Small parameters in infrared Quantum Chromodynamics

M. Peláez[1], U. Reinosa[2], J. Serreau[3],
M. Tissier[4] and N. Wschebor[1]

[1]Instituto de Física, Facultad de Ingeniería, Udelar,
Montevideo, Uruguay,
[2]Centre de Physique Théorique, Ecole Polytechnique, Palaiseau, France,
[3]APC, Université Paris Diderot, France,
[4]LPTMC, Université de Paris VI, France.

December 2017, GDR, Saclay

The Euclidean Lagrangian

Why the Euclidean space?

- Lattice simulations are done in the Euclidean space.

The Euclidean Lagrangian with $SU(N)$ symmetry is

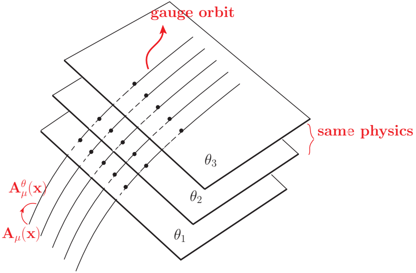
$$\mathcal{L}_{\text{inv}} = \underbrace{\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}}_{\substack{\text{dynamic of the gluon field} \\ \text{gluon self interaction}}} + \underbrace{\sum_{i=1}^{N_f} \bar{\psi}_i(-\gamma_\mu D_\mu + M_i)\psi_i}_{\substack{\text{dynamic of the quark field} \\ \text{quark-gluon interaction}}}$$

Invariant under infinitesimal **gauge transformations**:

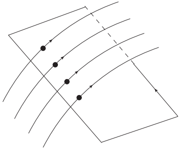
- $\psi(x) \to \psi(x) - ig\theta^a(x)t^a\psi(x)$ **for the fermion field**
- $A^a_\mu(x) \to A^a_\mu(x) - \left(\partial_\mu\theta^a(x) + gf^{abc}A^b_\mu(x)\theta^c(x)\right)$ **for the gluon field**

Problems of the gauge symmetry

- $$\boxed{\langle \mathcal{O}_{\text{inv}} \rangle = \frac{\int \mathcal{D}\phi \; \mathcal{O}_{\text{inv}} e^{-S_{\text{inv}}}}{\int \mathcal{D}\phi \; e^{-S_{\text{inv}}}}}$$
- The gauge symmetry introduces an infinite factor in the integrals bringing an **indetermination of the kind** $\frac{\infty}{\infty}$.



- The challenge: how to factorize this factors out of the integrals and consider an integral taking into account **only one representative of each orbit**



Faddeev-Popov

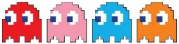
- The standard procedure: Faddeev-Popov

$$\frac{\int \mathcal{D}\phi\, \mathcal{O}_{\text{inv}} e^{-S_{\text{inv}}}}{\int \mathcal{D}\phi\, e^{-S_{\text{inv}}}} = \frac{\int \mathcal{D}\phi\, \mathcal{O}_{\text{inv}} e^{-(S_{\text{inv}}+S_{GF}+S_{FP})}}{\int \mathcal{D}\phi\, e^{-(S_{\text{inv}}+S_{GF}+S_{FP})}}$$

- Landau gauge condition: $\boxed{\partial_\mu A_\mu^a = 0}$

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \sum_{i=1}^{N_f} \bar{\psi}_i(-\gamma_\mu D_\mu + M_i)\psi_i + \underbrace{ih^a \partial_\mu A_\mu^a}_{\text{Landau gauge}} + \underbrace{\partial_\mu \bar{c}^a (D_\mu c)^a}_{\text{Ghosts!}}.$$

$$(D_\mu c)^a = \partial_\mu c^a + g f^{abc} A_\mu^b c^c.$$

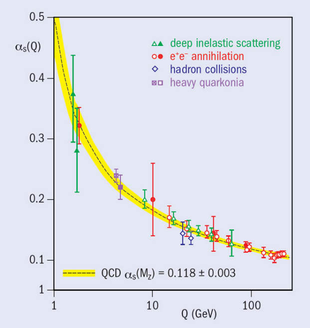


Asymptotic freedom

Asymptotic freedom

- **Asymptotic freedom:** The coupling constant decreases with the momentum scale. At large momentum the coupling constant goes to zero. Nobel 2004: D. Politzer, D. Gross, F. Wilczek
- However, at low momentum the coupling constant, computed with perturbation theory, goes to infinity: **Landau pole**.

$$\alpha_{\text{QCD}} = \frac{g^2}{4\pi}$$

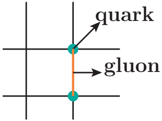


Infrared regime

Problem:

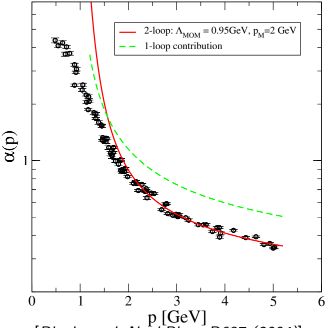
We can not use standard perturbation theory at low momentum!

- Alternatives: Dyson-Schwinger equations [*Alkofer (2001)*], NPRG [*Pawlowski (2007)*].
- Lattice simulations



$$\langle \mathcal{O}\rangle = \underbrace{\frac{1}{\ }_{\text{conf}} \sum_{\text{conf}} \mathcal{O}[\text{conf}]}_{\text{conf probability } e^{-S[\text{conf}]}}$$

- Observation: **Finite coupling constant.**



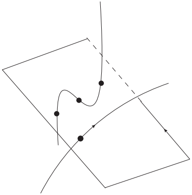
[*Bloch et al, Nucl.Phys. B687 (2004)*]

So, why does perturbation theory not work at all?

The Faddeev-Popov procedure is not justified at low momentum.

Gribov copies

- **Existence Gribov copies** [*Gribov (1978)*].



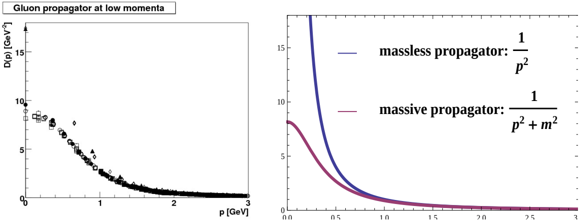
- Studies trying to restrict the integrals to a region without Gribov copies: Gribov-Zwanziger action and refined-Gribov-Zwanziger action. [*Zwanziger (1989)*, *Dudal et al (2008)*]

Landau lattice simulations:

- Several Gribov copies are found by lattice simulations.
- Fortunately, Lattice simulations are able to choose one Gribov copy for each orbit.

Massive gluons

- Moreover, lattice simulations find a finite gluon propagator in the infrared.



Lattice data from [*A. Cucchieri, A. Maas and T. Mendes, Phys.Lett. D77, 2008*]

Gluons behave as if they were massive in the infrared!

Lattice simulations

Lattice simulations:

- Finite coupling constant. Some kind of perturbation theory should be possible.
- Massive gluons in the infrared.
- Massless ghosts.
- They succeed in choosing one representative configuration of each gauge orbit.

However

The equivalent gauge fixed action in the continuum is not known.

The model: Massive gluons

What is the simplest Lagrangian that allows us to do perturbation theory reproducing lattice results?

- Let's try just adding a gluon mass term:

$$\mathcal{L} = \mathcal{L}_{\text{inv}} + ih^a \partial_\mu A^a_\mu + \partial_\mu \bar{c}^a (D_\mu c)^a + \frac{\mathbf{m^2}}{\mathbf{2}} \mathbf{A^a_\mu A^a_\mu}$$

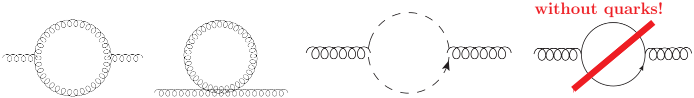
[*Curci-Ferrari (1975)*]

- Problem: **This term breaks BRST symmetry (symmetry of FP Lagrangian).** [*Becchi, Rouet, Stora (1975) and Tyutin (1975)*]
  But it still has a modified-BRST symmetry which allows to prove renormalizability.
- **This modification does not change the results in the ultraviolet.**

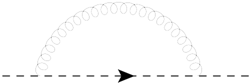
We need to verify if the perturbative analysis at first order reproduces the lattice data also in the infrared

Quenched ghost and gluon propagators.

- One-loop diagrams contributing to the **gluon propagator**.



- One-loop diagrams contributing to the **ghost propagator**.



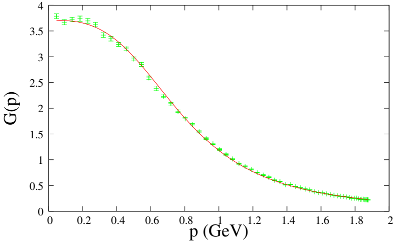
- $\boxed{\textbf{Only two parameters to fit!}}$:

$$\boxed{g(1\text{ GeV})\text{ and }m(1\text{ GeV})}$$

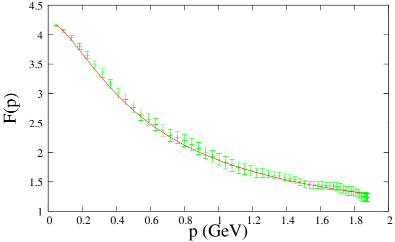
Fit of ghost and gluon propagators for $SU(2)$ and $d = 4$.

- $d = 4$

Gluon propagator



Ghost dressing function



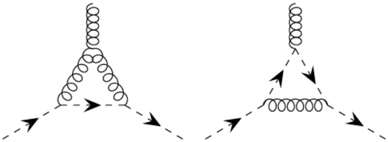
$g(1\ \mathrm{GeV}) = 7{,}5$ and $m(1\ \mathrm{GeV}) = 0{,}77$ GeV

Lattice data from [*A. Cucchieri, A. Maas and T. Mendes, Phys.Lett. D77, 2008*]. Results from [*M. Tissier and N. Wschebor, Phys.Rev. D84, 2011*]

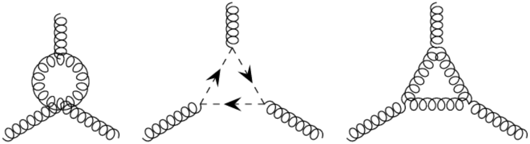
Dressing function $= p^2 \times$ the propagator

Ghost-gluon vertex and three-gluon vertex.

- One-loop diagrams contributing to the **Ghost-gluon vertex**.



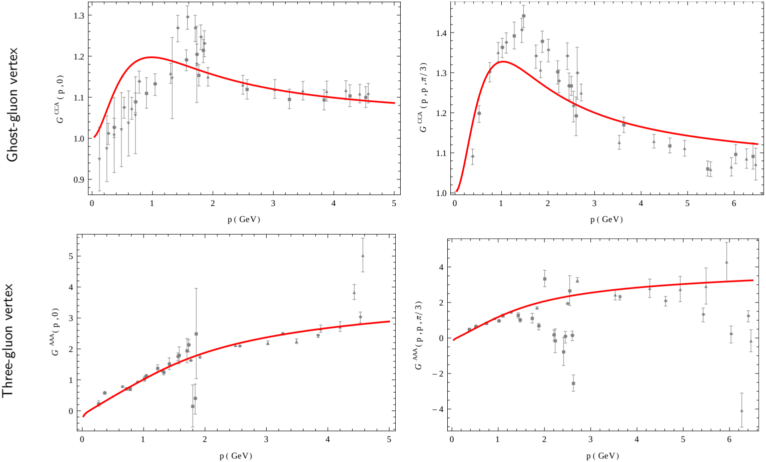
- One-loop diagrams contributing to the **Three-gluon vertex**.



- Results presented in [*M. Peláez, M. Tissier, N. Wschebor, Phys. Rev. D88, (2013)*].

Vertices $SU(2)$ and $d = 4$

- **Without any extra fitting.**

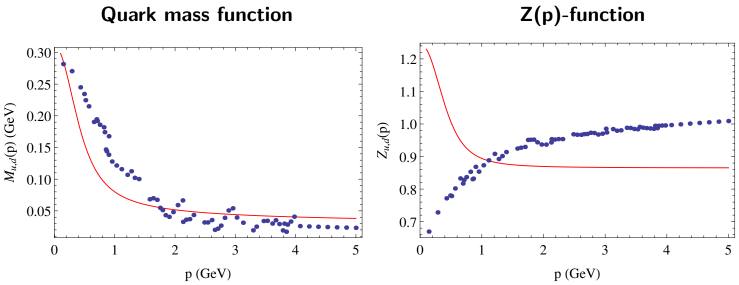


Configurations: gluon vanishing momentum and all equal momenta.

Lattice data from [*A. Cucchieri, A. Maas and T. Mendes, Phys.Lett. D77, 2008*]. Results from [*M. Peláez, M. Tissier and N. Wschebor, Phys.Rev. D88, 2013*]

Quark sector

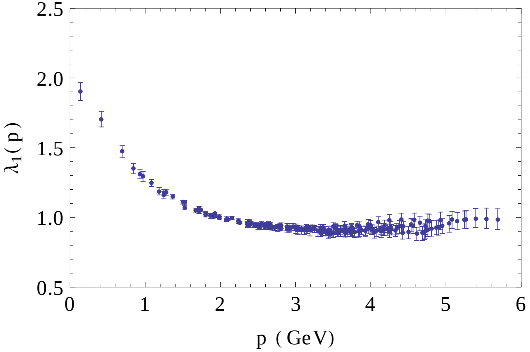
- Yang-Mills one loop calculation gives very accurate results for two and three correlation funcions.
- However, in the **quark sector** results are not as good as Yang-Mills ones.



The points are lattice data of [*Bowman et al, Phys.Rev. D70 (2004)*] [*M. Peláez, M. Tissier, N. Wschebor, Phys. Rev D90 (2014)*].

Quark-Gluon coupling VS Ghost-Gluon coupling

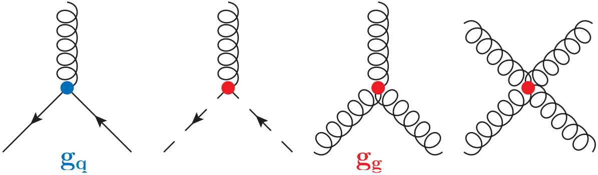
- Quark-gluon coupling constant not too small.
  $g_q(\mu) = g_g(\mu)\lambda_1(\mu)$



Data from [*Skullerud et al. JHEP* **0304**, *047 (2003)*]

Quark-Gluon coupling VS Ghost-Gluon coupling

- As the quark-gluon g_q and YM g_g running coupling constants are different in the infrared, we treat them separetly,



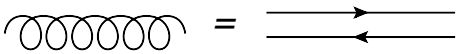
- g_g is considered as small parameter. Yang-Mills sector can be studied perturbately in the infrared.
- g_q is not a small parameter.

Large N_c limit

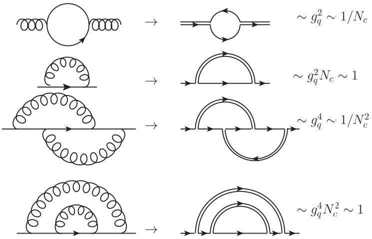
- Large N_c limit shows the same general features of QCD.

[*G. 't Hooft, Nucl. Phys. B* **75**, *461 (1974). Witten, Nucl. Phys. B* **160**, *57 (1979)*]

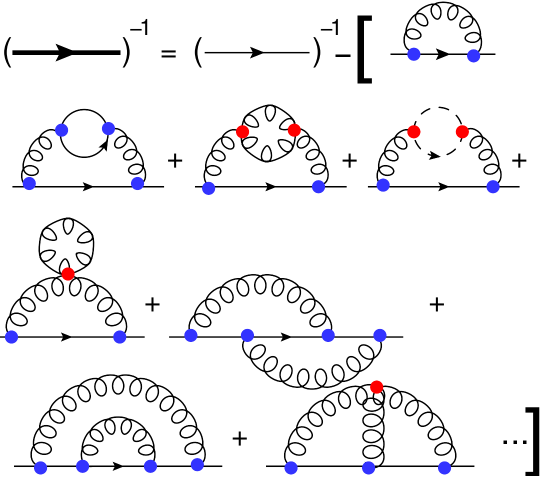
In the large N_c limit, gluon propagators can be replaced by double color lines and



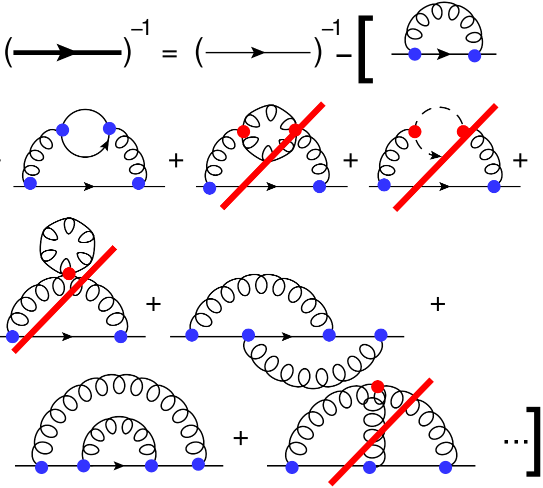
$g_q \sim 1/\sqrt{N_c}$



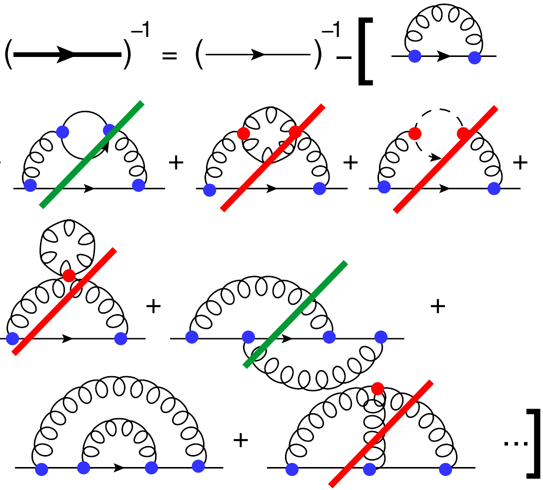
Quark propagator



Quark propagator

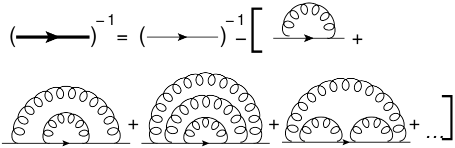


Quark propagator

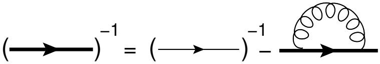


Rainbow equation

- Only Rainbow diagrams survive



- They can be resummed in:



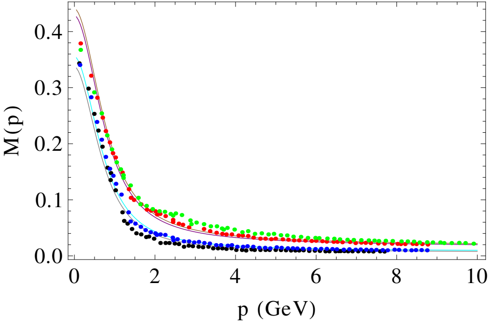
which is the well-known Rainbow approximation for the quark propagator.

see e.g. [*Johnson et al, PRB (1964). Maskawa, PTP (1975). Atkinson et al, PRD (1988). Miransky et al, PRC (2004).*]

[Maris et al, IJMP (2003). Roberts et al,EPJST (2007). Eichman et al, PRC (2008).]

- We use the **ultraviolet one-loop running of the coupling constant** for N_f flavors properly regularised in the infrared.

$$g_q^2(\mu) = \frac{g_0^2}{1 + \beta_0 g_0^2 \ln\left(\frac{\mu^2 + x^2 m_0^2}{x^2 m_0^2}\right)}$$



Comparision with lattice data from [*Oliveira et al. arxiv:1605.09632*] for $M(p)$ for $M(10GeV) = 0{,}008, 0{,}01, 0{,}02, 0{,}022$. Parametres: $N_f = 2$, $N_c = 3$, $m_0 = 0{,}4$ GeV, $g_0 = 7$ and $x = 5$.

Conclusions

To summarize

- We use a modification of FP Lagrangian including massive gluons.
- **The mass of the gluon regularizes the theory in the infrared** allowing the use of perturbation theory.
- **Two and three point correlation functions in Yang-Mill sector** have shown to be **well reproduced by a one loop analysis** using a massive Lagrangian.

Small parameters: g_g and $\mathbf{1/N_c}$

- At leading order, this scheme reproduces the well-known **rainbow approximation**.
- It allows for a systematic study of higher order corrections.
- We are able to implement a **consistent renormalization group** improvement.

Conclusions and perspectives

Perspectives

We are begining to use the present scheme to calculate mesonic properties such as the mass spectrum or decay rates.

Thanks