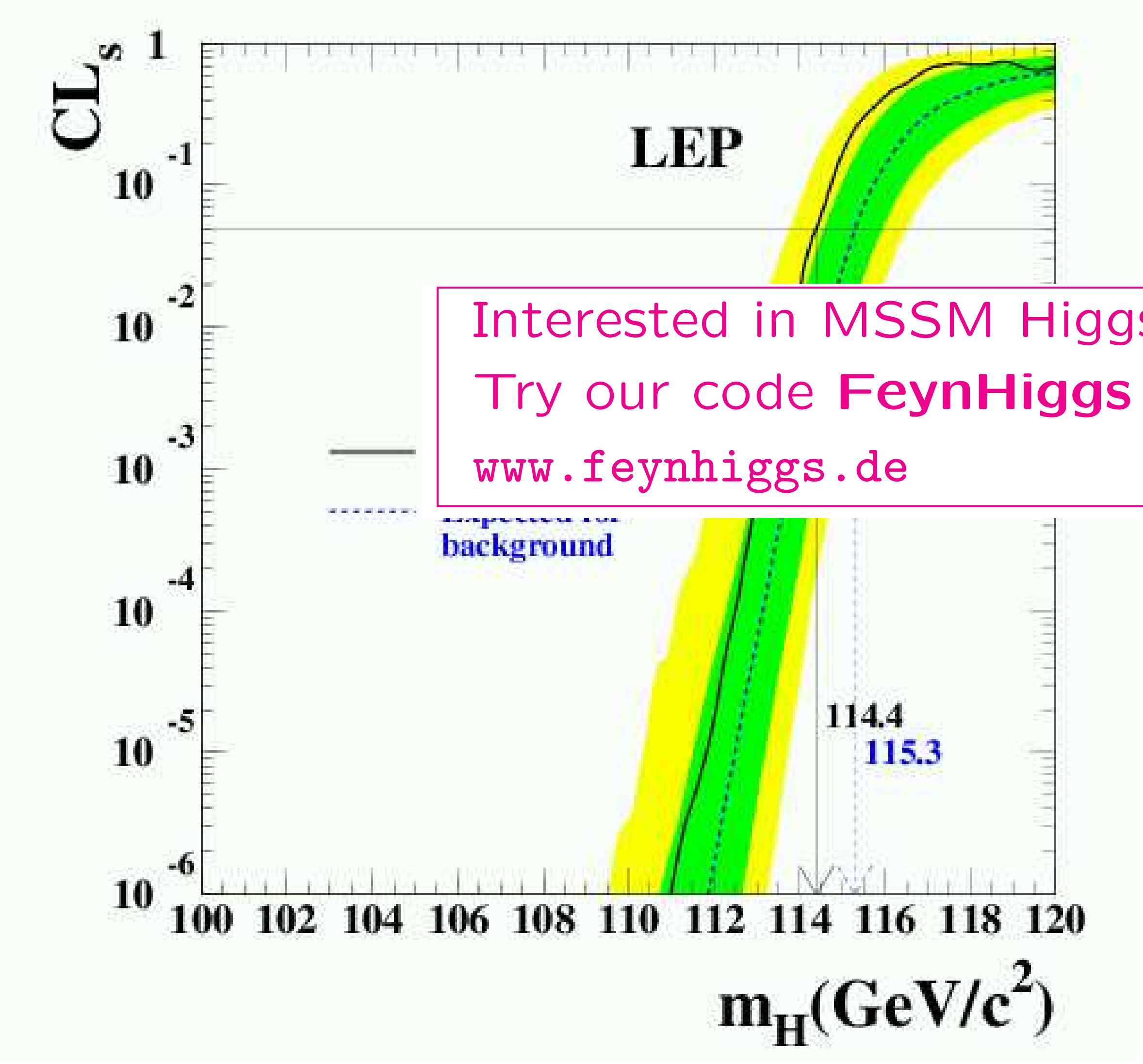
at 95% C.L.

$\delta M_h^{\text{intr.}} \approx 3$ GeV

We use *FeynHiggs*

In FUT:

SM bound of M_H search can be used [*LEP Higgs Working Group '03*]



Interested in MSSM Higgs physics?

Try our code **FeynHiggs**

`www.feynhiggs.de`

$M_h > 114.4$ GeV

$\delta M_h^{\text{intr.}} \approx 3$ GeV

We use *FeynHiggs*

2E) Cold Dark Matter

Cold Dark Matter exists:

⇒ It all fits together

$$\Omega_{\rm tot} \approx 1$$

$$\Omega_M h^2 = 0.135^{+0.008}_{-0.009}$$

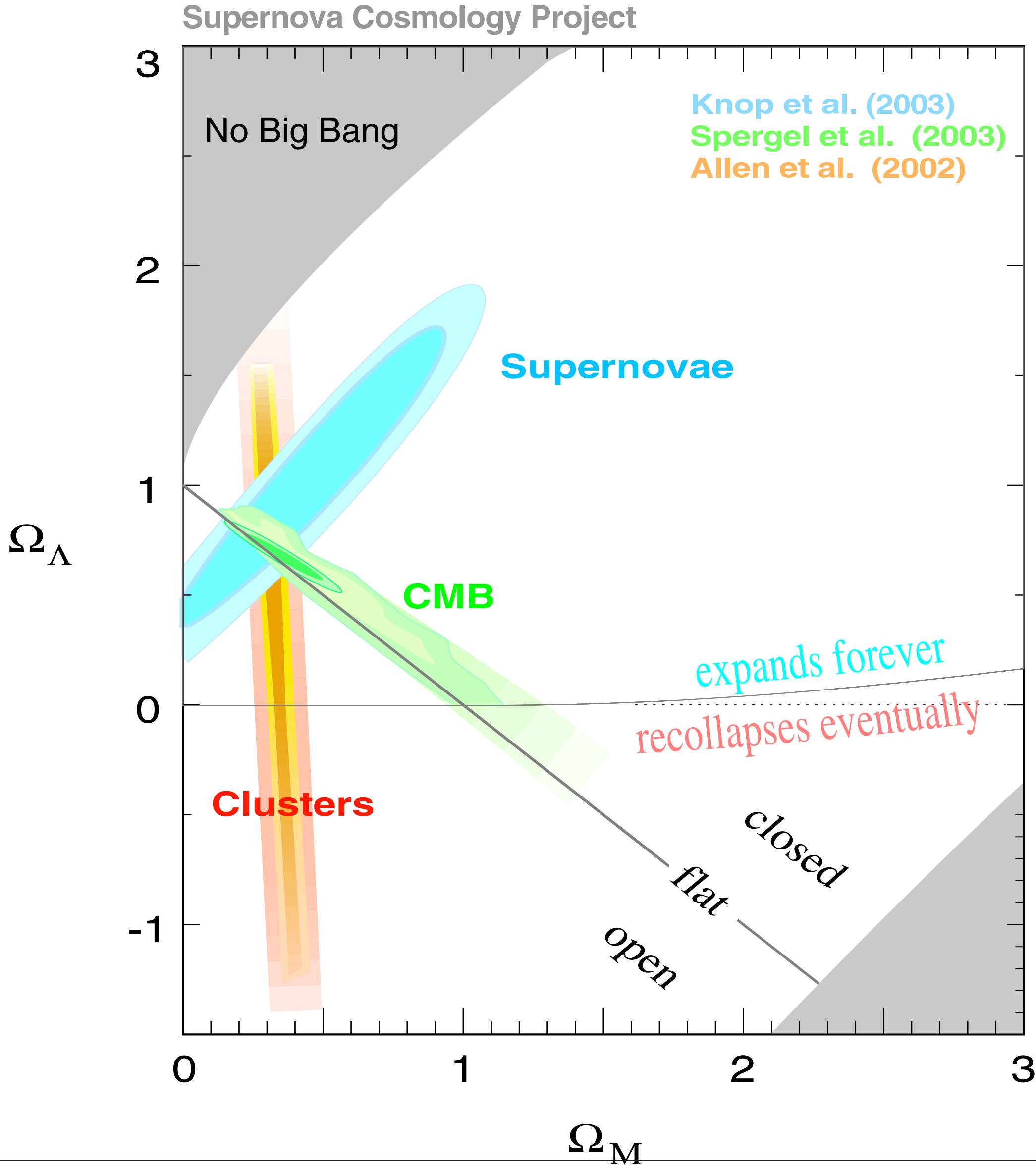
$$\Omega_B h^2 = 0.0224 \pm 0.0009$$

$$\Omega_\chi h^2 = 0.112 \pm 0.018$$

$$\Omega_\Lambda \approx 0.73$$

Ω_χ ⇒ dark matter

Ω_Λ ⇒ dark energy . . .



Experimental result ($2\,\sigma$ range):

[*WMAP et al.*]

$$0.094 \; < \; \Omega_{\text{CDM}} h^2 \; < \; 0.129$$

We apply a more loose bound:

$$\Omega_{\text{CDM}} h^2 \; < \; 0.3$$

→ takes into account (larger) uncertainties in RGE running,
possibly other CDM candidate

Keep in mind:

FUT has a “natural extension” with bi-linear R-parity violation

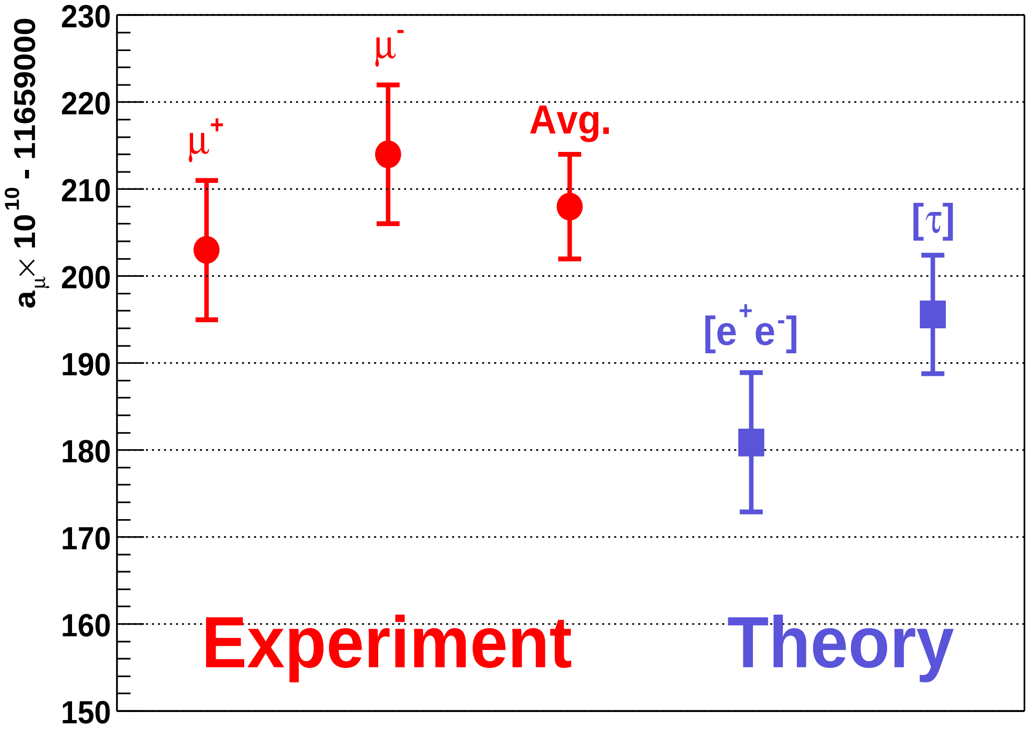
⇒ CDM would be no constraint at all anymore

2F) The anomalous magnetic moment of the muon: $(g-2)_\mu$

Coupling of muon to magnetic field : $\mu - \mu - \gamma$ coupling

$$\bar{u}(p') \left[\gamma^\mu F_1(q^2) + \frac{i}{2m_\mu} \sigma^{\mu\nu} q_\nu F_2(q^2) \right] u(p) A_\mu \qquad F_2(0) = (g-2)_\mu = 2a_\mu$$

Overview about the current experimental and SM (theory) result:
[g-2 Collaboration, hep-ex/0602035]



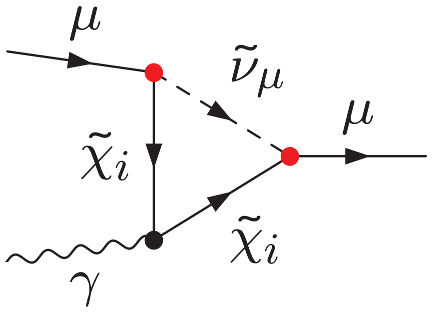
Possible deviation from the SM:

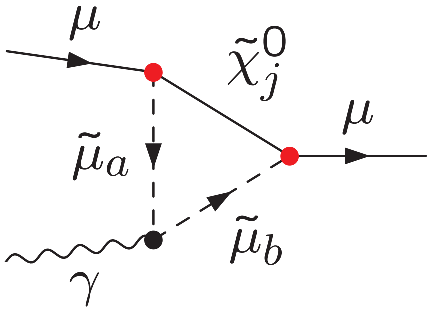
e^+e^- data : $a_\mu^{\text{exp}} - a_\mu^{\text{theo,SM}} \approx (28 \pm 8) \times 10^{-10}$

τ data : agreement with SM

currently: e^+e^- data favored

Feynman diagrams for MSSM 1L corrections:





– Diagrams with chargino/sneutrino exchange

– Diagrams with neutralino/smuon exchange

SUSY corrections at 1L:

$$a_\mu^{\text{SUSY,1L}} \approx 13 \times 10^{-10} \left(\frac{100\ \text{GeV}}{M_{\text{SUSY}}}\right)^2 \tan\beta\, \text{sign}(\mu)$$

$M_{\text{SUSY}}(= m_{\tilde{\mu}} = m_{\tilde{\nu}} = m_{\tilde{\chi}})$: generic SUSY mass scale

$$\begin{aligned} a_\mu^{\text{SUSY,1L}} &= (-100 \ldots +100) \times 10^{-10} \\ e^+e^-\ data : a_\mu^{\text{exp}} - a_\mu^{\text{theo,SM}} &\approx (28 \pm 8) \times 10^{-10} \end{aligned}$$

⇒ SUSY could easily explain the “discrepancy” for $\mu > 0$

⇒ SUSY gives no contribution for heavy sleptons/gauginos
then SUSY is “as good” as the SM
possibly other sources needed to reconcile $(g-2)_\mu$

The plan:

Apply successively the constraints

Step 1:

check which of the four models

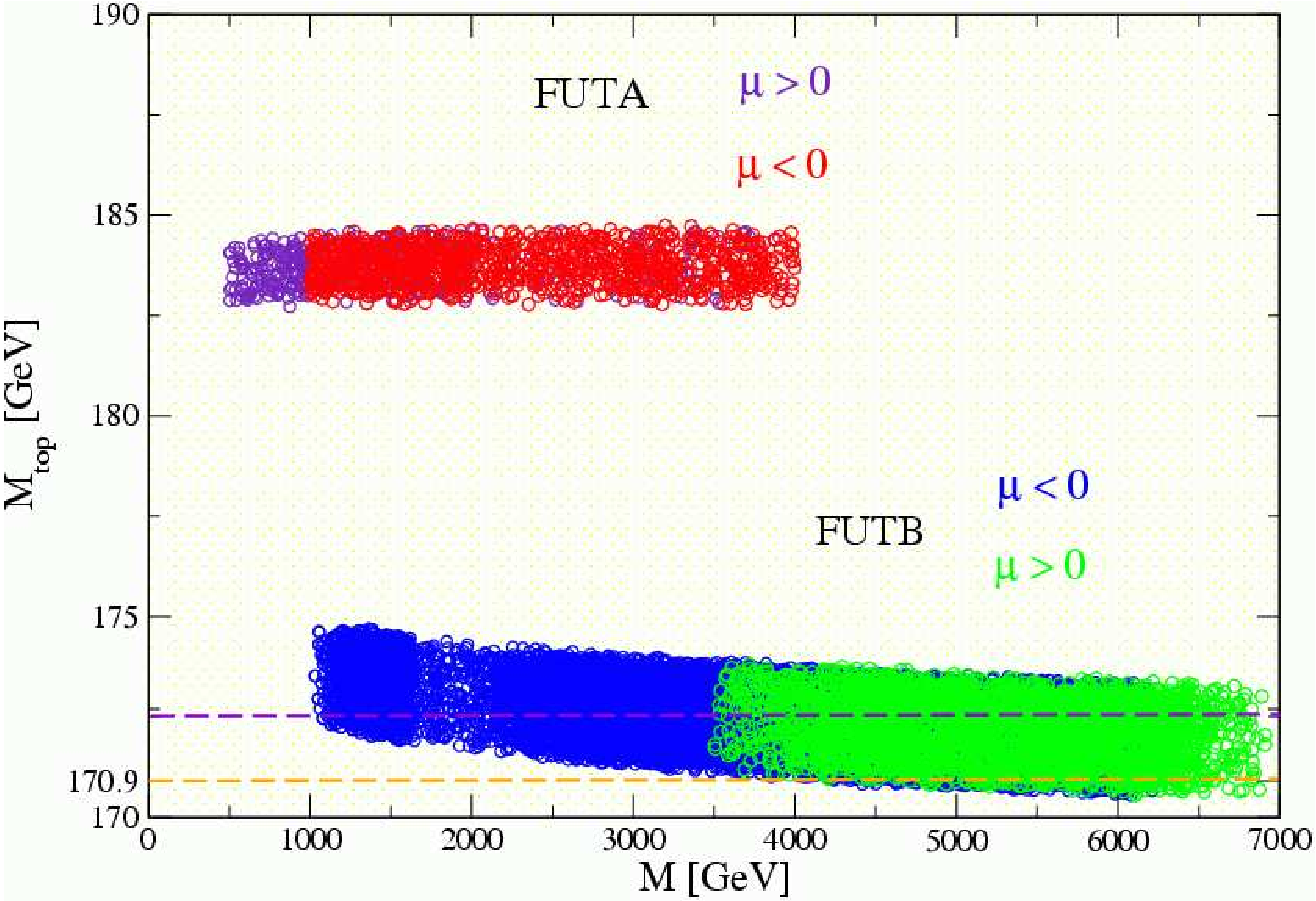
FUTA+, FUTA−, FUTB+, FUTB− ($\pm : \mu \gtrless 0$)

survives (or performs best)

Step 2:

evaluate prediction for the SUSY/FUT parameter space
⇒ collider phenomenology

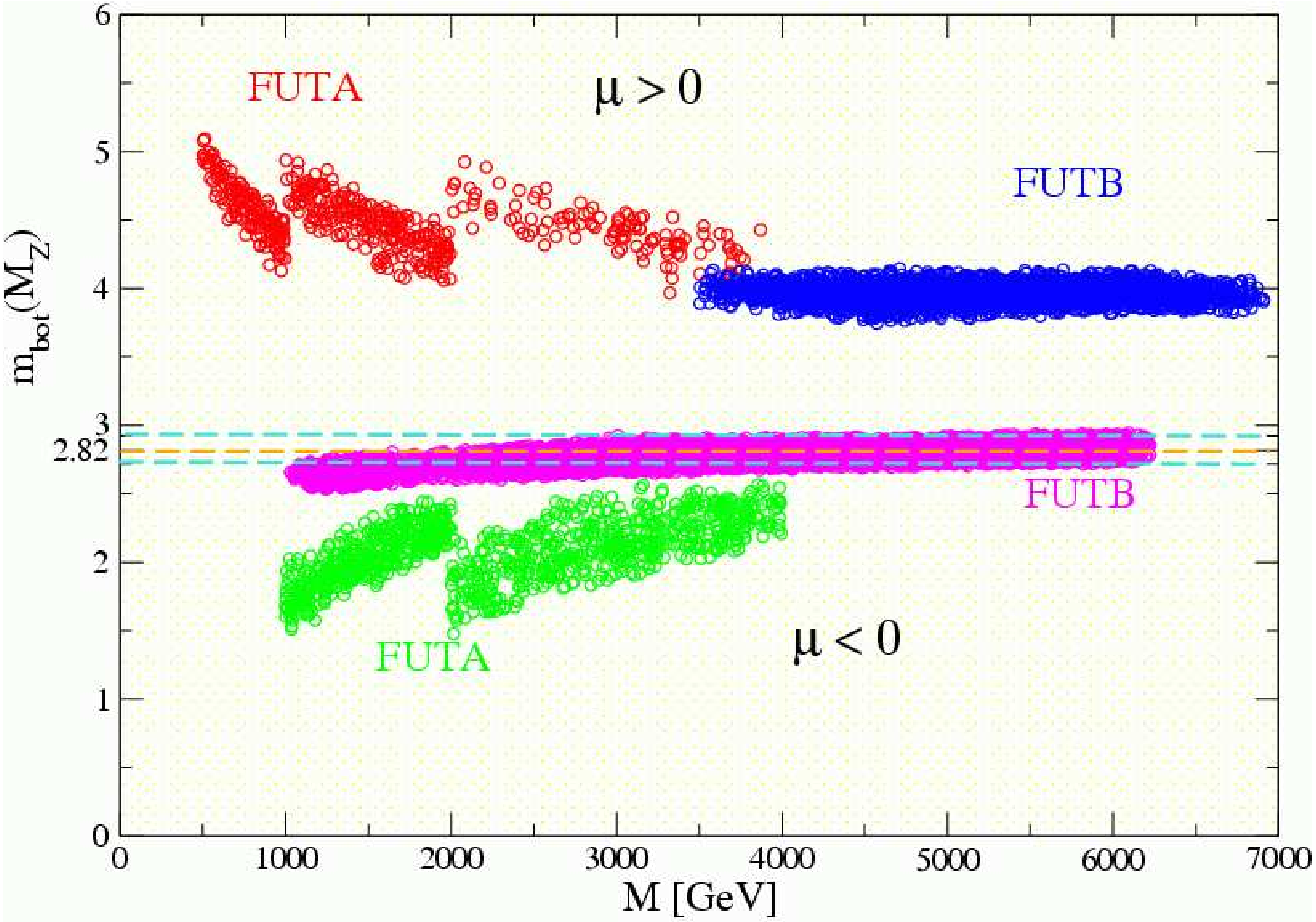
Application of m_t^{pole}:



⇒ FUTB gives the correct prediction for m_t

⇒ FUTA is ruled out experimentally

Application of $m_b(M_Z)$:



$\Rightarrow \mu < 0$ strongly favored

$\Rightarrow \mu > 0$ experimentally excluded

Summary of heavy quark constraints:

$\Rightarrow$ FUTB gives the correct prediction for m_t

$\Rightarrow$ FUTA is ruled out experimentally

$\Rightarrow$ $\mu < 0$ strongly favored

$\Rightarrow$ $\mu > 0$ experimentally excluded

only FUTB– survives

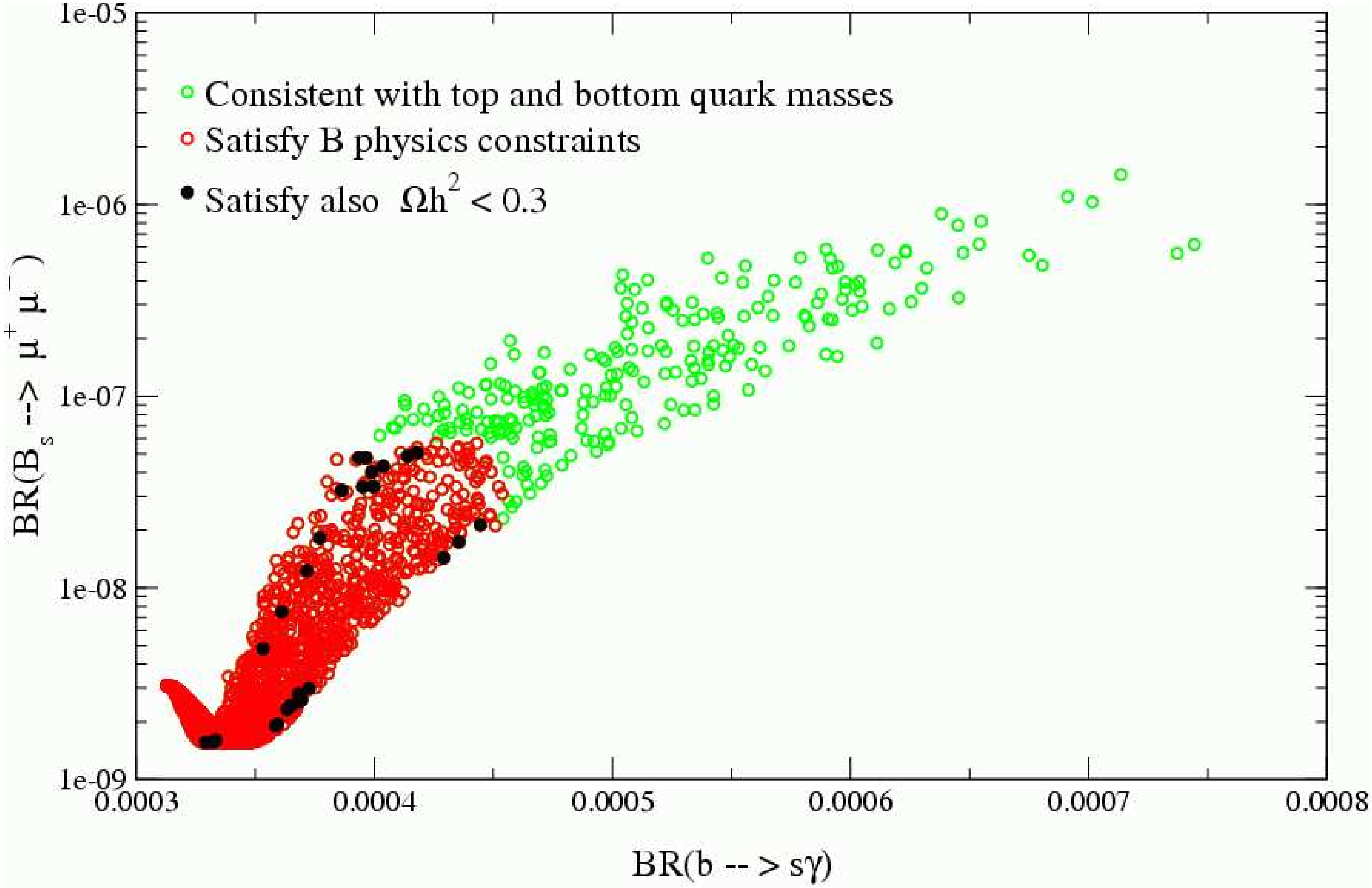
$\Rightarrow$ concentrate on FUTB– from now on

$\Rightarrow$ evaluate prediction for parameter space taking into account the other constraints

Conflict with $(g-2)_\mu$?

We will see later: FUT is as good as SM!

Application of B physics and CDM constraints on FUTB−

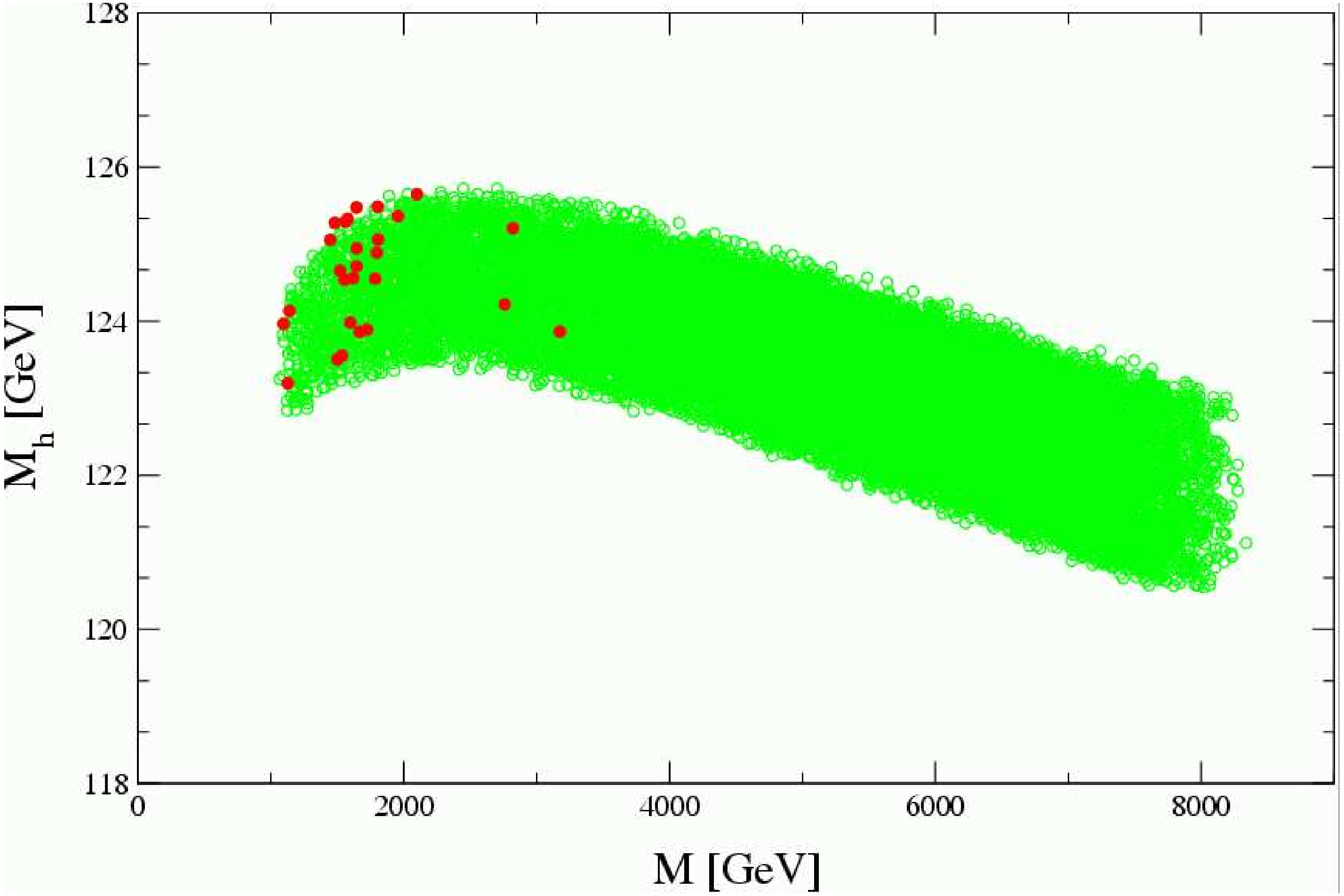


green: inconsistent

red: consistent with B physics constraints

black: agreement with (loose) CDM bound ⇒ FUTB− looks ok

Final check: the lightest Higgs boson mass M_h



green: consistent with B physics constraints

red: agreement with (loose) CDM bound

⇒ 120 GeV $\leq M_h \leq$ 126 GeV (no theory error incl. yet)

⇒ M_h automatically satisfied (contrary to mSUGRA, GMSB, AMSB, . . .)

3. The predictions

Particle content of the MSSM:

Superpartners for Standard Model particles

$$\left[u, d, c, s, t, b\right]_{L,R} \quad \left[e, \mu, \tau\right]_{L,R} \quad \left[\nu_{e,\mu,\tau}\right]_{L} \qquad \text{Spin } \tfrac{1}{2}$$

$$\left[\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}\right]_{L,R} \quad \left[\tilde{e}, \tilde{\mu}, \tilde{\tau}\right]_{L,R} \quad \left[\tilde{\nu}_{e,\mu,\tau}\right]_{L} \qquad \text{Spin } 0$$

$$g \quad \underbrace{W^{\pm}, H^{\pm}} \quad \underbrace{\gamma, Z, H_1^0, H_2^0} \qquad \text{Spin 1 / Spin 0}$$

$$\tilde{g} \quad \tilde{\chi}_{1,2}^{\pm} \quad \tilde{\chi}_{1,2,3,4}^{0} \qquad \text{Spin } \frac{1}{2}$$

Enlarged Higgs sector: Two Higgs doublets

Problem in the MSSM: many scales

Enlarged Higgs sector: Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2(\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$

$$+ \underbrace{\frac{g'^2 + g^2}{8}}(H_1\bar{H}_1 - H_2\bar{H}_2)^2 + \underbrace{\frac{g^2}{2}}\,|H_1\bar{H}_2|^2$$

gauge couplings, in contrast to SM

physical states: $h^0, H^0, A^0, H^\pm$

Goldstone bosons: $G^0, G^\pm$

Input parameters: (to be determined experimentally)

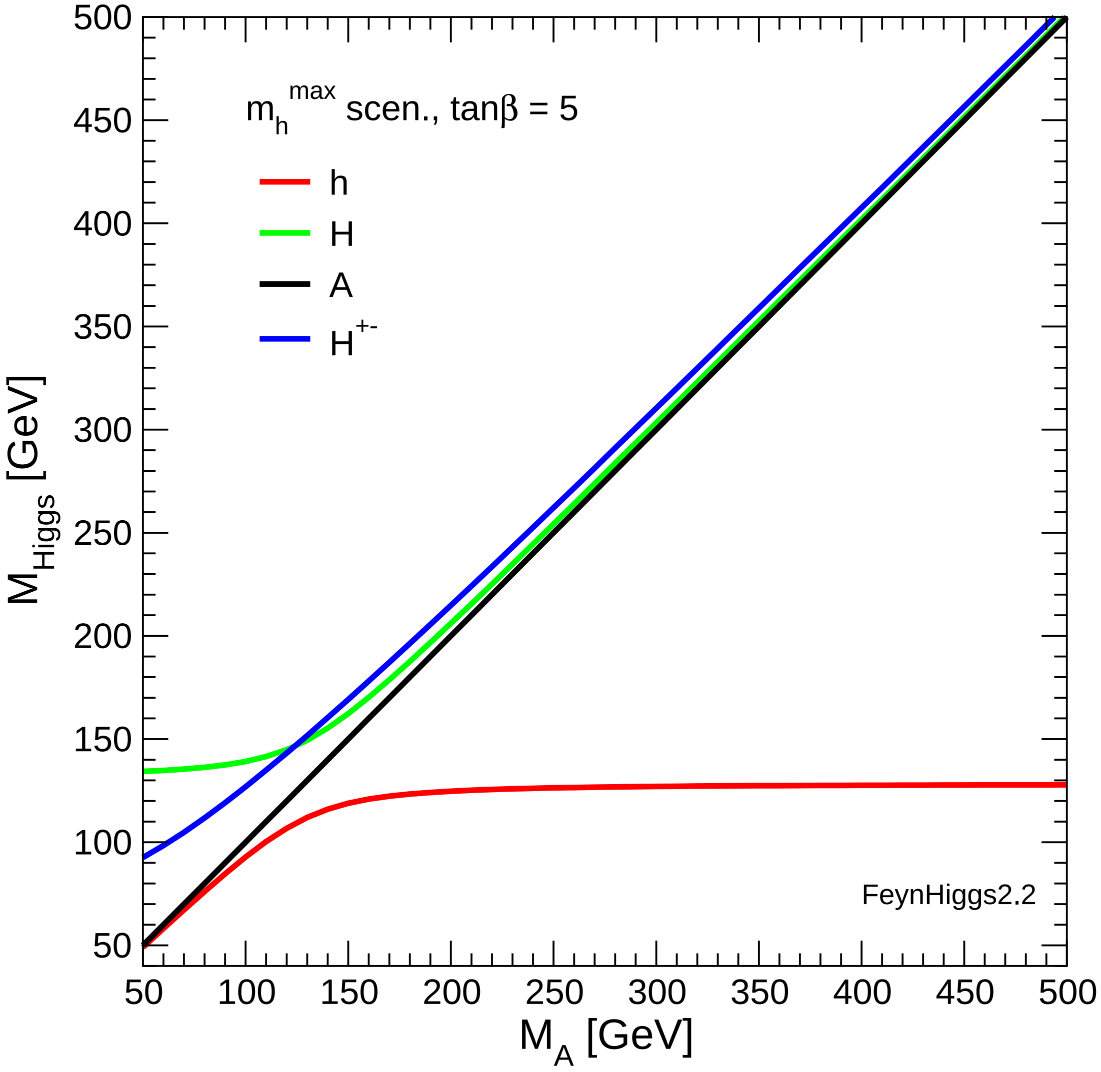
$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2(\tan\beta + \cot\beta)$$

The decoupling limit:

For $M_A \gtrsim 150$ GeV:

The lightest MSSM Higgs is SM-like

The heavy MSSM Higgses:
$M_A \approx M_H \approx M_{H^\pm}$



of course there are exceptions . . .

The predictions

A) Predictions for the lightest observables SUSY particle (LOSP)

B) Predictions for squarks and gluinos

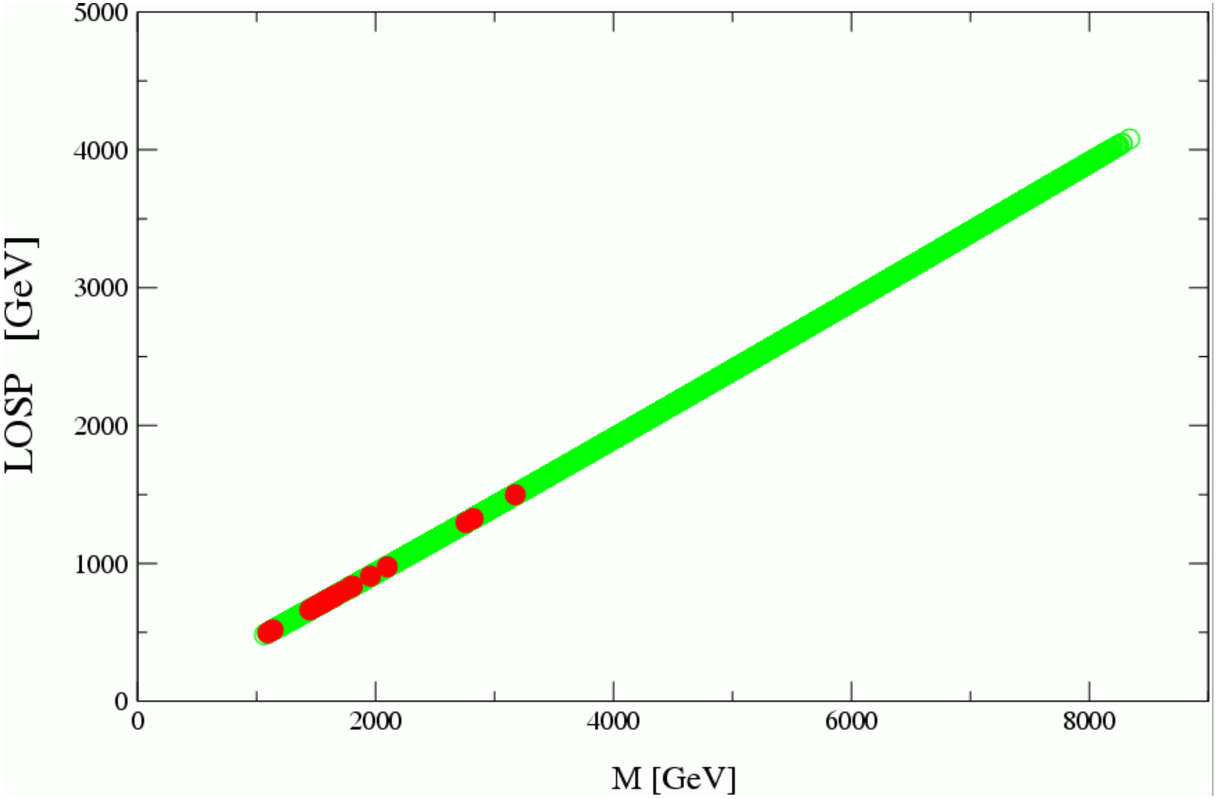
C) Predictions for heavy Higgs bosons

D) Predictions for the light Higgs boson

E) . . .

3A) Predictions for the LOSP

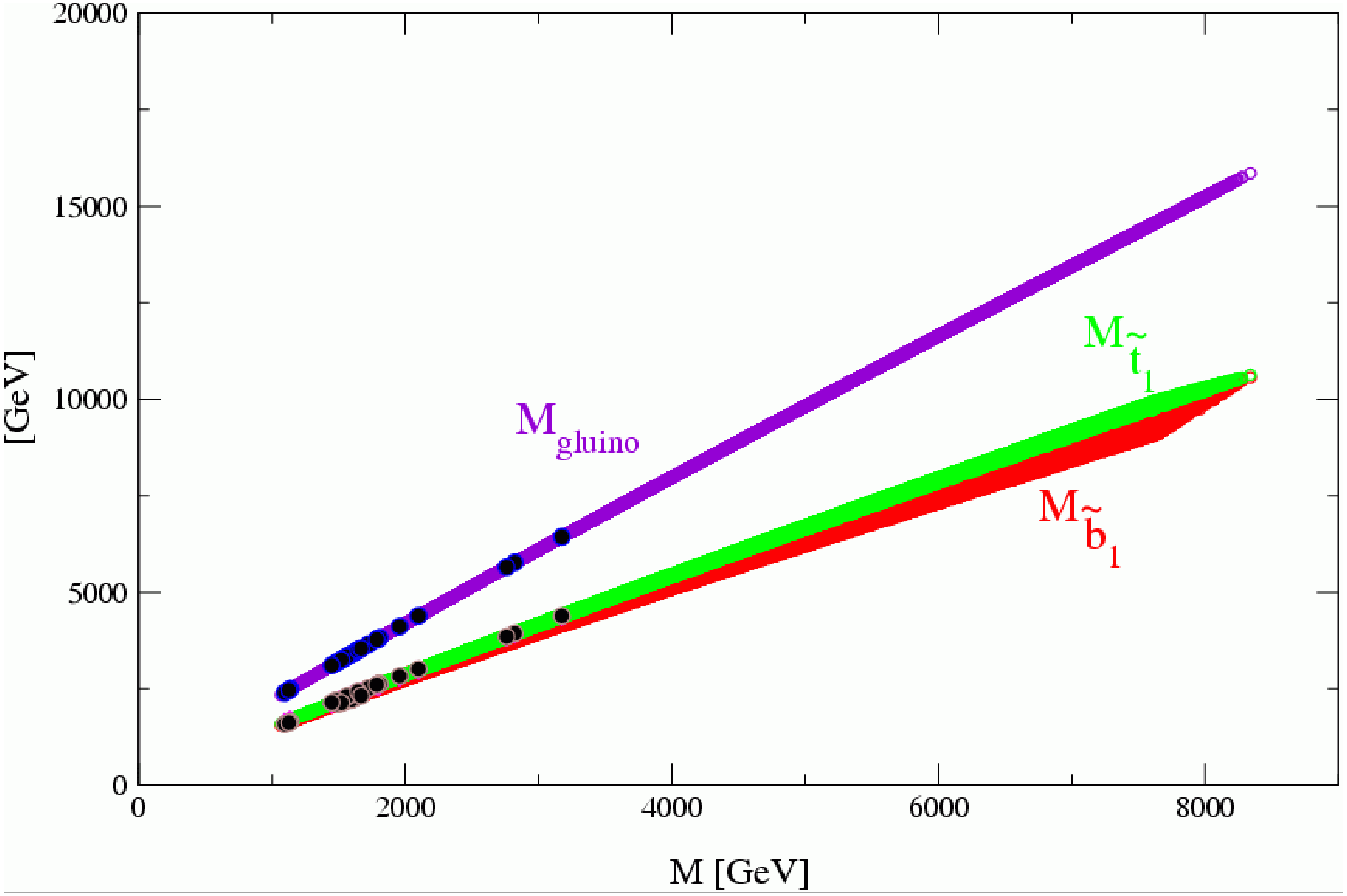
LOSP: lighter scalar tau or second lightest neutralino
(nearly mass degenerate with lightest chargino)



green: consistent with B physics constraints
red: agreement with (loose) CDM bound

$\Rightarrow$ very challenging for LHC (possibly in cascades?)

3B) Predictions for squarks and gluinos



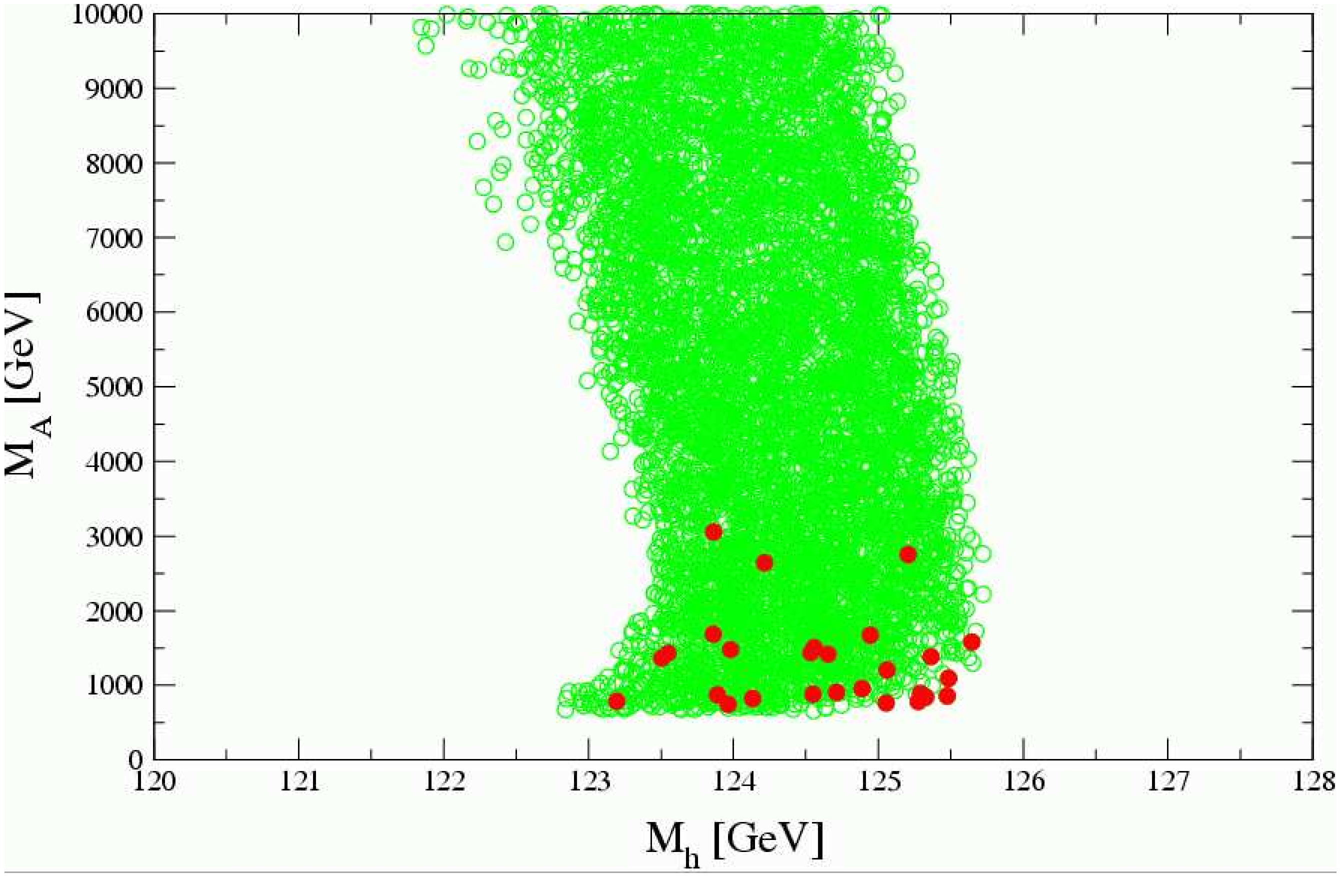
colored: consistent with B physics constraints

black: agreement with (loose) CDM bound

⇒ good chances if CDM is fulfilled

3C) Predictions for heavy Higgs bosons

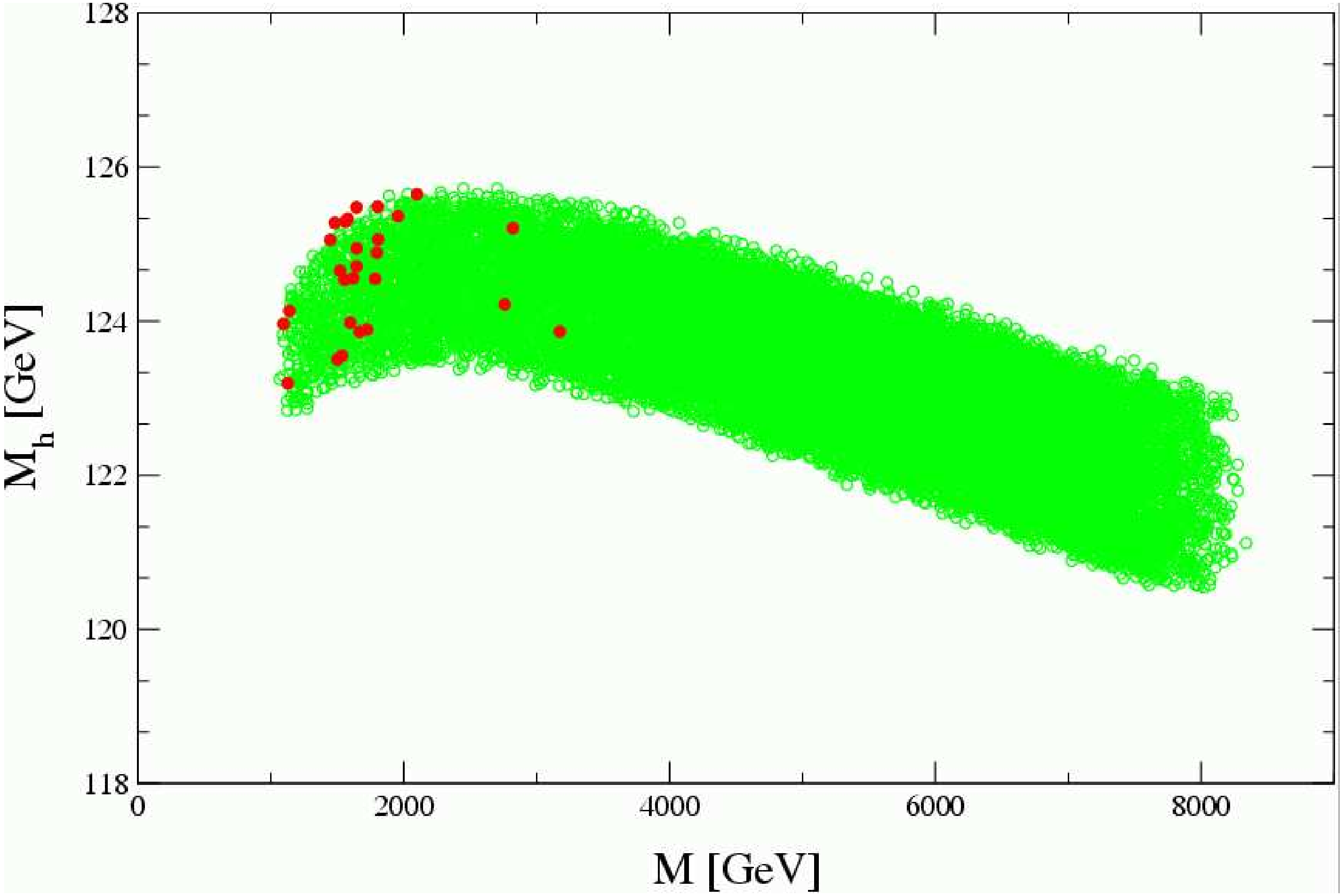
heavy Higgs bosons: mass degenerate



green: consistent with B physics constraints
red: agreement with (loose) CDM bound

⇒ very challenging for LHC
note: $\tan\beta$ large, $\Delta_b < 0$ ⇒ enhanced $Ab\bar{b}$ coupling ⇒ should be checked

3D) Predictions for the light Higgs boson

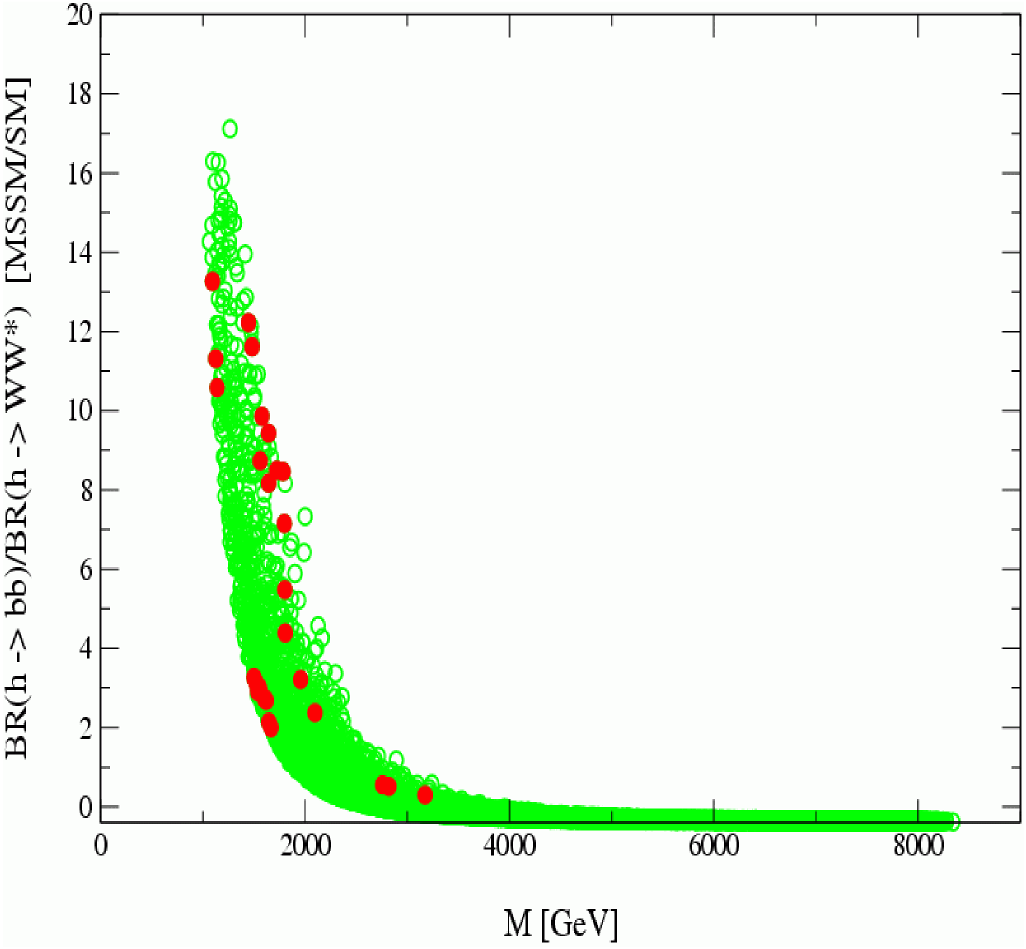


green: consistent with B physics constraints
red: agreement with (loose) CDM bound

$$118 \text{ GeV} \le M_h \le 129 \text{ GeV} \quad \text{(incl. theor. unc.)}$$

⇒ “easy” to find for LHC (but “only” SM-like . . .)

Difference to SM via decay modes?



green: consistent with B physics constraints

red: agreement with (loose) CDM bound

Best discriminator:

$$\frac{\mathrm{BR}(h \to b\bar{b})}{\mathrm{BR}(h \to WW^*)}$$

$1\,\sigma$ = 1.5% resolution (ILC)

LHC resolution?

If CDM is fulfilled:

good prospects for ILC

prospects for LHC?

Typical mass spectrum for FUTB- :

m_t	172	$\overline{m_b}(M_Z)$	2.7
$\tan\beta =$	46	α_s	0.116
$m_{\tilde{\chi}_1^0}$	796	$m_{\tilde{\tau}_2}$	1268
$m_{\tilde{\chi}_2^0}$	1462	$m_{\tilde{\nu}_3}$	1575
$m_{\tilde{\chi}_3^0}$	2048	μ	-2046
$m_{\tilde{\chi}_4^0}$	2052	B	4722
$m_{\tilde{\chi}_1^\pm}$	1462	M_A	870
$m_{\tilde{\chi}_2^\pm}$	2052	$M_{H^\pm}$	875
$m_{\tilde{t}_1}$	2478	M_H	869
$m_{\tilde{t}_2}$	2804	M_h	124
$m_{\tilde{b}_1}$	2513	M_1	796
$m_{\tilde{b}_2}$	2783	M_2	1467
$m_{\tilde{\tau}_1}$	798	M_3	3655

4. Conclusinos

- Finite Unified Theories (FUT) are an attractive realization of SUSY

  Four models analyzed: FUTA±, FUTB±

- Constraints:
  heavy quark masses, BR($b \to s\gamma$), BR($B_s \to \mu^+\mu^-$), M_h, CDM

- Heavy quark masses ⇒ only FUTB- is a viable model

- B physics observables and CDM "easily" fulfilled
  ⇒ M_h automatically fulfilled

- Predictions:
  LOSP (stau or 2nd neutralino/1st chargino): very challenging
  Stops, sbottoms, gluinos: good chances if CDM is fulfilled
  Heavy Higgs bosons: very challenging (tbc!)
  Light Higgs: 118 GeV $\leq M_h \leq$ 129 GeV (incl. theor. unc.)
  "easy" for the LHC, but "only" SM-like