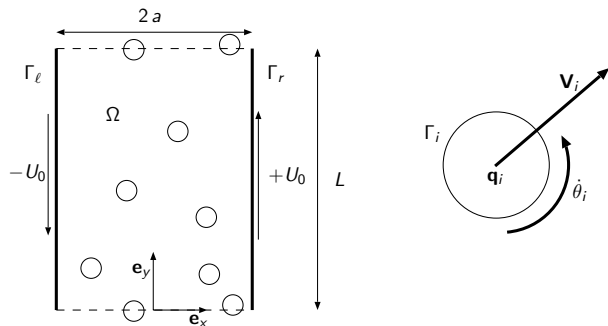


Simulation d'écoulements fluide-particules

B. Maury

13 mai 2008

OBJECTIFS



$\mathbf{q}_i = \mathbf{q}_i(t)$: centre de la particule i

Phase solide $\omega = \omega(t) = \bigcup_{i=1}^N \overline{B(\mathbf{q}_i(t), r)}$.

Domaine fluide : $\Omega_F(t) = \Omega \setminus \overline{\omega}(t)$.

Equations de Navier-Stokes dans le domaine mobile $\Omega_F(t)$,

$$\begin{cases} \frac{D\mathbf{u}}{Dt} - \mu\Delta\mathbf{u} + \nabla p = 0, \\ \nabla \cdot \mathbf{u} = 0, \end{cases}$$

PFD pour les particules

$$\frac{d\mathbf{V}_i}{dt} = - \int_{\Gamma_i} \sigma \cdot \mathbf{n} + \mathbf{F}_i \quad \left(+ \text{equation sur } \ddot{\theta}_i \right)$$

avec $\sigma = \mu(\nabla\mathbf{u} + {}^t\nabla\mathbf{u}) - p \text{ Id}$
 + condition de non-glissement

$$\mathbf{u} = \mathbf{V}_i + \dot{\theta}_i \times \mathbf{r}_i \text{ on } \Gamma_i \text{ for } 1 \leq i \leq N.$$

DIFFICULTÉ : MAUVAIS CONDITIONNEMENT

$$-\Delta u = f \quad \longrightarrow \quad Au = b.$$

Conditionnement : $\kappa = \lambda_N / \lambda_1$

Mauvais conditionnement : $-\Delta \in \mathcal{L}(L^2)$ est non-borné :

Les **Petites** valeurs propres correspondent aux valeurs propres “physiques” de $-\Delta$. Elles dépendent de la géométrie.

Les **Grandes** v.p. (non bornées pour l’opérateur continu) dépendent la discrétisation : carré de la plus haute fréquence capturée par le maillage $\sim 1/h^2$.

Conditionnement $\kappa = \frac{1}{\lambda_1 h^2} \sim \frac{L^2}{h^2}$

Mesure la stabilité de l'inversion :

$$A(u + \delta u) = b + \delta b \longrightarrow \frac{|\delta u|}{|u|} \leq \frac{|\delta b|}{|b|}$$

Intervient dans la vitesse de convergence des méthodes itératives

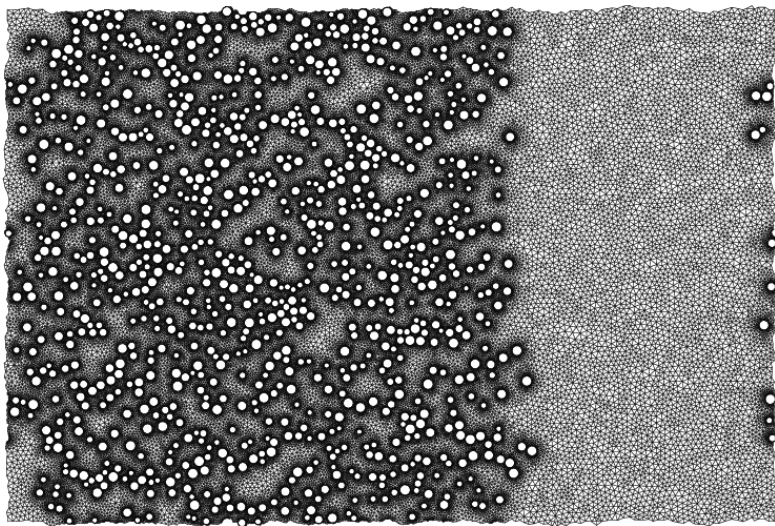
Ex. Algorithme de gradient conjugué : $e_k = (1 - 2/\sqrt{\kappa})^k$.

Nombre d'itérations pour atteindre une précision ε donnée

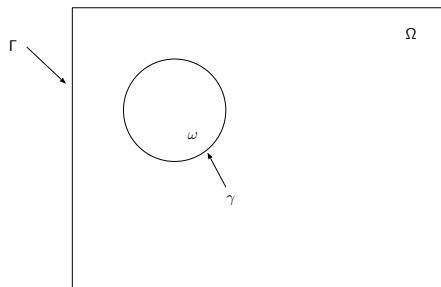
$$k \approx \sqrt{\kappa} \log \varepsilon = \frac{L}{h} \log \varepsilon.$$

Coût d'une itération (produit matrice vecteur) $\approx (L/h)^d$.

Coût total $\approx (L/h)^{d+1}$.



Version scalaire du pb fluide particule (avec une inclusion)



$$\left\{ \begin{array}{ll} -\Delta u = \mathbf{f} & \text{in } \Omega \setminus \bar{\omega} \\ u = 0 & \text{on } \Gamma = \partial\Omega \\ u = U \in \mathbb{R} & \text{on } \gamma = \partial\omega \\ \int_{\gamma} \partial u / \partial n = 0 \end{array} \right.$$

Revient à minimiser

$$J(v) = \frac{1}{2} \int_{\Omega} |\nabla v|^2 - \int_{\Omega} f v$$

sur

$$K = \{ v, \exists W \in \mathbb{R}, v = W \text{ a.e. in } \omega \}.$$

Bien posé : \exists une solution unique $u \in V = H_0^1(\Omega)$.

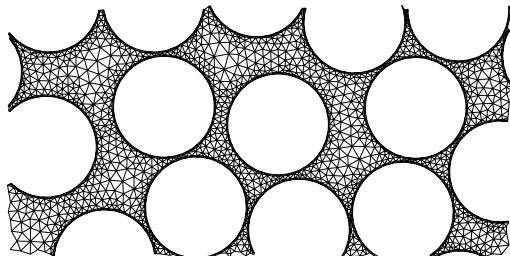
Rq : $u \notin H^2(\Omega)$.

APPROCHE DIRECTE

(H. Hu, B.M., T. Tezduyar)

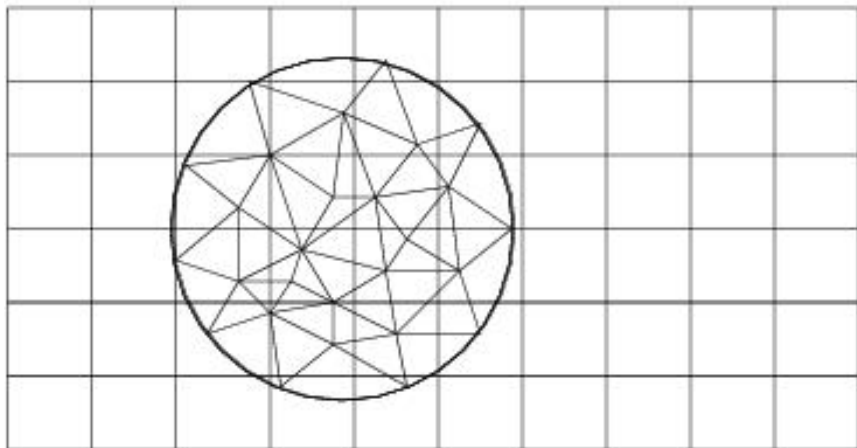
K_h espace d'approximation pour K : maillage conforme

Comme $\omega = \omega(t)$, $K_h = K_h(t)$: approche ALE



→20

DOMAINES FICTIFS



MEAN VALUE CONSTRAINT FORMULATION

(with J.B. Apoung Kamga, C. Jammes)

New formulation of the constraint

$$\int_{\partial\omega} \mu (u - m(u)) = 0 \quad \forall \mu \in L^2(\partial\omega), \text{ with } m(u) = \frac{\int_{\partial\omega} u}{|\partial\omega|}.$$

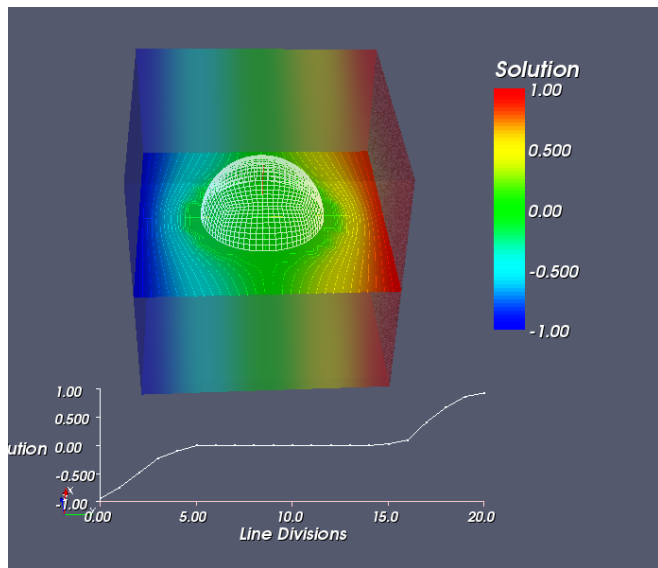
Saddle-point formulation

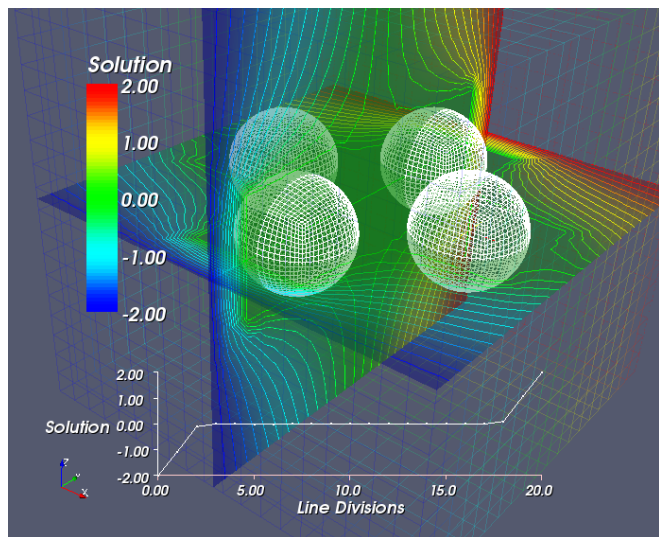
$$\int_{\Omega} \nabla u \cdot \nabla v + \int_{\gamma} \lambda (v - m(v)) = \int f v \quad \forall v \in H_0^1(\Omega)$$

$$\int_{\gamma} \mu (u - m(u)) = 0 \quad \forall \mu \in L^2(\partial\omega)$$

Algorithm :

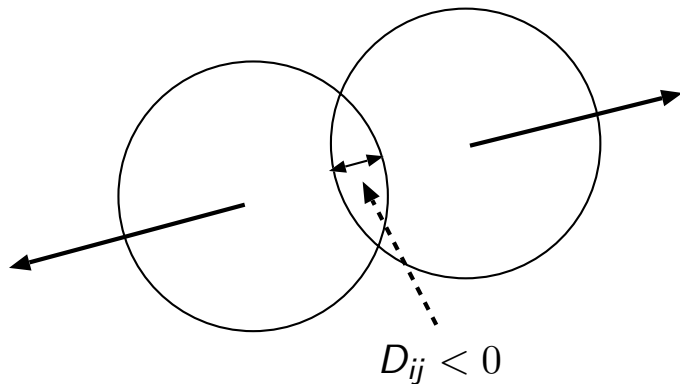
$$\begin{aligned} \int_{\Omega} \nabla u^{k+1} \cdot \nabla v + \int_{\gamma} \lambda^k (v - m(v)) &= \int f v \quad \forall v \in H_0^1(\Omega) \\ \lambda^{k+1} &= \lambda^k + \rho (u^k - m(u^k)) \end{aligned}$$





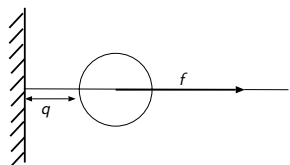
HANDLING OF CONTACTS

Approach I (Molecular Dynamics) : short range repulsive force
Most commonly used (Glowinski, most granular flow solvers)



Approach II (Contact Dynamics) : Non elastic collisions

Body wall problem



$$u(0) = 0,$$

$$q(t) = \int_0^t u(s) ds \quad \forall t \in I,$$

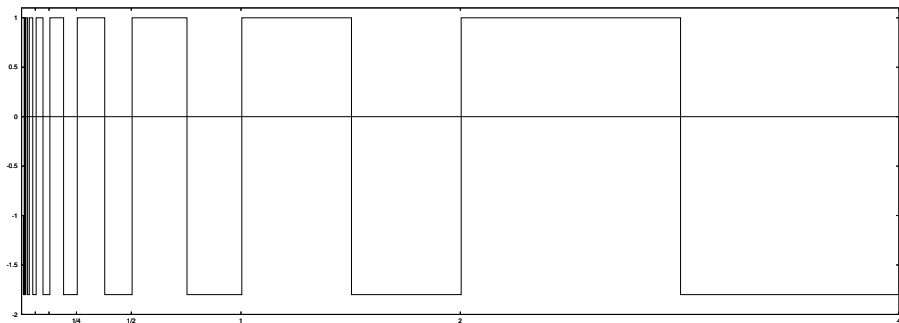
$$\dot{u}(t) = f(t) + \mu,$$

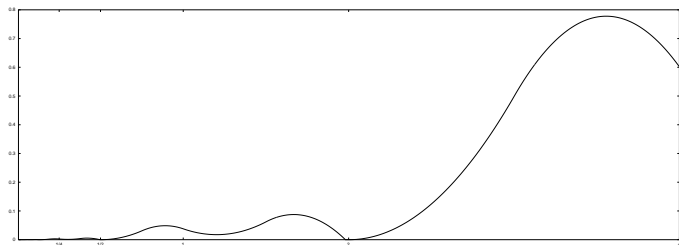
$$\text{supp}(\mu) \subset \{t, q(t) = 0\},$$

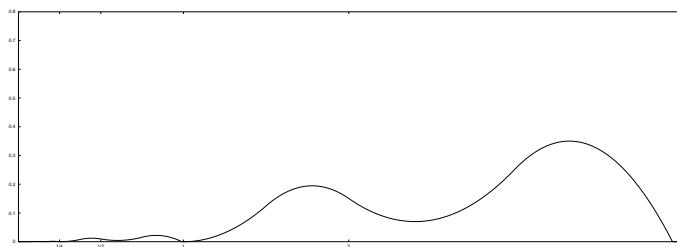
$$u^+ = P_{C_q} u^- \quad \forall t \in I,$$

C_q is \mathbb{R} whenever $q > 0$, and \mathbb{R}^+ as soon as $q = 0$.

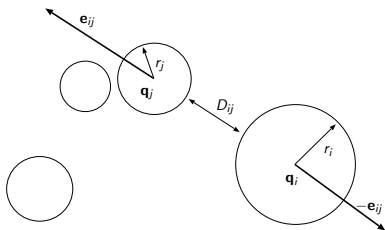
Remark : Ill-posed problem







Multibody formulation

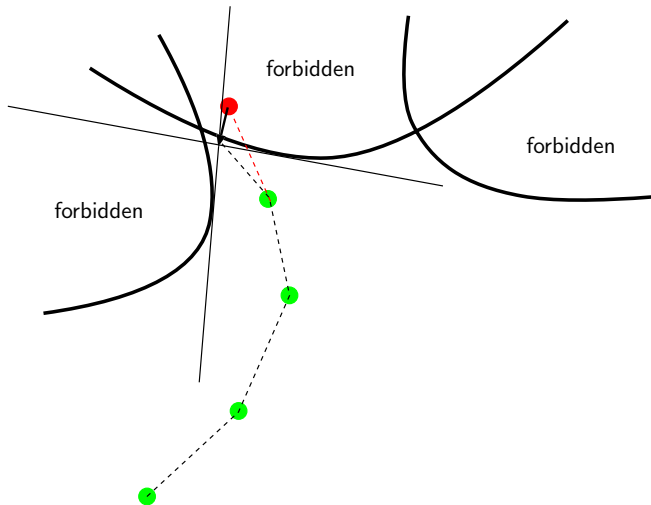


$$\dot{\mathbf{u}} = (\dot{\mathbf{u}}_1, \dots, \dot{\mathbf{u}}_N) = \mathbf{f}(\mathbf{q}, t) + \sum_{i < j} \mu_{ij} \mathbf{G}_{ij}(\mathbf{q}(t)),$$

$$\mathbf{u}^+(t) = P_{C_q} \mathbf{u}^-(t) \quad \forall t \in I,$$

$$\mathbf{G}_{ij} = \nabla D_{ij},$$

$$C_q = \{ \mathbf{v}, D_{ij} = 0 \implies \mathbf{G}_{ij} \cdot \mathbf{v} \geq 0 \} .$$



→run grains

NUMERICAL SCHEME

$$\mathbf{f}_h^{n+1}(\mathbf{q}) = \int_{t^n}^{t^{n+1}} \mathbf{f}(\mathbf{q}, t).$$

1 Initialization

$$(\mathbf{q}_h^0, \mathbf{u}_h^0) = (\mathbf{q}_0, \mathbf{u}_0).$$

2 Compute \mathbf{u}_h^{n+1} as the solution to the constrained minimization problem

$$\min_{\mathbf{u} \in C_h(\mathbf{q}_h^n)} \frac{1}{2} |\mathbf{u} - \mathbf{u}_h^n - h\mathbf{f}_h^{n+1}(\mathbf{q}_h^n)|^2$$

with

$$C_h(\mathbf{q}_h^n) = \left\{ \mathbf{u}, \underbrace{D_{ij}(\mathbf{q}_h^n) + h\mathbf{G}_{ij}(\mathbf{q}_h^n)}_{\approx D_{ij}(\mathbf{q}_h^n + h\mathbf{u})} \cdot \mathbf{u} \geq 0 \right\}.$$

3 Update the positions

$$\mathbf{q}_h^{n+1} = \mathbf{q}_h^n + h\mathbf{u}_h^{n+1}.$$

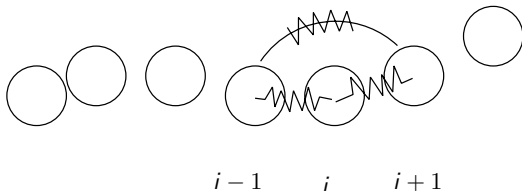
COMPLEX BODIES

$$\mathbf{q} = (\mathbf{q}_1, \dots, \mathbf{q}_N).$$

Interaction force : $\mathbf{F} = -\nabla_{\mathbf{q}} V$,

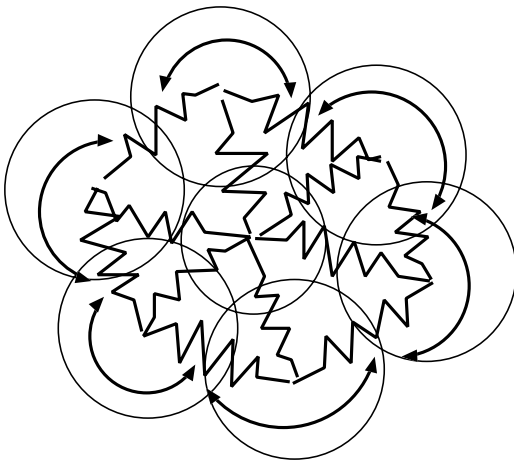
Elongated structure

$$V = \frac{k}{2} \sum (|\mathbf{q}_{i+1} - \mathbf{q}_i| - \ell)^2 + \frac{k_a}{2} \sum \frac{\mathbf{q}_{i+1} - \mathbf{q}_i}{|\mathbf{q}_{i+1} - \mathbf{q}_i|} \cdot \frac{\mathbf{q}_{i-1} - \mathbf{q}_i}{|\mathbf{q}_{i-1} - \mathbf{q}_i|}.$$



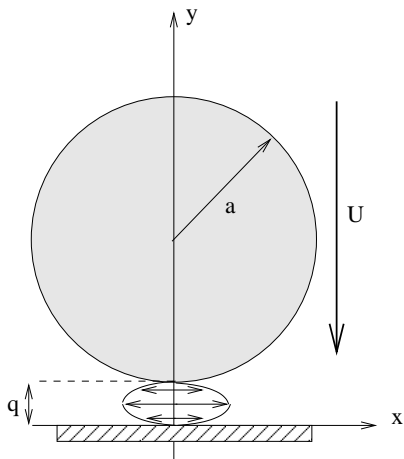
→60-62

Red cell

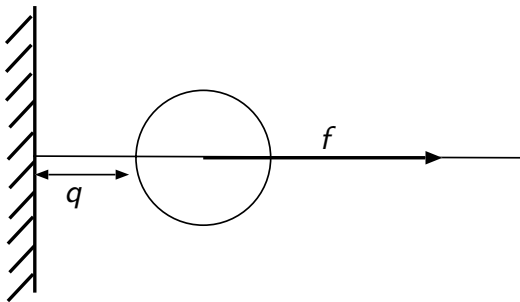


→65

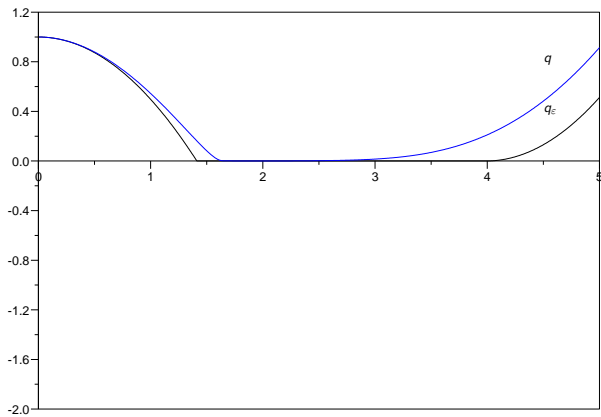
FORCES DE LUBRIFICATION (AVEC A. LEFEBVRE)

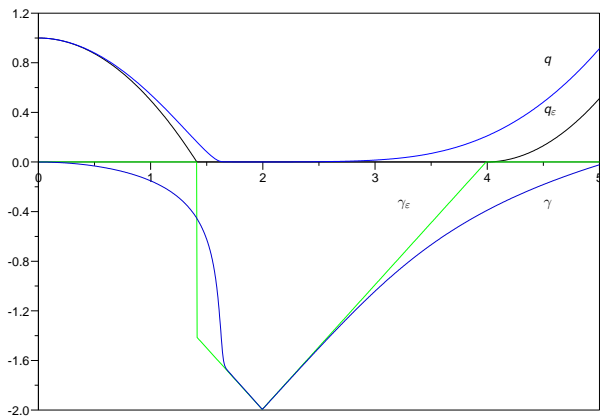


$$F_{lub} \sim -6\pi\mu a^2 \frac{U}{q} \mathbf{e}_y \quad (\text{Brenner ou Kim})$$



$$\left| \begin{array}{l} \ddot{q}_\varepsilon = -\varepsilon \frac{\dot{q}_\varepsilon}{q_\varepsilon} + f(t), \\ q_\varepsilon(0) = q^0 > 0, \quad \dot{q}_\varepsilon(0) = u^0, \end{array} \right.$$





COMPORTEMENT ASYMPTOTIQUE

Trouver $q \in W^{1,\infty}(I)$ avec $\dot{q} \in BV(I)$, $\gamma \in BV(I)$, $\mu \in \mathcal{M}(I)$, t.q.

$$\ddot{q} = f + \lambda \text{ in } \mathcal{M}(I),$$

$$\text{supp}(\lambda) \in \{t, q(t) = 0\},$$

$$\dot{q}^+ = P_{C_q} \dot{q}^-,$$

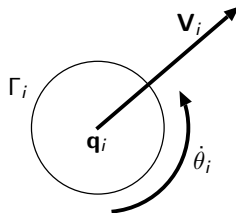
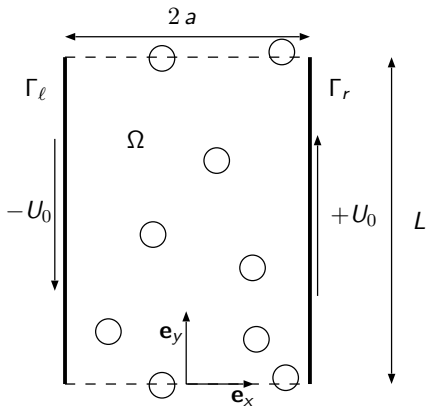
$$\dot{\gamma} = -\lambda, \quad \gamma \leq 0, \quad q \geq 0, \quad q\gamma = 0 \quad \text{p.p. sur } I,$$

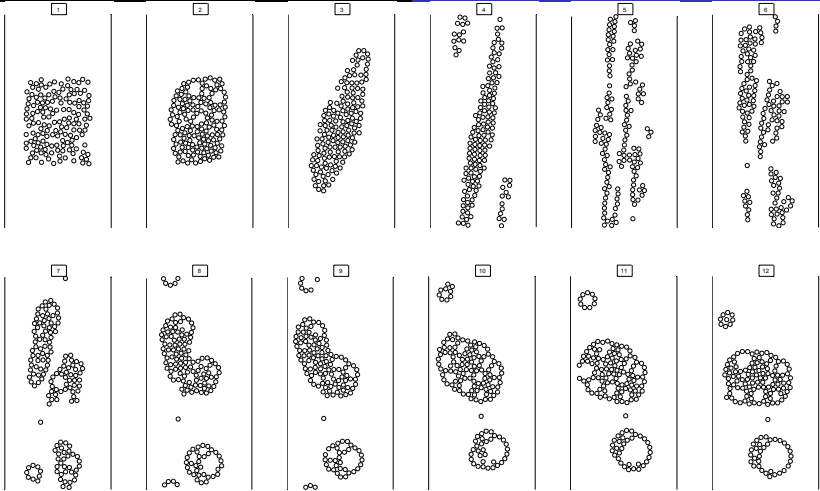
$$q(0) = q^0 > 0, \quad \dot{q}(0) = u^0,$$

$$\text{with } C_q = \begin{cases} \mathbb{R} & \text{if } q > 0, \\ \mathbb{R}^+ & \text{if } q = 0 \text{ and } \gamma^- = 0, \\ \{0\} & \text{if } q = 0 \text{ and } \gamma^- < 0. \end{cases}$$

→80-81-83

ESTIMATION DE LA VISCOSITÉ APPARENTE D'UNE SUSPENSION





Evolution de la viscosité apparente

