

# Neutrino Physics Prospects of Neutrinoless Double-Beta Decay

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# Compelling Evidences for $\nu$ -Oscillations

$-\nu_{\text{atm}}$ : SK UP-DOWN ASYMMETRY

$\theta_Z$ -,  $L/E$ - dependences of  $\mu$ -like events

Dominant  $\nu_{\mu} \rightarrow \nu_{\tau}$  K2K; MINOS, CNGS (OPERA)

$-\nu_{\odot}$ : Homestake, Kamiokande, SAGE, GALLEX/GNO

Super-Kamiokande, SNO; KamLAND

Dominant  $\nu_e \rightarrow \nu_{\mu,\tau}$  BOREXINO, ..., LowNu

- LSND

Dominant  $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e$  MiniBOONE

$$\nu_{lL} = \sum_{j=1} U_{lj} \nu_{jL} \quad l = e, \mu, \tau. \quad (1)$$

B. Pontecorvo, 1957; 1958; 1967;

Z. Maki, M. Nakagawa, S. Sakata, 1962;

## PMNS Matrix: Standard Parametrization

$$U = V \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix} \quad (2)$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13}e^{i\delta} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}e^{i\delta} \end{pmatrix} \quad (3)$$

- $s_{ij} \equiv \sin \theta_{ij}$ ,  $c_{ij} \equiv \cos \theta_{ij}$ ,  $\theta_{ij} = [0, \frac{\pi}{2}]$ ,
- $\delta$  - Dirac CP-violation phase,  $\delta = [0, 2\pi]$ ,
- $\alpha_{21}$ ,  $\alpha_{31}$  - the two Majorana CP-violation phases.

S.M. Bilenky, J. Hosek, S.T.P., 1980

- $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2 \cong 8.0 \times 10^{-5} \text{ eV}^2 > 0$ ,  $\sin^2 \theta_{12} \cong 0.30$ ,  $\cos 2\theta_{12} \gtrsim 0.28$  ( $2\sigma$ ),
- $|\Delta m_{\text{atm}}^2| \equiv |\Delta m_{31}^2| \cong 2.5 \times 10^{-3} \text{ eV}^2$ ,  $\sin^2 2\theta_{23} \cong 1$ ,
- $\theta_{13}$  - the CHOOZ angle:  $\sin^2 \theta_{13} < 0.027$  ( $0.041$ )  $2\sigma$  ( $3\sigma$ ).

A.Bandyopadhyay, S.Choubey, S.Goswami, S.T.P., D.P.Roy, hep-ph/0406328 (updated);

T. Schwetz, hep-ph/0606060.

- $\sqrt{\Delta m_\odot^2} \sin^2 \theta_{12} \cong 3.0 \times 10^{-3}$  eV ( $\pm$ )  $\sqrt{|\Delta m_{\text{atm}}^2|} \sin^2 \theta_{13} \lesssim 2.2 \times 10^{-3}$  eV;
- $\sqrt{|\Delta m_{\text{atm}}^2|} \cong 5 \times 10^{-2}$  eV;  $\sqrt{|\Delta m_{\text{atm}}^2|} \cos 2\theta_{12} \gtrsim 1.4 \times 10^{-2}$  eV ( $\cos 2\theta_{12} \gtrsim 0.28$ )
- $m_0$ :  $m_0^2 \gg \Delta m_\odot^2, |\Delta m_{\text{atm}}^2|$ ,  $m_0 \gtrsim 0.1$  eV
- $\text{sgn}(\Delta m_{\text{atm}}^2) = \text{sgn}(\Delta m_{31}^2)$  not determined

$\Delta m_{\text{atm}}^2 \equiv \Delta m_{31}^2 > 0$ , normal mass ordering

$\Delta m_{\text{atm}}^2 \equiv \Delta m_{32}^2 < 0$ , inverted mass ordering

Convention:  $m_1 < m_2 < m_3$  - NMO,  $m_3 < m_1 < m_2$  - IMO

### • Majorana phases $\alpha_{21}, \alpha_{31}$ :

-  $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$  not sensitive;

S.M. Bilenky, J. Hosek, S.T.P., 1980;  
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

- $|\langle m \rangle|$  in  $(\beta\beta)_{0\nu}$ -decay depends on  $\alpha_{21}, \alpha_{31}$ ;
- $\Gamma(\mu \rightarrow e + \gamma)$  etc. in SUSY theories depend on  $\alpha_{21,31}$ ;
- BAU, leptogenesis scenario:  $\alpha_{21,31}$  ?

## Future Progress

- Determination of the nature - Dirac or Majorana, of  $\nu_j$  .
- Determination of  $\text{sgn}(\Delta m_{\text{atm}}^2)$ , type of  $\nu$ - mass spectrum

$$m_1 \ll m_2 \ll m_3, \quad \text{NH},$$

$$m_3 \ll m_1 < m_2, \quad \text{IH},$$

$$m_1 \cong m_2 \cong m_3, \quad m_{1,2,3}^2 \gg \Delta m_{\text{atm}}^2, \quad \text{QD}; \quad m_j \gtrsim 0.10 \text{ eV}.$$

- Determining, or obtaining significant constraints on, the absolute scale of  $\nu_j$ -masses, or  $\min(m_j)$ .
- Status of the CP-symmetry in the lepton sector: violated due to  $\delta$  (Dirac), and/or due to  $\alpha_{21}$ ,  $\alpha_{31}$  (Majorana)?
- Measurement of, or improving by at least a factor of (5 - 10) the existing upper limit on,  $\sin^2 \theta_{13}$ .
- High precision determination of  $\Delta m_{\odot}^2$ ,  $\theta_{\odot}$ ,  $\Delta m_{\text{atm}}^2$ ,  $\theta_{\text{atm}}$ .
- Searching for possible manifestations, other than  $\nu_l$ -oscillations, of the non-conservation of  $L_l$ ,  $l = e, \mu, \tau$ , such as  $\mu \rightarrow e + \gamma$ ,  $\tau \rightarrow \mu + \gamma$ , etc. decays.

- Understanding at fundamental level the mechanism giving rise to the  $\nu$ - masses and mixing and to the  $L_l$ -non-conservation. Includes understanding
  - the origin of the observed patterns of  $\nu$ -mixing and  $\nu$ -masses ;
  - the physical origin of  $CPV$  phases in  $U_{\text{PMNS}}$  ;
  - Are the observed patterns of  $\nu$ -mixing and of  $\Delta m^2_{21,31}$  related to the existence of a new symmetry?
  - Is there any relations between  $q$ -mixing and  $\nu$ - mixing? Is  $\theta_{12} + \theta_c = \pi/4$  ?
  - Is  $\theta_{23} = \pi/4$ , or  $\theta_{23} > \pi/4$  or else  $\theta_{23} < \pi/4$ ?
  - Is there any correlation between the values of  $CPV$  phases and of mixing angles in  $U_{\text{PMNS}}$ ?
- Progress in the theory of  $\nu$ -mixing might lead to a better understanding of the origin of the BAU.

## $(\beta\beta)_{0\nu}$ -Decay Experiments:

- Majorana nature of  $\nu_j$
- Type of  $\nu$ -mass spectrum (NH, IH, QD)
- Absolute neutrino mass scale

$^3\text{H}$   $\beta$ -decay, cosmology:  $m_\nu$  (QD, IH)

- CPV due to Majorana CPV phases

$\nu_j$ - Dirac or Majorana particles, fundamental problem

$\nu_j$ -Dirac: **conserved lepton charge exists**,  $L = L_e + L_\mu + L_\tau$ ,  $\nu_j \neq \bar{\nu}_j$

$\nu_j$ -Majorana: **no lepton charge is exactly conserved**,  $\nu_j \equiv \bar{\nu}_j$

The observed patterns of  $\nu$ -mixing and of  $\Delta m_{\text{atm}}^2$  and  $\Delta m_{\odot}^2$  can be related to Majorana  $\nu_j$  and an approximate symmetry:

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

See-saw mechanism:  $\nu_j$ - Majorana

Establishing that  $\nu_j$  are Majorana particles would be as important as the discovery of  $\nu$ - oscillations.

If  $\nu_j$  – Majorana particles,  $U_{\text{PMNS}}$  contains (3- $\nu$  mixing)

$\delta$ -Dirac,  $\alpha_{21}$ ,  $\alpha_{31}$  - Majorana **physical CPV phases**

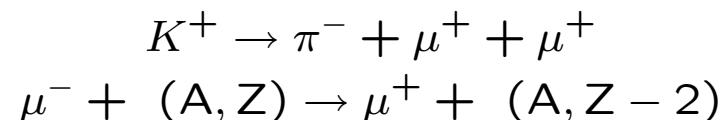
$\nu$ -oscillations  $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}, l, l' = e, \mu, \tau,$

- are not sensitive to the nature of  $\nu_j$ ,

S.M. Bilenky, J. Hosek, S.T.P., 1980;  
P. Langacker et al., 1987

- provide information on  $\Delta m_{jk}^2 = m_j^2 - m_k^2$ , but not on the absolute values of  $\nu_j$  masses.

The Majorana nature of  $\nu_j$  can manifest itself in the existence of  $\Delta L = \pm 2$  processes:



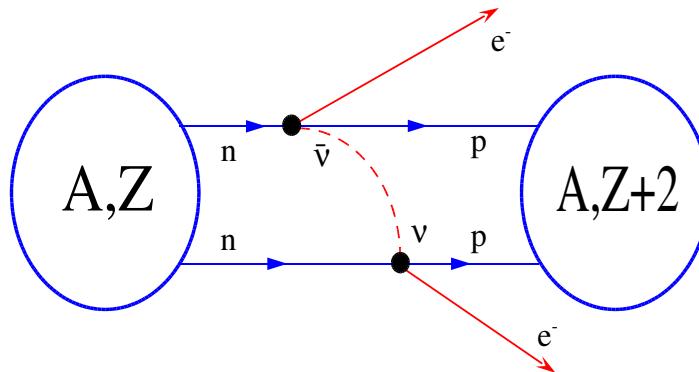
The process most sensitive to the possible Majorana nature of  $\nu_j$  –  $(\beta\beta)_{0\nu}$ -decay



of the even-even nuclei,  $^{48}\text{Ca}$ ,  $^{76}\text{Ge}$ ,  $^{82}\text{Se}$ ,  $^{100}\text{Mo}$ ,  $^{116}\text{Cd}$ ,  $^{130}\text{Te}$ ,  $^{136}\text{Xe}$ ,  $^{156}\text{Nd}$ .

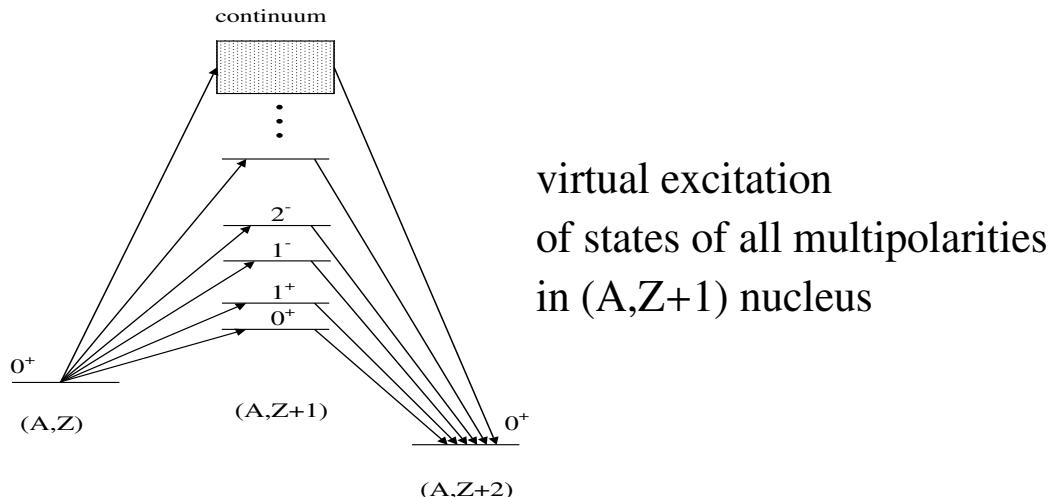
$2n$  from  $(A, Z)$  exchange a virtual Majorana  $\nu_j$  (via the CC weak interaction) and transform into  $2p$  of  $(A, Z+2)$  and two free  $e^-$ .

## Nuclear $0\nu\beta\beta$ -decay



strong in-medium modification of the basic process

$$dd \rightarrow uue^-e^-(\bar{\nu}_e\bar{\nu}_e)$$



$A(\beta\beta)_{0\nu} \sim \langle m \rangle$  M(A,Z),      M(A,Z) - NME,

$$\begin{aligned} |\langle m \rangle| &= |m_1|U_{e1}|^2 + m_2|U_{e2}|^2 e^{i\alpha_{21}} + m_3|U_{e3}|^2 e^{i\alpha_{31}}| \\ &= |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_{21}} + m_3 s_{13}^2 e^{i\alpha_{31}}|, \quad \theta_{12} \equiv \theta_\odot, \theta_{13} - \text{CHOOZ} \end{aligned}$$

$\alpha_{21}, \alpha_{31}$  - the two Majorana CPVP of the PMNS matrix.

**CP-invariance:**  $\alpha_{21} = 0, \pm\pi, \alpha_{31} = 0, \pm\pi$ ;

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1$$

**relative CP-parities of  $\nu_1$  and  $\nu_2$ , and of  $\nu_1$  and  $\nu_3$** .

L. Wolfenstein, 1981;

S.M. Bilenky, N. Nedelcheva, S.T.P., 1984;

B. Kayser, 1984.

Best sensitivity: Heidelberg-Moscow  $^{76}\text{Ge}$  experiment.

Claim for a positive signal at  $> 3\sigma$ :

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$$|\langle m \rangle| = (0.1 - 0.9) \text{ eV} \text{ (99.73% C.L.)}.$$

IGEX  $^{76}\text{Ge}$ :  $|\langle m \rangle| < (0.33 - 1.35) \text{ eV}$  (90% C.L.).

Taking data - NEMO3 ( $^{100}\text{Mo}$ ), CUORICINO ( $^{130}\text{Te}$ ):

$$|\langle m \rangle| < (0.7 - 1.2) \text{ eV}, |\langle m \rangle| < (0.18 - 0.90) \text{ eV} \text{ (90% C.L.)}.$$

Large number of projects:  $|\langle m \rangle| \sim (0.01 - 0.05) \text{ eV}$

CUORE -  $^{130}\text{Te}$ ,

GERDA -  $^{76}\text{Ge}$ ,

SuperNEMO -  $^{82}\text{Se}$ ,

EXO -  $^{136}\text{Xe}$ ,

MAJORANA -  $^{76}\text{Ge}$ ,

MOON -  $^{100}\text{Mo}$ ,

CANDLES -  $^{48}\text{Ca}$ ,

XMASS -  $^{136}\text{Xe}$ .

$|<m>| : m_j, \theta_\odot \equiv \theta_{12}, \theta_{13}, \alpha_{21,31}$

$m_{1,2,3}$  - in terms of  $\min(m_j)$ ,  $\Delta m_{\text{atm}}^2$ ,  $\Delta m_\odot^2$

S.T.P., A.Yu. Smirnov, 1994

Convention:  $m_1 < m_2 < m_3$  - NMO,  $m_3 < m_1 < m_2$  - IMO

$$\Delta m_\odot^2 \equiv \Delta m_{21}^2, \quad m_2 = \sqrt{m_1^2 + \Delta m_\odot^2},$$

while either

$$\Delta m_{\text{atm}}^2 \equiv \Delta m_{31}^2 > 0, \quad m_3 = \sqrt{m_1^2 + \Delta m_{\text{atm}}^2}, \quad \text{normal mass ordering, or}$$

$$\Delta m_{\text{atm}}^2 \equiv \Delta m_{32}^2 < 0, \quad m_1 = \sqrt{m_3^2 + |\Delta m_{\text{atm}}^2| - \Delta m_\odot^2}, \quad \text{inverted mass ordering}$$

The neutrino mass spectrum –

*Normal hierarchical (NH)* if  $m_1 \ll m_2 \ll m_3$ ,

*Inverted hierarchical (IH)* if  $m_3 \ll m_1 \cong m_2$ ,

*Quasi-degenerate (QD)* if  $m_1 \cong m_2 \cong m_3 = m$ ,  $m_j^2 \gg |\Delta m_{\text{atm}}^2|$ ;  $m_j \gtrsim 0.1$  eV

Given  $|\Delta m_{\text{atm}}^2|$ ,  $\Delta m_\odot^2$ ,  $\theta_\odot$ ,  $\theta_{13}$ ,

$$|<m>| = |<m>| (m_{\min}, \alpha_{21}, \alpha_{31}; S), \quad S = \text{NO(NH)}, \text{IO(IH)}.$$

$$A(\beta\beta)_{0\nu} \sim \langle m \rangle \text{ M(A,Z), } \quad \text{M(A,Z) - NME,}$$

$$|\langle m \rangle| \cong \left| \sqrt{\Delta m_{\odot}^2} \sin^2 \theta_{12} e^{i\alpha} + \sqrt{\Delta m_{31}^2} \sin^2 \theta_{13} e^{i\beta_M} \right|, \quad m_1 \ll m_2 \ll m_3 \text{ (NH),}$$

$$|\langle m \rangle| \cong \sqrt{m_3^2 + \Delta m_{23}^2} |\cos^2 \theta_{12} + e^{i\alpha} \sin^2 \theta_{12}|, \quad m_3 < (\ll) m_1 < m_2 \text{ (IH),}$$

$$|\langle m \rangle| \cong m |\cos^2 \theta_{12} + e^{i\alpha} \sin^2 \theta_{12}|, \quad m_{1,2,3} \cong m \gtrsim 0.10 \text{ eV (QD),}$$

$$\theta_{12} \equiv \theta_{\odot}, \theta_{13}\text{-CHOOZ}; \quad \alpha \equiv \alpha_{21}, \beta_M \equiv \alpha_{31}.$$

**CP-invariance:**  $\alpha = 0, \pm\pi, \beta_M = 0, \pm\pi;$

$$|\langle m \rangle| \lesssim 5 \times 10^{-3} \text{ eV, NH;}$$

$$\sqrt{\Delta m_{23}^2} \cos 2\theta_{12} \cong 0.013 \text{ eV} \lesssim |\langle m \rangle| \lesssim \sqrt{\Delta m_{23}^2} \cong 0.055 \text{ eV, IH;}$$

$$m \cos 2\theta_{12} \lesssim |\langle m \rangle| \lesssim m, \quad m \gtrsim 0.10 \text{ eV, QD.}$$

## Oscillation Parameters

$$\Delta m_{\odot}^2 = 8.0 \times 10^{-5} \text{ eV}^2, \quad 3\sigma(\Delta m_{\odot}^2) = 12\%,$$

$$\sin^2 \theta_{\odot} = 0.30, \quad 3\sigma(\sin^2 \theta_{\odot}) = 27\%,$$

$$|\Delta m_{\text{atm}}^2| = 2.5 \times 10^{-3} \text{ eV}, \quad 3\sigma(|\Delta m_{\text{atm}}^2|) = 28\%.$$

## Future:

3 kTy KamLAND:  $3\sigma(\Delta m_{\odot}^2) = 7\%$ ,  $3\sigma(\sin^2 \theta_{\odot}) = 18\%$  ;

A. Bandyopadhyay et al., hep-ph/0410283

SK-Gd (0.1% Gd:  $43 \times (\text{KL } \bar{\nu}_e \text{ rate})$ ), 3y:  $3\sigma(\Delta m_{\odot}^2) \cong 4\%$

S. Choubey, S.T.P., hep-ph/0404103;

J. Beacom and M. Vagins, hep-ph/0309300

KL type reactor  $\bar{\nu}_e$  detector,  $L \sim 60$  km,  $\sim 60$  GW kTy:  $3\sigma(\sin^2 \theta_{\odot}) \cong 12\%$

A. Bandyopadhyay et al., hep-ph/0410283 and hep-ph/0302243;

H. Minakata et al., hep-ph/0407326

T2K (SK):  $3\sigma(|\Delta m_{\text{atm}}^2|) \cong 6\%$

$\text{sgn}(\Delta m_{\text{atm}}^2)$ :  $\nu_{\text{atm}}$  experiments, studying the subdominant  $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$  and  $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$  oscillations; LBL  $\nu$ -oscillation experiments (T2K, NO $\nu$ A);  $\nu$ -factory.

$\sin^2 \theta_{13}$ : reactor  $\bar{\nu}_e$  experiments,  $L \sim (1 - 2)$  km: Double CHOOZ, Daya-Bay, KASKA, ... - factor (5 - 10).

# Absolute Neutrino Mass Measurements

The Troitzk and Mainz  ${}^3\text{H}$   $\beta$ -decay experiments

$$m_{\nu_e} < 2.3 \text{ eV} \quad (95\% \text{ C.L.})$$

There are prospects to reach sensitivity

$$\text{KATRIN : } m_{\nu_e} \sim 0.2 \text{ eV}$$

Cosmological and astrophysical data: the WMAP result combined with data from large scale structure surveys (2dFGRS, SDSS)

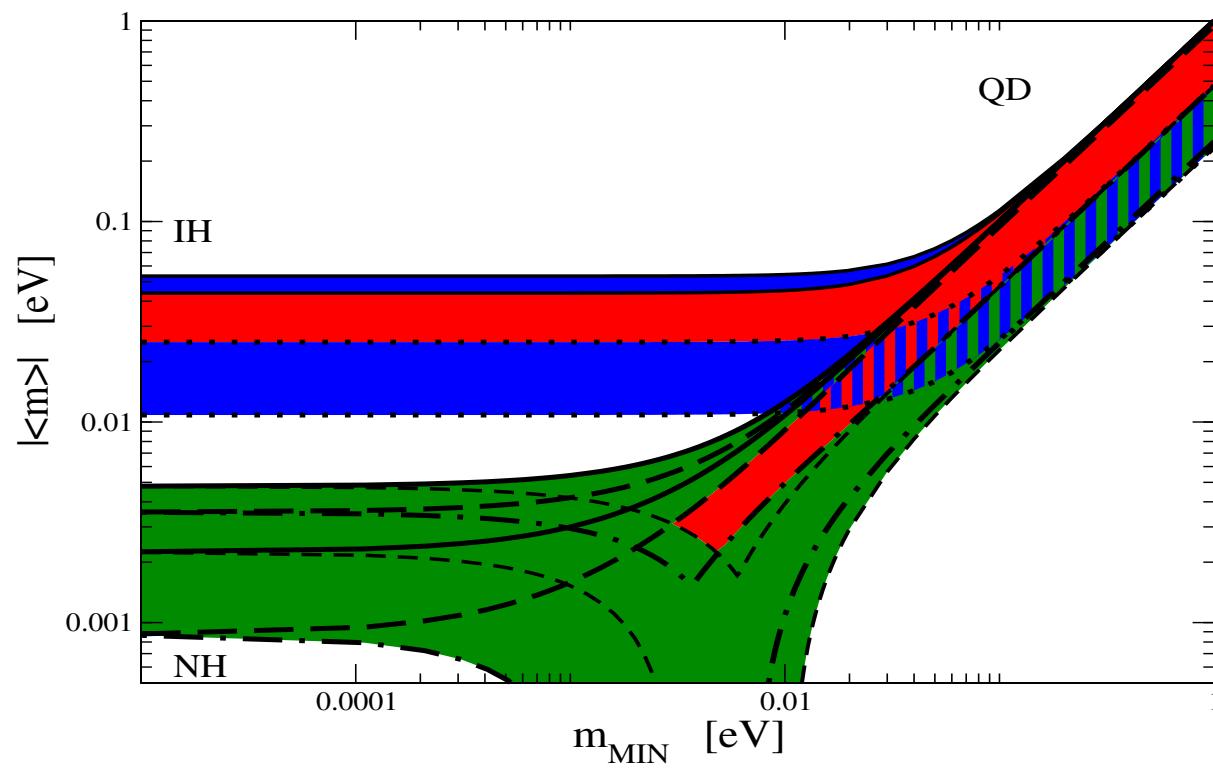
$$\sum_j m_j \equiv \Sigma < (0.4 - 1.7) \text{ eV}$$

The WMAP and future PLANCK experiments can be sensitive to

$$\sum_j m_j \cong 0.4 \text{ eV}$$

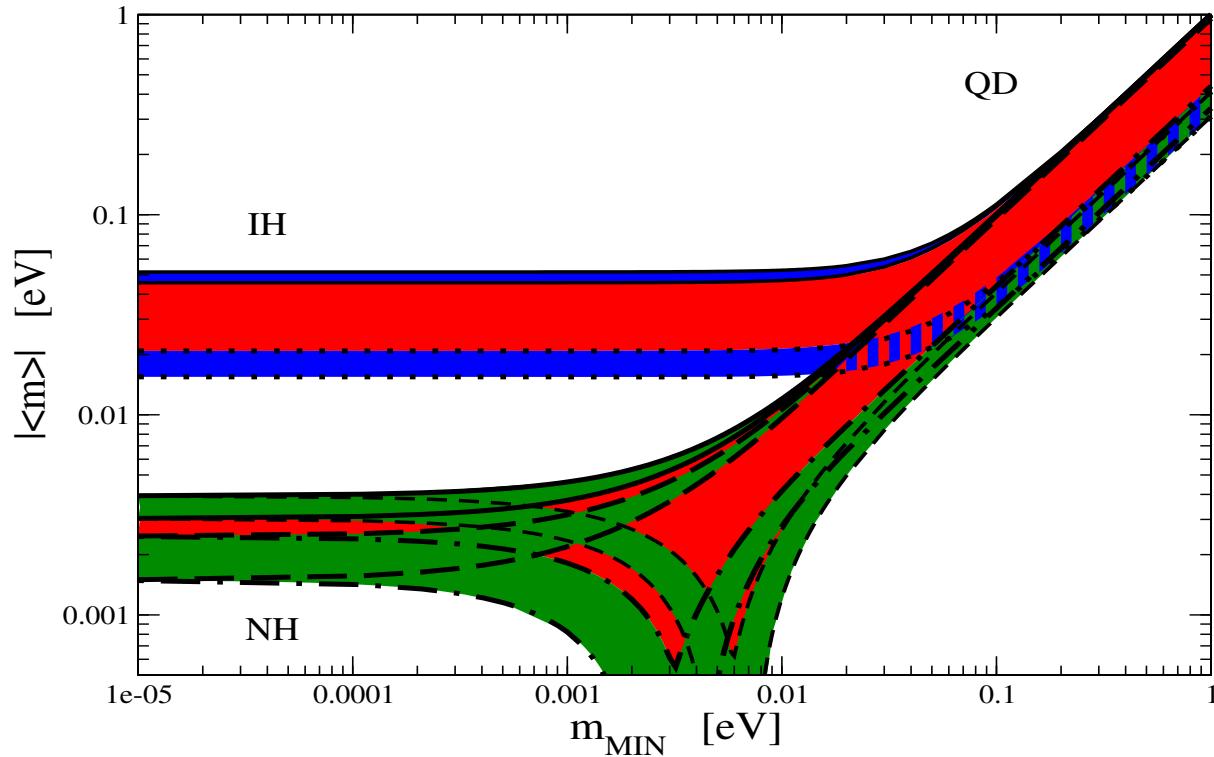
Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and PLANCK experiments may allow to determine

$$\sum_j m_j : \quad \delta \cong 0.04 \text{ eV.}$$



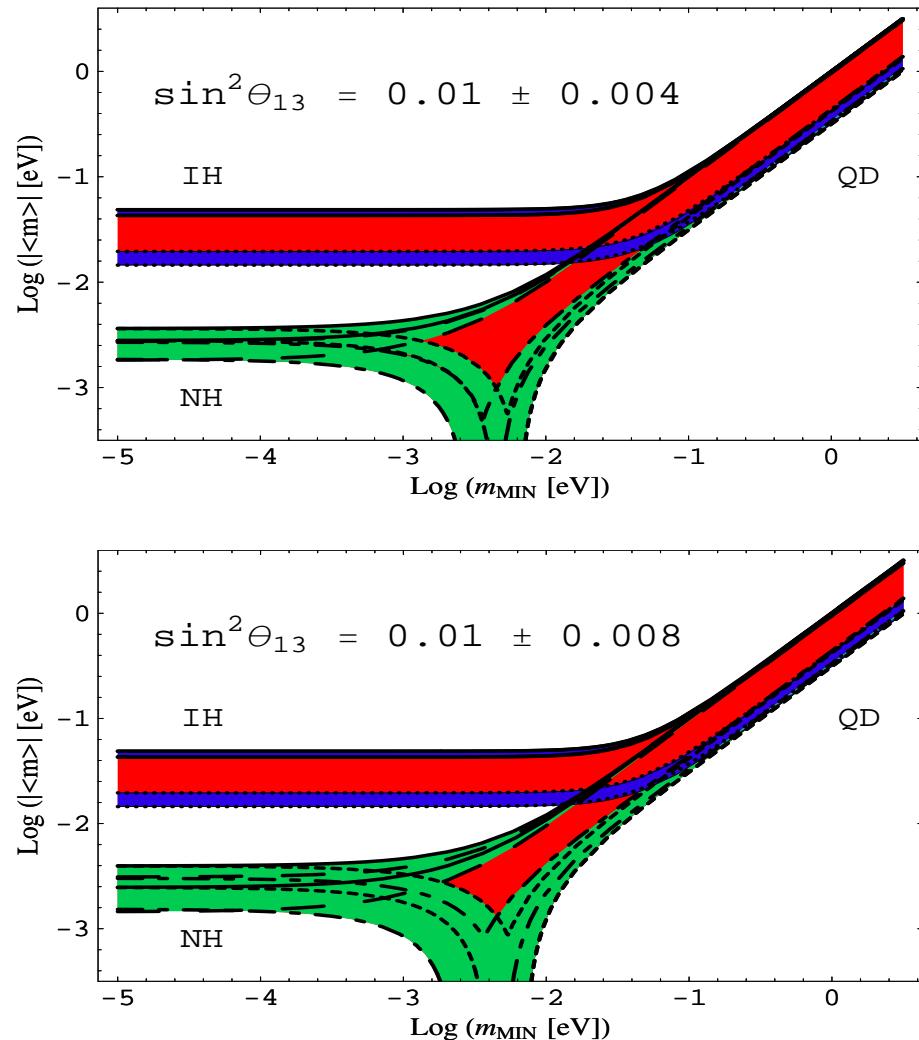
S. Pascoli, S.T.P., 2006

The current  $2\sigma$  ranges of values of the parameters used.



S. Pascoli, S.T.P., 2006

$\sin^2 \theta_{13} = 0.015 \pm 0.006$ ;  $1\sigma(\Delta m_{\odot}^2) = 4\%$ ,  $1\sigma(\sin^2 \theta_{\odot}) = 4\%$ ,  $1\sigma(|\Delta m_{\text{atm}}^2|) = 6\%$ ;  
 $2\sigma(|\langle m \rangle|)$  used.



S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

$3\sigma(\Delta m_{\odot}^2) = 6\%$ ,  $3\sigma(\sin^2 \theta_{\odot}) = 12\%$ ,  $3\sigma(|\Delta m_{\text{atm}}^2|) = 18\%$ .

# Nuclear Matrix Element Uncertainty

$$|\langle m \rangle| = \zeta ((|\langle m \rangle|_{\text{exp}})_{\text{MIN}} \pm \Delta), \quad \zeta \geq 1,$$

$(|\langle m \rangle|_{\text{exp}})_{\text{MIN}}$  – obtained with the **maximal physically allowed value of NME**.

A measurement of the  $\beta\beta_{0\nu}$ -decay half-life time

$$(|\langle m \rangle|_{\text{exp}})_{\text{MIN}} - \Delta \leq |\langle m \rangle| \leq \zeta ((|\langle m \rangle|_{\text{exp}})_{\text{MIN}} + \Delta).$$

The estimated range of  $\zeta^2$ :

$^{48}\text{Ca}, \quad \zeta^2 \simeq 3.5$

$^{76}\text{Ge}, \quad \zeta^2 \simeq 10$

$^{82}\text{Se}, \quad \zeta^2 \simeq 10$

$^{130}\text{Te}, \quad \zeta^2 \simeq 38.7$

S. Elliot, P. Vogel, 2002

NH vs IH (QD):

$$\zeta |\langle m \rangle|_{\text{max}}^{\text{NH}} < |\langle m \rangle|_{\text{min}}^{\text{IH(QD)}}, \quad \zeta \geq 1.$$

IH vs QD:

$$\zeta |\langle m \rangle|_{\text{max}}^{\text{IH}} < |\langle m \rangle|_{\text{min}}^{\text{QD}}, \quad \zeta \geq 1.$$

## Method of Analysis

$$\Gamma_{\text{th}} = G |\mathcal{M}|^2 (|\langle m \rangle|(\mathbf{x}))^2, \quad \mathbf{x} = (\mathbf{x}_{\text{osc}}, \mathbf{x}_{\beta\beta}^{0\nu})$$

$$\mathbf{x}_{\text{osc}} = (\theta_{12}, \theta_{13}, |\Delta m_{31}^2|, \Delta m_{21}^2),$$

$$\mathbf{x}_{\beta\beta}^{0\nu} = (m_0, \text{sgn}(\Delta m_{31}^2), \alpha_{21}, \alpha_{31}).$$

$$|\langle m \rangle|^{\text{obs}} \equiv \sqrt{\frac{\Gamma_{\text{obs}}}{G}} \frac{1}{|\mathcal{M}_0|}, \quad \sigma_{\beta\beta} = \frac{1}{2} \frac{1}{\sqrt{\Gamma_{\text{obs}} G}} \frac{1}{|\mathcal{M}_0|} \sigma(\Gamma_{\text{obs}}),$$

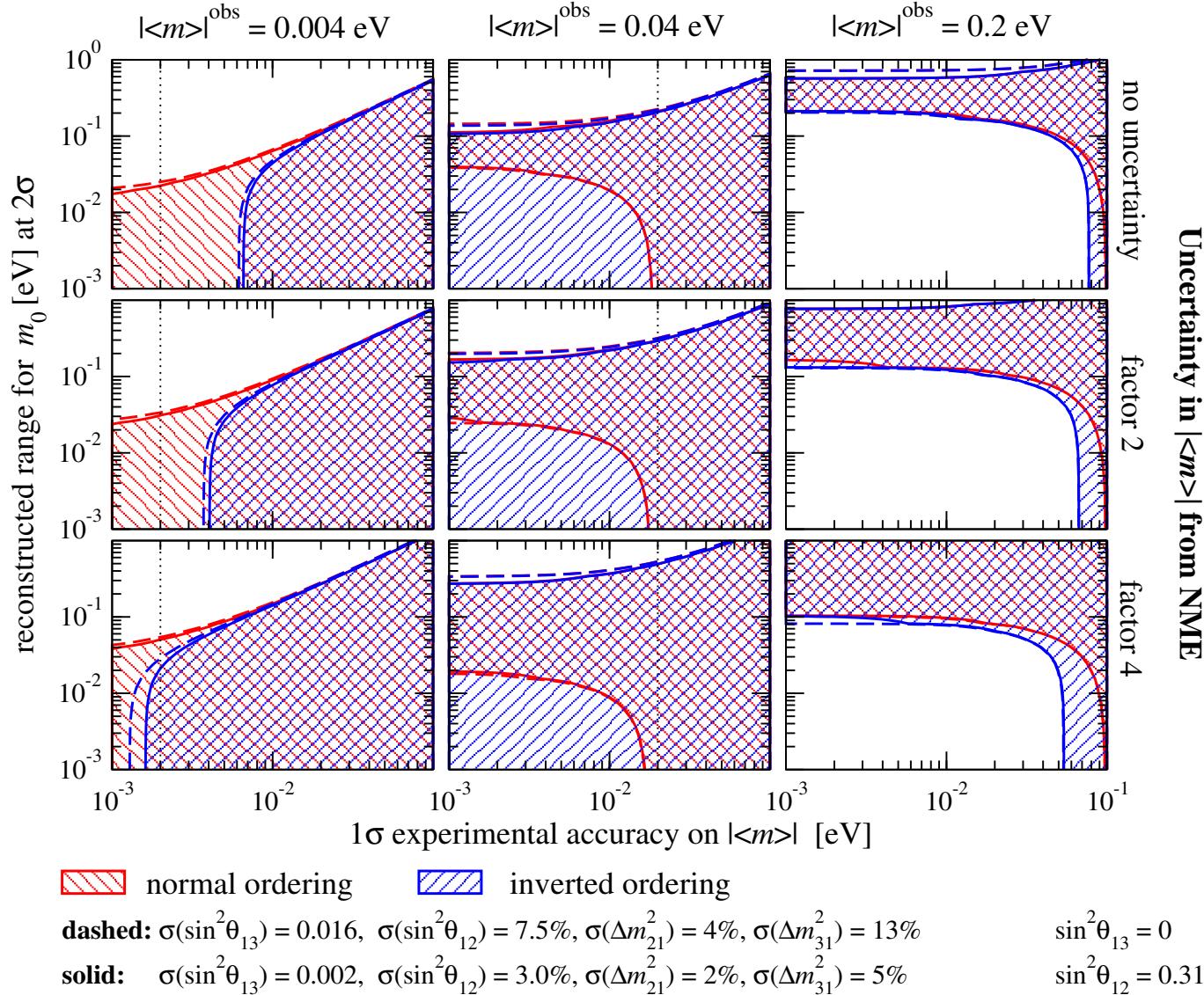
$|\mathcal{M}_0|$  is some nominal value of the NME.

$$\chi^2(\mathbf{x}_{\beta\beta}^{0\nu}, \mathbf{F}) = \min_{\xi \in [1/\sqrt{F}, \sqrt{F}]} \frac{\left[ \xi |\langle m \rangle|(\mathbf{x}) - |\langle m \rangle|^{\text{obs}} \right]^2}{\sigma_{\beta\beta}^2 + \xi^2 \sigma_{\text{th}}^2}.$$

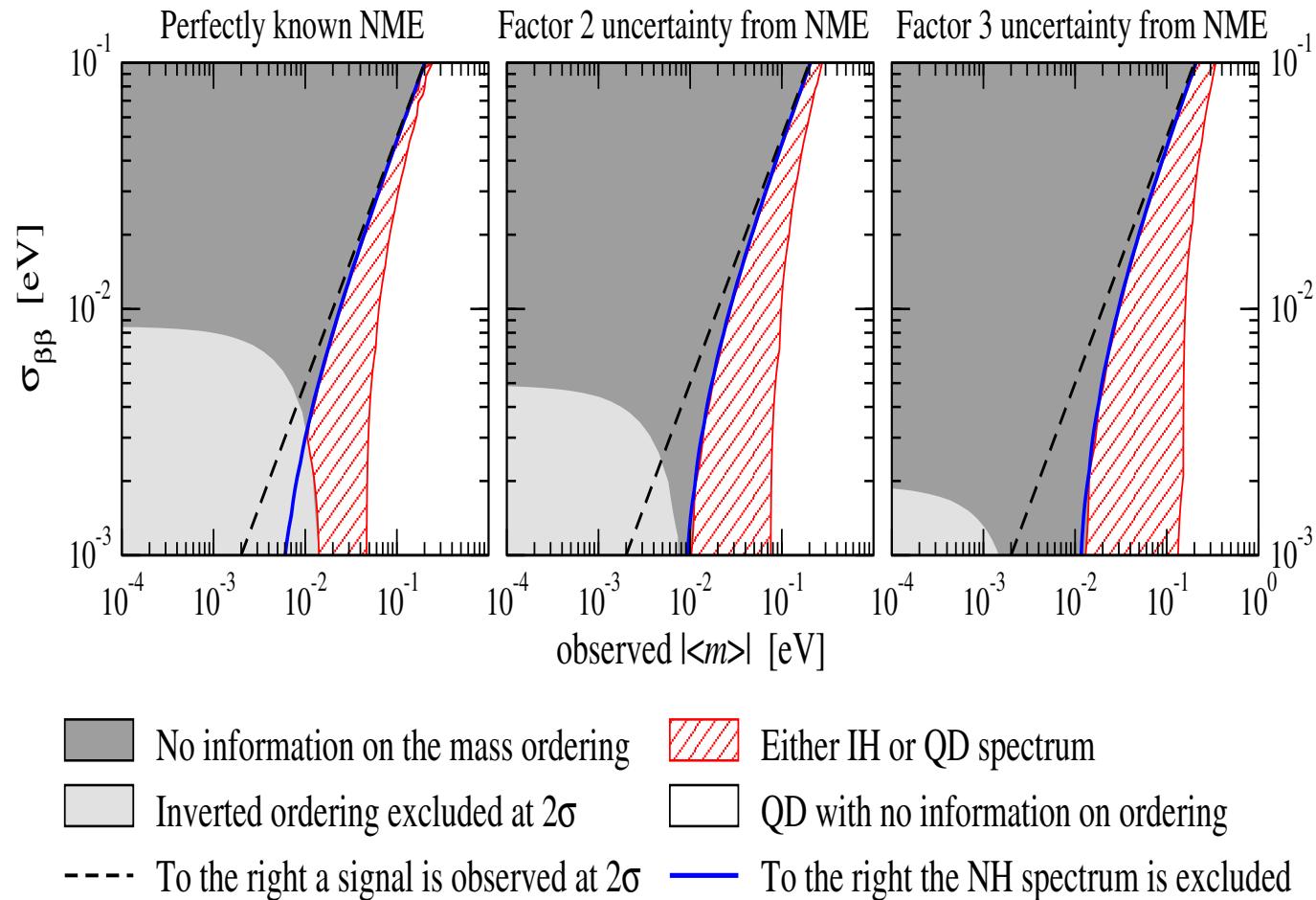
$$\xi \equiv \frac{|\mathcal{M}|}{|\mathcal{M}_0|}, \quad \xi = [1/\sqrt{F}, \sqrt{F}], \quad F \geq 1,$$

$|\mathcal{M}|$  is the *true* value of the NME.

# Absolute Neutrino Mass Scale



# Distinguishing Between Different Spectra



$$\sin^2 \theta_{13} = 0.03 \pm 0.006, \sin^2 \theta_{12} = 0.31 \pm 3\%, \Delta m_{21}^2 = 8 \times 10^{-5} \pm 2\%, |\Delta m_{31}^2| = 2.2 \times 10^{-3} \pm 3\%$$

## Majorana CPV Phases and $|\langle m \rangle|$

IH spectrum:  $m_{\min} < 0.01$  eV,  $\sin^2 \theta -$  negligible

$$\sqrt{\Delta m_{\text{atm}}^2} |\cos 2\theta_{\odot}| \leq |\langle m \rangle| \leq \sqrt{\Delta m_{\text{atm}}^2}.$$

“Just CP-violating” region:

$$(|\langle m \rangle|_{\text{exp}})_{\text{MAX}} < \sqrt{(\Delta m_{\text{atm}}^2)_{\text{MIN}}} ,$$

$$(|\langle m \rangle|_{\text{exp}})_{\text{MIN}} > \sqrt{(\Delta m_{\text{atm}}^2)_{\text{MAX}}} (\cos 2\theta_{\odot})_{\text{MAX}} ,$$

$$|\langle m \rangle| = \zeta ((|\langle m \rangle|_{\text{exp}})_{\text{MIN}} \pm \Delta) , \quad \zeta \geq 1$$

Necessary condition for establishing CP-violation:

$$1 \leq \zeta < \frac{\sqrt{(\Delta m_{\text{atm}}^2)_{\text{MIN}}}}{\sqrt{(\Delta m_{\text{atm}}^2)_{\text{MAX}}} (\cos 2\theta_{\odot})_{\text{MAX}} + 2\Delta} \simeq \frac{1}{(\cos 2\theta_{\odot})_{\text{MAX}}}$$

QD spectrum,  $m_{1,2,3} \simeq m_0 \gtrsim 0.20$  eV - similar condition:  $\Delta m_{\text{atm}}^2 \rightarrow m_0^2$ .

CPV can be established provided

- $|\langle m \rangle|$  measured with  $\Delta \lesssim 15\%$  ;
- $\Delta m_{\text{atm}}^2$  (IH) or  $m_0$  (QD) measured with  $\delta \lesssim 10\%$  ;
- $\xi \lesssim 1.5$  ;
- $\alpha_{21}$  (QD): in the interval  $\sim [\frac{\pi}{4} - \frac{3\pi}{4}]$ , or  $\sim [\frac{5\pi}{4} - \frac{3\pi}{2}]$  ;
- $\tan^2 \theta_\odot \gtrsim 0.40$  .

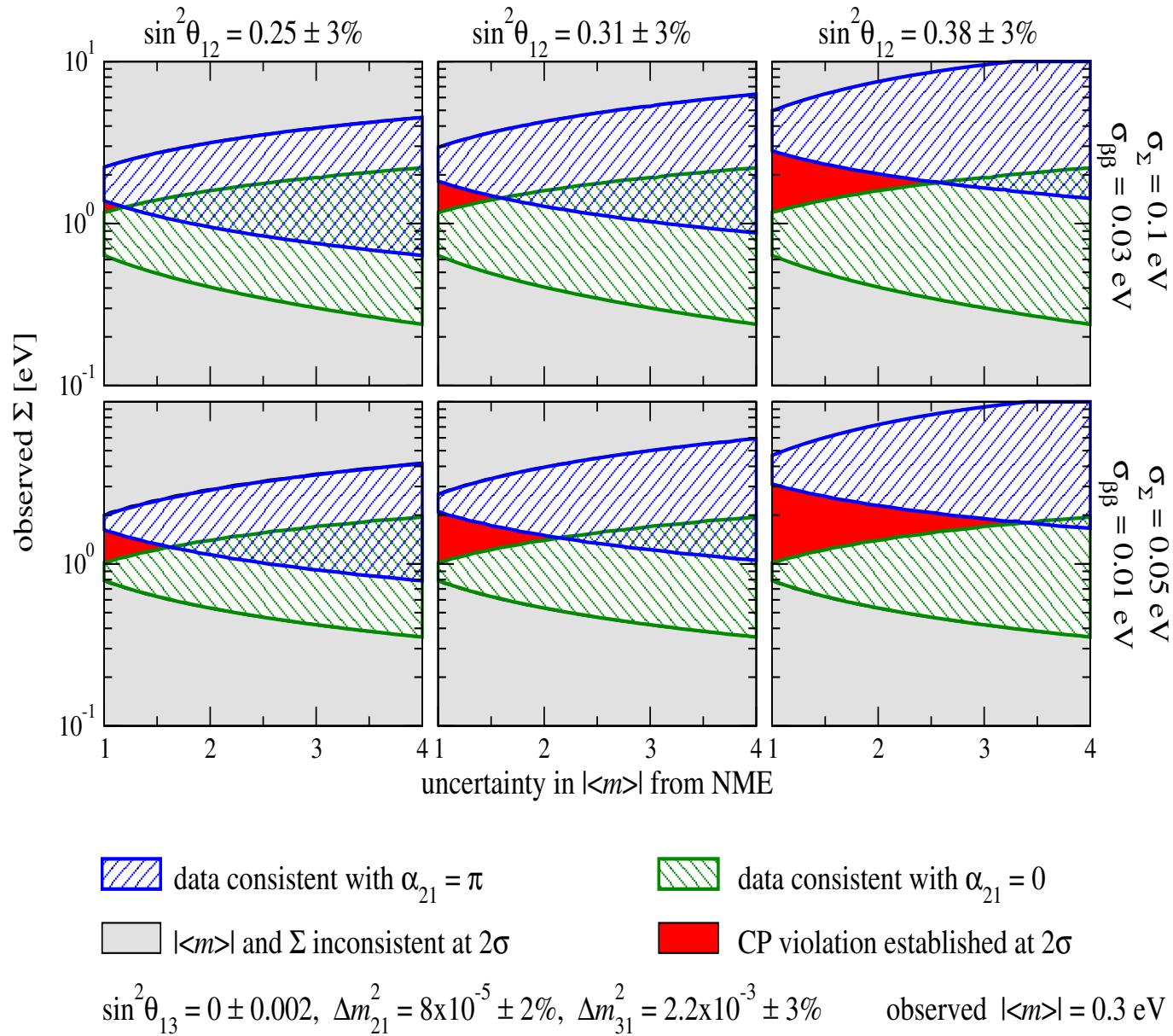
S. Pascoli, S.T.P., W. Rodejohann, 2002

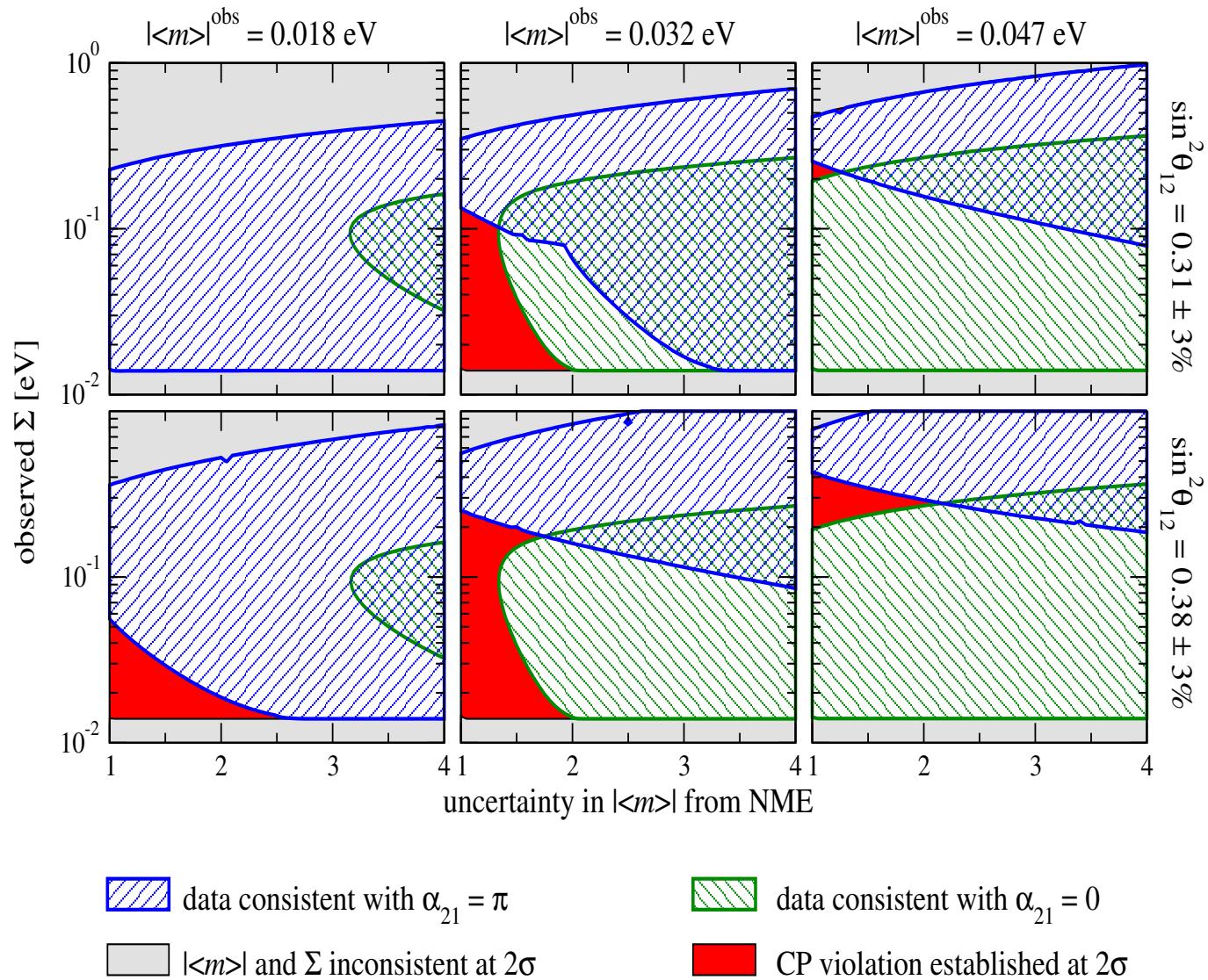
S. Pascoli, S.T.P., L. Wolfenstein, 2002

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

No “No-go for detecting CP-Violation via  $(\beta\beta)_{0\nu}$ -decay”

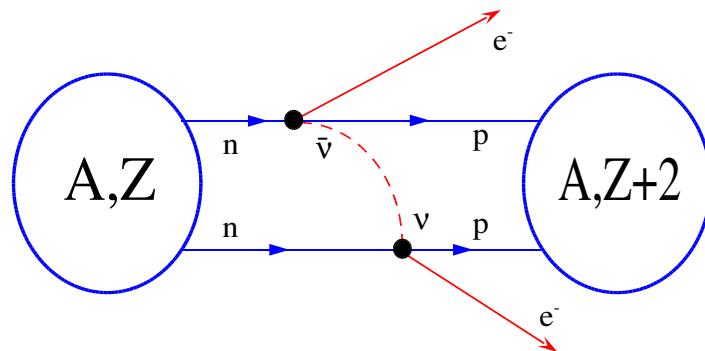
V. Barger *et al.*, 2002





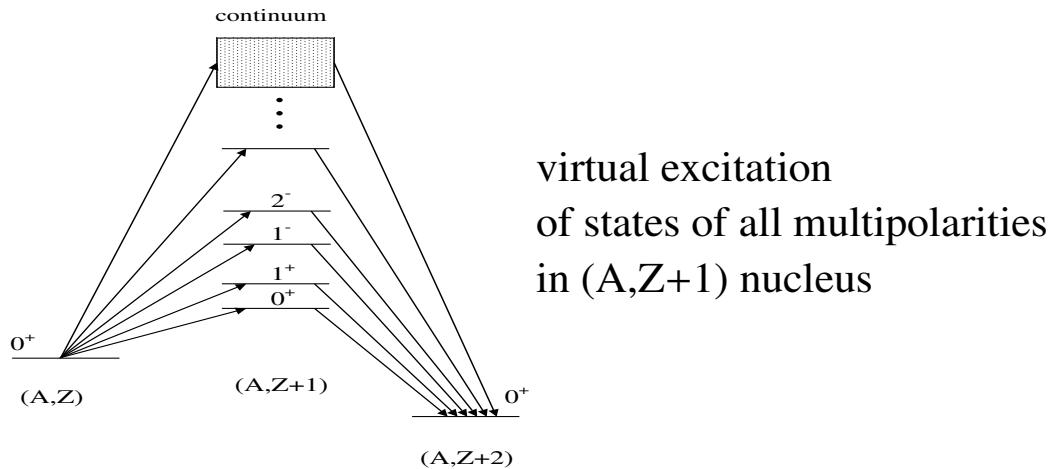
$$\sin^2 \theta_{13} = 0 \pm 0.002, \Delta m_{21}^2 = 8 \times 10^{-5} \pm 2\%, \Delta m_{31}^2 = -2.2 \times 10^{-3} \pm 3\%, \sigma_{\beta\beta} = 0.004 \text{ eV}, \sigma_\Sigma = 0.04 \text{ eV}$$

## Nuclear $0\nu\beta\beta$ -decay



strong in-medium modification of the basic process

$$dd \rightarrow uue^-e^-(\bar{\nu}_e\bar{\nu}_e)$$



# On the NME Uncertainties

The  $(\beta\beta)_{0\nu}$ -decay half-life

$$(T_{1/2}^{0\nu}(A, Z))^{-1} = |\langle m \rangle|^2 |M^{0\nu}(A, Z)|^2 G^{0\nu}(E_0, Z),$$

$G^{0\nu}(E_0, Z)$ ,  $E_0$  - known phase-space factor and energy release.

If we use a model  $M$  of the calculation of NME,

$$|\langle m \rangle|_M^2(A, Z) = \frac{1}{T_{1/2}^{0\nu}(A, Z) |M_M^{0\nu}(A, Z)|^2 G^{0\nu}(E_0, Z)}.$$

Suppose  $(\beta\beta)_{0\nu}$ -decay of several nuclei is observed.

$|\langle m \rangle|$  cannot depend on parent nucleus  $(A_j, Z_j)$ .

If the light Majorana  $\nu$ -exchange - dominant mechanism of  $(\beta\beta)_{0\nu}$ -decay, **model  $M$  for NME can be correct only if**

$$|\langle m \rangle|_M^2(A_1, Z_1) \simeq |\langle m \rangle|_M^2(A_2, Z_2) = \dots$$

For different models and the same nucleus  $(A, Z)$ ,

$$|\langle m \rangle|_{M_1}^2(A, Z) |M_{M_1}^{0\nu}(A, Z)|^2 = |\langle m \rangle|_{M_2}^2(A, Z) |M_{M_2}^{0\nu}(A, Z)|^2 = \dots,$$

$$|\langle m \rangle|_{M_2}^2(A, Z) = \eta^{M_2; M_1}(A, Z) |\langle m \rangle|_{M_1}^2(A, Z) ,$$

$$\eta^{M_2; M_1}(A, Z) = \frac{|M_{M_1}^{0\nu}(A, Z)|^2}{|M_{M_2}^{0\nu}(A, Z)|^2} .$$

Nucleus	$\eta^{M_2; M_1}$	$\eta^{M_3; M_1}$	$\eta^{M_2; M_3}$
$^{76}\text{Ge}$	0.37	0.19	1.93
$^{82}\text{Se}$	—	0.38	—
$^{100}\text{Mo}$	—	—	6.56
$^{130}\text{Te}$	0.74	0.10	7.32
$^{136}\text{Xe}$	0.53	0.02	22.42

$M_1$  (SM): E. Caurier et al., 1999;  $M_2$  (QRPA): V. Rodin et al., 2003;  
 $M_3$  (QRPA): O. Civatarese and J. Suhonen, 2003.

The observation of  $(\beta\beta)_{0\nu}$ -decay of at least 3 nuclei would be important for the solution of the problem of NME.

Table 2 suggests:  $^{76}\text{Ge}$ ,  $^{130}\text{Te}$ ,  $^{136}\text{Xe}$ .

If for some model  $M$

$$|\langle m \rangle|_M^2(A_1, Z_1) \simeq |\langle m \rangle|_M^2(A_2, Z_2) = \dots \equiv |\langle m \rangle|_0^2,$$

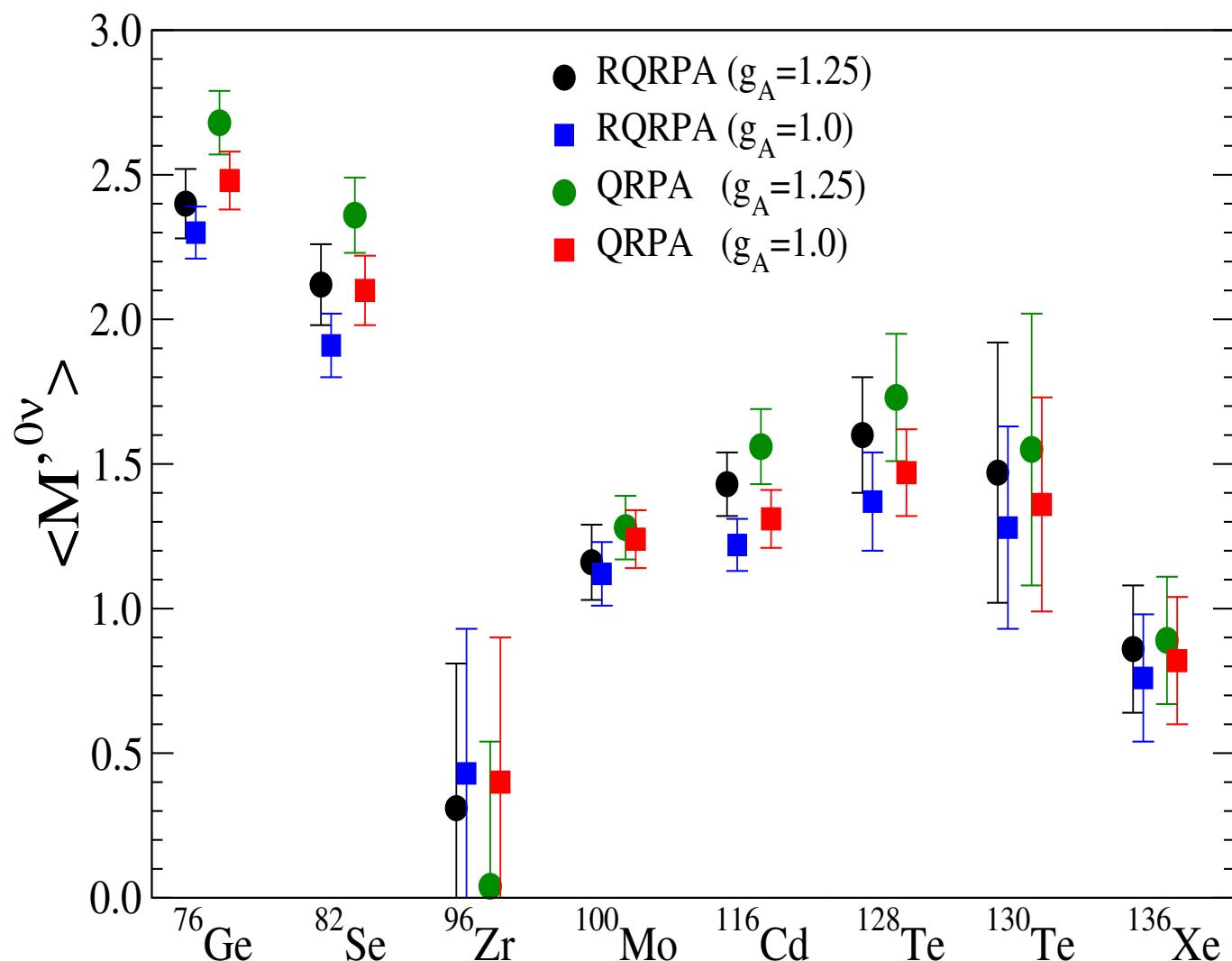
$|\langle m \rangle|_0$  - the true value (most likely).

Strong dependence of NME on  $(A, Z)$  - crucial for the test.

S. M. Bilenky, S.T.P., 2004

Encouraging results on the problem of calculating the NME ( $\xi \lesssim 1.5$ ) have been obtained recently in

V. A. Rodin, A. Faessler, F. Simkovic, P. Vogel, nucl-th/0503063



V. A. Rodin *et al.*, nucl-th/0503063

The errors have no statistical origin, just illustrate the degree of the variation of the results by changing the basis size. The “systematic error” of the QRPA (due to neglecting many-particle configurations):  $(3 \div 5) \times 10\%$ , can vary from one nucleus to another.

## Alternative Mechanisms of $(\beta\beta)_{0\nu}$ -Decay

- Light neutrino exchange
- R-parity violating SUSY
- Heavy neutrino exchange
- Right-handed weak currents

**Majorana CP-Violating Phases,  
Leptogenesis and  $(\beta\beta)_{0\nu}$ -Decay**

# $M_\nu$ from the See-Saw Mechanism

P. Minkowski, 1977.

M. Gell-Mann, P. Ramond, R. Slansky, 1979;

T. Yanagida, 1979;

R. Mohapatra, G. Senjanovic, 1980.

- Explains the smallness of  $\nu$ -masses.
- Through **leptogenesis theory** links the  $\nu$ -mass generation to the generation of baryon asymmetry of the Universe  $Y_B$ .  
S. Fukugita, T. Yanagida, 1986.
- In SUSY GUT's with see-saw mechanism of  $\nu$ -mass generation, the LFV decays

$$\mu \rightarrow e + \gamma, \quad \tau \rightarrow \mu + \gamma, \quad \tau \rightarrow e + \gamma, \text{ etc.}$$

are predicted to take place with rates within the reach of present and future experiments.

F. Borzumati, A. Masiero, 1986.

- The  $\nu_j$  are **Majorana particles**;  $(\beta\beta)_{0\nu}$ -decay is allowed.

See-Saw: Dirac  $\nu$ -mass  $m_D$  + Majorana mass  $M_R$  for  $N_R$

# Leptogenesis

$$Y_B = \frac{n_B - n_{\bar{B}}}{S} \sim 6 \times 10^{-10}$$

$$Y_B \cong -10^{-2} \quad \kappa \varepsilon$$

W. Buchmüller, M. Plümacher, 1998;

W. Buchmüller, P. Di Bari, M. Plümacher, 2004

$\kappa$ - efficiency factor;  $\kappa \sim 10^{-1} - 10^{-3}$ :  $\varepsilon \gtrsim 10^{-7}$ .

$\varepsilon$ :  $CP-$ ,  $L-$  violating asymmetry generated in out of equilibrium  $N_{Rj}$ -decays in the early Universe,

$$\begin{aligned} \varepsilon_1 &= \frac{\Gamma(N_1 \rightarrow \Phi^- \ell^+) - \Gamma(N_1 \rightarrow \Phi^+ \ell^-)}{\Gamma(N_1 \rightarrow \Phi^- \ell^+) + \Gamma(N_1 \rightarrow \Phi^+ \ell^-)} \\ &\simeq \frac{1}{8\pi} \frac{1}{(Y_\nu Y_\nu^\dagger)_{11}} \sum_{j=2,3} \text{Im}(Y_\nu Y_\nu^\dagger)_{1j}^2 \left( f(M_j^2/M_1^2) + g(M_j^2/M_1^2) \right) . \end{aligned}$$

$$f(x) = \sqrt{x} \left( 1 - (1+x) \ln \left( \frac{1+x}{x} \right) \right) , \quad g(x) = \frac{\sqrt{x}}{1-x}$$

M.A. Luty, 1992;

L. Covi, E. Roulet, and F. Vissani, 1996

$$\frac{1}{\kappa} \simeq \frac{3.3 \times 10^{-3} \text{ eV}}{\tilde{m}_1} + \left( \frac{\tilde{m}_1}{0.55 \times 10^{-3} \text{ eV}} \right)^{1.16} , \quad \tilde{m}_1 \equiv \frac{v_u^2}{M_1} (Y_\nu Y_\nu^\dagger)_{11}$$

G. F. Giudice *et al.*, 2004

Assume:

- $M_{SUSY} \sim (100 - 600)$  GeV (LHC); SUSY broken at  $M_X > M_R$ .
- $M_R$ :  $M_1 \ll M_2 \ll M_3$ ,  $M_3 \gtrsim 5 \times 10^{13}$  GeV (GUT).  
( $m_\nu \cong m_D^2/M_R$ ;  $m_D \sim 175$  GeV,  $m_\nu \sim 5 \times 10^{-2}$  eV, then  $M_R \sim 6 \times 10^{14}$  GeV.)

Impose:

- $BR(\mu \rightarrow e + \gamma) < 1.2 \times 10^{-11}$  (MEGA)
- Leptogenesis successful: BAU correctly reproduced.

$\omega$  - leptogenesis CPV parameter;  $\omega$  - complex.

IH spectrum:

A.  $\mathbf{Y}_{\nu 21} = 0$ :

$$\tan \omega = e^{-i\alpha/2} \tan \theta_{12}.$$

B.  $\mathbf{Y}_{\nu 22} \cong 0$ , neglecting  $s_{13}$ :

$$\tan \omega = -e^{-i\alpha/2} \cot \theta_{12}.$$

Leptogenesis:  $\omega$ -complex; thus  $\alpha \neq 0, \pi$ , CP-violating values

$$|\langle m \rangle| \cong \sqrt{\Delta m_{13}^2} |\cos^2 \theta_{12} + e^{i\alpha} \sin^2 \theta_{12}|$$

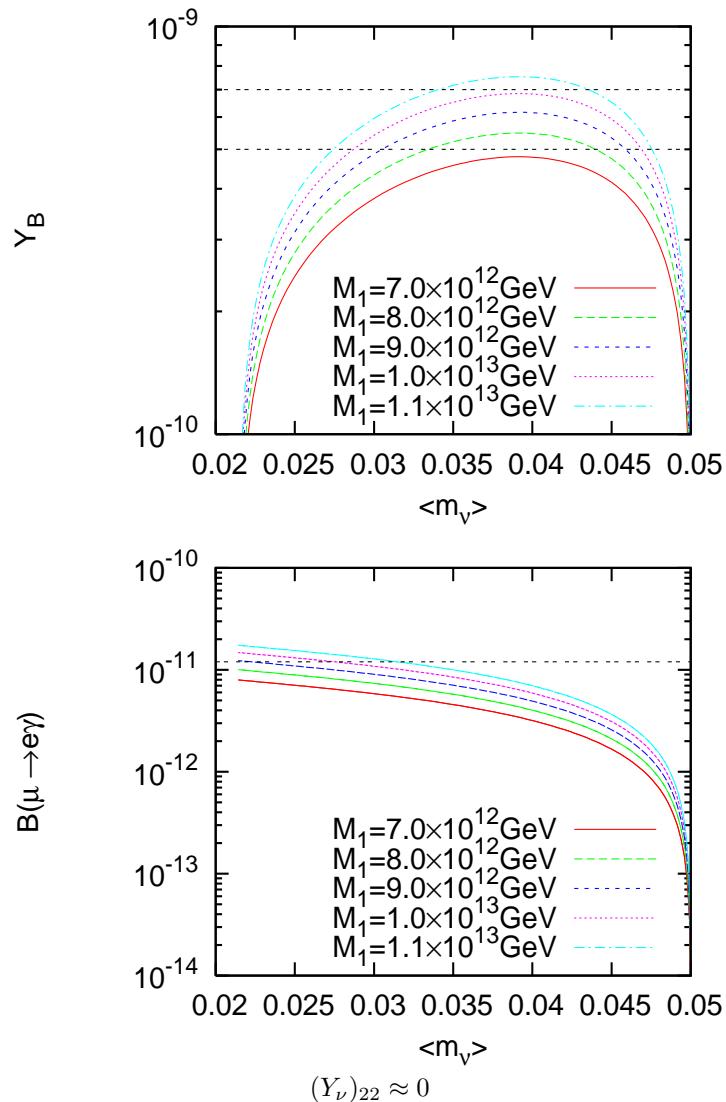
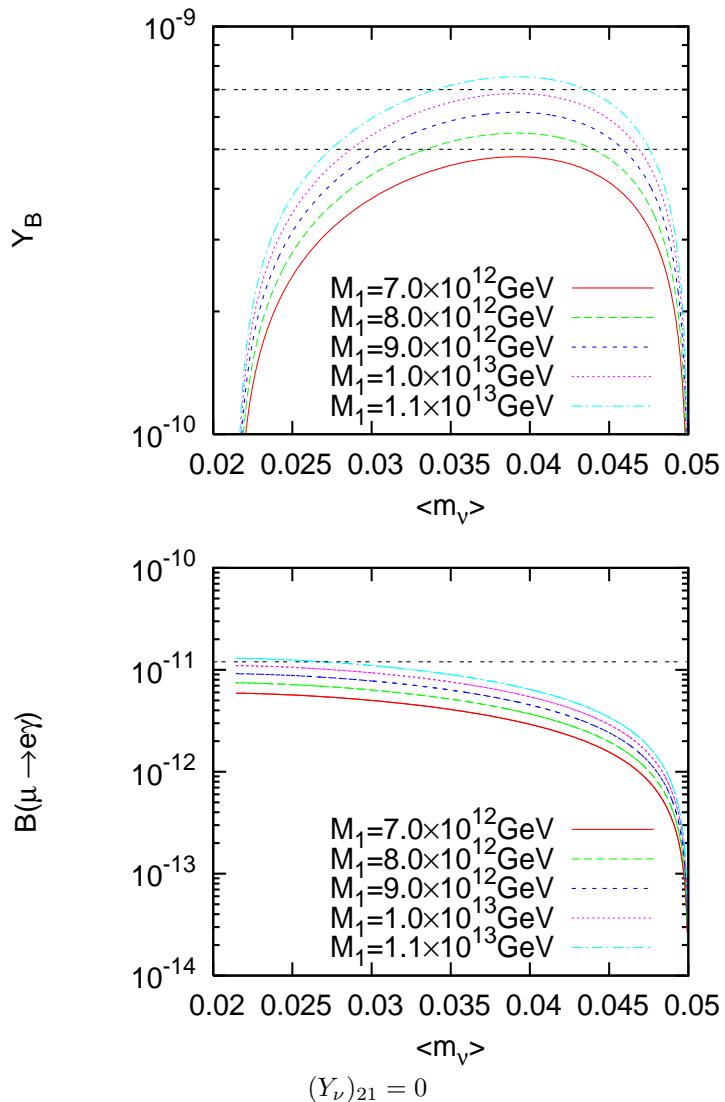


Figure 6: Predicted values of  $Y_B$  and  $B(\mu \rightarrow e\gamma)$  for  $s_{13} = 0$ . The SUSY parameters are fixed as  $m_0 = m_{1/2} = 450$ ,  $A_0 = 0$ , and  $\tan \beta = 5$ .

## Conclusions

$(\beta\beta)_{0\nu}$ -decay experiments have remarkable physics potential:

- Can establish the Majorana nature of  $\nu_j$
- Can provide unique information on the  $\nu$  mass spectrum
- Can provide unique information on the absolute scale of  $\nu$  masses
- Can provide information on the Majorana CPV phases

The knowledge of the values of the relevant  $(\beta\beta)_{0\nu}$ -decay NME with a sufficiently small uncertainty is crucial for obtaining quantitative information on the neutrino mass and mixing parameters from a measurement of  $\Gamma(\beta\beta)_{0\nu}$ .

The precision in the measurement of  $\Gamma(\beta\beta)_{0\nu}$  will also be very important for the quantitative interpretation of the data.

**Coordinated attack on the NME problem needed.**

There are relatively few theorists working in  $\beta\beta$  decay, and their efforts have been fragmented.

More collaborations, postdoctoral and Ph.D projects, meetings, etc., would make progress faster.

How to reduce the uncertainty in  $M^{0\nu}$  to  $\approx 10 \div 20\%$ ?

- The accuracy is not reachable in the present nuclear models.
- New developments in the nuclear structure theory are needed.
- Long-term theoretical R&D programme.
- Diversification of independent research groups working in the field would be very useful.
- Any monopoly on the NME calculations must be avoided.
- Support from experimental study of charge-exchange reactions, muon capture, etc., is HIGHLY REQUIRED.