Observing Gravitational Waves from Spinning Neutron Stars

Reinhard Prix (Albert-Einstein-Institut)

for the LIGO Scientific Collaboration

Orsay, 28 June 2006

Outline

- Astrophysical Motivation
 - Gravitational Waves from Neutron Stars?
 - Emission Mechanisms (Mountains, Precession, Oscillations, Accretion)
 - Gravitational Wave Astronomy of NS
- Detecting Gravitational Waves from NS
 - Status of LIGO (+GEO600)
 - Data-analysis of continous waves
 - Observational Results

Outline

- Astrophysical Motivation
 - Gravitational Waves from Neutron Stars?
 - Emission Mechanisms (Mountains, Precession, Oscillations, Accretion)
 - Gravitational Wave Astronomy of NS
- Detecting Gravitational Waves from NS
 - Status of LIGO (+GEO600)
 - Data-analysis of continous waves
 - Observational Results

Quadrupole formula (Einstein 1916). GW luminosity (ϵ : deviation from axisymmetry):

$$L_{\rm GW} \sim \frac{G}{c^5} \ \epsilon^2 \ \left(\frac{M \ V^3}{R}\right)^2$$

Quadrupole formula (Einstein 1916). GW luminosity (ϵ : deviation from axisymmetry):

$$\mathcal{O}(10^{-53})$$
 $L_{\rm GW} \sim \frac{G}{c^5} \epsilon^2 \left(\frac{M V^3}{R}\right)^2$

Quadrupole formula (Einstein 1916). GW luminosity (ϵ : deviation from axisymmetry):

$$\mathcal{O}(10^{-53}) \qquad L_{\text{GW}} \sim \frac{G}{c^5} \epsilon^2 \left(\frac{M V^3}{R}\right)^2$$

$$= \frac{c^5}{G} \epsilon^2 \left(\frac{R_s}{R}\right)^2 \left(\frac{V}{c}\right)^6$$

Schwarzschild radius $R_s = 2GM/c^2$

Quadrupole formula (Einstein 1916). GW luminosity (ϵ : deviation from axisymmetry):

$$\mathcal{O}(10^{-53}) \qquad L_{\text{GW}} \sim \frac{G}{c^5} \epsilon^2 \left(\frac{M V^3}{R}\right)^2$$

$$\mathcal{O}(10^{59}) \frac{\text{erg}}{\text{s}} = \frac{c^5}{G} \epsilon^2 \left(\frac{R_s}{R}\right)^2 \left(\frac{V}{c}\right)^6$$

Schwarzschild radius $R_s = 2GM/c^2$

Quadrupole formula (Einstein 1916). GW luminosity (ϵ : deviation from axisymmetry):

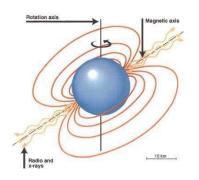
$$\mathcal{O}(10^{-53}) \qquad L_{\text{GW}} \sim \frac{G}{c^5} \epsilon^2 \left(\frac{M V^3}{R}\right)^2$$

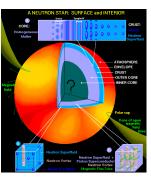
$$\mathcal{O}(10^{59}) \stackrel{\text{erg}}{=} \frac{c^5}{G} \epsilon^2 \left(\frac{R_s}{R}\right)^2 \left(\frac{V}{c}\right)^6$$

Schwarzschild radius $R_s = 2GM/c^2$

Need compact objects in relativistic motion: Black Holes, Neutron Stars, White Dwarfs

What is a neutron star?





Mass: $M \sim 1.4 M_{\odot}$

 $R\sim 10~\mathrm{km}$ Radius:

> ⇒ density: $\bar{\rho} \gtrsim \rho_{\rm nucl}$

 $\frac{R_s}{R} = \frac{2GM}{c^2R} \sim 0.4$ ⇒ relativistic:

Rotation:

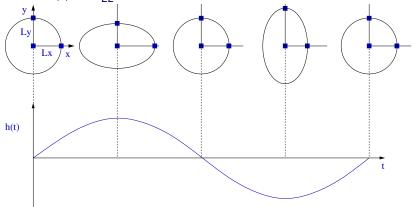
Magnetic field:

 $\nu \lesssim 700 \ s^{-1}$ $B \sim 10^{12} - 10^{14} \ G$

Gravitational Wave Strain h(t)

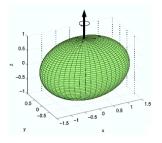
Plane gravitational wave $h_{\mu\nu}^+$ along *z*-direction:

Strain
$$h(t) \equiv \frac{L_x - L_y}{2I}$$
:



Rotating neutron star:

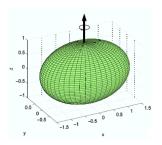
- non-axisymmetric $\epsilon = \frac{I_{xx} I_{yy}}{I_{zz}}$
- rotation rate ν



Rotating neutron star:

- non-axisymmetric $\epsilon = \frac{I_{xx} I_{yy}}{I_{zz}}$
- rotation rate ν
- GW with frequency $f = 2\nu$ Strain-amplitude h_0 on earth:

$$h_0 = \frac{16\pi^2 G}{c^4} \frac{\epsilon I_{zz} \nu^2}{d}$$

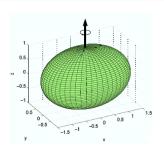


Rotating neutron star:

- non-axisymmetric $\epsilon = \frac{I_{xx} I_{yy}}{I_{zz}}$
- rotation rate ν
- GW with frequency $f = 2\nu$ Strain-amplitude h_0 on earth:

$$h_0 = \frac{16\pi^2 G}{c^4} \frac{\epsilon I_{zz} \nu^2}{d}$$

$$= 4 \times 10^{-25} \left(\frac{\epsilon}{10^{-6}}\right) \left(\frac{I_{zz}}{10^{45} \,\mathrm{g \, cm}^2}\right) \left(\frac{\nu}{100 \,\mathrm{Hz}}\right)^2 \left(\frac{100 \,\mathrm{pc}}{d}\right)$$



Rotating neutron star:

- non-axisymmetric $\epsilon = \frac{I_{xx} I_{yy}}{I_{zz}}$
- rotation rate ν
- GW with frequency $f = 2\nu$ Strain-amplitude h_0 on earth:

$$h_0 = \frac{16\pi^2 G}{c^4} \frac{\epsilon I_{zz} \nu^2}{d}$$

$$= 4 \times 10^{-25} \left(\frac{\epsilon}{10^{-6}}\right) \left(\frac{I_{zz}}{10^{45} \,\mathrm{g \, cm}^2}\right) \left(\frac{\nu}{100 \,\mathrm{Hz}}\right)^2 \left(\frac{100 \,\mathrm{pc}}{d}\right)$$

Current LIGO sensitivity (S5): $\sqrt{S_n} \sim 4 \times 10^{-23} \, \text{Hz}^{-1/2}$



Rotating neutron star:

- non-axisymmetric $\epsilon = \frac{I_{xx} I_{yy}}{I_{--}}$
- rotation rate ν
- \square GW with frequency $f = 2 \nu$ Strain-amplitude h_0 on earth:

rain-amplitude
$$h_0$$
 on earth:

$$h_0 = \frac{16\pi^2 G}{c^4} \frac{\epsilon I_{zz} \nu^2}{d}$$

$$= 4 \times 10^{-25} \left(\frac{\epsilon}{10^{-6}}\right) \left(\frac{I_{zz}}{10^{45} \, \mathrm{g \, cm}^2}\right) \left(\frac{\nu}{100 \, \mathrm{Hz}}\right)^2 \left(\frac{100 \, \mathrm{pc}}{d}\right)$$

Current LIGO sensitivity (S5): $\sqrt{S_n} \sim 4 \times 10^{-23} \, \mathrm{Hz}^{-1/2}$

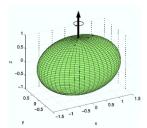
NS signals buried in the noise ⇒ need "matched filtering"



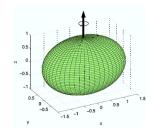
Outline

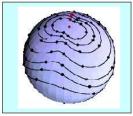
- Astrophysical Motivation
 - Gravitational Waves from Neutron Stars?
 - Emission Mechanisms (Mountains, Precession, Oscillations, Accretion)
 - Gravitational Wave Astronomy of NS
- Detecting Gravitational Waves from NS
 - Status of LIGO (+GEO600)
 - Data-analysis of continous waves
 - Observational Results

- "Mountains"
- Oscillations
- Free precession
- Accretion (driver)

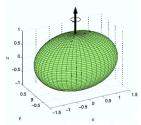


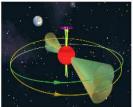
- "Mountains"
- Oscillations
- Free precession
- Accretion (driver)





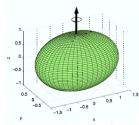
- "Mountains"
- Oscillations
- Free precession
- Accretion (driver)

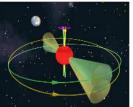


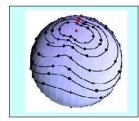




- "Mountains"
- Oscillations
- Free precession
- Accretion (driver)









- Conventional NS crustal shear mountains:
 - $\epsilon_{
 m crust} \lesssim 10^{-7} 10^{-6}$ (Ushomirsky, Cutler, Bildsten)
- Superfluid vortices: Magnus-strain deforming crust
- Exotic EOS: strange-quark solid cores

 ≤ 10⁻⁵ 10⁻⁴ (B Owen)
- Magnetic mountains:
 - large toroidal field $B_t \sim 10^{15} \text{ G} \perp$ to rotation:
 - accretion along *B*-lines \Longrightarrow "bottled" mountains $\epsilon_{\text{hottle}} \leq 10^{-6} 10^{-5}$ (Melatos, Payne)
 - non-aligned poloidal magnetic field $B \sim 10^{13}$ G, type-I or type-II superconducting interior, $\epsilon_B \leq 10^{-6}$ (Bonazzola&Gourgoulhon)

- Conventional NS crustal shear mountains:
 - $\epsilon_{crust} \lesssim 10^{-7} 10^{-6}$ (Ushomirsky, Cutler, Bildsten)
- Superfluid vortices: Magnus-strain deforming crust
 - $\epsilon_{
 m Magnus} \sim 5 imes 10^{-7}$ (D.I. Jones; Ruderman)
- Exotic EOS: strange-quark solid cores
 - $\epsilon_{\rm strange} \lesssim 10^{-5} 10^{-4}$ (B. Owen)
- Magnetic mountains:
 - large toroidal field $B_t \sim 10^{15} \; {\rm G} \perp {\rm to}$ rotation:
 - $\epsilon_{
 m toroidal} \sim 10^{-6}$ (C. Cutler)
 - accretion along *B*-lines \Longrightarrow "bottled" mountains
 - $\epsilon_{\text{bottle}} \lesssim 10^{-6} 10^{-5}$ (Melatos, Payne)
 - non-aligned poloidal magnetic field $B \sim 10^{13}$ G, type-I or type-II superconducting interior,



- Conventional NS crustal shear mountains:
 - $\epsilon_{crust} \lesssim 10^{-7} 10^{-6}$ (Ushomirsky, Cutler, Bildsten)
- Superfluid vortices: Magnus-strain deforming crust
 - $\epsilon_{Magnus} \sim 5 imes 10^{-7}$ (D.I. Jones; Ruderman)
- Exotic EOS: strange-quark solid cores
 - $\epsilon_{
 m strange} \lesssim 10^{-5} 10^{-4}$ (B. Owen)
- Magnetic mountains:
 - large toroidal field $B_t \sim 10^{15} \; {\rm G} \perp {\rm to}$ rotation:
 - $\epsilon_{
 m toroidal} \sim 10^{-6}$ (C. Cutler)
 - accretion along *B*-lines \Longrightarrow "bottled" mountains
 - $\epsilon_{
 m bottle} \lesssim 10^{-6} 10^{-5}$ (Melatos, Payne
 - non-aligned poloidal magnetic field $B \sim 10^{13}$ G, type-I or type-II superconducting interior,

- Conventional NS crustal shear mountains:
 - $\epsilon_{crust} \lesssim 10^{-7} 10^{-6}$ (Ushomirsky, Cutler, Bildsten)
- Superfluid vortices: Magnus-strain deforming crust $\epsilon_{Magnus} \sim 5 \times 10^{-7}$ (D.I. Jones; Ruderman)
- Exotic EOS: strange-quark solid cores $\epsilon_{\text{strange}} \lesssim 10^{-5} 10^{-4} \text{ (B. Owen)}$
- Magnetic mountains:
 - large toroidal field $B_t \sim 10^{15} \; \mathrm{G} \perp$ to rotation:
 - accretion along B-lines \Longrightarrow "bottled" mountains
 - non-aligned poloidal magnetic field $B \sim 10^{13}$ G, type-I or type-II superconducting interior.
 - $\epsilon_B \lesssim 10^{-6}$ (Bonazzola&Gourgoulhon)

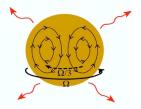
- Conventional NS crustal shear mountains:
 - $\epsilon_{crust} \lesssim 10^{-7} 10^{-6}$ (Ushomirsky, Cutler, Bildsten)
- Superfluid vortices: Magnus-strain deforming crust $\epsilon_{\text{Magnus}} \sim 5 \times 10^{-7}$ (D.I. Jones; Ruderman)
- Exotic EOS: strange-quark solid cores $\epsilon_{\text{strange}} \leq 10^{-5} 10^{-4}$ (B. Owen)
- Magnetic mountains:
 - large toroidal field $B_t \sim 10^{15} \; {\rm G} \perp$ to rotation:
 - $\epsilon_{
 m toroidal} \sim 10^{-6}$ (C. Cutler)
 - accretion along *B*-lines \Longrightarrow "bottled" mountains $\stackrel{\text{local}}{=}$ $\stackrel{<}{=}$ $10^{-6} 10^{-5}$ (Melatos, Payne)
 - non-aligned poloidal magnetic field $B \sim 10^{13}$ G, type-I or type-II superconducting interior,

- Conventional NS crustal shear mountains:
 - $\epsilon_{crust} \lesssim 10^{-7} 10^{-6}$ (Ushomirsky, Cutler, Bildsten)
- Superfluid vortices: Magnus-strain deforming crust
 - $\epsilon_{Magnus} \sim 5 imes 10^{-7}$ (D.I. Jones; Ruderman)
- Exotic EOS: strange-quark solid cores
 - $\epsilon_{
 m strange} \lesssim 10^{-5} 10^{-4}$ (B. Owen)
- Magnetic mountains:
 - large toroidal field $B_t \sim 10^{15} \; {\rm G} \perp$ to rotation:
 - $\epsilon_{
 m toroidal} \sim 10^{-6}$ (C. Cutler)
 - accretion along B-lines ⇒ "bottled" mountains
 - $\epsilon_{
 m bottle} \lesssim 10^{-6} 10^{-5}$ (Melatos, Payne)
 - non-aligned poloidal magnetic field $B \sim 10^{13}$ G, type-I or type-II superconducting interior,



- Conventional NS crustal shear mountains:
 - $\epsilon_{crust} \lesssim 10^{-7} 10^{-6}$ (Ushomirsky, Cutler, Bildsten)
- Superfluid vortices: Magnus-strain deforming crust $\epsilon_{\text{Magnus}} \sim 5 \times 10^{-7}$ (D.I. Jones; Ruderman)
- Exotic EOS: strange-quark solid cores $\epsilon_{\text{strange}} \leq 10^{-5} 10^{-4}$ (B. Owen)
- Magnetic mountains:
 - large toroidal field $B_t \sim 10^{15} \; {\rm G} \perp$ to rotation: • $\epsilon_{\rm toroidal} \sim 10^{-6} \; ({\rm C. \, Cutler})$
 - accretion along *B*-lines \Longrightarrow "bottled" mountains $\epsilon_{\text{bottle}} \lesssim 10^{-6} 10^{-5}$ (Melatos, Payne)
 - non-aligned poloidal magnetic field $B \sim 10^{13}$ G, type-I or type-II superconducting interior, $\epsilon_B \lesssim 10^{-6}$ (Bonazzola&Gourgoulhon)

Oscillation Modes

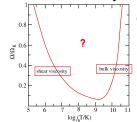


Chandrasekhar-Friedman-Schutz instability:

counter-rotating mode "dragged forward" ⇒negative energy and angular momentum

- emission of GW amplifies the mode
- counteracted by dissipation

r-mode instability window:



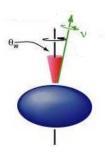
Open questions:

- Dissipation mechanisms: vortex friction, hyperons, crust-core coupling,...
- saturation amplitude, mode-mode coupling, evolution timescales



Free Precession

"Most general motion of a rigid body" (Landau&Lifshitz 1976)



NS are not rigid: coupled crust - core (viscosity + superfluid vortex pinning)

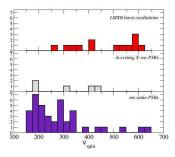
- likely to be damped rapidly
- no obvious instability or "pumping mechanism"

$$\textit{h}_{0} \sim 10^{-26} \left(\frac{\theta_{\textit{w}}}{0.1}\right) \left(\frac{100\,\text{pc}}{\textit{d}}\right) \left(\frac{\nu}{500\,\text{Hz}}\right)^{2}$$



Accretion





Breakup-limit $\nu_K \sim$ 1.5 kHz $\ ^{\ }$ What limits the NS-spin?

Bildsten, Wagoner: Accretion-torque = GW torque ($\propto \nu^5$)

Observed X-ray flux Sco X-1: $h_0 \sim 3 \times 10^{-26} (270 \, \text{Hz}/\nu)^{1/2}$

Outline

- Astrophysical Motivation
 - Gravitational Waves from Neutron Stars?
 - Emission Mechanisms (Mountains, Precession, Oscillations, Accretion)
 - Gravitational Wave Astronomy of NS
- Detecting Gravitational Waves from NS
 - Status of LIGO (+GEO600)
 - Data-analysis of continous waves
 - Observational Results

- NS are plausible sources for LIGO I, II or VIRGO
- Whether or not they are detectable depends on many poorly-understood aspects of NS physics
- Any GW-detection from rotating NS will be extremely valuable for NS physics
- Even the absence of detection can yield astrophysically interesting information (crust deformation, B, instabilities)
- NS physics producing GWs is very different and complementary to electromagnetic emission (bulk-mass motion vs magnetosphere-electron motion)

- NS are plausible sources for LIGO I, II or VIRGO
- Whether or not they are detectable depends on many poorly-understood aspects of NS physics
- Any GW-detection from rotating NS will be extremely valuable for NS physics
- Even the absence of detection can yield astrophysically interesting information (crust deformation, B, instabilities)
- NS physics producing GWs is very different and complementary to electromagnetic emission (bulk-mass motion vs magnetosphere-electron motion)

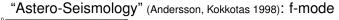


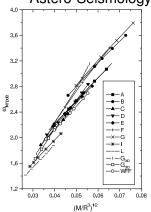
- NS are plausible sources for LIGO I, II or VIRGO
- Whether or not they are detectable depends on many poorly-understood aspects of NS physics
- Any GW-detection from rotating NS will be extremely valuable for NS physics
- Even the absence of detection can yield astrophysically interesting information (crust deformation, B, instabilities)
- NS physics producing GWs is very different and complementary to electromagnetic emission (bulk-mass motion vs magnetosphere-electron motion)

- NS are plausible sources for LIGO I, II or VIRGO
- Whether or not they are detectable depends on many poorly-understood aspects of NS physics
- Any GW-detection from rotating NS will be extremely valuable for NS physics
- Even the absence of detection can yield astrophysically interesting information (crust deformation, B, instabilities)
- NS physics producing GWs is very different and complementary to electromagnetic emission (bulk-mass motion vs magnetosphere-electron motion)

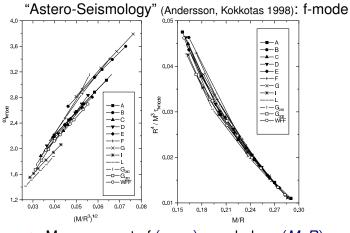
- NS are plausible sources for LIGO I, II or VIRGO
- Whether or not they are detectable depends on many poorly-understood aspects of NS physics
- Any GW-detection from rotating NS will be extremely valuable for NS physics
- Even the absence of detection can yield astrophysically interesting information (crust deformation, B, instabilities)
- NS physics producing GWs is very different and complementary to electromagnetic emission (bulk-mass motion vs magnetosphere-electron motion)

Gravitational Wave Astronomy



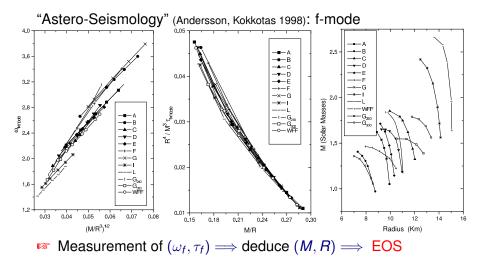


Gravitational Wave Astronomy



 \blacksquare Measurement of $(\omega_f, \tau_f) \Longrightarrow$ deduce (M, R)

Gravitational Wave Astronomy

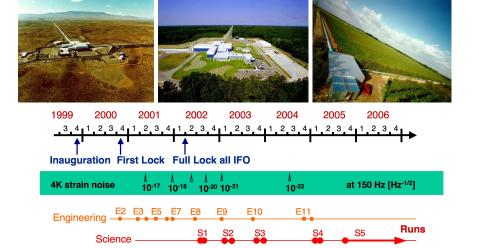


Outline

- Astrophysical Motivation
 - Gravitational Waves from Neutron Stars?
 - Emission Mechanisms (Mountains, Precession, Oscillations, Accretion)
 - Gravitational Wave Astronomy of NS
- Detecting Gravitational Waves from NS
 - Status of LIGO (+GEO600)
 - Data-analysis of continous waves
 - Observational Results

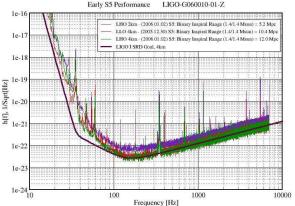
Status of LIGO (+GEO600) Data-analysis of continous wave Observational Results

LSC detectors: LIGO + GEO600



Current LIGO noise performance

Best Strain Sensitivities for the LIGO Interferometers



$$\textit{h}_0 = \frac{\Delta \textit{L}}{\textit{L}} \sim 3 \times 10^{-23} \Longrightarrow \Delta \textit{L} \sim 10^{-19} \ \textit{m} = 10^{-4} \ \textit{fm}!!$$

LIGO (H1, H2, L1) and GEO600 data analyzed within the LIGO Scientific Collaboration (LSC):

 \sim 40 institutions, \sim 320 authors (S3)

- Binary inspirals: short inspiral signals (modeled)
- Bursts: short unmodeled signals (supernovae, merger)
- Stochastic background: cosmological background GWs
- "Continuous waves": spinning NS signals (long-lived)

LIGO (H1, H2, L1) and GEO600 data analyzed within the LIGO Scientific Collaboration (LSC):

 \sim 40 institutions, \sim 320 authors (S3)

- Binary inspirals: short inspiral signals (modeled)
- Bursts: short unmodeled signals (supernovae, merger)
- Stochastic background: cosmological background GWs
- "Continuous waves": spinning NS signals (long-lived)



LIGO (H1, H2, L1) and GEO600 data analyzed within the LIGO Scientific Collaboration (LSC):

 \sim 40 institutions, \sim 320 authors (S3)

- Binary inspirals: short inspiral signals (modeled)
- Bursts: short unmodeled signals (supernovae, merger)
- Stochastic background: cosmological background GWs
- "Continuous waves": spinning NS signals (long-lived)

LIGO (H1, H2, L1) and GEO600 data analyzed within the LIGO Scientific Collaboration (LSC):

 \sim 40 institutions, \sim 320 authors (S3)

- Binary inspirals: short inspiral signals (modeled)
- Bursts: short unmodeled signals (supernovae, merger)
- Stochastic background: cosmological background GWs
- "Continuous waves": spinning NS signals (long-lived)



LIGO (H1, H2, L1) and GEO600 data analyzed within the LIGO Scientific Collaboration (LSC):

 \sim 40 institutions, \sim 320 authors (S3)

- Binary inspirals: short inspiral signals (modeled)
- Bursts: short unmodeled signals (supernovae, merger)
- Stochastic background: cosmological background GWs
- "Continuous waves": spinning NS signals (long-lived)

LIGO (H1, H2, L1) and GEO600 data analyzed within the LIGO Scientific Collaboration (LSC):

 \sim 40 institutions, \sim 320 authors (S3)

- Binary inspirals: short inspiral signals (modeled)
- Bursts: short unmodeled signals (supernovae, merger)
- Stochastic background: cosmological background GWs
- "Continuous waves": spinning NS signals (long-lived)

Outline

- Astrophysical Motivation
 - Gravitational Waves from Neutron Stars?
 - Emission Mechanisms (Mountains, Precession, Oscillations, Accretion)
 - Gravitational Wave Astronomy of NS
- Detecting Gravitational Waves from NS
 - Status of LIGO (+GEO600)
 - Data-analysis of continous waves
 - Observational Results

NS frame: monochromatic wave, slowly varying frequency

NS frame: monochromatic wave, slowly varying frequency

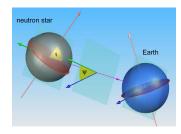
Phase
$$\Phi(\tau) = \phi_0 + 2\pi \left(\mathbf{f} \, \tau + \frac{1}{2} \dot{\mathbf{f}} \, \tau^2 + ... \right)$$

GW frequency for triaxial NS: $f = 2\nu$, r-modes: $f = 4/3\nu$, precession: $f \approx \nu$

NS frame: monochromatic wave, slowly varying frequency

Phase
$$\Phi(\tau)=\phi_0+2\pi\left(\mathbf{f}\,\tau+\frac{1}{2}\dot{\mathbf{f}}\,\tau^2+\ldots\right)$$
 GW frequency for triaxial NS: $\mathbf{f}=2\,\nu$, r-modes: $\mathbf{f}=4/3\,\nu$, precession: $\mathbf{f}\approx\nu$

2 polarization amplitudes: A_+, A_\times



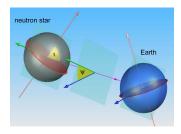
NS frame: monochromatic wave, slowly varying frequency

Phase
$$\Phi(\tau)=\phi_0+2\pi\left(\mathbf{f}\,\tau+\frac{1}{2}\dot{\mathbf{f}}\,\tau^2+\ldots\right)$$
 GW frequency for triaxial NS: $\mathbf{f}=2\,\nu$, r-modes: $\mathbf{f}=4/3\,\nu$, precession: $\mathbf{f}\approx\nu$

2 polarization amplitudes: A_+, A_\times

$$h_{\times}(\tau) = A_{+} \cos \Phi(\tau)$$

 $h_{+}(\tau) = A_{\times} \sin \Phi(\tau)$



NS frame: monochromatic wave, slowly varying frequency

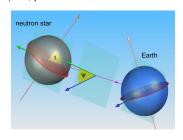
Phase
$$\Phi(\tau) = \phi_0 + 2\pi \left(\mathbf{f} \tau + \frac{1}{2} \dot{\mathbf{f}} \tau^2 + ... \right)$$

GW frequency for triaxial NS: $\mathbf{f} = 2\nu$, r-modes: $\mathbf{f} = 4/3\nu$, precession: $\mathbf{f} \approx \nu$

2 polarization amplitudes: A_+, A_\times

$$h_{\times}(\tau) = A_{+} \cos \Phi(\tau)$$

 $h_{+}(\tau) = A_{\times} \sin \Phi(\tau)$



- □ Detector frame t: sky-position (α, δ) dependent *modulations*:
 - Phase: Doppler-effect due to earth's motion $\tau = \tau(t; \alpha, \delta)$
 - Amplitude: rotating Antenna-pattern $F_{+,\times}(t,\psi;\alpha,\delta)$



NS frame: monochromatic wave, slowly varying frequency

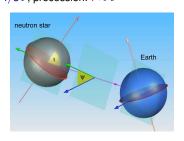
Phase
$$\Phi(\tau) = \phi_0 + 2\pi \left(\mathbf{f} \tau + \frac{1}{2} \dot{\mathbf{f}} \tau^2 + ... \right)$$

GW frequency for triaxial NS: $\mathbf{f} = 2\nu$, r-modes: $\mathbf{f} = 4/3\nu$, precession: $\mathbf{f} \approx \nu$

2 polarization amplitudes: A_{+}, A_{\times}

$$h_{\times}(\tau) = A_{+} \cos \Phi(\tau)$$

 $h_{+}(\tau) = A_{\times} \sin \Phi(\tau)$



- \square Detector frame t: sky-position (α, δ) dependent *modulations*:
 - Phase: Doppler-effect due to earth's motion $\tau = \tau(t; \alpha, \delta)$
 - Amplitude: rotating Antenna-pattern $F_{+,\times}(t,\psi;\alpha,\delta)$



NS frame: monochromatic wave, slowly varying frequency

Phase
$$\Phi(\tau) = \phi_0 + 2\pi \left(\mathbf{f} \tau + \frac{1}{2} \dot{\mathbf{f}} \tau^2 + ... \right)$$

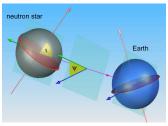
GW frequency for triaxial NS: $\mathbf{f} = 2\nu$, r-modes: $\mathbf{f} = 4/3\nu$, precession: $\mathbf{f} \approx \nu$

The requestion triaxial No. $I = 2\nu$, 1-modes. $I = 4/3\nu$, precession. $I \sim I$

2 polarization amplitudes: A_+, A_\times

$$h_{\times}(\tau) = A_{+} \cos \Phi(\tau)$$

 $h_{+}(\tau) = A_{\times} \sin \Phi(\tau)$



- \square Detector frame t: sky-position (α, δ) dependent *modulations*:
 - Phase: Doppler-effect due to earth's motion $\tau = \tau(t; \alpha, \delta)$
 - Amplitude: rotating Antenna-pattern $F_{+,\times}(t,\psi;\alpha,\delta)$



GW strain at the detector:

$$h(t) = F_{+}(t) h_{+}(t) + F_{\times}(t) h_{\times}(t)$$

GW strain at the detector:

$$h(t) = F_{+}(t) h_{+}(t) + F_{\times}(t) h_{\times}(t)$$

Signal dependencies

$$h(t) = F_{+}(t, \psi; \alpha, \delta) A_{+} \cos \left[\phi_{0} + \phi(t; \alpha, \delta, f, \dot{f}, ..) \right]$$

+ $F_{\times}(t, \psi; \alpha, \delta) A_{\times} \sin \left[\phi_{0} + \phi(t; \alpha, \delta, f, \dot{f}, ..) \right]$

Signal parameters:

- 4 "Amplitude parameters": $A^{\mu} = A^{\mu} (A_+, A_{\times}, \psi, \phi_0)$
- ullet "Doppler parameters": $oldsymbol{\lambda} = \{ lpha, \delta, f, f, ...$ (+ orbital parameters) $\}$



GW strain at the detector:

$$h(t) = F_{+}(t) h_{+}(t) + F_{\times}(t) h_{\times}(t)$$

Signal dependencies

$$h(t) = F_{+}(t, \psi; \alpha, \delta) A_{+} \cos \left[\phi_{0} + \phi(t; \alpha, \delta, f, \dot{f}, ..) \right]$$

+ $F_{\times}(t, \psi; \alpha, \delta) A_{\times} \sin \left[\phi_{0} + \phi(t; \alpha, \delta, f, \dot{f}, ..) \right]$

Signal parameters:

- 4 "Amplitude parameters": $A^{\mu} = A^{\mu} (A_+, A_{\times}, \psi, \phi_0)$
- ullet "Doppler parameters": $oldsymbol{\lambda} = \{lpha, \delta, f, f, ...$ (+ orbital parameters)

GW strain at the detector:

$$h(t) = F_{+}(t) h_{+}(t) + F_{\times}(t) h_{\times}(t)$$

Signal dependencies

$$h(t) = F_{+}(t, \psi; \alpha, \delta) A_{+} \cos \left[\phi_{0} + \phi(t; \alpha, \delta, f, \dot{f}, ..) \right]$$

+ $F_{\times}(t, \psi; \alpha, \delta) A_{\times} \sin \left[\phi_{0} + \phi(t; \alpha, \delta, f, \dot{f}, ..) \right]$

Signal parameters:

- 4 "Amplitude parameters": $A^{\mu} = A^{\mu} (A_+, A_{\times}, \psi, \phi_0)$
- "Doppler parameters": $\lambda = \{\alpha, \delta, f, f, \dots$ (+ orbital parameters) $\}$



Measured strain:
$$\underbrace{x(t)}_{\text{data}} = \underbrace{n_{\text{oise}}}_{\text{noise}} + \underbrace{signal}_{\text{signal}}$$

Measured strain:
$$x(t) = noise n(t) + s(t; A, \lambda)$$

scalar product:
$$(x|y) \equiv \int \frac{\widetilde{x}(f)\widetilde{y}^*(f)}{S_n(f)} df$$

Measured strain:
$$x(t) = noise n(t) + s(t; A, \lambda)$$

scalar product:
$$(x|y) \equiv \int \frac{\widetilde{x}(f)\widetilde{y}^*(f)}{S_n(f)} df$$

pdf for Gaussian noise
$$n(t)$$
: $P(n(t)|S_n) = k e^{-\frac{1}{2}(n|n)}$

Measured strain:
$$x(t) = noise noise $x(t) + s(t; A, \lambda)$$$

scalar product:
$$(x|y) \equiv \int \frac{\ddot{x}(f)\ddot{y}^*(f)}{S_n(f)} df$$

pdf for Gaussian noise
$$n(t)$$
: $P(n(t)|S_n) = k e^{-\frac{1}{2}(n|n)}$

 \implies likelihood of x(t) in presence of signal $s(t; A, \lambda)$:

$$P(x(t)|A, \lambda; S_n) = k e^{-\frac{1}{2}(x|x)} e^{(x|s) - \frac{1}{2}(s|s)}$$

Measured strain:
$$x(t) = noise noise $x(t) + s(t; A, \lambda)$$$

scalar product:
$$(x|y) \equiv \int \frac{\widetilde{x}(f)\widetilde{y}^*(f)}{S_n(f)} df$$

pdf for Gaussian noise n(t): $P(n(t)|S_n) = k e^{-\frac{1}{2}(n|n)}$

 \implies likelihood of x(t) in presence of signal $s(t; A, \lambda)$:

$$P(x(t)|\mathcal{A}, \lambda; S_n) = k e^{-\frac{1}{2}(x|x)} e^{(x|s) - \frac{1}{2}(s|s)}$$

Bayesian posterior probability for signal $\{A, \lambda\}$ in data x(t):

$$P(A, \lambda | x(t); S_n) = k' \underbrace{P(A, \lambda)}_{\text{"prior" probability}} e^{(x|s) - \frac{1}{2}(s|s)}$$

Measured strain:
$$\underbrace{x(t)}_{\text{data}} = \underbrace{n_{\text{oise}}}_{\text{noise}} + \underbrace{signal}_{\text{signal}}$$

scalar product:
$$(x|y) \equiv \int \frac{\widetilde{x}(f)\widetilde{y}^*(f)}{S_n(f)} df$$

pdf for Gaussian noise n(t): $P(n(t)|S_n) = k e^{-\frac{1}{2}(n|n)}$

 \implies likelihood of x(t) in presence of signal $s(t; A, \lambda)$:

$$P(x(t)|\mathcal{A}, \lambda; S_n) = k e^{-\frac{1}{2}(x|x)} e^{(x|s) - \frac{1}{2}(s|s)}$$

Bayesian posterior probability for signal $\{A, \lambda\}$ in data x(t):

$$P(A, \lambda | x(t); S_n) = k' \underbrace{P(A, \lambda)}_{\text{"prior"probability}} e^{(x|s) - \frac{1}{2}(s|s)}$$

detection statistic:
$$Q(A, \lambda) \equiv (x|s) - \frac{1}{2}(s|s)$$

find maximum of Q in parameter space $\{A, \lambda\}$.

detection statistic:
$$Q(A, \lambda) \equiv (x|s) - \frac{1}{2}(s|s)$$

find maximum of Q in parameter space $\{A, \lambda\}$.

$$s(t; A, \lambda) = \sum_{\mu=1}^{4} A^{\mu} h_{\mu}(t; \lambda)$$
 (Jaranowski, Krolak, Schutz, PRD 1998)

detection statistic:
$$Q(A, \lambda) \equiv (x|s) - \frac{1}{2}(s|s)$$

find maximum of Q in parameter space $\{A, \lambda\}$.

$$s(t; A, \lambda) = \sum_{\mu=1}^{4} A^{\mu} h_{\mu}(t; \lambda)$$
 (Jaranowski, Krolak, Schutz, PRD 1998)

 \Longrightarrow analytically maximize Q over \mathcal{A}^{μ} : $\frac{\partial Q}{\partial \mathcal{A}^{\mu}} = 0 \Longrightarrow \mathcal{A}^{\mu}_{_{\mathrm{MLE}}}$

detection statistic: $Q(A, \lambda) \equiv (x|s) - \frac{1}{2}(s|s)$

find maximum of Q in parameter space $\{A, \lambda\}$.

$$s(t; A, \lambda) = \sum_{\mu=1}^{4} A^{\mu} h_{\mu}(t; \lambda)$$
 (Jaranowski, Krolak, Schutz, PRD 1998)

 \Longrightarrow analytically maximize Q over \mathcal{A}^{μ} : $\frac{\partial Q}{\partial \mathcal{A}^{\mu}}=0\Longrightarrow \mathcal{A}^{\mu}_{_{\mathrm{MLE}}}$

Definition of the " \mathcal{F} -statistic": $\mathcal{F} = Q(\mathcal{A}_{\scriptscriptstyle{\mathrm{MLE}}}, \lambda)$

$$2\mathcal{F}(\frac{\lambda}{\lambda}) = x_{\mu} \, \mathcal{M}^{\mu\nu} \, x_{\nu}$$

where $x_{\mu}(\lambda) \equiv (x|h_{\mu}(\lambda))$, and $\mathcal{M}^{\mu\nu}(\lambda) = (h_{\mu}(\lambda)|h_{\nu}(\lambda))^{-1}$

detection statistic: $Q(A, \lambda) \equiv (x|s) - \frac{1}{2}(s|s)$

find maximum of Q in parameter space $\{A, \lambda\}$.

$$s(t; A, \lambda) = \sum_{\mu=1}^{4} A^{\mu} h_{\mu}(t; \lambda)$$
 (Jaranowski, Krolak, Schutz, PRD 1998)

 \Longrightarrow analytically maximize Q over \mathcal{A}^{μ} : $\frac{\partial Q}{\partial \mathcal{A}^{\mu}}=0\Longrightarrow \mathcal{A}^{\mu}_{_{\mathrm{MLE}}}$

Definition of the " \mathcal{F} -statistic": $\mathcal{F} = Q(\mathcal{A}_{\text{\tiny MLE}}, \lambda)$

$$2\mathcal{F}(\frac{\lambda}{\lambda}) = x_{\mu} \, \mathcal{M}^{\mu\nu} \, x_{\nu}$$

where
$$x_{\mu}(\lambda) \equiv (x|h_{\mu}(\lambda))$$
, and $\mathcal{M}^{\mu\nu}(\lambda) = (h_{\mu}(\lambda)|h_{\nu}(\lambda))^{-1}$

find maximum of \mathcal{F} in reduced parameter-space $\{\lambda\}$.



Matched filtering III: multi-detector generalization

multi-detector vector $\{\boldsymbol{x}(t)\}^{X} = x^{X}(t)$ with $X \in \{H1, L1, V1...\}$

multi-detector vector $\{\boldsymbol{x}(t)\}^{X} = \boldsymbol{x}^{X}(t)$ with $X \in \{H1, L1, V1...\}$

$$(\boldsymbol{x}|\boldsymbol{y}) = \int \widetilde{\boldsymbol{\chi}}^{\mathrm{X}}(f) \, \mathcal{S}_{\mathrm{XY}}^{-1} \, \widetilde{\boldsymbol{y}}^{\mathrm{Y*}}(f) \, df$$

multi-detector vector $\{\boldsymbol{x}(t)\}^{X} = \boldsymbol{x}^{X}(t)$ with $X \in \{H1, L1, V1...\}$

$$(\boldsymbol{x}|\boldsymbol{y}) = \int \widetilde{\boldsymbol{x}}^{\mathrm{X}}(f) \, S_{\mathrm{XY}}^{-1} \, \widetilde{\boldsymbol{y}}^{\mathrm{Y*}}(f) \, df$$

$$egin{aligned} x_{\mu}(\pmb{\lambda}) &= (\pmb{x}|\pmb{h_{\mu}}), \quad \mathcal{M}^{\mu
u}(\pmb{\lambda}) &= (\pmb{h_{\mu}}|\pmb{h_{
u}})^{-1} \ \Longrightarrow & 2\mathcal{F}(\pmb{\lambda}) &= x_{\mu} \, \mathcal{M}^{\mu
u} \, x_{
u} \qquad ext{(Cutler \& Schutz, PRD 2005)} \end{aligned}$$

multi-detector vector $\{\boldsymbol{x}(t)\}^{X} = \boldsymbol{x}^{X}(t)$ with $X \in \{H1, L1, V1...\}$

$$(\boldsymbol{x}|\boldsymbol{y}) = \int \widetilde{\boldsymbol{x}}^{\mathrm{X}}(f) \, \mathcal{S}_{\mathrm{XY}}^{-1} \widetilde{\boldsymbol{y}}^{\mathrm{Y*}}(f) \, df$$

$$egin{aligned} x_{\mu}(\pmb{\lambda}) &= (\pmb{x}|\pmb{h_{\mu}}), & \mathcal{M}^{\mu
u}(\pmb{\lambda}) &= (\pmb{h_{\mu}}|\pmb{h_{
u}})^{-1} \ \Longrightarrow & 2\mathcal{F}(\pmb{\lambda}) &= x_{\mu}\,\mathcal{M}^{\mu
u}\,x_{
u} & ext{(Cutler \& Schutz, PRD 2005)} \end{aligned}$$

Signal-to-noise ratio @ perfect match

$$\mathrm{SNR} = \sqrt{(\boldsymbol{s}|\boldsymbol{s})} \propto \frac{h_0}{\sqrt{\mathcal{S}_n}} \, \sqrt{\mathcal{T}\,\mathcal{N}} \qquad \begin{array}{c} \mathcal{T} \; ... \; \text{observation time} \\ \mathcal{N} \; ... \; \text{equal-noise detectors} \end{array}$$

multi-detector vector $\{\boldsymbol{x}(t)\}^{X} = \boldsymbol{x}^{X}(t)$ with $X \in \{H1, L1, V1...\}$

$$(\boldsymbol{x}|\boldsymbol{y}) = \int \widetilde{\boldsymbol{x}}^{\mathrm{X}}(f) \, \mathcal{S}_{\mathrm{XY}}^{-1} \widetilde{\boldsymbol{y}}^{\mathrm{Y*}}(f) \, df$$

$$egin{aligned} x_{\mu}(\pmb{\lambda}) &= (\pmb{x}|\pmb{h}_{\pmb{\mu}}), & \mathcal{M}^{\mu\nu}(\pmb{\lambda}) &= (\pmb{h}_{\pmb{\mu}}|\pmb{h}_{\pmb{\nu}})^{-1} \ \Longrightarrow & 2\mathcal{F}(\pmb{\lambda}) &= x_{\mu}\,\mathcal{M}^{\mu\nu}\,x_{
u} & ext{(Cutler\&Schutz, PRD 2005)} \end{aligned}$$

Signal-to-noise ratio @ perfect match

$${
m SNR} = \sqrt{({m s}|{m s})} \propto rac{h_0}{\sqrt{S_n}} \, \sqrt{T\, {\cal N}} \qquad {
m T ... observation time} \ {\cal N} \, ... \, {
m equal-noise detectors}$$

$$h_0/\sqrt{S_n}\ll 1$$
 reed long T (and many detectors \mathcal{N})



The covering problem

Choose a finite number N_p of "templates" $\lambda_{(k)}$, such that

- ① never lose more than a fraction m at closest template $\lambda_{(k)}$
- 2 N_p is the smallest possible number satisfying 1

The covering problem

Choose a finite number N_p of "templates" $\lambda_{(k)}$, such that

- never lose more than a fraction m at closest template $\lambda_{(k)}$
- 2 N_p is the smallest possible number satisfying 1

The *covering problem*

Choose a finite number N_p of "templates" $\lambda_{(k)}$, such that

- never lose more than a fraction m at closest template $\lambda_{(k)}$
- 2 N_p is the smallest possible number satisfying 1

The *covering problem*

Choose a finite number N_p of "templates" $\lambda_{(k)}$, such that

- never lose more than a fraction m at closest template $\lambda_{(k)}$
- 2 N_p is the smallest possible number satisfying 1

Relative loss in mismatched $\mathcal{F}(\lambda)$ at $\lambda = \lambda_{\text{sig}} + \Delta \lambda$:

The covering problem

Choose a finite number N_p of "templates" $\lambda_{(k)}$, such that

- never lose more than a fraction m at closest template $\lambda_{(k)}$
- \bigcirc N_p is the smallest possible number satisfying 1

Relative loss in mismatched $\mathcal{F}(\lambda)$ at $\lambda = \lambda_{\text{sig}} + \Delta \lambda$:

$$\mathcal{F}(\lambda) = \mathcal{F}(\lambda_{\text{sig}}) \left(1 - g_{ij} \Delta \lambda^{i} \Delta \lambda^{j} + .. \right) \implies \text{"metric" } g_{ij}$$

The covering problem

Choose a finite number N_p of "templates" $\lambda_{(k)}$, such that

- never lose more than a fraction m at closest template $\lambda_{(k)}$
- 2 N_p is the smallest possible number satisfying 1

Relative loss in mismatched $\mathcal{F}(\lambda)$ at $\lambda = \lambda_{\text{sig}} + \Delta \lambda$:

$$\mathcal{F}(\lambda) = \mathcal{F}(\lambda_{ ext{sig}}) \left(1 - g_{ij} \, \Delta \lambda^i \Delta \lambda^j + ..
ight) \implies ext{"metric" } g_{ij}$$
 $N_p \propto \int_{\{\lambda\}} \sqrt{\det g_{ij}} \, \, d^n \lambda$

The covering problem

Choose a finite number N_p of "templates" $\lambda_{(k)}$, such that

- never lose more than a fraction m at closest template $\lambda_{(k)}$
- 2 N_p is the smallest possible number satisfying 1

Relative loss in mismatched $\mathcal{F}(\lambda)$ at $\lambda = \lambda_{\text{sig}} + \Delta \lambda$:

$$\mathcal{F}(\lambda) = \mathcal{F}(\lambda_{ ext{sig}}) \left(1 - g_{ij} \, \Delta \lambda^i \Delta \lambda^j + ..
ight) \implies ext{"metric" } g_{ij}$$
 $N_p \propto \int_{\{\lambda\}} \sqrt{\det g_{ij}} \, \, d^n \lambda$

isolated NS $\lambda^i = (\alpha, \delta, f, \dot{f})$:

$$N_p \propto T^5$$



The covering problem

Choose a finite number N_p of "templates" $\lambda_{(k)}$, such that

- never lose more than a fraction m at closest template $\lambda_{(k)}$
- 2 N_p is the smallest possible number satisfying 1

Relative loss in mismatched $\mathcal{F}(\lambda)$ at $\lambda = \lambda_{\text{sig}} + \Delta \lambda$:

$$\mathcal{F}(\lambda) = \mathcal{F}(\lambda_{ ext{sig}}) \left(1 - g_{ij} \, \Delta \lambda^i \Delta \lambda^j + ..
ight) \implies ext{"metric" } g_{ij}$$
 $N_p \propto \int_{\{\lambda\}} \sqrt{\det g_{ij}} \, \, d^n \lambda$

isolated NS $\lambda^i = (\alpha, \delta, f, \dot{f})$:

$$N_p \propto T^5$$



The covering problem

Choose a finite number N_p of "templates" $\lambda_{(k)}$, such that

- never lose more than a fraction m at closest template $\lambda_{(k)}$
- 2 N_p is the smallest possible number satisfying 1

Relative loss in mismatched $\mathcal{F}(\lambda)$ at $\lambda = \lambda_{\text{sig}} + \Delta \lambda$:

$$\mathcal{F}(\lambda) = \mathcal{F}(\lambda_{ ext{sig}}) \left(1 - g_{ij} \, \Delta \lambda^i \Delta \lambda^j + ..
ight) \implies ext{"metric" } g_{ij}$$
 $N_p \propto \int_{\{\lambda\}} \sqrt{\det g_{ij}} \, d^n \lambda$

isolated NS $\lambda^i = (\alpha, \delta, f, \dot{f})$:

 $N_p \propto T^5$... but NO scaling with $\mathcal{N}!$ (R. Prix, gr-qc/0606088)

The covering problem

Choose a finite number N_p of "templates" $\lambda_{(k)}$, such that

- never lose more than a fraction m at closest template $\lambda_{(k)}$
- 2 N_p is the smallest possible number satisfying 1

Relative loss in mismatched $\mathcal{F}(\lambda)$ at $\lambda = \lambda_{\text{sig}} + \Delta \lambda$:

$$\mathcal{F}(\lambda) = \mathcal{F}(\lambda_{ ext{sig}}) \left(1 - g_{ij} \, \Delta \lambda^i \Delta \lambda^j + ..
ight) \implies ext{"metric" } g_{ij}$$
 $N_p \propto \int_{\{\lambda\}} \sqrt{\det g_{ij}} \, \, d^n \lambda$

isolated NS $\lambda^i = (\alpha, \delta, f, f)$:

 $N_p \propto T^5$... but NO scaling with $\mathcal{N}!$ (R. Prix, gr-qc/0606088)

Computing "cost": $C_p \propto \mathcal{N} T^6$

$$N_p \propto T^5$$

$$C_p \propto \mathcal{N} T^6$$

$$SNR \propto \sqrt{\mathcal{N} T}$$

$$N_p \propto T^5$$
 $C_p \propto \mathcal{N} \ T^6$ SNR $\propto \sqrt{\mathcal{N} \ T}$

Det	Т	SNR	C_p
H1+L1	T_0	ρ_0	C_0
H1+L1+V1	T_0	1.22 ρ_0	1.5 <i>C</i> ₀
H1+L1	$\frac{3}{2} T_0$	1.22 ρ_0	11.4 <i>C</i> ₀
V1	2 T ₀	ρ_0	32 C ₀
V1	3 T ₀	$1.22 \rho_0$	364 <i>C</i> ₀

$$N_p \propto T^5$$
 $C_p \propto \mathcal{N} \ T^6$ SNR $\propto \sqrt{\mathcal{N} \ T}$

Det	T	SNR	C_p
H1+L1	T_0	$ ho_0$	C_0
H1+L1+V1	T_0	1.22 ρ_0	1.5 <i>C</i> ₀
H1+L1	$\frac{3}{2} T_0$	1.22 ρ_0	11.4 <i>C</i> ₀
V1	2 T ₀	$ ho_0$	32 C ₀
V1	3 <i>T</i> ₀	1.22 ρ_0	364 C ₀

$$N_p \propto T^5$$
 $C_p \propto \mathcal{N} \ T^6$ SNR $\propto \sqrt{\mathcal{N} \ T}$

Det	Т	SNR	C_p
H1+L1	T_0	$ ho_0$	<i>C</i> ₀
H1+L1+V1	<i>T</i> ₀	1.22 ρ_0	1.5 <i>C</i> ₀
H1+L1	$\frac{3}{2} T_0$	$1.22 \rho_0$	11.4 C ₀
111721	2 70	1.22 00	11.400
V1	2 T ₀	ρ_0	32 C ₀

$$N_p \propto T^5$$
 $C_p \propto \mathcal{N} \ T^6$ SNR $\propto \sqrt{\mathcal{N} \ T}$

Det	T	SNR	C_p
H1+L1	T_0	$ ho_0$	C_0
H1+L1+V1	T_0	1.22 ρ_0	1.5 <i>C</i> ₀
H1+L1	$\frac{3}{2} T_0$	1.22 ρ_0	11.4 <i>C</i> ₀
H1+L1 V1	$\frac{3}{2} T_0$ 2 T_0	1.22 ρ_0	11.4 <i>C</i> ₀

$$N_p \propto T^5$$
 $C_p \propto \mathcal{N} \ T^6$ SNR $\propto \sqrt{\mathcal{N} \ T}$

Det	T	SNR	C_p
H1+L1	T_0	$ ho_0$	C_0
H1+L1+V1	T_0	1.22 ρ_0	1.5 <i>C</i> ₀
H1+L1	$\frac{3}{2} T_0$	1.22 ρ_0	11.4 <i>C</i> ₀
V1	2 T ₀	$ ho_0$	32 <i>C</i> ₀
V1	3 T ₀	$1.22 \rho_0$	364 C ₀

$$N_p \propto T^5$$
 $C_p \propto \mathcal{N} \ T^6$ SNR $\propto \sqrt{\mathcal{N} \ T}$

Det	T	SNR	C_p
H1+L1	T_0	$ ho_0$	C_0
H1+L1+V1	T_0	1.22 ρ_0	1.5 <i>C</i> ₀
H1+L1	$\frac{3}{2} T_0$	1.22 ρ_0	11.4 <i>C</i> ₀
V1	2 T ₀	$ ho_0$	32 <i>C</i> ₀
V1	3 T ₀	1.22 ρ_0	364 <i>C</i> ₀

Assume similar sensitivity H1 \sim L1 \sim V1

$$N_{\!
ho} \propto T^5$$
 $C_{\!
ho} \propto {\cal N} \, T^6$ SNR $\propto \sqrt{{\cal N} \, T}$

Det	Т	SNR	C_p
H1+L1	T_0	$ ho_0$	C_0
H1+L1+V1	T_0	1.22 ρ_0	1.5 <i>C</i> ₀
H1+L1	$\frac{3}{2} T_0$	1.22 ρ_0	11.4 <i>C</i> ₀
V1	2 T ₀	$ ho_0$	32 <i>C</i> ₀
V1	3 T ₀	1.22 ρ_0	364 <i>C</i> ₀

Combining (similar-sensitivity) detectors is the computationally cheapest way to increase sensitivity!

(at fixed computing power ⇒ highest sensitivity)

☐ Wide-parameter searches for unknown NS:

 \Box Targeted searches for known pulsars ($f = 2\nu$)

□ Wide-parameter searches for unknown NS: Need to scan space of Doppler-parameters λ (but not A) e.g. isolated NS (α, δ, f, f) : number of templates $N_p \propto T^5$

 \Box Targeted searches for known pulsars ($f = 2\nu$)

- □ Wide-parameter searches for unknown NS: Need to scan space of Doppler-parameters λ (but not A) e.g. isolated NS (α, δ, f, f) : number of templates $N_p \propto T^5$
 - Fully coherent: F-statistic (Einstein@Home T ≤ 30 hours)
 optimal sensitivity @ infinite computing power

☐ Targeted searches for known pulsars ($f = 2\nu$)

- □ Wide-parameter searches for unknown NS: Need to scan space of Doppler-parameters λ (but not A) e.g. isolated NS $(\alpha, \delta, f, \dot{f})$: number of templates $N_p \propto T^5$
 - Fully coherent: F-statistic (Einstein@Home T ≤ 30 hours)
 □ optimal sensitivity @ infinite computing power
 - 2 Semi-coherent: Hough, StackSlide, PowerFlux ($T \sim \text{data}$) sub-optimal but fast

■ Targeted searches for known pulsars ($f = 2\nu$)

- Wide-parameter searches for unknown NS: Need to scan space of Doppler-parameters λ (but not A) e.g. isolated NS (α, δ, f, f) : number of templates $N_p \propto T^5$
 - If Fully coherent: \mathcal{F} -statistic (Einstein@Home $T \leq 30$ hours) optimal sensitivity @ infinite computing power
 - Semi-coherent: Hough, StackSlide, PowerFlux ($T \sim \text{data}$) sub-optimal but fast
 - Mierarchical search: combine 1 + 2, will run on E@H A optimal sensitivity @ finite computing power

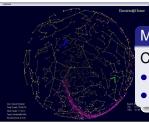


 \Box Targeted searches for known pulsars ($f = 2\nu$)

- Wide-parameter searches for unknown NS: Need to scan space of Doppler-parameters λ (but not A) e.g. isolated NS (α, δ, f, f) : number of templates $N_p \propto T^5$
 - If Fully coherent: \mathcal{F} -statistic (Einstein@Home $T \leq 30$ hours) optimal sensitivity @ infinite computing power
 - Semi-coherent: Hough, StackSlide, PowerFlux ($T \sim \text{data}$) sub-optimal but fast
 - Mierarchical search: combine 1 + 2, will run on E@H A optimal sensitivity @ finite computing power



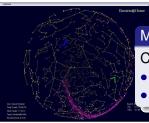
 \Box Targeted searches for known pulsars ($f = 2\nu$) only one template $\lambda_0 = \{\alpha, \delta, f, f, ...\}$ from radio/X-ray Fully coherent, not computationally limited ($T \sim \text{data}$), ⇒ most sensitive search!



Maximize available computing power

- Send workunits $\Delta \lambda$ to participating hosts
- Hosts return finished work and request next
- Public distributed computing project, launched Feb. 2005
- Currently ∼120,000 active participants, ~50Tflops
- runs on GNU/Linux, Mac OSX, Windows,...
- Search for isolated neutron stars $f \in [50, 1500]$ Hz
- Aiming for detection, not upper limits
- Analyzed data from S3, S4, just started: S5

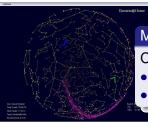




Maximize available computing power

- Send workunits Δλ to participating hosts
- Hosts return finished work and request next
- Public distributed computing project, launched Feb. 2005
- Currently \sim 120,000 active participants, \sim 50Tflops
- runs on GNU/Linux, Mac OSX, Windows,...
- Search for isolated neutron stars $f \in [50, 1500]$ Hz
- Aiming for detection, not upper limits
- Analyzed data from S3, S4, just started: S5

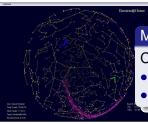




Maximize available computing power

- Send workunits Δλ to participating hosts
- Hosts return finished work and request next
- Public distributed computing project, launched Feb. 2005
- Currently \sim 120,000 active participants, \sim 50Tflops
- runs on GNU/Linux, Mac OSX, Windows,...
- Search for isolated neutron stars $f \in [50, 1500]$ Hz
- Aiming for detection, not upper limits
- Analyzed data from S3, S4, just started: S5

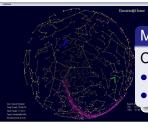




Maximize available computing power

- Send workunits $\Delta \lambda$ to participating hosts
- Hosts return finished work and request next
- Public distributed computing project, launched Feb. 2005
- Currently ~120,000 active participants, ~50Tflops
- runs on GNU/Linux, Mac OSX, Windows,...
- Search for isolated neutron stars $f \in [50, 1500]$ Hz
- Aiming for detection, not upper limits
- Analyzed data from S3, S4, just started: S5

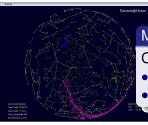




Maximize available computing power

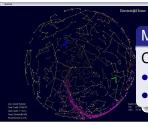
- Send workunits $\Delta \lambda$ to participating hosts
- Hosts return finished work and request next
- Public distributed computing project, launched Feb. 2005
- Currently ~120,000 active participants, ~50Tflops
- runs on GNU/Linux, Mac OSX, Windows,...
- Search for isolated neutron stars $f \in [50, 1500]$ Hz
- Aiming for detection, not upper limits
- Analyzed data from S3, S4, just started: S5





Maximize available computing power

- Send workunits $\Delta \lambda$ to participating hosts
- Hosts return finished work and request next
- Public distributed computing project, launched Feb. 2005
- Currently \sim 120,000 active participants, \sim 50Tflops
- runs on GNU/Linux, Mac OSX, Windows,...
- Search for isolated neutron stars $f \in [50, 1500]$ Hz
- Aiming for detection, not upper limits
- Analyzed data from S3, S4, just started: S5



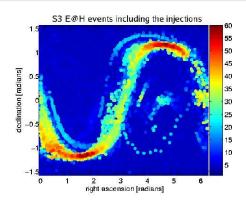
Maximize available computing power

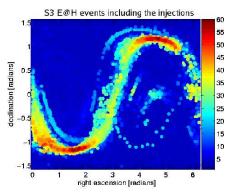
- Send workunits $\Delta \lambda$ to participating hosts
- Hosts return finished work and request next
- Public distributed computing project, launched Feb. 2005
- Currently \sim 120,000 active participants, \sim 50Tflops
- runs on GNU/Linux, Mac OSX, Windows,...
- Search for isolated neutron stars $f \in [50, 1500]$ Hz
- Aiming for detection, not upper limits
- Analyzed data from S3, S4, just started: S5



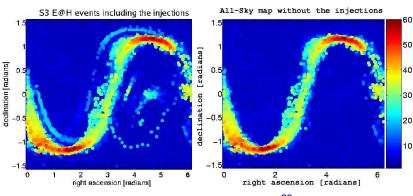
Outline

- Astrophysical Motivation
 - Gravitational Waves from Neutron Stars?
 - Emission Mechanisms (Mountains, Precession, Oscillations, Accretion)
 - Gravitational Wave Astronomy of NS
- Detecting Gravitational Waves from NS
 - Status of LIGO (+GEO600)
 - Data-analysis of continous waves
 - Observational Results

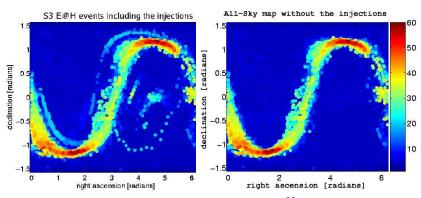




 \Box correctly identified injections ($h_0 \sim 10^{-23}$)



 \Box correctly identified injections ($h_0 \sim 10^{-23}$)



- \Box correctly identified injections ($h_0 \sim 10^{-23}$)
- □ all "outliers" either on $\mathbf{r}(t) \cdot \mathbf{n} = 0$ circles (stationary lines), or ruled out by follow-up studies (S4)

 \blacksquare Fully coherent (\mathcal{F} -statistic) searches [gr-qc/0605028]:

Semi-coherent searches:

 \blacksquare Fully coherent (\mathcal{F} -statistic) searches [gr-qc/0605028]:

S2 Sco X-1 (unknown
$$f$$
, a_p , \bar{T}), using $T=6~h$ of S2 $h_0^{95\%} \sim 2 \times 10^{-22}$

Semi-coherent searches:

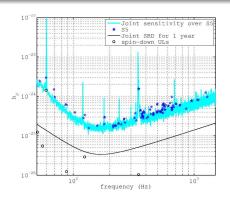
- □ Fully coherent (𝒯-statistic) searches [gr-qc/0605028]:
 - S2 Sco X-1 (unknown f, a_p , \bar{T}), using T = 6 h of S2 $h_0^{95\%} \sim 2 \times 10^{-22}$
 - S2 All-sky, isolated NS, ($f \in [160, 728] \text{ Hz}$), using T = 10 h of S2 $h_0^{95\%} \sim 7 \times 10^{-23}$
- Semi-coherent searches:

- □ Fully coherent (𝒯-statistic) searches [gr-qc/0605028]:
 - S2 Sco X-1 (unknown f, a_p , \bar{T}), using T=6~h of S2 $h_p^{95\%} \sim 2 \times 10^{-22}$
 - S2 All-sky, isolated NS, ($f \in [160, 728] \text{ Hz}$), using T = 10 h of S2 $h_0^{95\%} \sim 7 \times 10^{-23}$
- Semi-coherent searches:
 - S2 Hough-transform: all-sky, isolated NS ($f \in [200, 400] \text{ Hz}$) $h_0^{95\%} \sim 4.5 \times 10^{-23}$

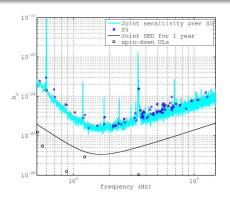
- □ Fully coherent (𝒯-statistic) searches [gr-qc/0605028]:
 - S2 Sco X-1 (unknown f, a_p , \bar{T}), using T = 6 h of S2 $h_0^{95\%} \sim 2 \times 10^{-22}$
 - S2 All-sky, isolated NS, ($f \in [160, 728] \text{ Hz}$), using T = 10 h of S2 $h_0^{95\%} \sim 7 \times 10^{-23}$
- Semi-coherent searches:
 - S2 Hough-transform: all-sky, isolated NS ($f \in [200, 400]$ Hz) $h_0^{95\%} \sim 4.5 \times 10^{-23}$
 - S4 StackSlide: all-sky, isolated NS ($f \in [50, 225]$ Hz) $h_0^{95\%} \sim 4.5 \times 10^{-24}$ (preliminary)

- □ Fully coherent (𝒯-statistic) searches [gr-qc/0605028]:
 - S2 Sco X-1 (unknown f, a_p , \bar{T}), using T=6~h of S2 $h_0^{95\%} \sim 2 \times 10^{-22}$
 - S2 All-sky, isolated NS, ($f \in [160, 728] \text{ Hz}$), using T = 10 h of S2 $h_0^{95\%} \sim 7 \times 10^{-23}$
- Semi-coherent searches:
 - S2 Hough-transform: all-sky, isolated NS ($f \in [200, 400]$ Hz) $h_0^{95\%} \sim 4.5 \times 10^{-23}$
 - S4 StackSlide: all-sky, isolated NS ($f \in [50, 225]$ Hz) $h_0^{95\%} \sim 4.5 \times 10^{-24}$ (preliminary)
- Early S5 PowerFlux: all-sky, isolated NS ($f \in [40,700]$ Hz) $h_0^{95\%} \sim 2 \times 10^{-24}$ (preliminary)

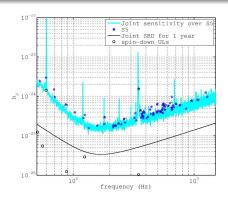




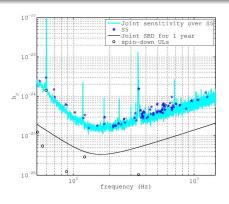
- Targeted 73 pulsars ($f = 2\nu$): 32 isolated, 41 binary (29 in GCs)
- first 2 months of S5
- all 3 detectors: H1, H2, L1
- Best 95% upper limits: $h_0 \lesssim 2 \times 10^{-25}$ (PSR J1603-7202) $\epsilon \lesssim 4 \times 10^{-7}$ (PSR J2124-3358)



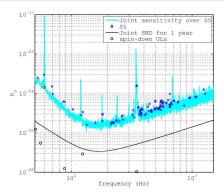
- Targeted 73 pulsars ($f = 2\nu$): 32 isolated, 41 binary (29 in GCs)
- first 2 months of S5
- all 3 detectors: H1, H2, L1
- Best 95% upper limits: $h_0 \lesssim 2 \times 10^{-25}$ (PSR J1603-7202 $\epsilon \lesssim 4 \times 10^{-7}$ (PSR J2124-3358)



- Targeted 73 pulsars ($f = 2\nu$): 32 isolated, 41 binary (29 in GCs)
- first 2 months of S5
- all 3 detectors: H1, H2, L1
- Best 95% upper limits: $h_0 \lesssim 2 \times 10^{-25}$ (PSR J1603-7202) $\epsilon \lesssim 4 \times 10^{-7}$ (PSR J2124-3358)



- Targeted 73 pulsars ($f = 2\nu$): 32 isolated, 41 binary (29 in GCs)
- first 2 months of S5
- all 3 detectors: H1, H2, L1
- Best 95% upper limits: $h_0 \lesssim 2 \times 10^{-25}$ (PSR J1603-7202) $\epsilon \lesssim 4 \times 10^{-7}$ (PSR J2124-3358)



- Targeted 73 pulsars ($f = 2\nu$): 32 isolated, 41 binary (29 in GCs)
- first 2 months of S5
- all 3 detectors: H1, H2, L1
- Best 95% upper limits: $h_0 \lesssim 2 \times 10^{-25}$ (PSR J1603-7202) $\epsilon \lesssim 4 \times 10^{-7}$ (PSR J2124-3358)

Upper-limits well above spindown-limit (except in GCs)

But: Crab-pulsar is only a factor 2.1 away from spindown-limit will (most likely) be able to beat spindown-limit during S5!

Published results

Published LSC results of neutron-star searches:

- S1 Setting upper limits on the strength of periodic gravitational waves from PSR J1939 + 2134 using the first science data from the GEO 600 and LIGO detectors, B. Abbott et al. (LSC), Phys. Rev. D 69, 082004 (2004)
- S2 Limits on gravitational wave emission from selected pulsars using LIGO data, B. Abbott et al. (LSC), Phys. Rev. Lett. 94, 181103 (2005)
- S2 First all-sky upper limits from LIGO on the strength of periodic gravitational waves using the Hough transform,
 B. Abbott et al. (LSC), Phys. Rev. D 72, 102004 (2005)
- S2 Coherent searches for periodic gravitational waves from unknown isolated sources and Scorpius X-1: results from the second LIGO science run, to be submitted, [gr-qc/0605028]
- S3 Online report on Einstein@Home results for S3 search: http://einstein.phys.uwm.edu/PartialS3Results/

- No GW detection so far, but none expected
 setting upper limits on h₀ and ϵ
- S5 upper-limits are approaching astrophysically relevant regimes (S Crab, EOS-limits on ε)
- LIGO S5 operating at design-sensitivity, will collect one year's worth of data (duration \sim 1.5 years)
- Einstein@Home: Started analyzing S5.
 Developing a fully hierarchical search most sensitive possible search for unknown NS
- NS detection with LIGO-I not very likely, but not impossible ("Expect the unexpected!")
- The future is bright: S6, VIRGO, LIGO-II, GEO-HF, ...



- No GW detection so far, but none expected
 setting upper limits on h₀ and ε
- LIGO S5 operating at design-sensitivity, will collect one year's worth of data (duration ~1.5 years)
- Einstein@Home: Started analyzing S5.
 Developing a fully hierarchical search most sensitive possible search for unknown NS
- NS detection with LIGO-I not very likely, but not impossible ("Expect the unexpected!")
- The future is bright: S6, VIRGO, LIGO-II, GEO-HF, ...



- No GW detection so far, but none expected
 setting upper limits on h₀ and ε
- LIGO S5 operating at design-sensitivity, will collect one year's worth of data (duration \sim 1.5 years)
- Einstein@Home: Started analyzing S5.
 Developing a fully hierarchical search most sensitive possible search for unknown NS
- NS detection with LIGO-I not very likely, but not impossible ("Expect the unexpected!")
- The future is bright: S6, VIRGO, LIGO-II, GEO-HF, ...



- No GW detection so far, but none expected
 setting upper limits on h₀ and ε
- LIGO S5 operating at design-sensitivity, will collect one year's worth of data (duration ~1.5 years)
- Einstein@Home: Started analyzing S5.
 Developing a fully hierarchical search most sensitive possible search for unknown NS
- NS detection with LIGO-I not very likely, but not impossible ("Expect the unexpected!")
- The future is bright: S6, VIRGO, LIGO-II, GEO-HF, ...

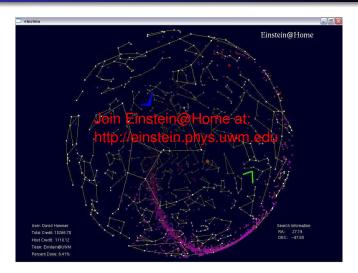


- No GW detection so far, but none expected
 setting upper limits on h₀ and ε
- LIGO S5 operating at design-sensitivity, will collect one year's worth of data (duration ~1.5 years)
- Einstein@Home: Started analyzing S5.
 Developing a fully hierarchical search most sensitive possible search for unknown NS
- NS detection with LIGO-I not very likely, but not impossible ("Expect the unexpected!")
- The future is bright: S6, VIRGO, LIGO-II, GEO-HF, ...



- No GW detection so far, but none expected
 setting upper limits on h₀ and ε
- LIGO S5 operating at design-sensitivity, will collect one year's worth of data (duration ~1.5 years)
- Einstein@Home: Started analyzing S5.
 Developing a fully hierarchical search most sensitive possible search for unknown NS
- NS detection with LIGO-I not very likely, but not impossible ("Expect the unexpected!")
- The future is bright: S6, VIRGO, LIGO-II, GEO-HF, ...

You can help us find Gravitational Waves!



The "spindown-limit" (for known pulsars)

Energy lost in GW: $\frac{dE_{\rm GW}}{dt} \propto \nu^6 \ \emph{I}_{\it ZZ}^2 \ \epsilon^2$

Rotational energy: $\frac{dE_{\rm rot}}{dt} \propto I_{zz} \underbrace{\nu \dot{\nu}}_{\rm observed}$

Spindown limit

$$\frac{dE_{\text{GW}}}{dt} \leq \frac{dE_{\text{rot}}}{dt} \implies \text{upper limit on } \epsilon \text{ and } h_0$$

imit on deformation ϵ and amplitude h_0 :

$$\epsilon_{\mathrm{sd}}^2 \propto rac{1}{I_{zz}} rac{\dot{
u}}{
u^5} \quad , \qquad h_{\mathrm{sd}} \propto rac{\sqrt{I_{zz}}}{d} \sqrt{rac{\dot{
u}}{
u}}$$

