

DBD-NME in the QRPA method: a critical review

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*some of the results presented in this talk have been obtained in
collaboration with J. Suhonen. Univ. of Jyväskylä .

Topics and Motivations

Basic notions about the QRPA

Nuclear matrix elements in the QRPA

Parameters and dependence upon them

Beyond the QRPA

Some results (symmetry aspects)

Conclusions

- Nuclear structure effects on the matrix elements governing the mass sector of the neutrinoless double beta decay (g_{pp} -dependence)

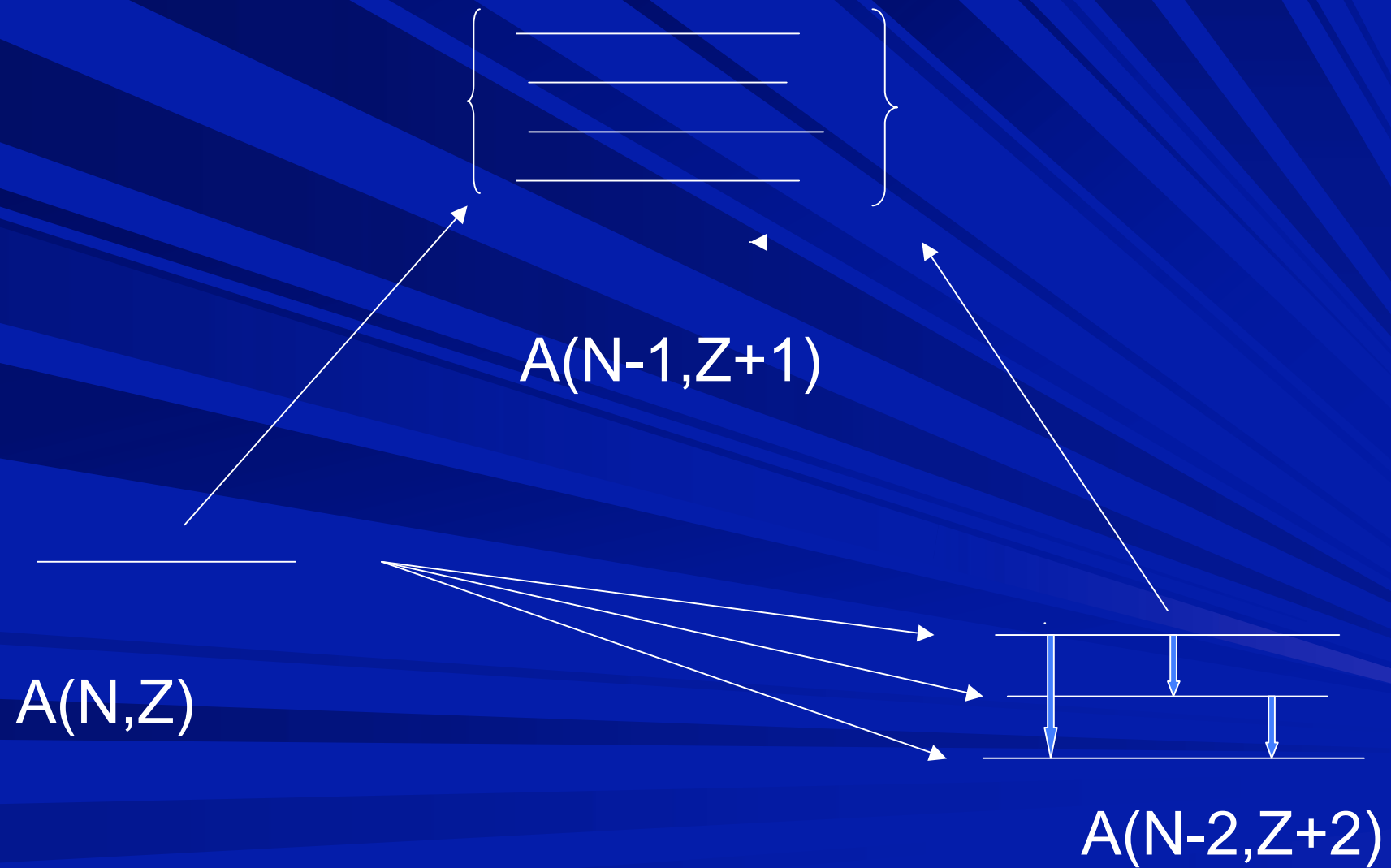
- Effective parameters: single-beta decay and double beta decay?
two-neutrino or zero-neutrino modes?

- Universality (if any) of the matrix elements

- * as a rule:

- if a certain approximation does not work in a simple model then there is certainly a problem with it in realistic cases.*

Schematic view



2-neutrino mode

$A(N,Z) \rightarrow A(N-2,Z+2) + 2 \text{ electrons} + 2 \text{ neutrinos}$

$$\left[t_{1/2}^{(2\nu)} (0_I^+ \rightarrow 0_F^+) \right]^{-1} = G^{(2\nu)} \left| M_{\text{GT}}^{(2\nu)} \right|^2$$

- a) Lepton number is conserved
- b) Suppressed by kinematics (four leptons in the final state)
- c) Independent of neutrino properties

$$\left[t_{1/2}^{(2\nu)} (0_I^+ \rightarrow 0_F^+) \right]^{-1} = G^{(2\nu)} \left| M_{\text{GT}}^{(2\nu)} \right|^2$$

$$M_{\text{GT}}^{(2\nu)} = \sum_n \frac{(0_F^+ \parallel \sum_j \sigma(j) t_j^- \parallel 1_n^+) (1_n^+ \parallel \sum_j \sigma(j) t_j^- \parallel 0_I^+)}{(\frac{1}{2} Q_{\beta\beta} + E_n - M_I)/m_e + 1}$$

0-neutrino mode

$A(N,Z) \rightarrow A(N-2,Z+2) + 2 \text{ electrons}$

$$\left[t_{1/2}^{0\nu}(J_f) \right]^{-1} = C_{mm}^{(0\nu)} \frac{\langle m_\nu \rangle^2}{m_e^2}$$

$$C_{mm}^{(0\nu)} = G_1^{(0\nu)} \left[(M_{GT}^{(0\nu)}) (1 - \chi_F) \right]^2$$

a) lepton number is not conserved

b) not suppressed by kinematics (two leptons in the final state)

$$\left[t_{1/2}^{0\nu}(J_f)\right]^{-1} = C_{mm}^{(0\nu)} \frac{\langle m_\nu \rangle^2}{m_e^2}$$

$$C_{mm}^{(0\nu)} = G_1^{(0\nu)} \left[(M_{GT}^{(0\nu)}) (1 - \chi_F) \right]^2$$

$$\chi_F = \frac{M_F^{(0\nu)}}{M_{GT}^{(0\nu)}}$$

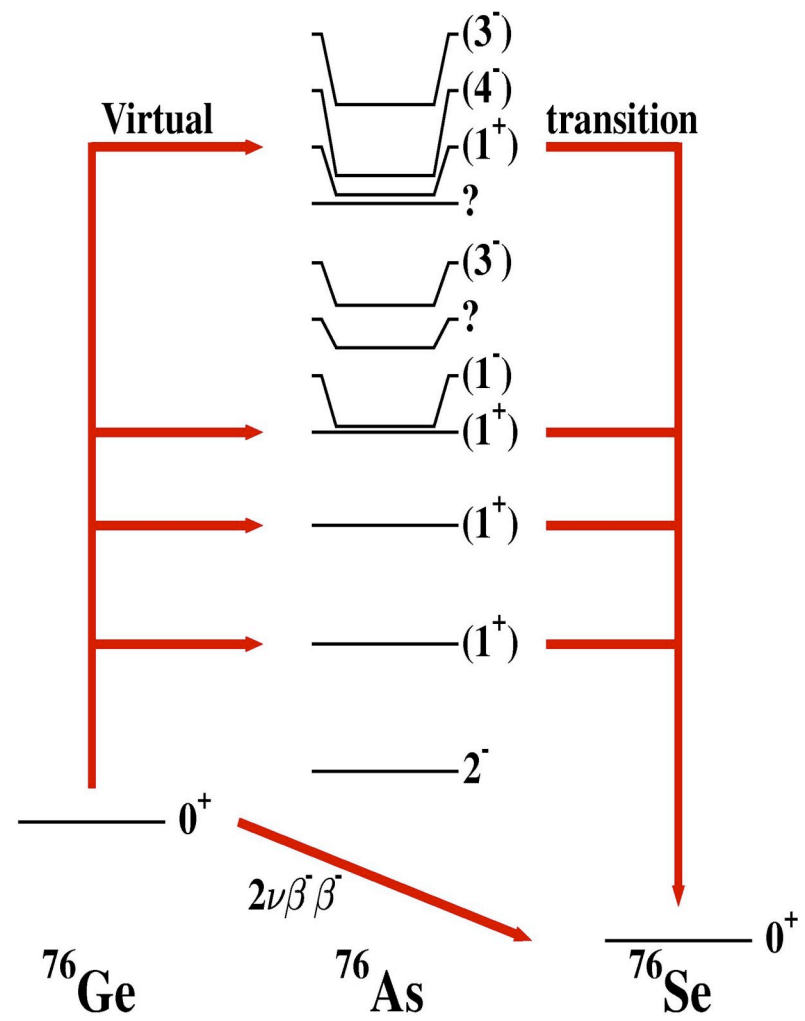
$$M_{GT}^{(0\nu)} = (m_e R)^{-2}$$

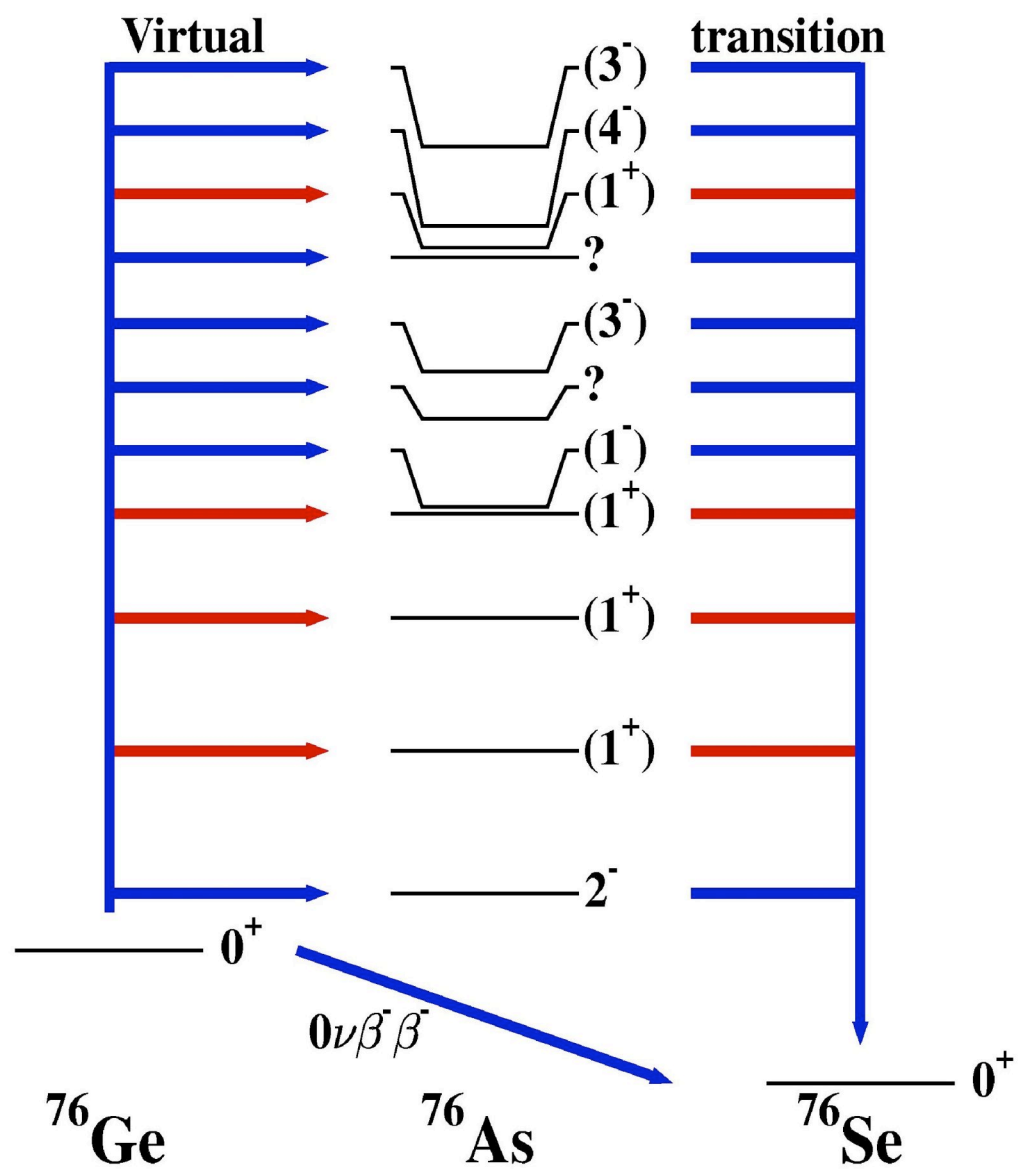
$$\sum_{ij} \sum_a \langle 0_F^+ || h_+(r_{ij}, E_a) \sigma(i) \sigma(j) \tau(i)^- \tau(j)^- || 0_I^+ \rangle$$

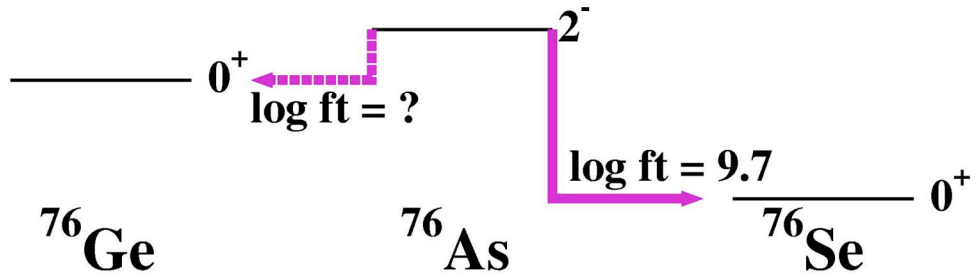
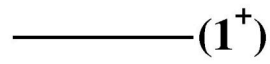
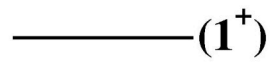
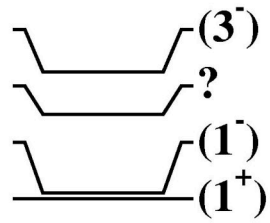
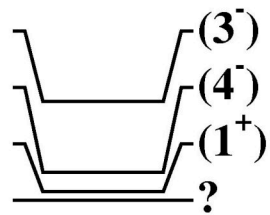
Basic Information (consistency test)

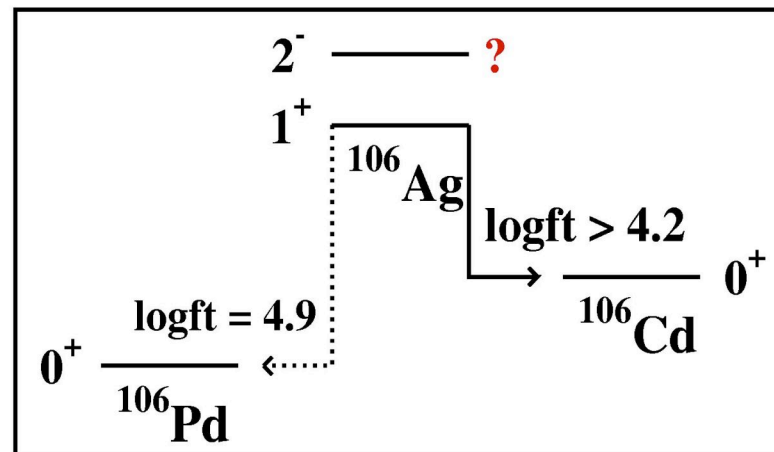
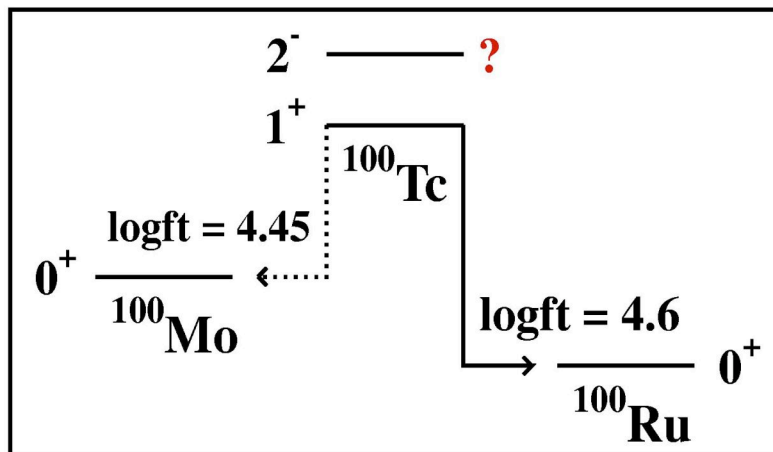
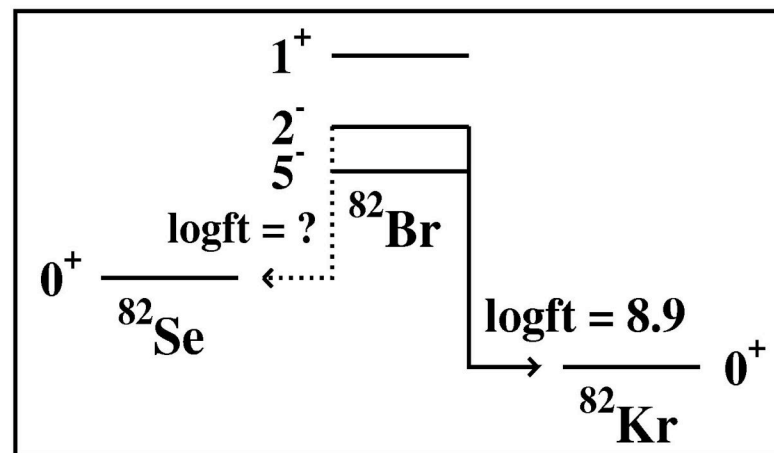
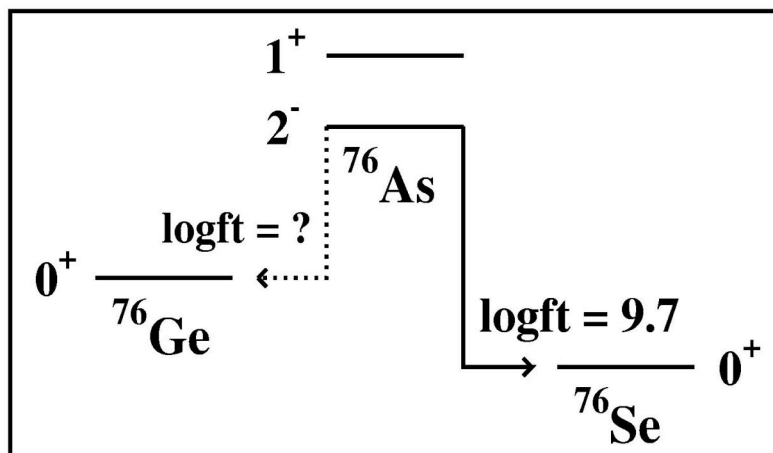
- Single beta decay and EC transitions
- Energy spectra, electromagnetic transitions
- Particle transfer and charge exchange
($^3\text{He}, t$), (p, n) data.

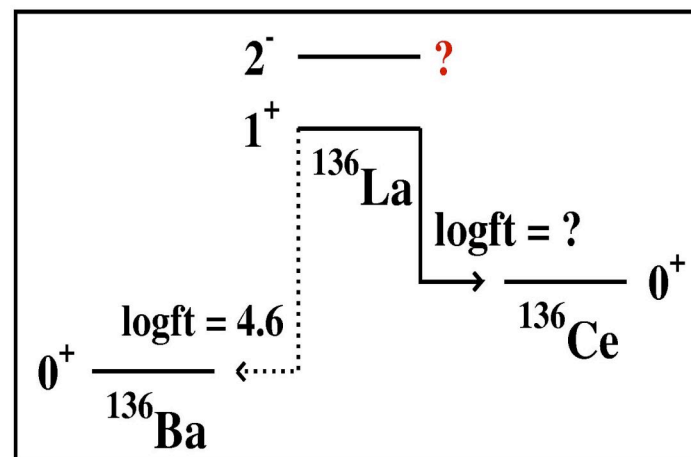
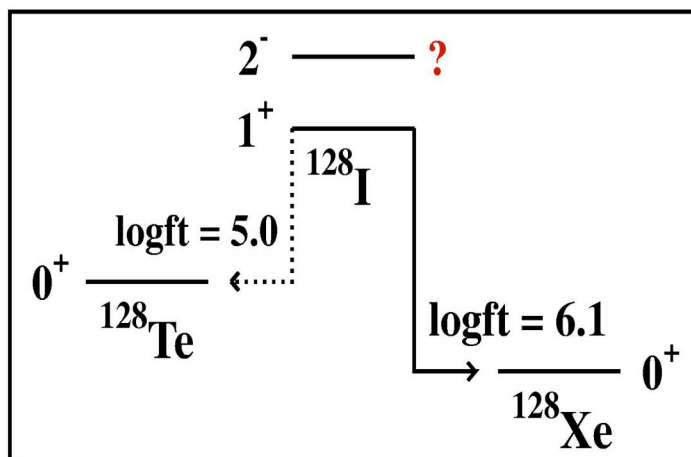
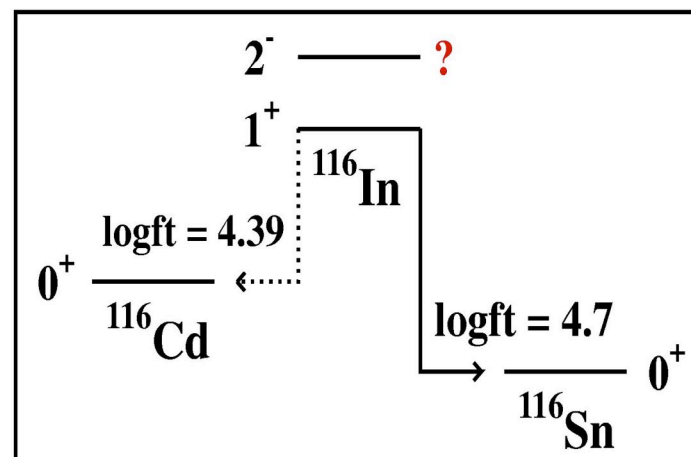
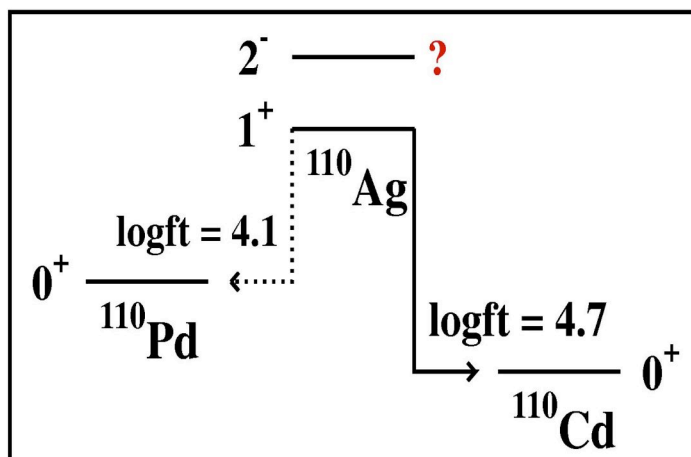
Practical rule: The information obtained by “theoretical reproducing” a single DBD transition does not mean anything, unless these related observables are also reproduced.







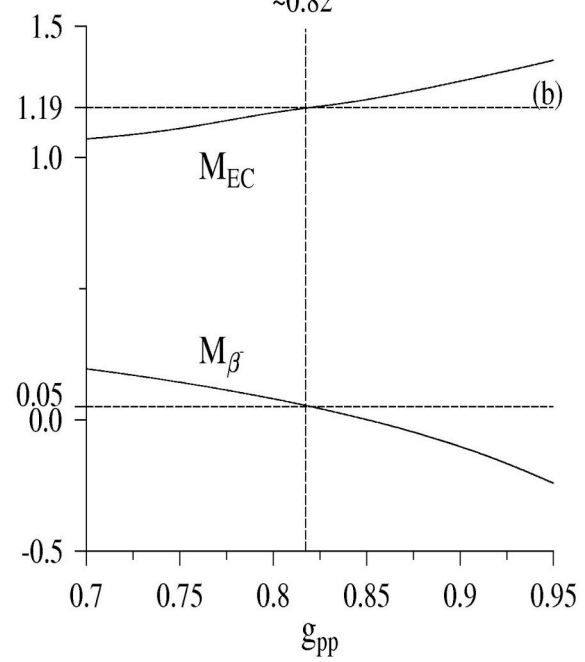
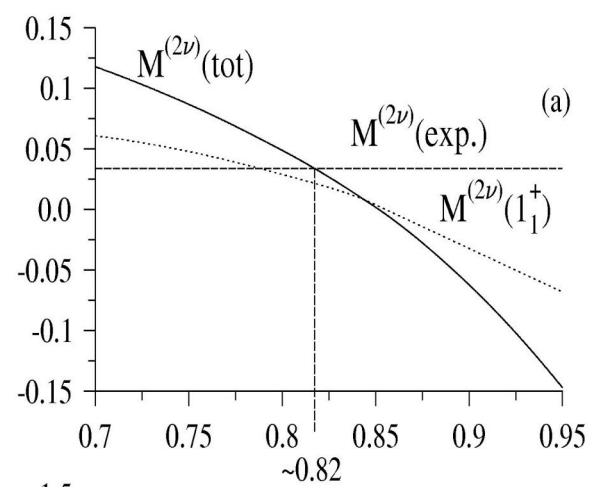




G_{pp} dependence

- Transformation from the single-particle to the quasi-particle basis (particle number is lost, isospin symmetry is lost).
- In open shells the inclusion of pair terms of the residual proton-neutron interaction induces attractive pairing-like effects which compete with repulsive spin-dependent terms. This effect is also present in shell model results.
- The renormalization of the particle-particle interactions may induce a symmetry breaking, which invalidates the qrpa approach
- Mean field terms and residual interactions (they do not talk to each other)

$$\begin{aligned}
H_{pn} = & \frac{1}{2J+1} \sum_{pn,M} < p || \emptyset(J) || n > < p' || \emptyset(J) || n' >^* \\
& \left\{ \chi \left([a_p^\dagger a_{\bar{n}}]^{JM} [a_{p'}^\dagger a_{\bar{n}'}]^\dagger \overline{J\overline{M}} + [a_p^\dagger a_{\bar{n}}]^\dagger \overline{J\overline{M}} [a_{p'}^\dagger a_{\bar{n}'}]^{JM} \right) \right. \\
& \left. - \kappa \left([a_p^\dagger a_{\bar{n}}^\dagger]^{JM} [a_{p'}^\dagger a_{\bar{n}'}^\dagger]^\dagger \overline{J\overline{M}} + [a_p^\dagger a_{\bar{n}}^\dagger]^\dagger \overline{J\overline{M}} [a_{p'}^\dagger a_{\bar{n}'}^\dagger]^{JM} \right) \right\},
\end{aligned}$$



Beyond the QRPA

- Inclusion of terms which go beyond the quasi-boson approximation: the procedure is not supported by self-consistency requirements and it introduces spurious effects.
- Among the attempts: “fully renormalized qrpa”

$$\begin{aligned} H &= E_p N_p + E_n N_n \\ &+ \lambda_1 A^\dagger A \\ &+ \lambda_2 (A^\dagger A^\dagger + A A) \\ &- \lambda_3 (A^\dagger B + B^\dagger A) \\ &- \lambda_4 (A^\dagger B^\dagger + B A) \\ &+ \lambda_5 B^\dagger B \\ &+ \lambda_6 (B^\dagger B^\dagger + B B) \end{aligned}$$

$$\Gamma = X \tilde{A}^\dagger - Y \tilde{A}$$

$$\tilde{A}^\dagger = D^{-1/2} A^\dagger,$$

$$\tilde{A} = D^{-1/2} A,$$

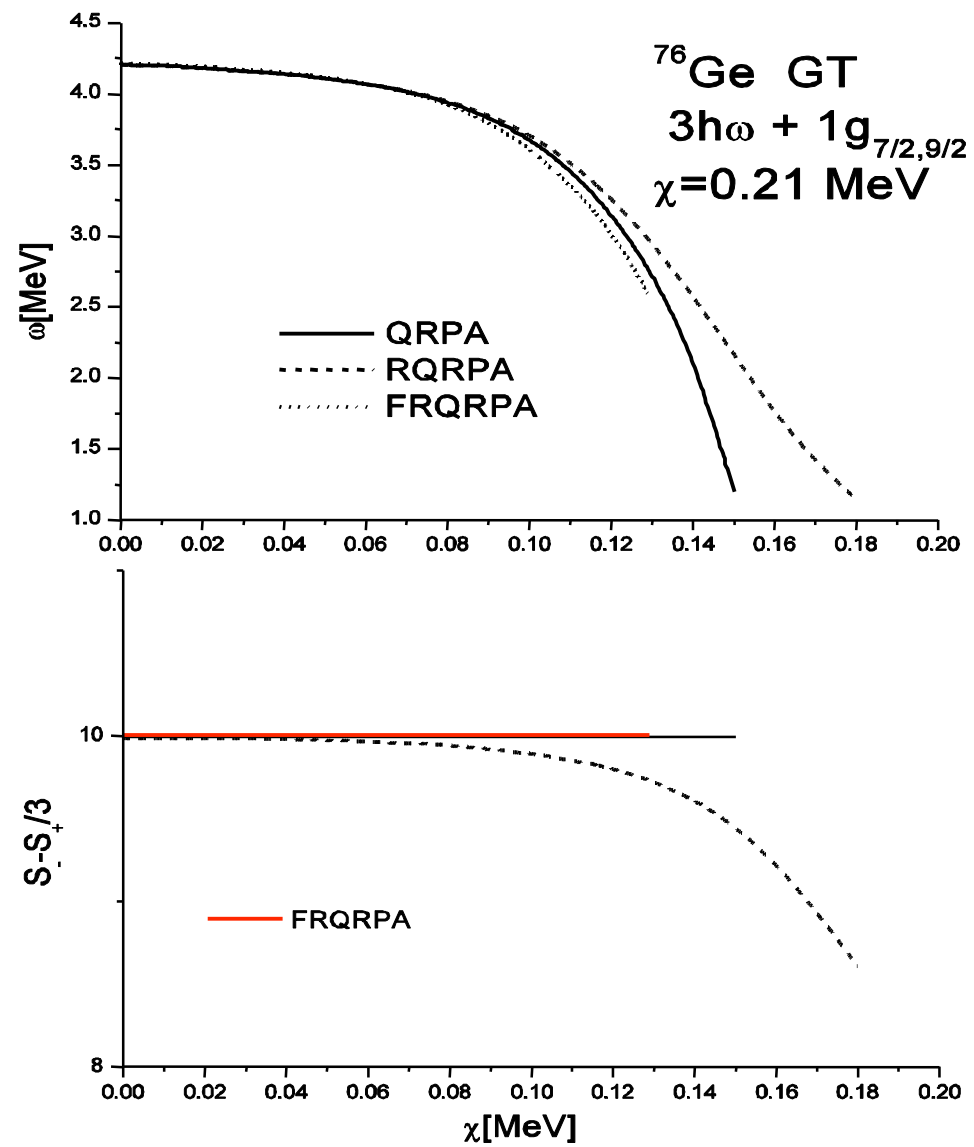
$$[\tilde{A}, \tilde{A}^\dagger] = 1,$$

$$D = \langle [A, A^\dagger] \rangle = 1 - \langle \frac{N_p + N_n}{2 \Omega} \rangle .$$

$$\overline{A}_{pn}^\dagger = \left(A_{pn}^\dagger + \frac{u_n v_n B_{pn}^\dagger - u_p v_p B_{pn}}{v_n^2 - v_p^2} \right) D_{pn}^{-1/2},$$

$$A_{pn}^\dagger(JM) \equiv [\alpha_p^\dagger \alpha_n^\dagger]^{JM}, \quad B_{pn}^\dagger(JM) = [\alpha_p^\dagger \alpha_{\bar{n}}]^{JM},$$

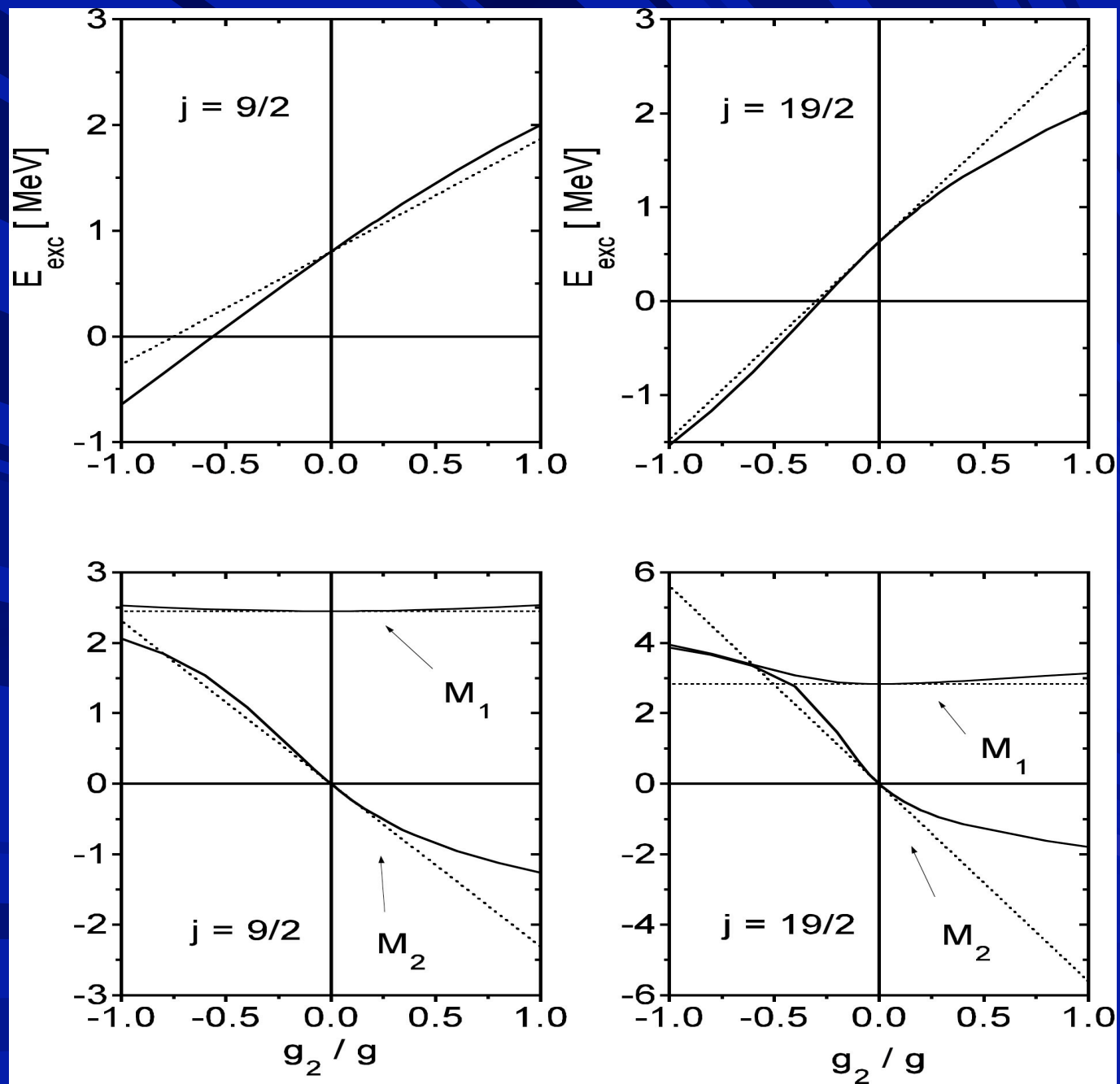
$$D_{pn} \equiv 1 - \langle 0 | \hat{N}_p + \hat{N}_n | 0 \rangle - (1 - v_p^2 - v_n^2) R_{np},$$

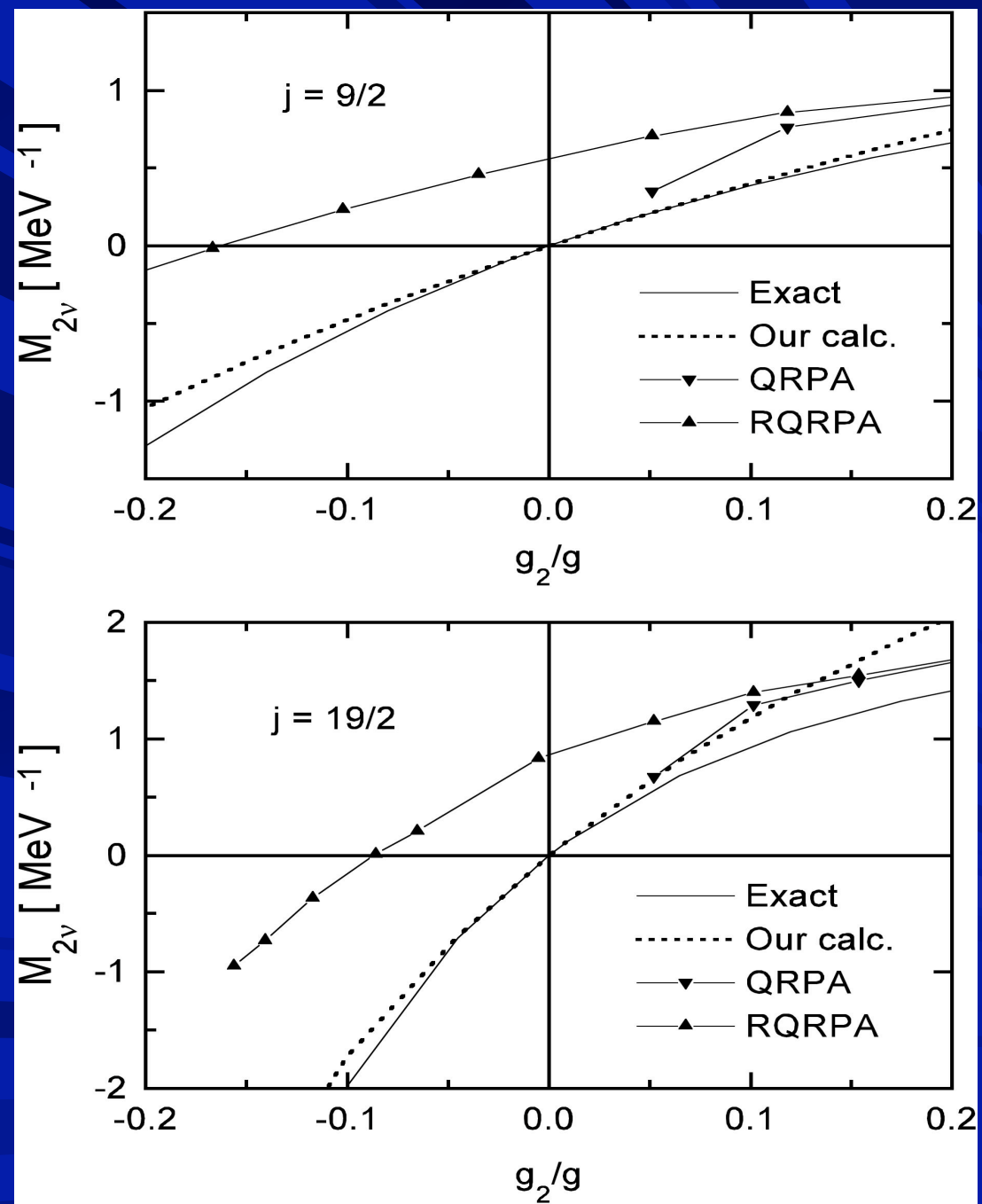


Symmetries

Isospin violations (mean field effects)

- 1) Mean field approximations (like BCS) break isospin and number symmetries.
- 2) The QRPA also break these symmetries, but in a different way.
- 3) Both mechanisms are not coupled.

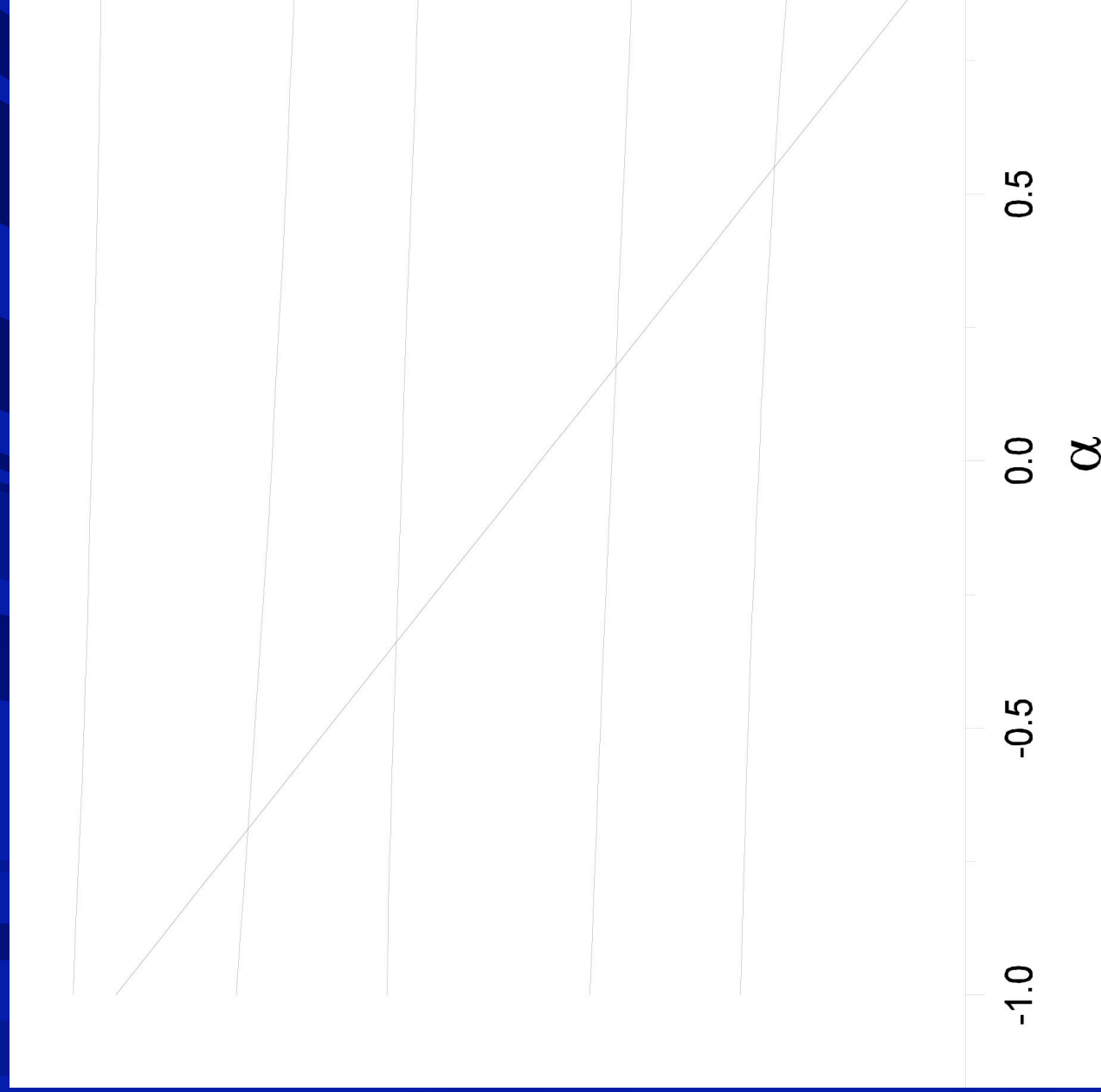


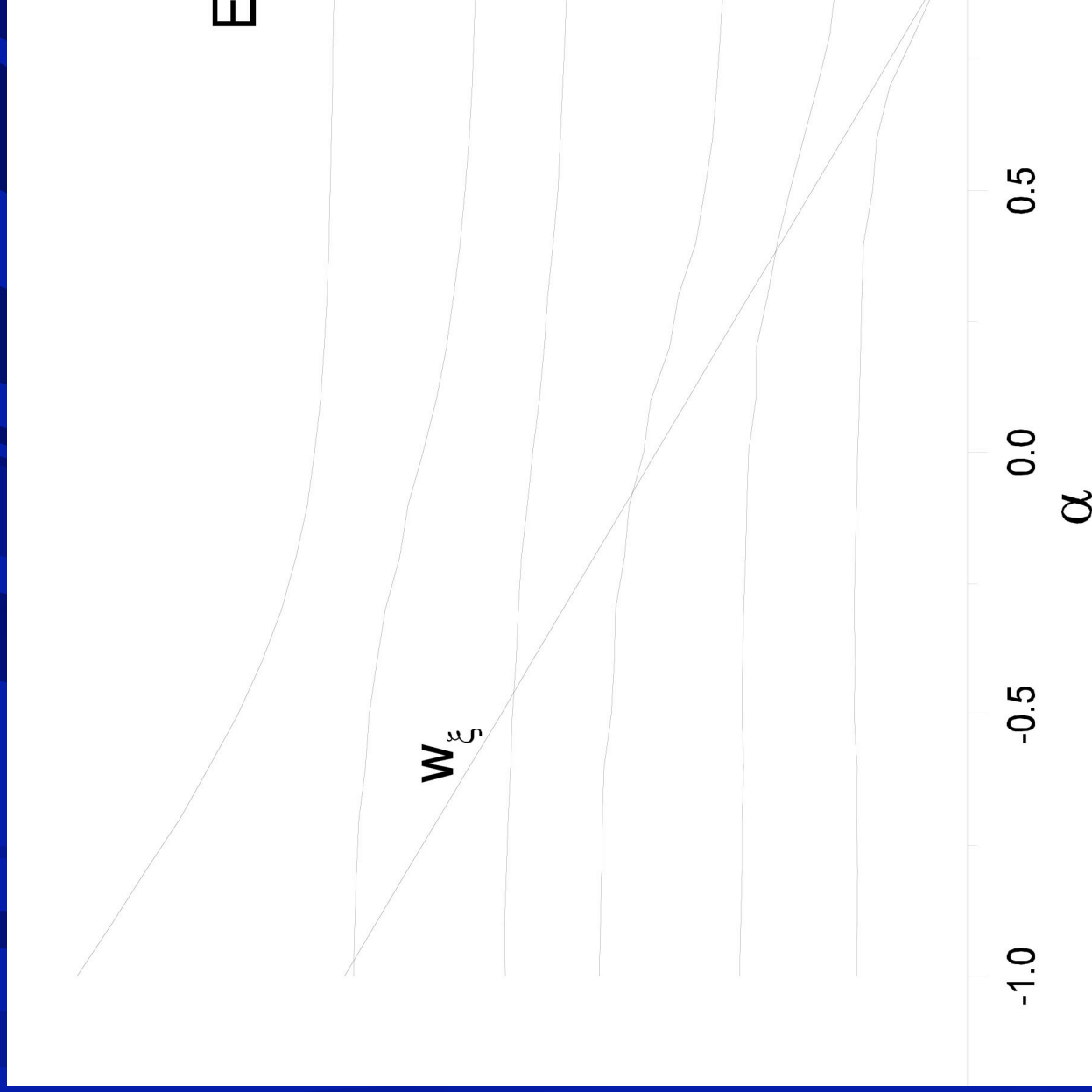


Spurious roots

The QRPA do have spurious roots (a zero energy root) which originates in the quasiparticle pair terms of symmetry operators (like number, isospin, or angular momentum in the deformed case). They represent collective rotations in the space of the symmetry variables.

The root is coupled to the “intrinsic” roots, sometimes it is mistaken as a real intrinsic root.





Results

Standard elements are:

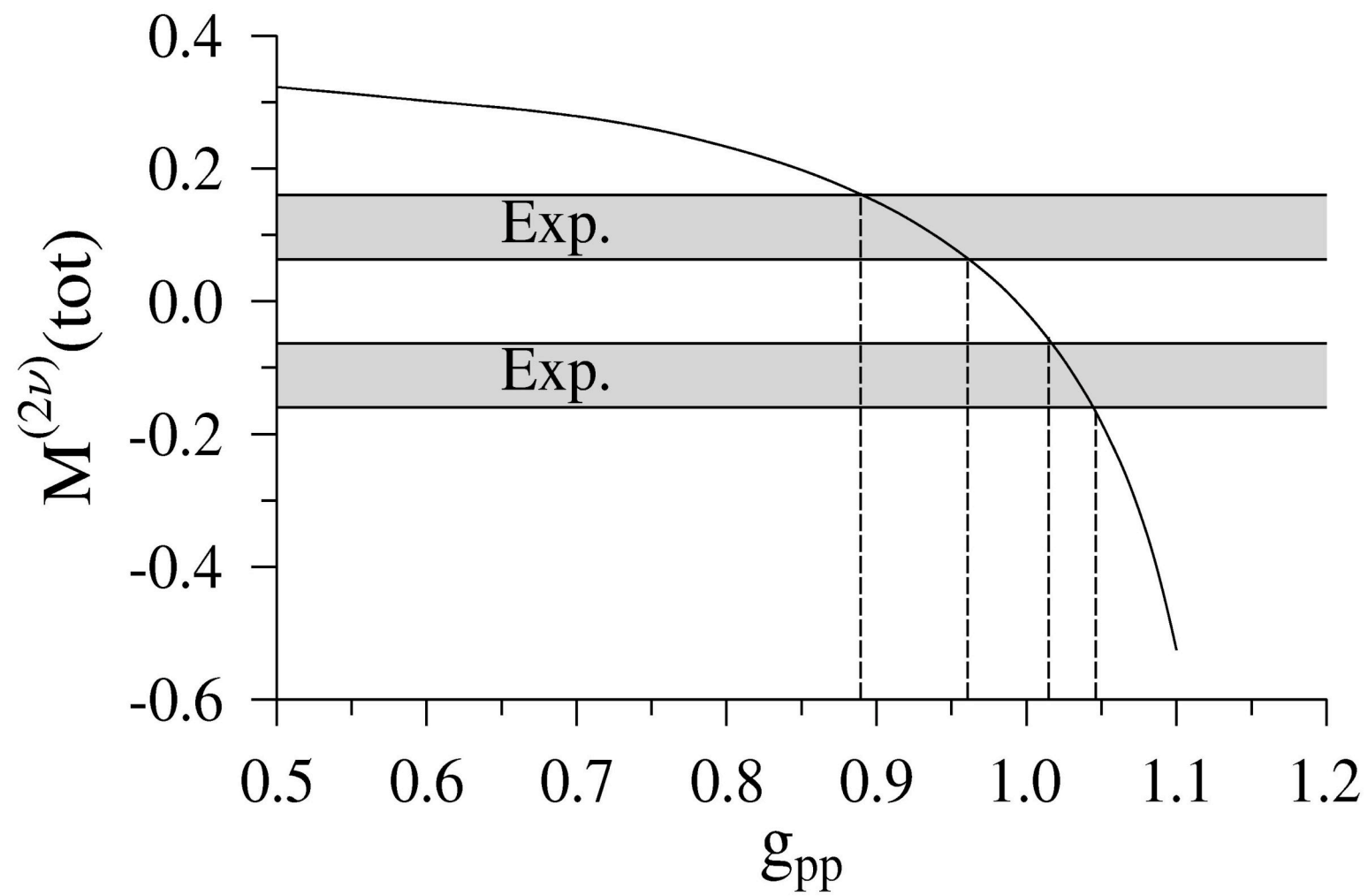
- Saxon-Woods or harmonic oscillator potentials.
- Effective interactions constructed from one pion exchange potentials.
- Gap parameters adjusted to reproduce odd-even mass differences around the double beta decay systems.
- Residual interactions adjusted to reproduced the Gamow-Teller resonance and other giant states if their energies are known.

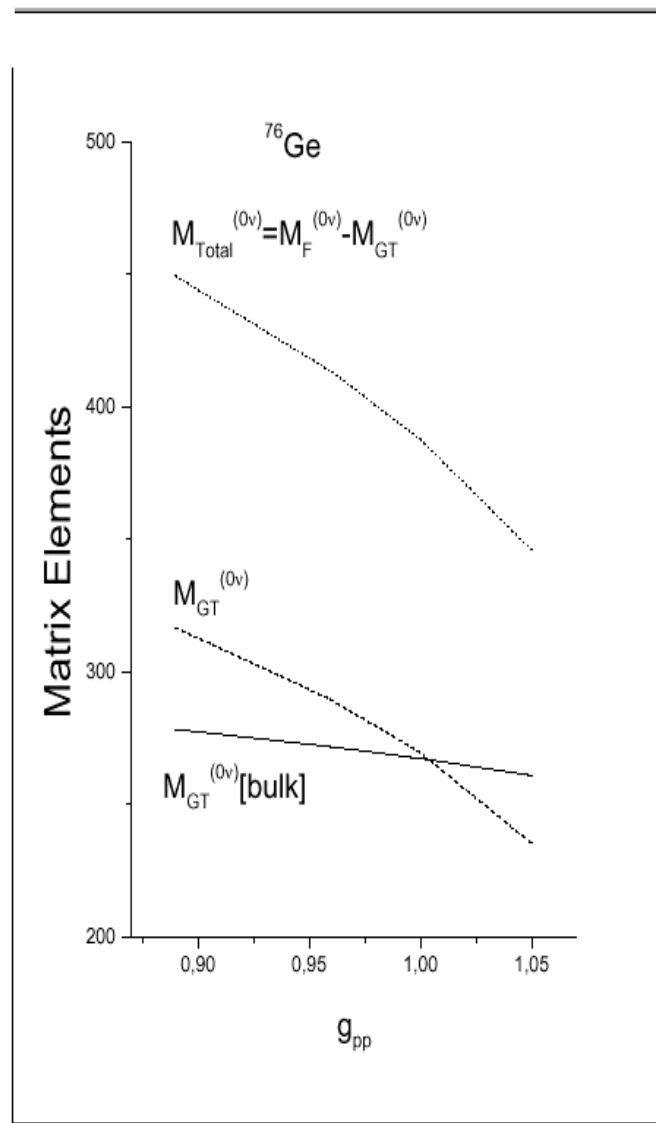
$$F_N = C_{mm}^{(0\nu)} T_{1/2}$$

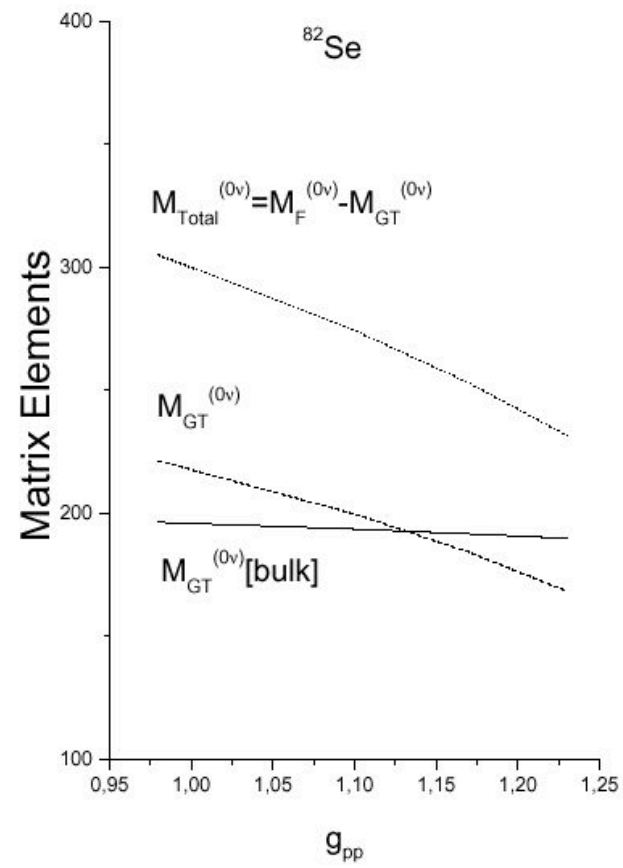
$$\left[t_{1/2}^{0\nu}(J_f) \right]^{-1} = C_{mm}^{(0\nu)} \frac{\langle m_\nu \rangle^2}{m_e^2}$$

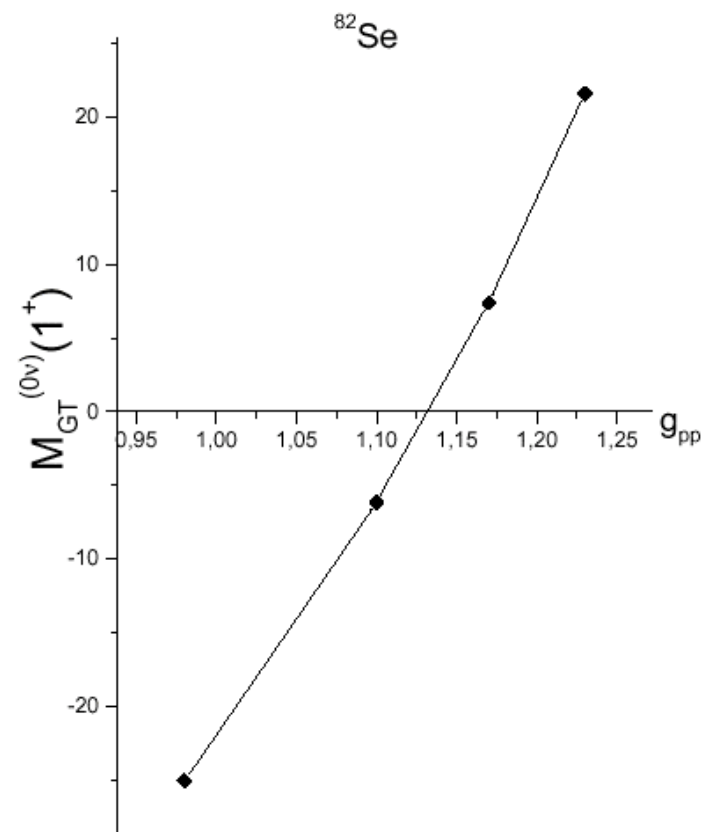
$$C_{mm}^{(0\nu)} = G_1^{(0\nu)} \left[(M_{GT}^{(0\nu)}) (1 - \chi_F) \right]^2$$

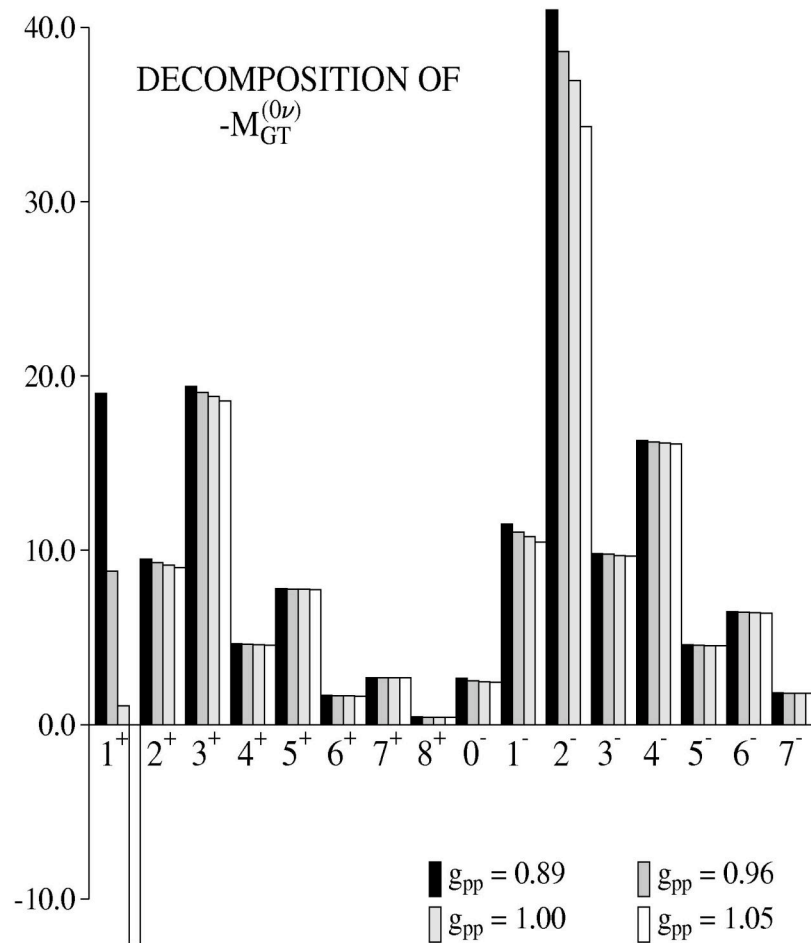
$C_{mm}^{(0)}$	$F_N \times 10^{-12}$	Theory
1.12×10^{-13}	2.80	pnQRPA
6.97×10^{-14}	1.74	pnQRPA
7.51×10^{-14}	1.88	pnQRPA (proj.)
7.33×10^{-14}	1.83	pnQRPA
1.42×10^{-14}	0.35	pnQRPA+ pn pairing
1.18×10^{-13}	2.95	pnQRPA
8.27×10^{-14}	2.07	pnQRPA
2.11×10^{-13}	5.27	RQRPA
6.19×10^{-14}	1.55	RQRPA+ q-dep.
$1.8 - 2.2 \times 10^{-14}$	0.45-0.55	pnQRPA
$5.5 - 6.3 \times 10^{-14}$	1.37-1.57	RQRPA
$2.7 - 3.2 \times 10^{-15}$	0.07-0.08	SCRQRPA
1.85×10^{-14}	0.46	pnQRPA
1.21×10^{-14}	0.30	RQRPA
3.63×10^{-14}	0.91	full-RQRPA
6.50×10^{-14}	1.62	SQRPA
2.88×10^{-13}	7.20	VAMPIR
1.58×10^{-13}	3.95	Shell Model
1.90×10^{-14}	0.47	Shell Model











Case	N.M.E.(extracted)	N.M.E.	$\langle m_\nu \rangle$
^{48}Ca	1.08-2.38		8.70-19.0
^{76}Ge	2.98-4.33	3.33	0.30-0.43
^{82}Se	2.53-3.98	3.44	4.73-7.44
^{96}Zr	2.74	3.55	19.1-24.7
^{100}Mo	0.77-4.67	2.97	1.38-8.42
^{116}Cd	1.09-3.46	3.75	2.37-8.18
^{128}Te	2.51-4.58		9.51-17.4
^{130}Te	2.10-3.59	3.49	1.87-3.20
^{136}Xe	1.61-1.90	4.64	0.79-2.29

SUMMARY

- *Results of the extensions of the QRPA and of the QRPA are not comparable.*
- *The effect of renormalized 1^+ contributions to the NME of the neutrinoless mode is minor.*
- *The bulk of the NME, for the neutrinoless mode, is nearly insensitive to g_{pp}*
- *Single beta decay, particle transfer, $(3\text{He}, t)$ and (p, n) data are proper tools to extract g_{pp} values.*
- *It may be that g_{pp} is entirely fixed by symmetry, i.e. : not need of “ad-hoc” procedures which go beyond the mean field + qrpa approach.*