

# Fluctuations du CMB et weak lensing causés par des cordes cosmiques

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# Outline

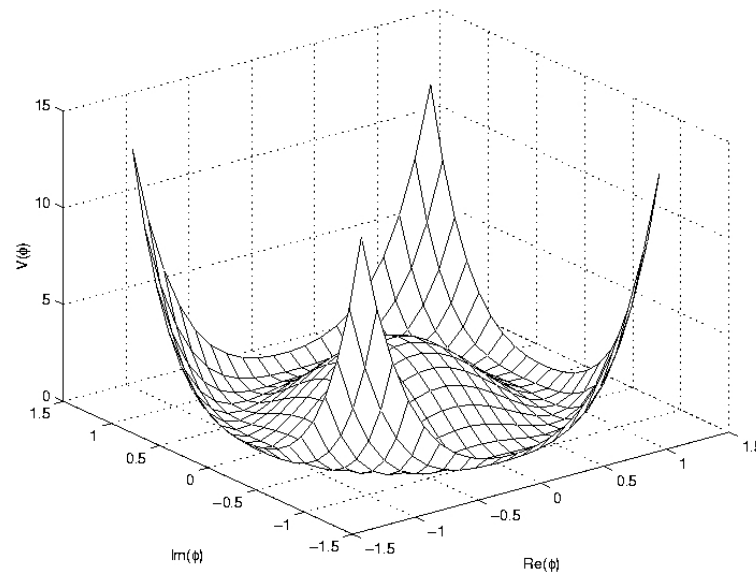
- What are cosmic strings?
- Historical perspective
- CMB Fluctuations
- Weak Lensing
- Outlook

# What are cosmic strings?

Cosmic strings are one of four types of topological (the others being monopoles, domain walls and textures).

They are formed during a phase transition, after the symmetry between the fundamental interactions is spontaneously broken.

The “Mexican Hat” potential for a massless complex scalar field leads to the formation of cosmic strings.



## Nambu-Goto strings

When a cosmic string's curvature radius is much larger than its correlation length, it can be treated as a 1D object. A network can then be treated by the Nambu-Goto action

$$\mathcal{S} = -\mu \int d^2\zeta \sqrt{-\gamma},$$

from which we can derive the equations of motion

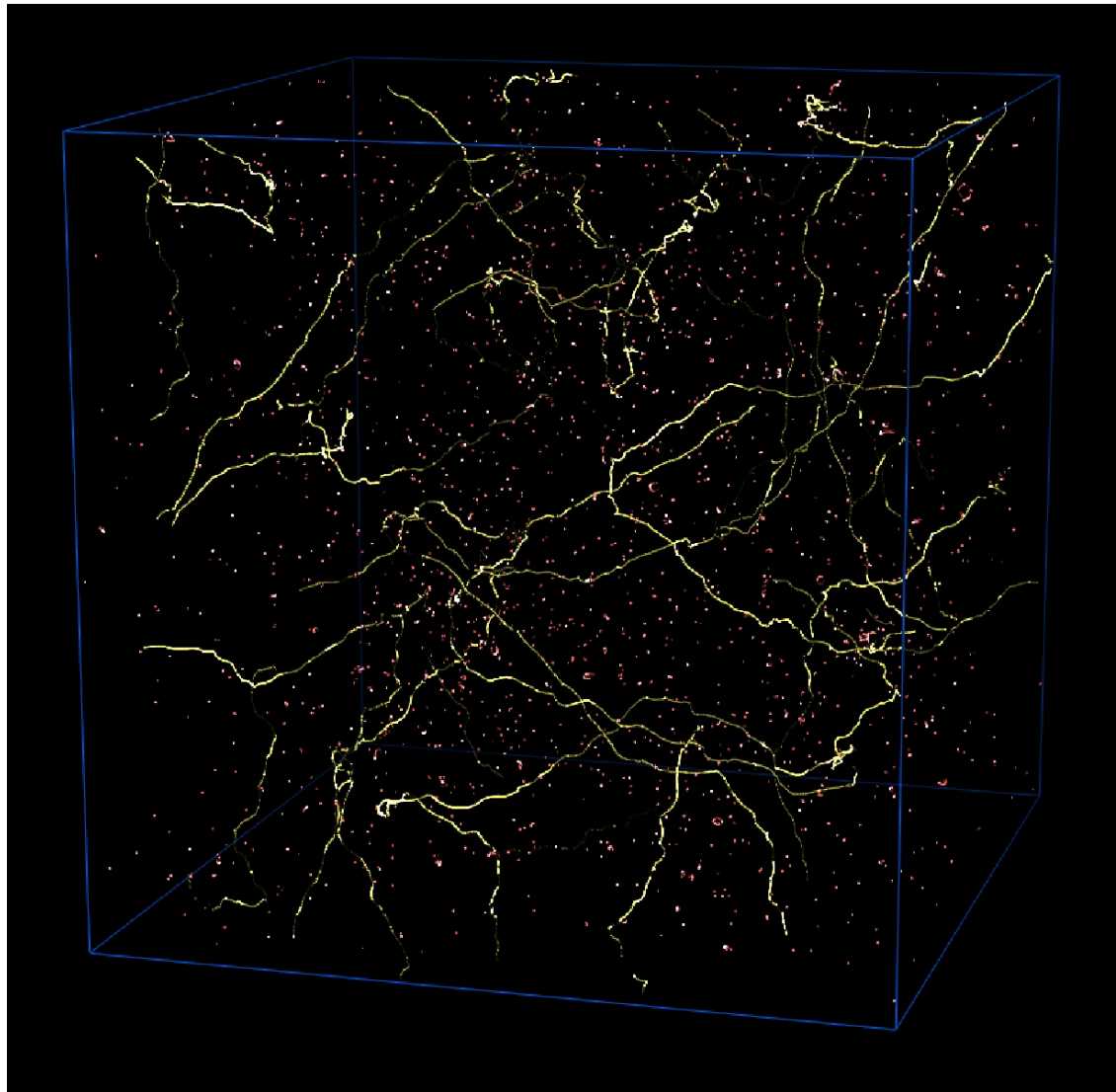
$$\ddot{\mathbf{x}} + 2\frac{\dot{a}}{a}(1 - \dot{\mathbf{x}}^2)\dot{\mathbf{x}} = \epsilon^{-1}(\epsilon^{-1}\mathbf{x}')'$$
$$\dot{\epsilon} = -2\frac{\dot{a}}{a}\epsilon\dot{\mathbf{x}}^2.$$

and the energy-momentum tensor of the strings

$$\Theta^{\mu\nu}\sqrt{-g} = \mu \int d\zeta (\epsilon \dot{x}_s^\mu \dot{x}_s^\nu - \epsilon^{-1} x_s'^\mu x_s'^\nu) \delta^{(3)}(\mathbf{x} - \mathbf{x}_s).$$

There are only two successful numerical implementation of this effective treatment (Bennett & Bouchet 1988 and Allen & Shellard 1990).

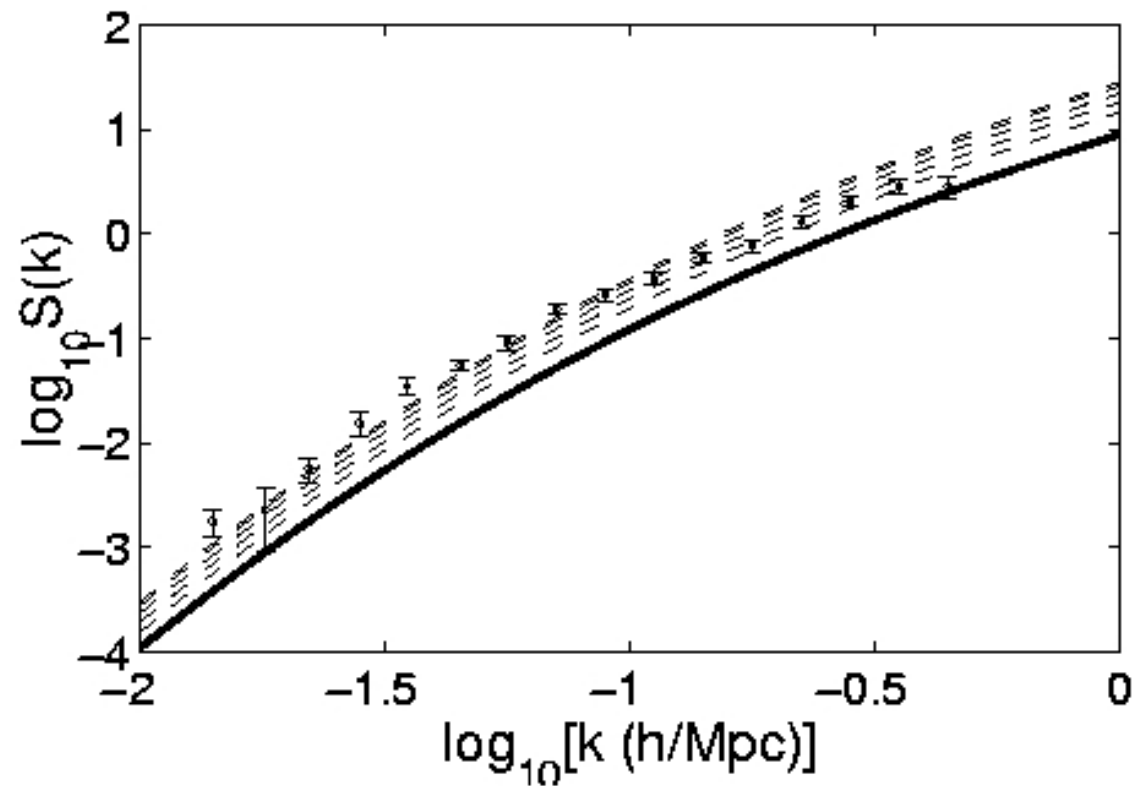
Transforming the problem from 3D QFT to a 1D effective action enables a longer dynamic range.



Cosmic string network in the matter dominated era

## The good old days

In the 1980's and early 90's, cosmic strings on their own could account for the observed matter power spectrum. They were thus seen as alternatives to inflation.

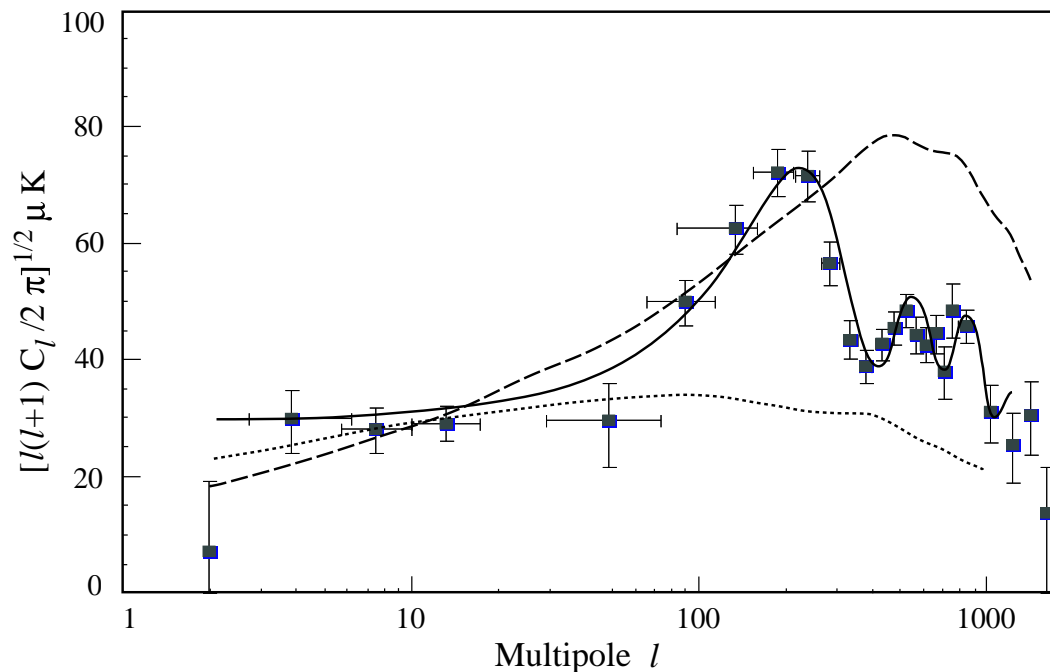


$P(k)$  for cosmic strings (Avelino et al. 1999) along with LSS data (Peacock & Dodds 1994)

# The dark ages

New observational data (especially of the CMB) of increasing quality disfavoured such models, resulting in a loss of interest in cosmic strings.

The remaining few who retained interest claimed that strings could be produced after inflation and could thus be a subdominant source of cosmological perturbations.



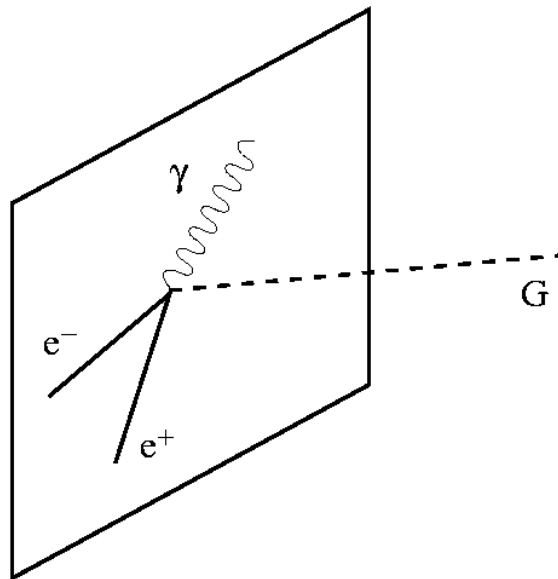
Global textures (dotted), Durrer et al. (1999); cosmic strings (dashed), Battye et al. (1998); hybrid defect (18%) - inflation (solid), Bouchet et al (2002)

# The Renaissance

## Fundamental physics

It was shown that all cosmologically viable SUSY GUTs predicts the formation of strings after inflation; Jeannerot, Rocher & Sakellariadou (2003).

In M-theory, there has been much interest in an alternative to compactification to get rid of extra dimensions, namely, the braneworld scenarios, in which our Universe is a hypersurface embedded in a higher dimensional “bulk”.



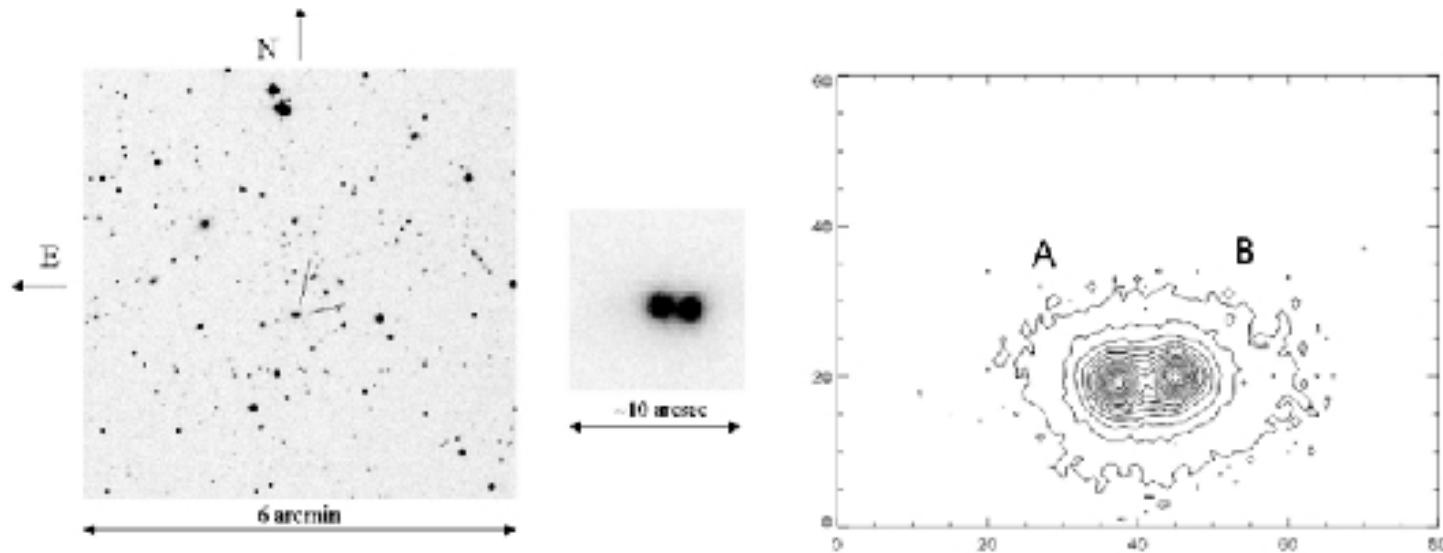
Braneworld inflation models predict the copious production of cosmic strings on our brane at the end of the inflationary phase; e.g. Sarangi & Tye (2002), Copeland, Myers & Polchinski (2004).



## Observational cosmology

Increasing resolution of CMB observations renders the detection of strings, especially through their non-Gaussian signature, not unrealistic, e.g. Planck, AMI, ACT, Clover, ...

A lensing event was claimed to be explainable only if the lensing object is a string; Sazhin et al. (2003). Follow-up observations have shown that the two objects have identical spectra and that there was an anomalously high number of other lenses in the vicinity.

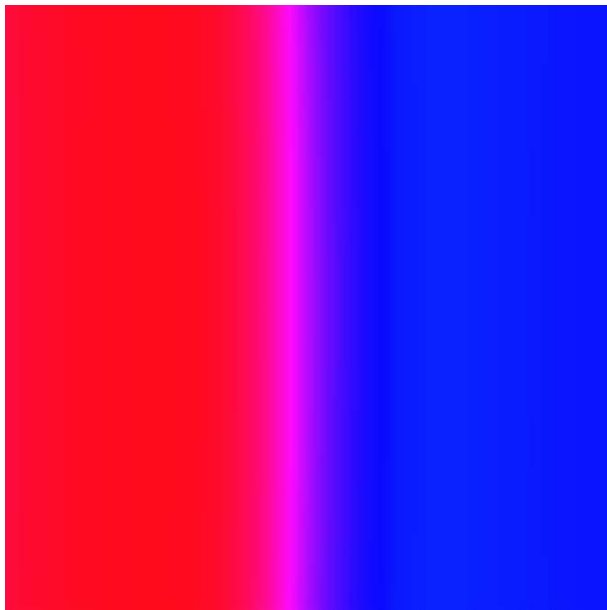


R-band image of object CSL-1 (left and centre); contours of the near IR image (right)

# CMB Fluctuations

A moving cosmic string will produce a discontinuity in the CMB temperature. This is known as the [Kaiser-Stebbins](#) effect. In Minkowski space, the value of the jump, when matter is neglected, is

$$\frac{\Delta T}{T} = 8\pi G\mu v\gamma.$$



However, in an expanding Universe, with a complete treatment of matter, this discontinuity is smeared, making it more difficult to locate: Landriau & Shellard (2003)

# CMB Maps from Cosmic Strings

ML & EPS Shellard, Phys. Rev. **D67**, 103512 (2003)

- Our goal was to produce realistic maps from high resolution cosmic string simulations using a full Boltzmann code.
- The cosmic string simulations used in our work are based on the Nambu-Goto action. The energy-momentum tensor of the strings is interpolated onto a 3D grid for the Boltzmann evolution.
- The scalar, vector and tensor systems of ODE's can each be written as

$$\dot{y}(\mathbf{k}, \eta) = \mathcal{A}(k, \eta)y(\mathbf{k}, \eta) + q(\mathbf{k}, \eta)$$

with initial conditions  $y(\mathbf{k}, 0) = c(\mathbf{k})$ , where  $y$  is the vector whose components are the metric and matter perturbations and  $q$  contains the relevant components of the strings' EM tensor.

- The solution to this equation is:

$$y(\mathbf{k}, \eta) = Y(k, \eta) \mathbf{c}(\mathbf{k}, 0) + Y(k, \eta) \int_0^\eta Y^{-1}(k, \eta') q(\mathbf{k}, \eta') d\eta'$$

where  $Y$  is the solution to

$$\dot{Y}(k, \eta) = \mathcal{A}(k, \eta) Y(k, \eta)$$

with initial conditions  $Y(k, 0) = \mathcal{I}$ .

- Thus, after FFT the relevant perturbation grids back to real space, we can compute the temperature and polarization distributions:

$$\frac{\delta T}{T} = \int_0^{\eta_0} \left( \dot{\tau} e^{-\tau} \left( \frac{\delta_\gamma}{4} - \mathbf{v}_B \cdot \hat{\mathbf{n}} + \Pi_{ij}^I \hat{n}_i \hat{n}_j \right) - \frac{1}{2} e^{-\tau} \dot{h}_{ij} \hat{n}_i \hat{n}_j \right) d\eta$$

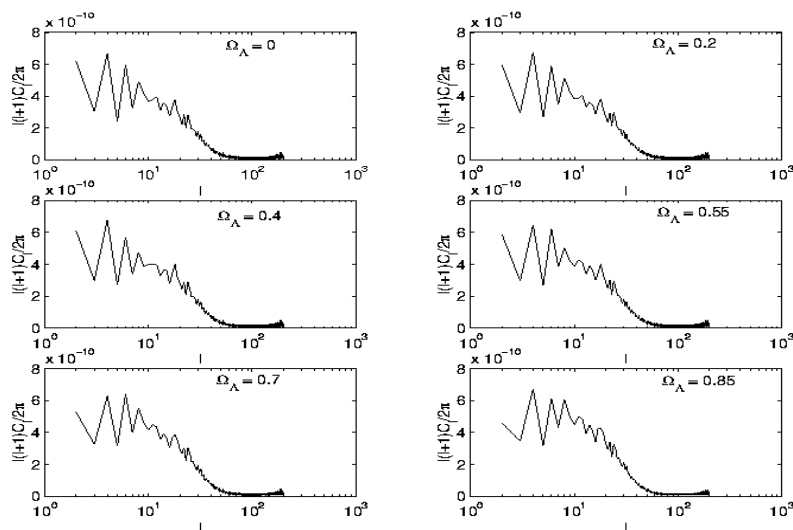
$$Q = \int_0^{\eta_0} \dot{\tau} e^{-\tau} \Pi_{ij}^Q \hat{n}_i \hat{n}_j d\eta$$

$$U = \int_0^{\eta_0} \dot{\tau} e^{-\tau} \Pi_i^U \hat{n}_i d\eta$$

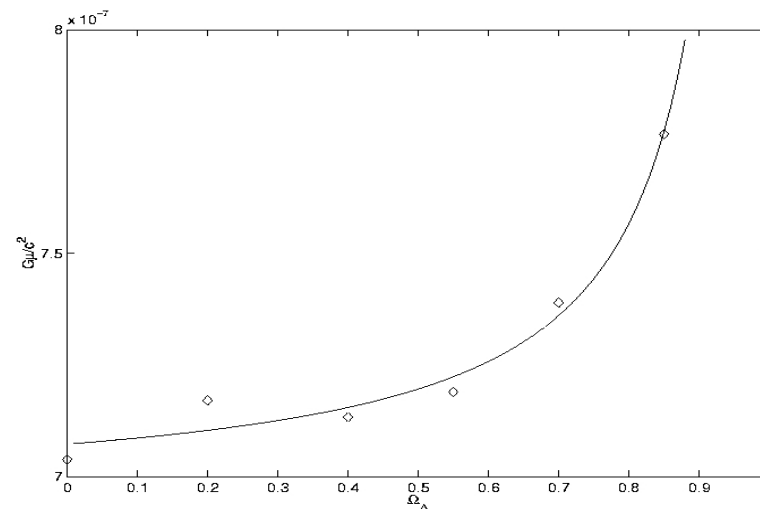
# Large Angle Results

ML & Shellard, Phys. Rev. **D69**, 023003 (2004)

- First study to consider the effect of the cosmological constant on the normalization of Nambu-Goto strings and made use of the highest resolution simulations at the time
- $h = 0.72$ ,  $\Omega_b h^2 = 0.02$ ,  $\Omega_{tot} = 1$
- $\Omega_\Lambda = 0.2, 0.4, 0.55, 0.7, 0.85$
- The normalization we find is  $\frac{G\mu}{c^2} = \left(0.695 + \frac{0.012}{1-\Omega_\Lambda}\right) \times 10^{-6}$



Average power spectrum for each  $\Omega_\Lambda$

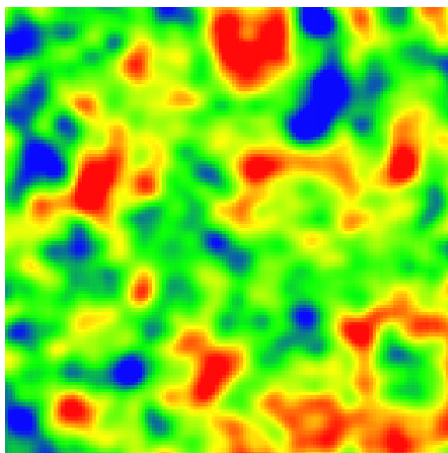


String linear density vs  $\Omega_\Lambda$

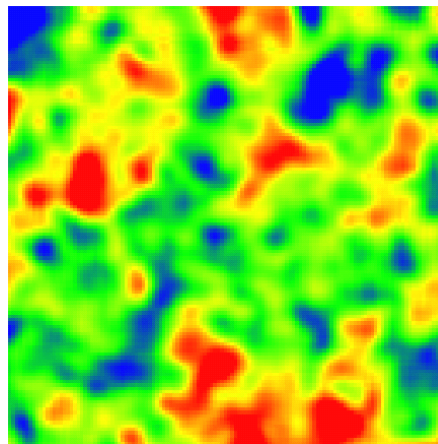
# Small Angle Results

ML & Shellard, astro-ph/0310229

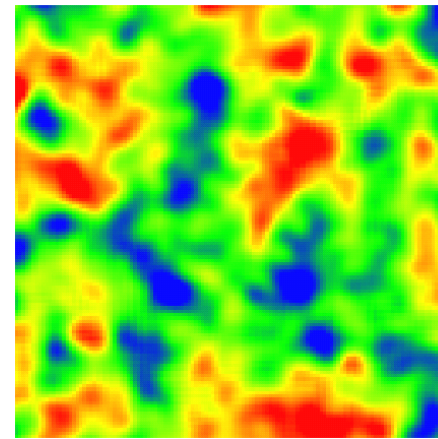
$$h = 0.72, \Omega_b h^2 = 0.02, \Omega_\Lambda, \Omega_{tot} = 1$$



$$6.4^\circ \times 6.4^\circ; \eta_{rec} \rightarrow 4\eta_{rec}$$

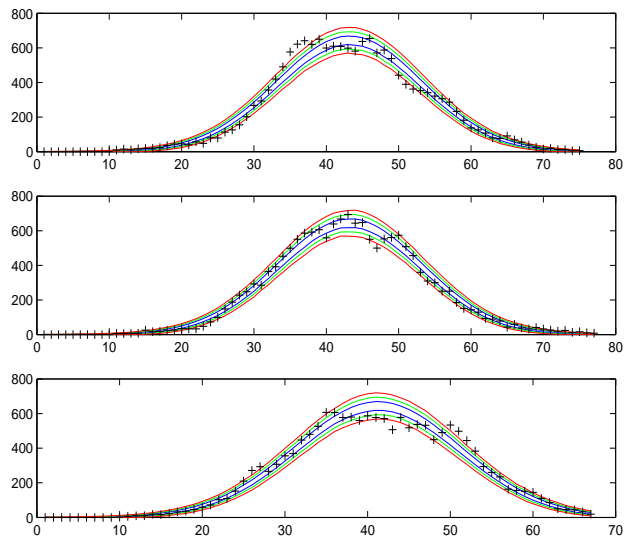


$$12.8^\circ \times 12.8^\circ; 2\eta_{rec} \rightarrow 8\eta_{rec}$$

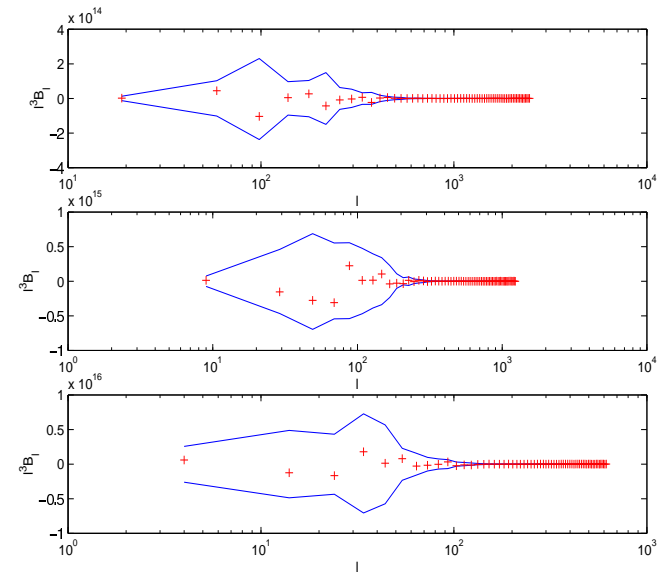


$$25.6^\circ \times 25.6^\circ; 4\eta_{rec} \rightarrow 16\eta_{rec}$$

# Non-Gaussianities



Pixel temperature distribution



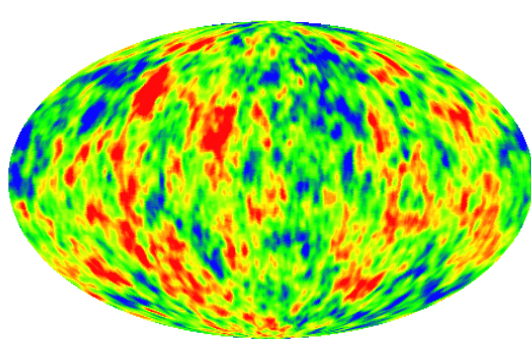
Diagonal bispectrum

# Work in Progress

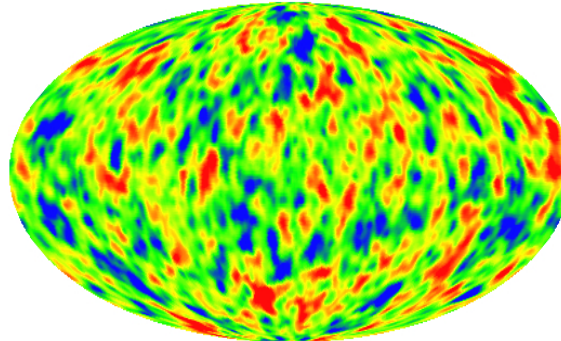
ML & EPS Shellard, in preparation

ML, EPS Shellard & E Komatsu, in preparation

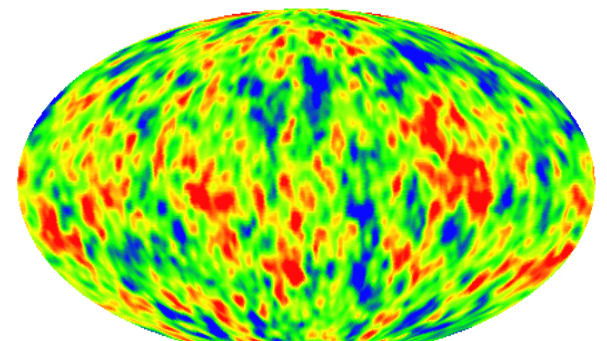
- Late time simulations rescaled to cover recombination epoch.
- Preliminary results indicate strong B-mode polarization signal.



Temperature

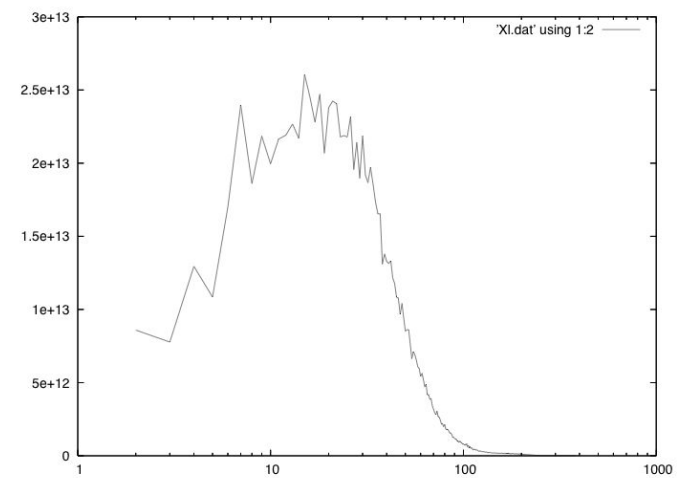
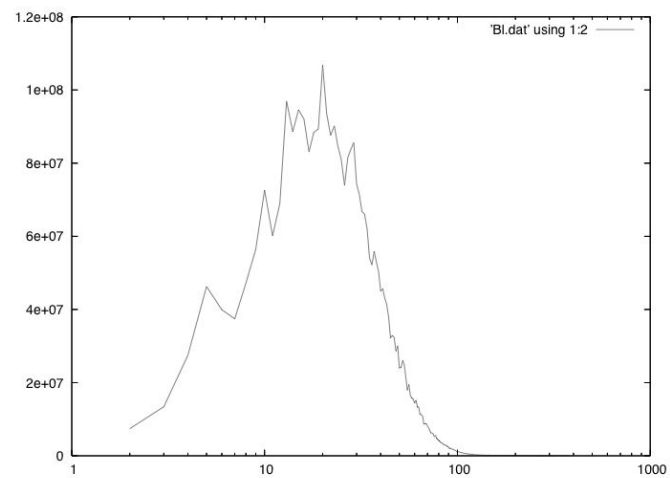
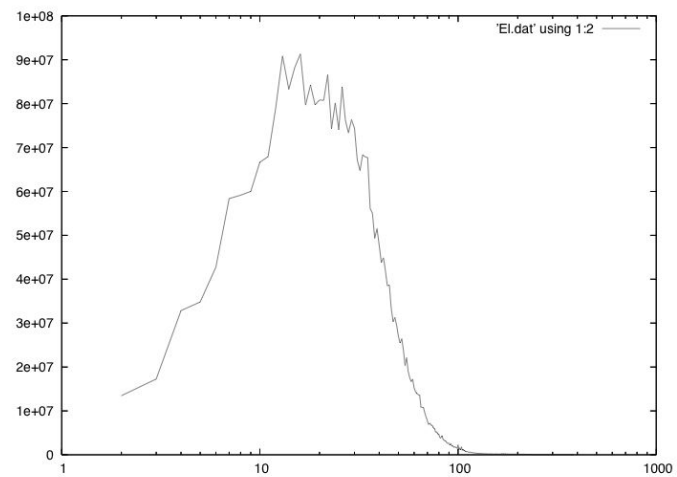
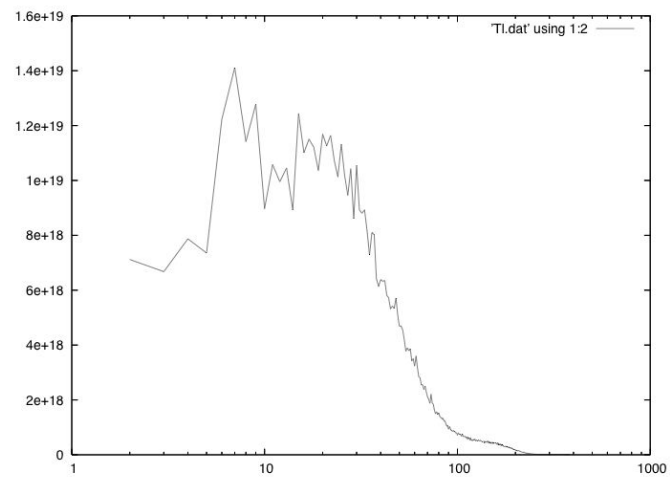


Q



U





# Weak Lensing

ML, E Komatsu & EPS Shellard, in preparation

- The method initially developed to produce CMB maps can be very easily extended to study weak lensing.
- Weak lensing signal from strings may be important on scales where the CMB is more difficult to observe.
- Combination of several astrophysical signature increases chances of detection and ensures we are not confronted with a fluke.

The metric of a (spatially flat) FRW Universe is given by

$$g_{\mu\nu} = a(\eta)^2(\eta_{\mu\nu} + h_{\mu\nu}(\eta, \mathbf{x})),$$

Given a null geodesic  $g : x^\mu(\lambda)$ , where  $\lambda$  is the affine parameter, the Minkowski space geodesic is identical, but with affine parameters related by  $d\lambda = a^2 d\hat{\lambda}$ .

The tangent vector to the geodesic is given by:

$$p^\mu = \frac{dx^\mu}{d\lambda} = \frac{1}{a^2} \frac{dx^\mu}{d\hat{\lambda}}.$$

At the position of the observer, we can define two spacelike vectors  $n_a$ ,  $a = 1, 2$ , which obey the following relations:

$$n_a^\mu n_{b\mu} = \delta_{ab} \quad \text{and} \quad n_a^\mu u_\mu = n_a^\mu p_\mu = 0.$$

Choosing an observer at rest ( $u^\mu = \frac{1}{a}(1, \mathbf{0})$ ), the above constraints impose the following form for the basis vectors:

$$p^\mu = \frac{1}{a^2}(1, \mathbf{n}) \quad \text{and} \quad n_a^\mu = \frac{1}{a}(0, \mathbf{e}_a),$$

with  $\mathbf{e}_a \cdot \mathbf{e}_b = \delta_{ab}$  and  $\mathbf{n} = \mathbf{e}_1 \times \mathbf{e}_2$ .

Our goal to compute maps and power spectrum of the shear field,  $\gamma = (\gamma_1, \gamma_2)$ , which is obtained from the amplification matrix:

$$\mathcal{A}_{ab} \equiv \frac{\mathcal{D}_{ab}(\lambda_S)}{\lambda_S} = \begin{pmatrix} 1 - \kappa - \gamma_1 & \gamma_2 \\ \gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix},$$

where  $\kappa$  is the convergence and the matrix  $\mathcal{D}$  obeys the following evolution equation:

$$\mathcal{D}_{ab}'' = \mathcal{R}_a^c \mathcal{D}_{cb}$$

and obeys the following initial conditions:

$$\mathcal{D}_{ab}(\lambda_O) = 0 \quad \text{and} \quad \mathcal{D}_{ab}'(\lambda_O) = \mathcal{I}_{ab},$$

while  $\mathcal{R}_{ab}$  is obtained from the Riemann tensor in the following way:

$$\mathcal{R}_{ab} = R_{\mu\nu\alpha\beta} p^\nu p^\alpha n_a^\nu n_b^\beta$$

To zeroth order in the perturbation, we have, as in the Minkowski case,  $\mathcal{R}_{ab} = 0$ . The evolution equation then gives us

$$\mathcal{D}_{ab}^{(0)} = \lambda \mathcal{I}_{ab}.$$

Using this result, we obtain the following equation to first order:

$$\mathcal{D}_{ab}^{(1)'} = \lambda \mathcal{R}_{ab}^{(1)},$$

to which the solution is

$$\mathcal{D}_{ab}^{(1)} = \int_0^{\lambda_S} \lambda (\lambda_S - \lambda) \mathcal{R}_{ab}^{(1)} d\lambda$$

Choosing the conformal time for the affine parameter  $\hat{\lambda} = \eta$  and the synchronous gauge  $h_{0\mu} = 0$ , the equation becomes, after integrating by parts and Fourier transforming:

$$\mathcal{D}_{ab}^{(1)} = \int_{\eta_S}^{\eta_0} \sum e^{-i\mathbf{k} \cdot \mathbf{x}} \left[ g_1 (k_l k_j \dot{h}_{ik} - k_l k_i \dot{h}_{jk} - k_j k_k \dot{h}_{il} + k_i k_k \dot{h}_{jl}) n^j n^k + (g_2 - g_3) \dot{h}_{li} \right] e_a^i e_b^l d\eta,$$

where the functions  $g_i$  are defined as:

$$\begin{aligned} g_1(\eta) &= \int_{\eta_S}^{\eta} \lambda (\lambda_S - \lambda) a^{-2} d\eta \\ g_2(\eta) &= \int_{\eta_S}^{\eta} \lambda (\lambda_S - \lambda) a^{-4} \left( \frac{\dot{a}}{a} \right)^2 d\eta \\ g_3(\eta) &= a^{-4} \frac{\dot{a}}{a} \end{aligned}$$

## Summary and Outlook

- Cosmic strings, are generically formed in many fundamental theories and a confirmed detection would thus provide invaluable information on physics beyond the Standard Model.
- String-induced CMB fluctuations have a distinct signal (e.g. non-Gaussianities, B-mode polarization) which could be observed on small scales.
- Study of the weak lensing signal from strings will complement the information from the CMB.
- Various observational signatures used in conjunction should be used to
  - Increase our chance of detection.
  - Make sure we are not confronted with a fluke.