# Modifying General Relativity

**Plan**

I General Relativity - its basic principles

II Why modify general relativity?

Quantum Gravity — UV $\longleftrightarrow$ IR — Accelerating Universe

The cosmological constant problem

Dark energy?

III Ways to modify gravity

Consistency checks

IV Xtra Dimensions

f(R) theories

Brans-Dicke Gravity

V) Solar constraints on BD.

VI) Conclusions.

Christos Charmousis
Laboratoire de Physique Théorique.

# General Relativity

## Basic Principles:

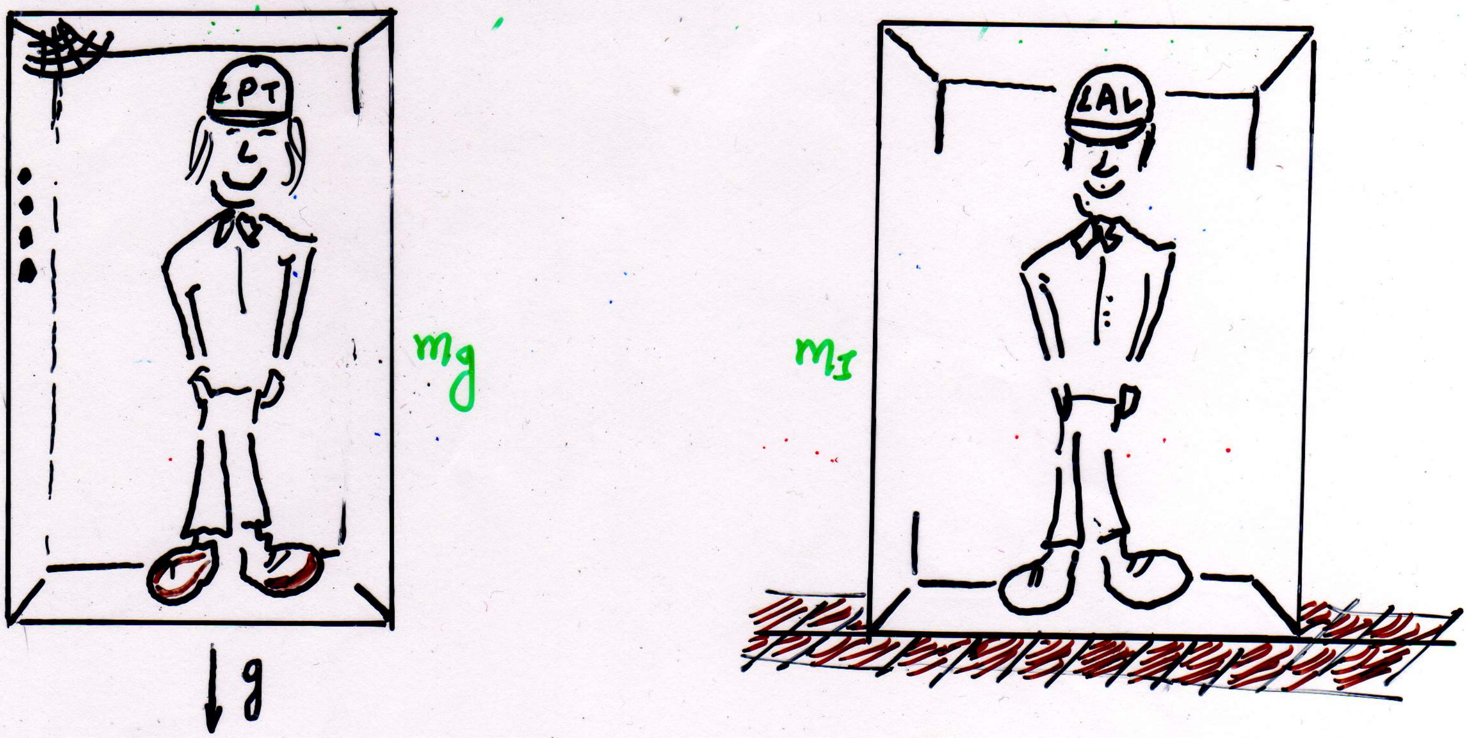
### Mach's Principle:

The presence of matter curves the geometry of spacetime.

Matter $\cong$ Geometry.

### Equivalence Principle:

- Locally a free-falling observer and an inertial observer are indistinguishable.




- $m_I \equiv m_G$ !

Gravitational and inertial mass are one and the same.

- Gravity is not a force it is a local condition of spacetime.
- Gravity sees all.
- In Newtonian gravity $m_I$ & $m_g$ "happen" to be the same

In GR it is a founding principle of the theory.

$$
\begin{array}{lll}
\text{Field variable} & g_{\mu\nu} & \text{metric tensor} \\
\text{Derivative.} & \Gamma^{\sigma}_{\mu\nu} & \text{Christophel symbols} \\
 & R_{\mu\nu\rho}{}^{\sigma} & \text{Curvature tensor}
\end{array}
$$

Einstein's Equations

$$R_{\mu\nu} = 8\pi G \; T_{\mu\nu}$$

Geometry. $\longleftrightarrow$ Matter

$$\nabla^{\mu} T_{\mu\nu} = 0.$$

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In GR it is a founding principle of the theory.

Field variable $g_{\mu\nu}$ metric tensor

Derivative.

$\Gamma^{\sigma}_{\mu\nu}$ Christophel symbols — If $\Gamma^{\sigma}_{\mu\nu} = 0 \leftarrow$ inertial frame and geodesics are lines.

$R_{\mu\nu\rho}{}^{\sigma}$ Curvature tensor — If $R_{\mu\nu\rho}{}^{\sigma} = 0 \leftarrow$ flat spacetime

Einstein-Hilbert action $S = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\, R + S_{matter}$

where $G = \frac{1}{m_{Pl}^2}$ and $\frac{1}{\sqrt{8\pi G}} \sim 2.4\ 10^{18}$ GeV.

Einstein's Equations

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\ T_{\mu\nu}$$

Geometry. $\longleftrightarrow$ Matter

$\nabla^{\mu} G_{\mu\nu} = 0$ !    $\nabla^{\mu} T_{\mu\nu} = 0$.

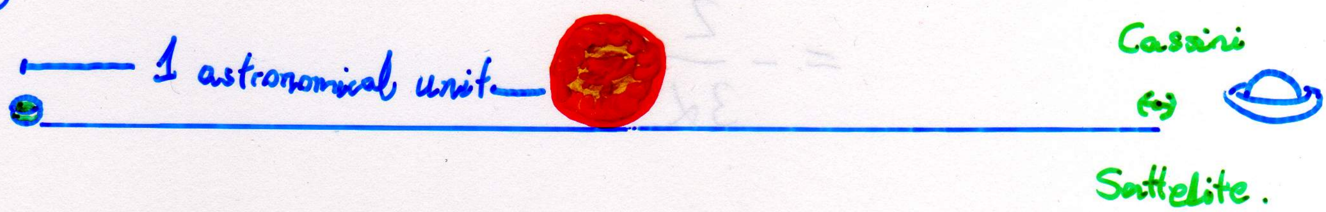
Q/ What is the experimental status of GR?

Solar system:

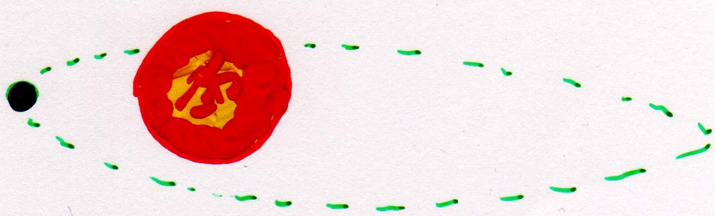
Bending of light

Time delay of light.

signal emitted from earth.




Advance in Perihelion of Mercury.

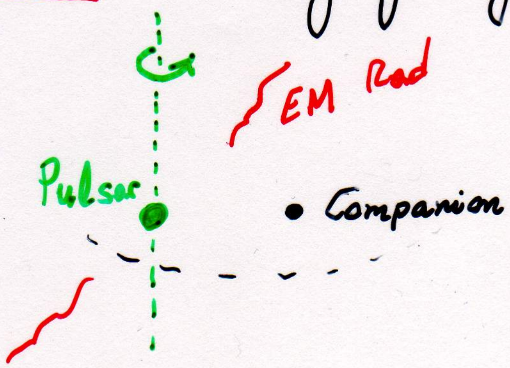


Planet trajectories.

Best Known Mars and Earth, Mercury Venus.

Laboratory tests of Newtonian gravity

Binary Pulsars : Strong gravity experiment




Taylor & Hulse PSR 1913+16

Binary Pulsar discovery

→ Indirect evidence for gravitational waves.

Excellent agreement with GR

Theoretical prediction: $\gamma = 1$, $\beta = 1$ for GR.

We get $\gamma = 1 \pm 10^{-5}$ $\beta = 1 \pm 10^{-5}$ from exp.

$\gamma$ & $\beta$ are Parametrised Post Newtonnian parameters PPN

For Newtonnian potential $\Phi = \frac{G M}{c^2 r}$ ← sun

← distance to the sun

Metric solution for a PPN expansion

$$ds^2 = -(1 - 2\underset{\substack{\uparrow \\ \text{Newtonnian Potential}}}{\Phi} + 2\beta \Phi^2) dt^2 + (1 + 2\gamma \Phi) dx^i dx^j \delta_{ij}$$

BUT typically distance scales are at a few astronomical units (A.U)

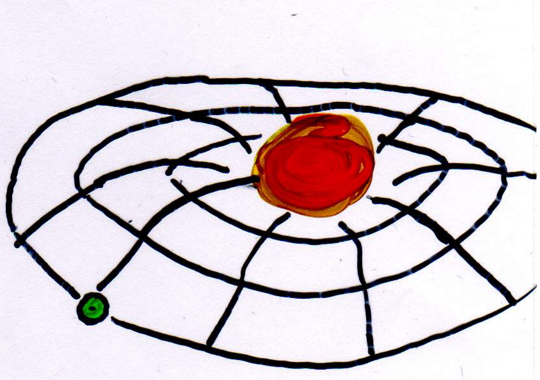
PPN is a good approximation as long as $|v_{source}| \ll c$

and $\frac{G m}{c^2 r} = \varepsilon$ is small.

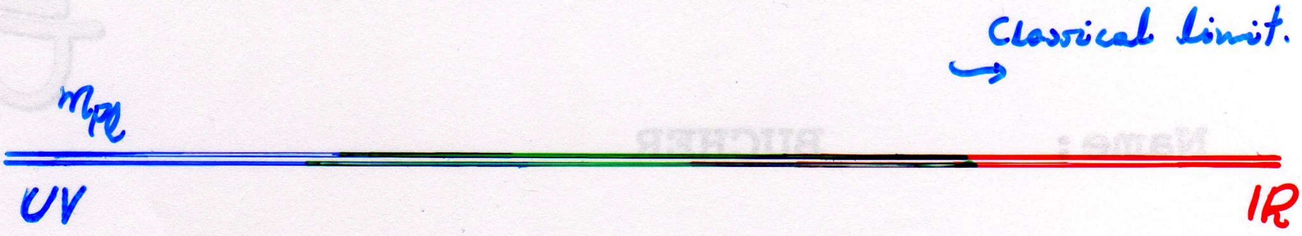
Typically $\varepsilon \sim 0.5$ B.H. or cosmology.

$\varepsilon \sim 0.2$ pulsars

$\varepsilon \lesssim 10^{-5}$ solar system



# II Why modify GR?




large Curvature

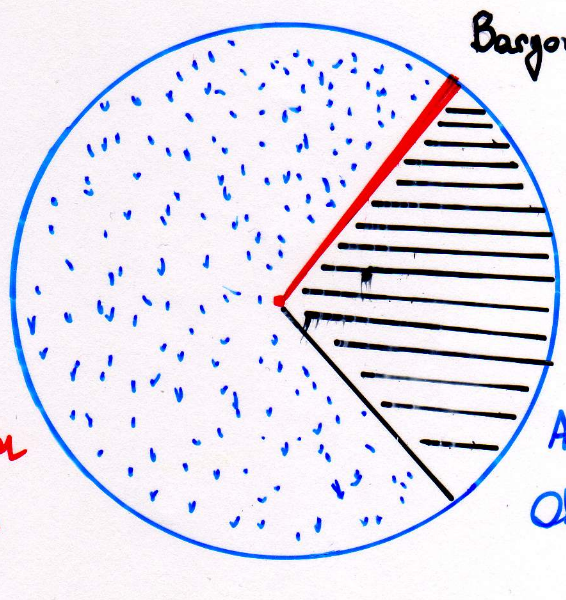
Big Bang Singularity
Black hole singularity } breakdown of GR which is a classical theory of gravity.

Quantum gravity
String theory
Extra dimensions etc.

Q • What is the matter content of the Universe today?

Assume $$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}$$

\+ Homogeneity and Isotropy




A/ Only a mere 4% has been discovered in the laboratory.

**Fact**: If we do not allow for exotic matter then cosmological and local experimental data do not agree.

Observational evidence from explosions of SN type 1a (complimented by CMB, large scale structure)

Universe is accelerating

Supernovae are very bright and standard candles.

therefore we can observe them way back in time and we can estimate their absolute luminosity magnitude.

Fact: This is most "easient" fitted with data if we assume the presence of a tiny cosmological constant $\Lambda$.

Q/ How tiny is that?

A/ As big as the Universe is today!

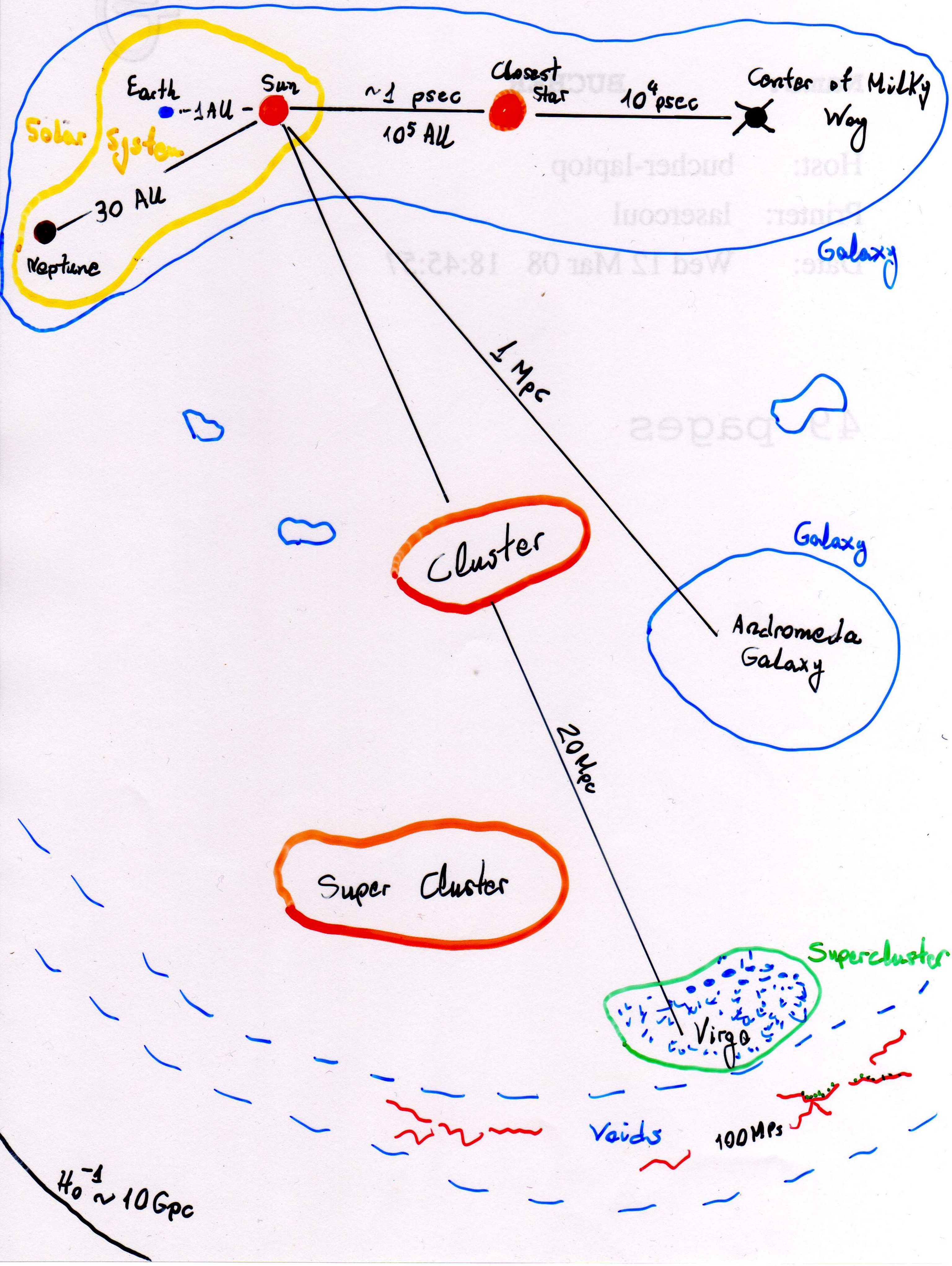
$$|\Lambda|_{obs} \lesssim (10^{-3} eV)^4$$

Cosmological scale $r_0 = H_0^{-1}$

Hence $\dfrac{\text{Solar System Scale} \sim 1 \text{ A.U.}}{\text{Cosmological Scale} \sim H_0^{-1}} = (1 \text{ A.U. } H_0) \sim 10^{-15}$

- $\Lambda_{obs}$ is so small that planet trajectories can allow a $10^{11}$ bigger value!

# From Astronomical to cosmological scales.



# III Cosmological Constant Problem

Weinberg

- From QFT we have contributions from vacuum energy at the ultraviolet cutoff of the theory

$$\Lambda_{vac} \sim m_{pl}^4 \quad \text{or} \quad \Lambda_{vac} \sim M_{SUSY}^4$$

$\Lambda^{vac}$

+ • Symmetry breaking contributes potential energy to cosmological constant

for example at E-W phase transition $\Lambda^{SM}$

$$\Lambda^{EW} \sim (200\,\text{GeV})^4 \quad \text{etc...}$$

+ • Bare gravitational cosmological constant $\Lambda^{bare}$

$$S = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\,[R - 2\Lambda^{bare}]$$

---

$$\Sigma \Lambda^{bare} + \Lambda^{vac} + \Lambda^{EW} \simeq 0 \quad \text{CC Problem}$$

1) Why such a discrepancy between theoretical and observed value?

# III Cosmological Constant Problem. Weinberg

$\Lambda_{obs} \lesssim (10^{-3} eV)^4$

Typical mass scale of neutrinos

- From QFT we have contributions from vacuum energy at the ultraviolet cutoff of the theory.

$\Lambda_{vac} \sim m_{Pl}^4$ or $\Lambda_{vac} \sim M_{SUSY}^4$ $\Lambda^{vac}$

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---

$\sum \Lambda^{bare} + \Lambda^{vac} + \Lambda^{EW} \simeq 0$ CC Problem

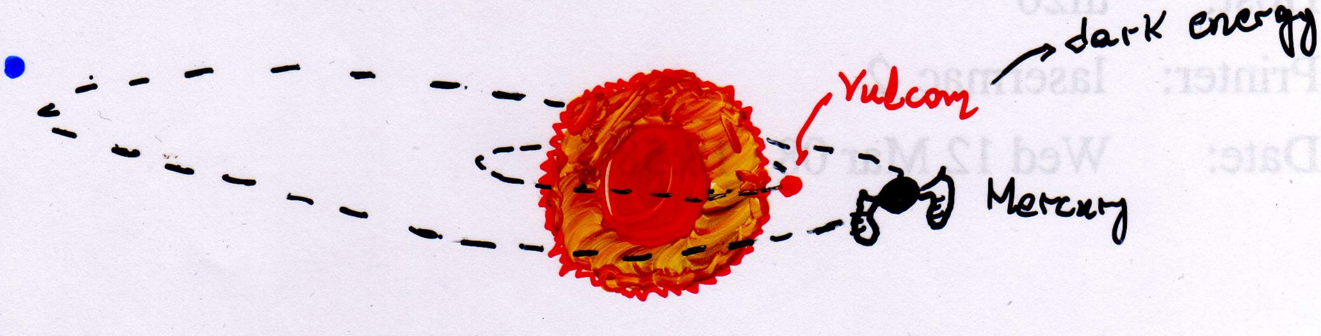
1) Why such a discrepancy between theoretical and observed value?

2) Why is $\Lambda_{obs}$ so small and not zero?

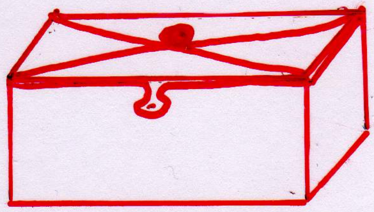
3) Why do we observe it now with $\Omega_m \sim \Omega_\Lambda$?

read R. Bousso, S. Carroll, Polchinski.

What if the need for exotic matter or for a cosmological constant is the sign for new gravitational physics at classical scales of energy.




- Remember $\Lambda_{bare}$ is free in GR. This is similar to the non-explained but observed equality $m_I = m_g$ in Newton's theory that becomes a principle in GR.



Modification of GR is hard because GR is very firmly established theoretically and well constrained experimentally. Moreover it is an extremely elegant theory and modifying it even slightly complicates things enormously.
Remember this is a classical modification, hence no hiding behind UV sector yet undiscovered.

# III Ways to modify gravity.

Many alternatives studied

- Modify Einstein-Hilbert action

$$\int d^4x \sqrt{-g}\, R \longrightarrow f(R),\ \mathrm{Riem}^2,\ f(GB)$$

with higher order tensors depending on the metric

- Add new degrees of freedom : scalar(s), vector, metric

  Brans-Dicke theory, 50's (Damour, Esposito-Farese)

  Nordvedt, more recently Jacobson

  bigravity theories Kogan et al. Damour et. al.

- Enlarge parameter space ie allow for Torsion, a non-trivial connexion or more commonly extra dimensions.

Extra dimensions GRS model, DGP model, Brane-Bulk energy xchange, stealth acceleration etc.

3 main consistency tests for a modified gravity theory

questions

Which principles should we throw out the window?

ghosts, tachyons

Lorentz invariance

**Theoretical consistency**

strong coupling

lab tests

PPN parameters

**Experiment**

binary pulsars

SNIa

matter perturbations and large scale structure

**Naturalness**

# of parameters and additional fields.

How do we compare to a cosmological constant?

What do we learn if anything?

Counterexamples: Quintessence, Bekenstein theory

Dark energy

modified gravity theory.

# Examples of gravity modification

i) Brans-Dicke theory 1961 (BD)

Q Which other gravity theories verify the equivalence principle?

Basic idea: Consider single extra degree of freedom

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \quad R \qquad\qquad - \mathcal{L}_{matter}.$$

# Examples of gravity modification

i) Brans-Dicke theory 1961 (BD)

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Basic idea: Consider single extra degree of freedom

$$S = \frac{1}{16\pi G}\int d^4x\, \sqrt{-g}\left[\Phi R - \frac{\omega_{BD}}{\Phi}(\nabla\Phi)^2\right] - \mathcal{L}_{matter}.$$

It verifies the weak but not the strong gravitational principle

{For multiple scalar fields and potential read Damour, Esposito-Farese 1992.}

- Extra dimensional theories predict scalar-tensor gravity
  String theory, braneworld, Kaluza-Klein compactification.

- Does not satisfy Birkhoff's theorem.
  A black hole metric does not describe a star metric!

Problem: Solar system tests:

We get $\gamma = \frac{1+\omega_{BD}}{2+\omega_{BD}}$ but remember $|\gamma - 1| \lesssim 10^{-5}$

therefore $\omega_{BD} > 40000$

ii) $f(R)$ theories

- Stick to 4 dimensions and add only metric scalars

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[ R + c_1 R_{ab} R^{ab} + c_2 R_{abcd} R^{abcd} + c_3 R^2 + \dots \right]$$

We get generically 4th order field equations

The flat vacuum is unstable to small perturbations

the theory is sick and therefore useless for physical experiment

Only two ways out: $f(R)$ theory, $f$ is a free function

and a particular combination $\underbrace{R^2 - 4 R_{ab} R^{ab} + R_{abcd} R^{abcd}}$

This is the Gauss-Bonnet invariant $\hat{G}$

Its variation with respect to the metric is trivial

ie. it gives no extra term in the field equations.

$$\int_M d^4x \sqrt{-g}\, \hat{G} = \chi(M) \quad \text{is a topological invariant.}$$

$$\int_M d^2x \sqrt{-g}\, R = \chi(M)$$

ii) $f(R)$ theories (Carroll et.al, Capozziello et.al. 2003)
(see also Starobinski 1980)

- Stick to 4 dimensions and add only metric scalars

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\, f(R) + \mathcal{L}_{matter}.$$

e.g. $f(R) = R - \dfrac{\mu^{2(1+n)}}{R^n}$, $n > 0$ and take $\mu^2 \sim 10^{-33}$ eV.
i.e. $\mu \sim H_0$

- We get late-time accelleration (even 2, 3, 4 is enough for $n$)
- Field equations are 4th order

But problems:

i) Actually $f(R)$ theory is a scalar-tensor theory in disguise (Starobinski, Whitt 1984)

$\hookrightarrow$ we get $\Phi(x) = f'(x)$ $\quad V(x) = \frac{1}{2}(x f'(x) - f(x))$ potential

and $\omega = 0$!

ii) no flat vacuum for $n > 0$.

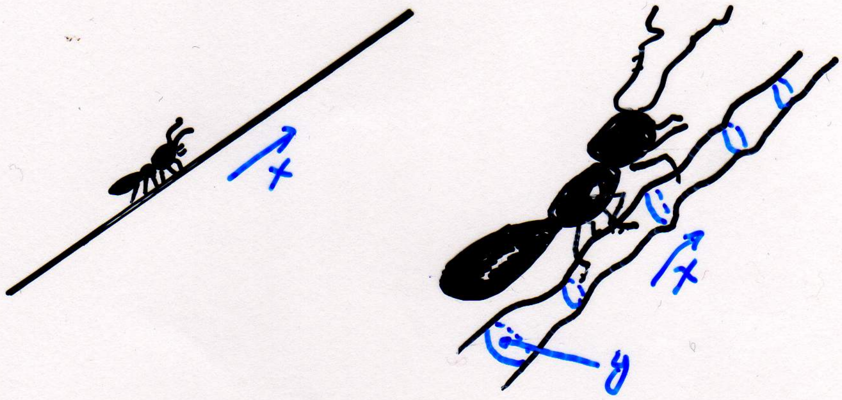
iii) $f(R)$ theory is strongly coupled and "influences" matter domination era. Matter does not cluster sufficiently
(Amendola, Polarski, Tsujikawa)
ruled out from CMB and large scale structure formation.

$$\int_M d^2x \sqrt{-g}\, R = \chi(M)$$

iii) Extra dimensions

Kaluza & Klein $\longrightarrow$ string theory $\rightarrow$ braneworld.

E-M is part of the gravitational sector in a 5 dimesional spacetime $\rightarrow$ Presence of dilaton spoils this




Q/ What if we live in a higher dimensional spacetime

$\hookrightarrow$ Gravity sees all

$$S = \frac{1}{16\pi G}\int d^4x \quad R \qquad + \int d^4x \sqrt{-g}\, \mathcal{L}_{matter}$$

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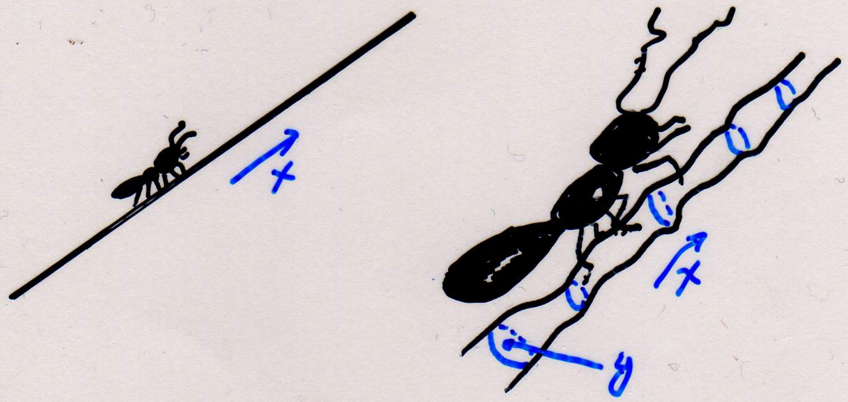
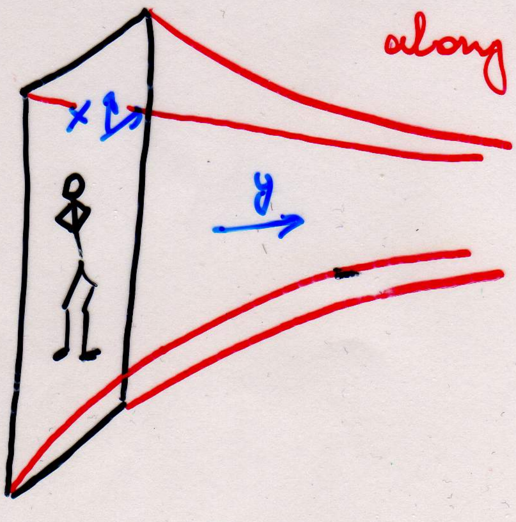



Q/ What if we live in a higher dimensional spacetime

↳ Gravity sees all

$$S = \frac{1}{16\pi G_5}\int d^4x\,dy\,R^{(5)} + \int d^4x\,\sqrt{-g}\,\mathcal{L}_{matter}$$

Braneworld : Gravity percieves all dimensions but not matter which is localised on a brane. Unlike KK gravity can be warped along "y"



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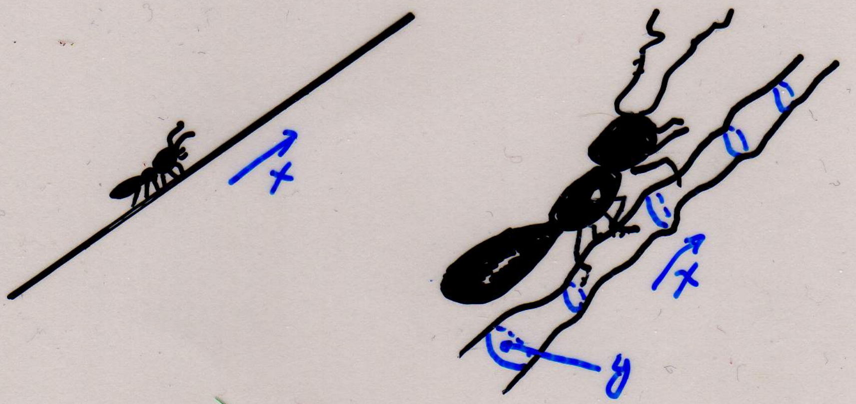


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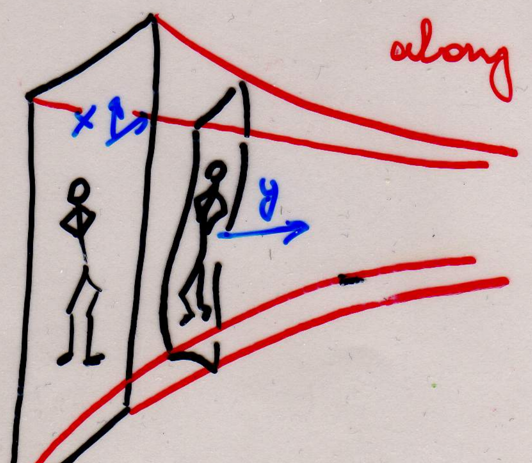
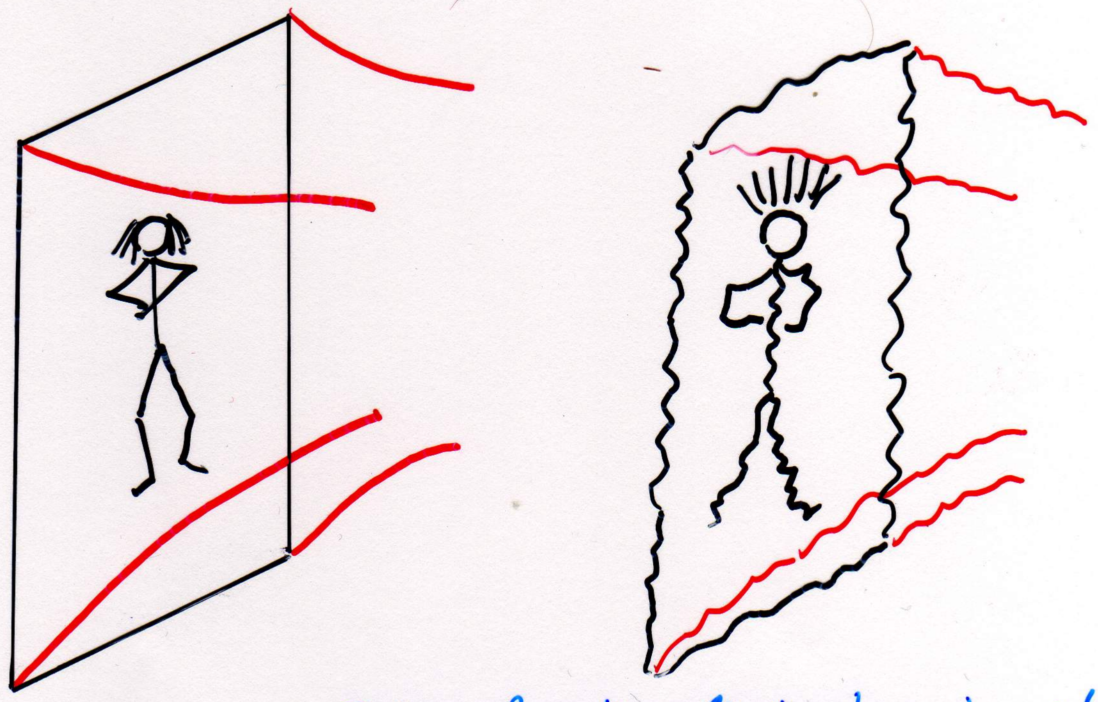


Two cases:

Either we have a zero mass 4 dimensional graviton mode which dominates at large distances. Gravity picks up modifications at the ultraviolet. Example Randall Sundrum model.

Take a solution ⟶ Fluctuate it in all directions



Solve for the fluctuations to pick up the effective 4 dimensional gravitational spectrum

Or we have an effective 4 dim graviton 0-mode that dissappears at a certain large scale. Beyond that gravity becomes 5 dimensional Example: DGP Dvali, Gabadadze Porratti model.

Basic idea of DGP

$$S = M_5^3 \int \sqrt{-g}\, d^4x\, dy\, R^{(5)} + m_{Pl}^2 \int \sqrt{-\gamma}\, d^4x\, R^{(4)} + \text{matter}$$

Define $r_c = \frac{m_{Pl}^2}{2M_5^3}$ and set $r_c \sim H_0^{-1} \sim 10^{33}\,eV$.

Hence we want $m_{Pl} >> M_5$, typically $M_5 \sim 40\,MeV$

For $r < r_c$ gravity is 4 dimensional.

4d $\qquad r_c \qquad$ 5d

crossover scale

For $r > r_c$ gravity is 5 dimensional.

For cosmology we get

$$\underbrace{3m_{Pl}^2 H^2}_{4\,\text{dim}} - \underbrace{6\varepsilon M_5^3 H}_{5-\text{dim}} = \frac{\rho}{2} \qquad \varepsilon = \pm 1$$

$$\hookrightarrow H = \frac{1}{2r_c}\left[1 + \varepsilon\sqrt{1 + \frac{\rho}{3r_c}}\right] \qquad \text{(Deffayet)}$$

self accelerating geometric vacuum is such that even if $\rho = 0$ we have $H_0 \neq 0$.

$\rightarrow$ indeed here $H_0 = \frac{1}{r_c} = \frac{m_{Pl}^2}{2M_5^3}$

therefore the effective cosmological constant results from the fact that gravity is 5 dimensional.

Unfortunately this solution is perturbatively unstable

Gorbunov, Koyama & Sibiryakov

Charmousis, Gregory, Kaloper & Padilla.

But self-acceleration is an interesting paradigm

* Higher codimension 
* Stealth acceleration (Charmousis, Gregory, Padilla)

Let's summarise:

- GR is a unique theory in 4 dimensions
- Scalars are problematic if they have no mass
- Braneworlds are promising but also present problems.

↪ For the moment a cosmological constant is still by far the best we can do.

Q/ Is GR a unique theory in higher dimension?

No it is not. The general theory is Lovelock theory

Q/ What is Lovelock theory?

- It is exactly GR in 4 dimensions
- It is the generalisation of GR in higher dimension?

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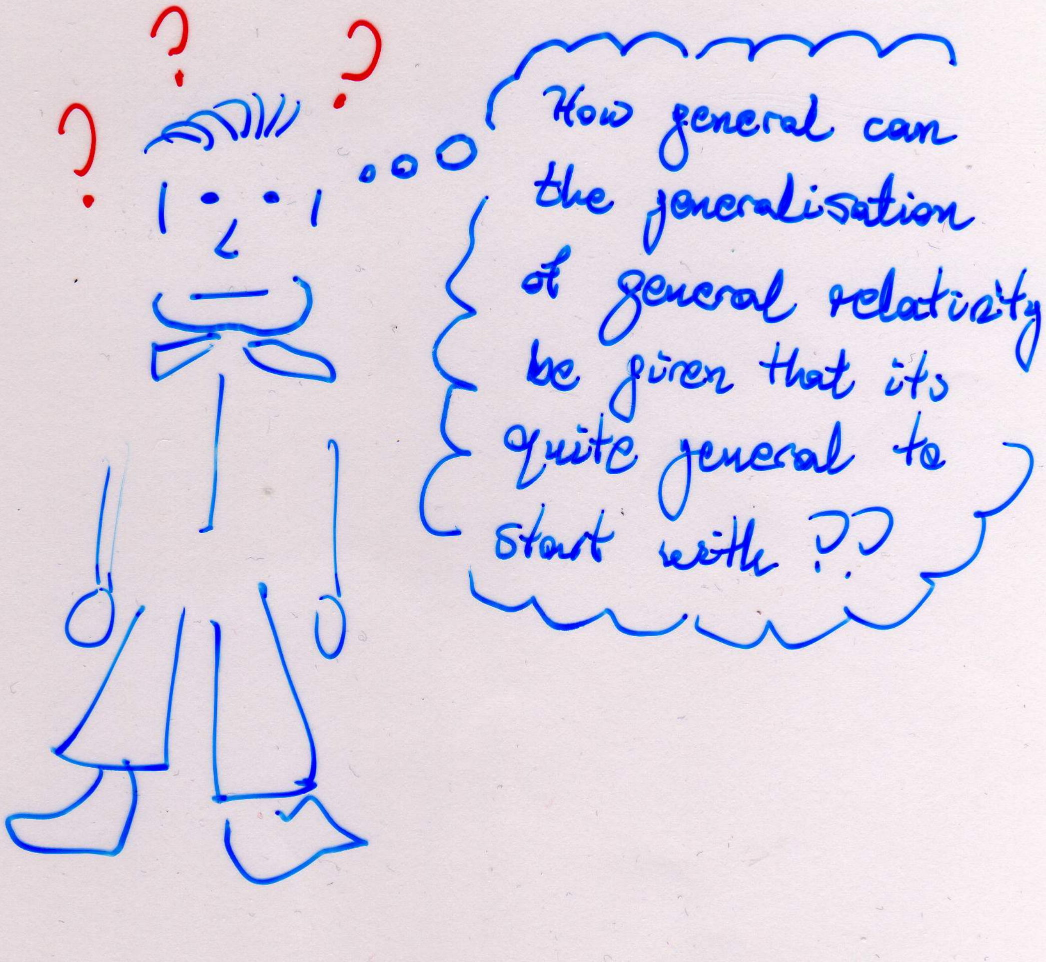

In 5 or 6 dimensions the theory is supplemented by a unique geometric term, the Gauss-Bonnet term:

$$S = M_5^3 \int \sqrt{-g}\, d^4x\, dy \left[ R + \alpha \underbrace{(R^2 - 4R_{ab}^2 + R_{abcd}^2)}_{\hat{G}} \right]$$

A topological invariant in lower dimensions gives a dynamical term in higher dimension.

As in GR we have:
- Energy conservation
- Second order field equations.
- Well-defined braneworlds
- ...

↳ Self-accelerating solutions for codimension 2 braneworlds. (Charmousis & Papazoglou)

- Interesting connections to string theory

Q/ Apart from braneworlds does it have an effect in 4 dimensions?

Back to 4 dimensions

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \quad R^{(4)} \quad + \alpha \quad \hat{G}^{(4)}$$

Back to 4 dimensions

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\, e^{-2\varphi}\Big[R^{(4)} - \omega(\nabla\varphi)^2 + \alpha\, \xi_1(\varphi)\hat{G}^{(4)} + \xi_2(\varphi)\, G^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi\Big]$$

the Gauss-Bonnet term does not affect GR but affects Brans-Dicke theory. It is a well-defined higher order correction.

What about solar system constraints?

Repeating the BD calculation for $\gamma$ for the higher order theory we find the general solution for $\xi_1(\varphi)$, $\xi_2(\varphi)$ parametrised by one free constant $\lambda$ and a free function $S(\varphi)$

Even in the extreme case of $\gamma = 1$ as in GR we can have $2\omega_{BD} \sim \frac{1}{\lambda}$

$ds^2 \sim -(1-2U)\,dt^2 + (1+2U)\,dx_3^2$ ← as in GR.

The higher order correction from the metric competes with the BD term giving seemingly GR like $\gamma$.

(Amendola, Charmousis & Davis)

## Conclusions

- The cosmological constant is still by far the most economic way to fit the late acceleration of the Universe. No real explanation as yet.
- Modification of GR at the IR is heavily constrained both from experiment and from theoretical principles.
- Extra dimensions have been up to now the most interesting way to modify gravity. A lot of work still to do.
- In 4 dimensions modifying GR will most probably entail some deep geometric modification. Remember $m_I$ and $m_G$ and how they become one by the equivalence principle. Could something similar be true for $\Lambda$?
- Hope to learn more in the near future using weak lensing experiment such as DUNE. Can we disentangle, dark energy, cosmological constant and modified gravity?