Scale-dependent hadrons light-front wave functions

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5th Dec, 2017

What are scale dependent hadrons light front wave functions?

It is the set of wave functions that
- describes hadron in the front form of dynamic,
- and depends on the renormalization scale.

Hadron can be described in terms of quarks and gluons.
For example the meson state can be written as:

$$\begin{aligned} |\text{Meson}\rangle \;=\; & \psi_{q\bar{q}}(\lambda)\,|q\bar{q};\lambda\rangle \\ & + \psi_{q\bar{q}g}(\lambda)\,|q\bar{q}\,g;\lambda\rangle \\ & + \psi_{q\bar{q}gg}(\lambda)\,|q\bar{q}\,gg;\lambda\rangle \\ & + \psi_{q\bar{q}q\bar{q}}(\lambda)\,|q\bar{q}\,q\bar{q};\lambda\rangle \\ & + \psi_{q\bar{q}q\bar{q}g}(\lambda)\,|q\bar{q}\,q\bar{q}\,g;\lambda\rangle \\ & + \ldots\,, \end{aligned}$$

where each state depends on λ, the renormalization group parameter.

M. Gell-Mann, Phys. Rev. 125 (1962) 1067-1084
M. Gell-Mann, Phys. Lett. 8 (1964) 214-215
G. Zweig, Developments in the Quark Theory of Hadrons (1964)

Schematic illustration of QCD coupling constant $g(\lambda)$

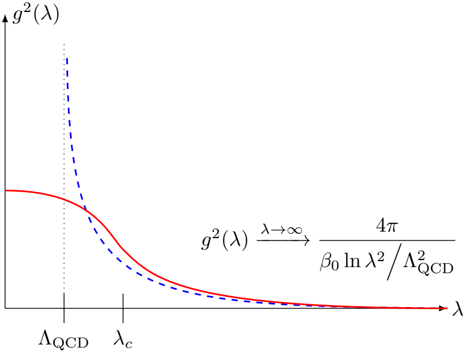

Figure: The dashed blue line it is the result originally obtained by Wilczek, Gross and Politzer. The solid red line shows possible dependence including the non-perturbation region.

D. Gross, F. Wilczek, Phys.Rev.Lett. 30 (1973) 1343-1346
H. Politzer, Phys.Rev.Lett. 30 (1973) 1346-1349

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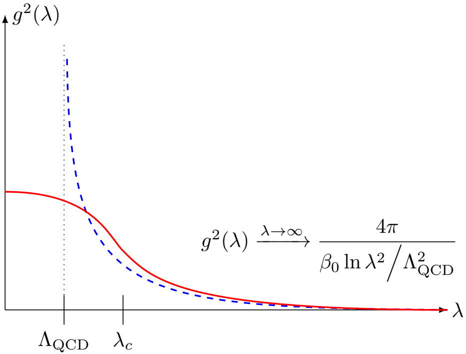
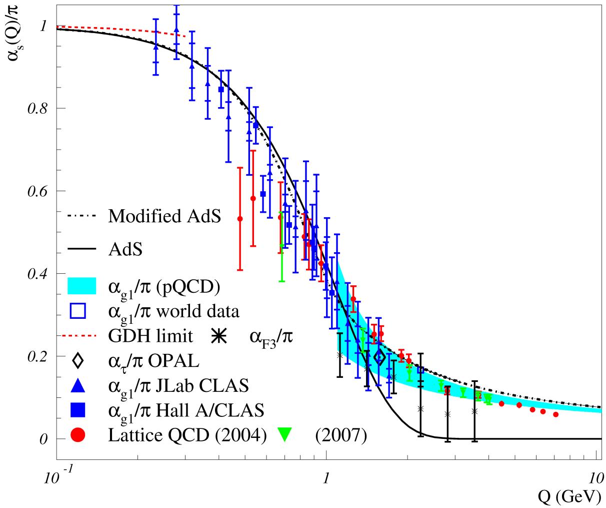


In the non-perturbation region it is likely that g stays constant for small λ. There it could be the scale, λ_c, where hadron is made of the constituent quarks.

S. Brodsky, G. de Téramond, A. Deur, Phys.Rev. D81 (2010) 096010

Constituent scale - λ_c



S. Brodsky, G. de Téramond, A. Deur, Phys.Rev. D81 (2010) 096010

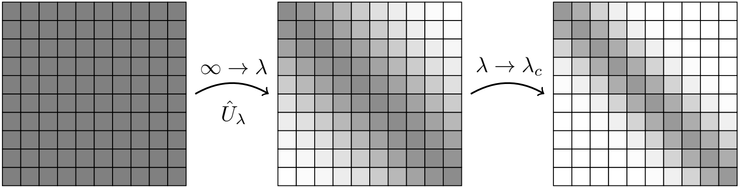
We make conjecture that

$$|\text{Meson}\rangle \approx \psi_{q\bar{q}}(\lambda_c)\,|q\bar{q};\lambda_c\rangle\,,$$
$$|\text{Barion}\rangle \approx \psi_{qqq}(\lambda_c)\,|qqq;\lambda_c\rangle\,.$$

The Renormalization Group Procedure for Effective Particles (RGPEP) provides the tool to calculate the hadron state for a general value of λ.

S. Głazek, Acta Phys.Polon. B42 (2011) 1933-2010
S. Głazek, Acta Phys.Polon. B43 (2012) 1843-1862

The Wilsonian renormalization procedure relies on a concept of a flow of equivalent Lagrangians or Hamiltonians described as functions of λ. Especially, the RGPEP diagonalizes Hamiltonian by a rotation parameterized by λ.



S. Głazek, K. Wilson, Phys.Rev. D48 (1993) 5863-5872
S. Głazek, K. Wilson, Phys.Rev. D49 (1994) 4214-4218
K. Wilson, *et al.*, Phys.Rev. D49 (1994) 6720-6766

Does H depend on λ?

• No. Hamiltonian is written in terms of creation and annihilation operators that depend on λ, but it is always **the same operator**.

What is the meaning of λ?

• λ limits the energy change in the interaction.

Who fixes λ?

• Nobody. λ is selected to describe physics in a simplest way possible.

It is easier to solve an eigenvalue equation for a small λ than for the large one. On the other hand, it is easier to solve scatterings problems for large value of λ than for the small one.

Pion example: LF wave function for λ_c

We model the pion light front wave function as:

$$\psi_{q\bar{q}/\pi}(x, k^{\perp}; \lambda_c) = \mathcal{N}\, \bar{u}\,(\not{p}_{\pi} + M)\,\gamma_5\, v\, \exp\left[-\frac{m_c^2 + (k^{\perp})^2}{2\,x(1-x)\,\varkappa^2}\right],$$

where in order to fix parameters m_c, M and $\varkappa$, the following conditions have been taken under consideration

1. π radius, $\langle r_{\pi}^2 \rangle = 0.45(1)$ fm^2, $r_{\pi}^2 = 0.44$ fm^2,
2. π decay constant, $\langle f_{\pi} \rangle = 130.4(2)$ MeV, $f_{\pi} = 130.7$ MeV,
3. the Gell-Mann-Okubo (GMO) formula. $\checkmark$.

$$m_c = 330 \text{ MeV}, \qquad \varkappa = 440 \text{ MeV}, \qquad M = -1.92 \text{ GeV}.$$

Particle Data Group Collaboration, Chin.Phys. C38 (2014) 090001
APT, Ph.D. thesis, University of Warsaw (2016)

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Particle Data Group Collaboration, Chin.Phys. C38 (2014) 090001
APT, Ph.D. thesis, University of Warsaw (2016)

Pion example: form factor

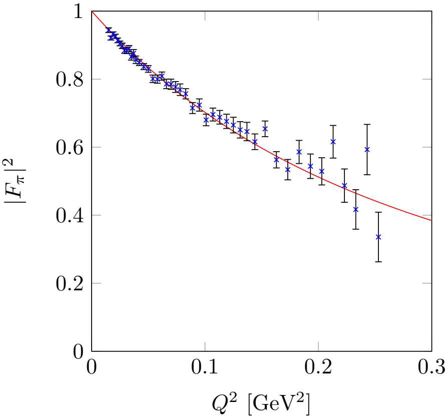
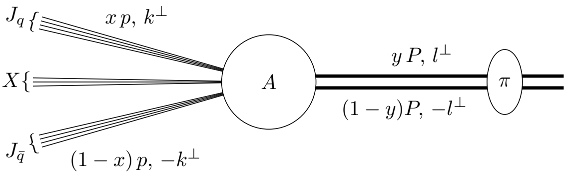


Figure: Solid red line represents the pion form-factor calculated using our wave function. Blue points show experimental data.

S. R. Amendolia *et al.* (NA7), Nucl. Phys. B277 (1986) 168
APT, Ph.D. thesis, University of Warsaw (2016)

The experiment E791 measured the transverse and the longitudinal momentum distribution of two jets emerging from scattering of π^- beam of energy 500 GeV on the platinum target.



We know how to describe π-A interaction on quark-gluon level. Thus, we need to rewrite the pion state for $\lambda \gg \lambda_c$.

E791 Collaboration, Phys.Rev.Lett. 86 (2001) 4768-4772
APT, Ph.D. thesis, University of Warsaw (2016)

Pion example: LF wave function for any λ

Starting from $|\pi\rangle = \psi_{q\bar{q}/\pi}(\lambda_c)\,|q\bar{q};\lambda_c\rangle$,

we can obtain the pion state for a general value of λ,

$$
\begin{aligned}
|\pi\rangle = \;& \psi_{q\bar{q}/\pi}(\lambda)\,|q\bar{q};\lambda\rangle \\
&+ \psi_{q\bar{q}g/\pi}(\lambda)\,|q\bar{q}\,g;\lambda\rangle \\
&+ \psi_{q\bar{q}gg/\pi}(\lambda)\,|q\bar{q}\,gg;\lambda\rangle \\
&+ \psi_{q\bar{q}q\bar{q}/\pi}(\lambda)\,|q\bar{q}\,q\bar{q};\lambda\rangle \\
&+ \psi_{q\bar{q}q\bar{q}g/\pi}(\lambda)\,|q\bar{q}\,q\bar{q}\,g;\lambda\rangle \\
&+ \ldots
\end{aligned}
$$

using the $\mathcal{W}$-transformation defined in the RGPEP.

S. Głazek, Acta Phys.Polon. B42 (2011) 1933-2010

Pion example: diffractive scattering

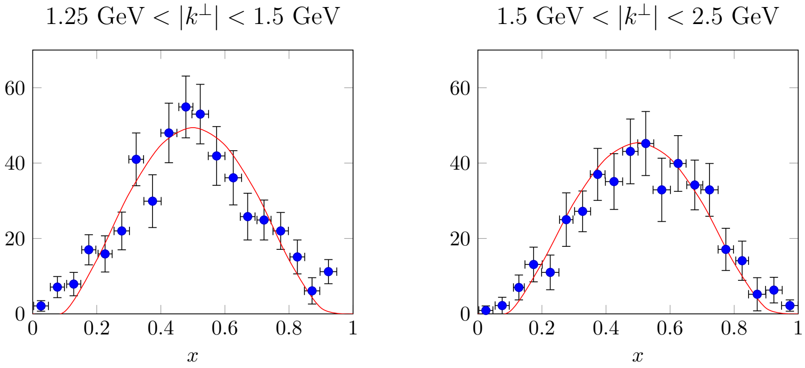


Figure: Blue points show experimental data. Red line represents our result.

E. M. Aitala *et al.* (E791), Phys. Rev. Lett. 86 (2001) 4768
APT, Ph.D. thesis, University of Warsaw (2016)

We model the proton light-front wave function as:

$$|p(P,\sigma)\rangle = f(p_1,p_2,p_3)\left[\sum_k a_k I_k(123)\right]\epsilon_{abc}\,\hat{u}_1^{a\,\dagger}\,\hat{u}_2^{b\,\dagger}\,\hat{d}_3^{c\,\dagger}|0\rangle\,,$$

where the scalar function f is given by,

$$f(p_1,p_2,p_3) = \exp\left\{-\frac{1}{6\varkappa^2}\left[\sum_{i=1}^{3}\frac{(p_i^{\perp})^2+m_c^2}{p_i^{+}/P^{+}} - \left(P^{\perp}\right)^2\right]\right\}\,,$$

and I_k denotes five Ioffe currents, among which three are independent.

S. D. Głazek, J.M. Namysłowski, Acta Phys.Polon. B19 (1988) 569
B.L. Ioffe, Nucl.Phys. B188 (1981) 317

Proton example: fitting procedure

The suggested model of the proton wave function has in total five free parameters: m_c, $\varkappa$, and coefficients a_1, a_2, a_3. In order to find them, we impose $e = \langle e \rangle$ and find the best fit minimizing

$$\left(\mu - \langle\mu\rangle\right)^2 + \left(r_E^2 - \langle r_E^2 \rangle\right)^2 + \left(r_M^2 - \langle r_M^2 \rangle\right)^2 ,$$

where, using proton form factors $G_E(Q^2)$ and $G_M(Q^2)$, we have

$e = G_E(0)$	*proton electric charge*
$\mu = G_M(0)$	*proton dipol moment*
$r_E^2 = -6 \left.\frac{d}{dQ^2}\right\vert_{Q^2=0} G_E$	*electric proton radius*
$r_M^2 = -6 \left.\frac{d}{dQ^2}\right\vert_{Q^2=0} G_M$	*magnetic proton radius*

Proton example: fitting result

The best found result reads,

$$\mu = 2.59 \qquad \langle \mu \rangle = 2.79\,,$$
$$r_E^2 = 20.94\ \ \mathrm{GeV}^{-2} \qquad \left\langle r_E^2 \right\rangle = 19.15\,\mathrm{GeV}^{-2}\,,$$
$$r_M^2 = 15.30\ \ \mathrm{GeV}^{-2} \qquad \left\langle r_M^2 \right\rangle = 15.05\,\mathrm{GeV}^{-2}\,.$$

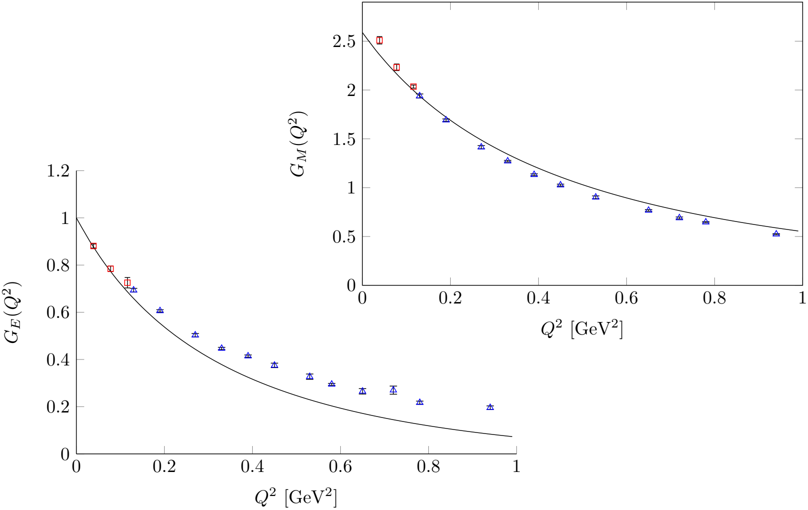
and was obtained for

$$m = 308\ \mathrm{MeV}, \qquad \varkappa = 310\ \mathrm{MeV}$$

$$a_1 = 1.944, \qquad a_2 = -0.279, \qquad a_3 = -4.112$$

APT, http://www.fuw.edu.pl/~trawinski/?page=PRELUDIUM.php

Proton example: form-factors



□ B. Dudelzak, G. Sauvage, P. Lehmann, Nuovo Cim. 28 (1963) 18

△ L.E. Price, et.al., Phys.Rev. D4 (1971) 45

Proton example: wave function of active constituent

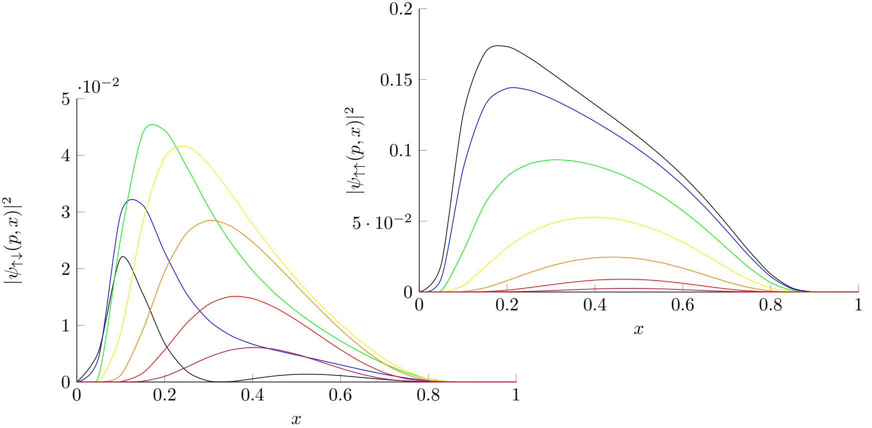


Figure: Square of the wave function of active constituent with spin parallel to the proton $|\psi_{\uparrow\uparrow}(p,x)|^2$, and anti-parallel $|\psi_{\uparrow\downarrow}(p,x)|^2$, in function of x for $p = |p^\perp|$: $p = 0$ GeV, $p = 0.1$ GeV, $p = 0.2$ GeV, $p = 0.3$ GeV, $p = 0.4$ GeV, $p = 0.5$ GeV, $p = 0.6$ GeV.

Proton example: active quark distribution

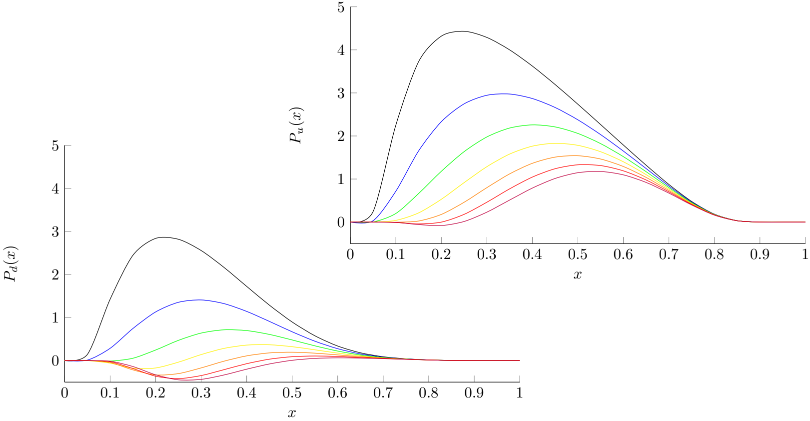


Figure: Unpolarized distribution of quarks u and d in proton in function of x, for $Q^2 = 0$ GeV, $Q^2 = 0.1$ GeV, $Q^2 = 0.2$ GeV, $Q^2 = 0.3$ GeV, $Q^2 = 0.4$ GeV, $Q^2 = 0.5$ GeV, $Q^2 = 0.6$ GeV.

Conclusions

The light-front wave functions for one λ can be transformed to the any other using the RGPEP.

The light-front wave functions give unique way to connect all experimental data.

The light-front wave functions have to satisfy many currently know facts.

Hard to find simple model.

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Backup slides

Difficulties in interpretation of the pion parity.

$$\bar{u}\,\not{p}_\pi \gamma^5 v = \begin{cases} \pm \dfrac{(k^\perp)^2 - m_c^2 - x(1-x)\,m_\pi^2}{\sqrt{x(1-x)}} & \text{for } \substack{(\uparrow\downarrow)\\(\downarrow\uparrow)} \\ 2\,\dfrac{k_1 \pm i k_2}{\sqrt{x(1-x)}} m_c & \text{for } \substack{(\uparrow\uparrow)\\(\downarrow\downarrow)} \end{cases}$$

$$M\,\bar{u}\gamma^5 v = \begin{cases} \pm \dfrac{m_c}{\sqrt{x(1-x)}} M & \text{for } \substack{(\uparrow\downarrow)\\(\downarrow\uparrow)} \\ \dfrac{k_1 \pm i k_2}{\sqrt{x(1-x)}} M & \text{for } \substack{(\uparrow\uparrow)\\(\downarrow\downarrow)} \end{cases}$$

P and S partial wave function are mixed in the pion model.

The QCD equation for λ_c

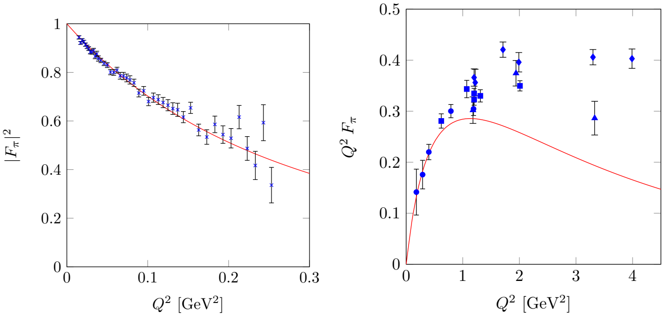
$$\left[\frac{(k^{\perp})^2 + m_q^2}{x} + \frac{(k^{\perp})^2 + m_{\bar{q}}^2}{1-x} + U_{\mathrm{QCD}}(\lambda_c)\right]\psi_{q\bar{q}}(x, k^{\perp}; \lambda_c) \approx M^2\,\psi_{q\bar{q}}(x, k^{\perp}; \lambda_c)$$

The quadratic effective potencial does not include the spin structure of the pion constituents quarks.

The quadratic effective potencial does not include short range interactions.

S.J. Brodsky, G.F. de Teramond, Phys.Rev. D77 (2008) 056007
APT *et al.*, Phys.Rev. D90 (2014) 7,074017

Wrong shape of the form-factor for Q^2 greater than 1 GeV^2.



The photon-quark vertex is not renormalized to the scale λ_c.

× S. R. Amendolia *et al.* (NA7), Nucl. Phys. B277 (1986) 168
□ C. J. Bebek *et al.*, Phys. Rev. D9 (1974) 1229
◊ C. J. Bebek *et al.*, Phys. Rev. D13 (1976) 25
APT, Ph.D. thesis, University of Warsaw (2016)

The scattering amplitude of the two hadrons A and B reads

$$\mathcal{M}_{AB\to\text{out}} = \langle\text{out}| \left(1 + \hat{U}\frac{1}{E_A + E_B - \hat{H} + i\epsilon}\right) \hat{U}|A, B\rangle \,,$$

where $\hat{H}$ is the full Hamiltonian, that ascribes energies,

$$\hat{H}|A\rangle = E_A|A\rangle \,, \qquad\qquad \hat{H}|B\rangle = E_B|B\rangle \,,$$

and it is written as a sum $\hat{H} = \hat{K} + \hat{U}$, and $\hat{K}$ ascribes the energies of hadrons separately, as if they did not interact,

$$\hat{K}|A, B\rangle = (E_A + E_B)|A, B\rangle \qquad\qquad \hat{H}|A, B\rangle \neq (E_A + E_B)|A, B\rangle \,,$$

thus $\hat{U}$ is the interaction Hamiltonian **between hadrons**.

M. Gell-Mann, M.L. Goldberger, Phys.Rev. 91 (1953) 398-408

We doesn't know $\hat{U}$.

However, in QCD hadrons are seen as states build from quarks and gluons, whose interactions are described by $\hat{H}_I$.

There is a difference between $\hat{H} = \hat{K} + \hat{U}$ and $\hat{H} = \hat{H}_0 + \hat{H}_I$.
One refers to hadrons and the other to quarks and gluons.

The goal is to express $\mathcal{M}_{AB \to out}$ in terms of the QCD interaction Hamiltonian, $\hat{H}_I$.

Theorem

If states of hadrons A and B read $|A\rangle = \hat{A}^{\dagger}|0\rangle$ *and* $|B\rangle = \hat{B}^{\dagger}|0\rangle$, *and operators* $\hat{A}^{\dagger}$ *and* $\hat{B}^{\dagger}$ *are expressible only in terms of creation operators, then*

$$\hat{U}|A, B\rangle = \left[\left[\hat{H}_I, \hat{A}^{\dagger}\right], \hat{B}^{\dagger}\right]|0\rangle \, .$$

- The double commutator in $\left[\left[\hat{H}_I, \hat{A}^{\dagger}\right], \hat{B}^{\dagger}\right]$ ensure that $\hat{H}_I$ mixes $\hat{A}^{\dagger}$, $\hat{B}^{\dagger}$.
- The physical meaning of the above theorem is that the interaction between the hadrons' states through interaction Hamiltonian $\hat{U}$ is equivalent to the interaction where a quark or gluon from one hadron interact with quark or gluon from the other hadron.

APT, Ph.D. thesis, University of Warsaw (2016)

In our model of pion–nucleus interaction we make following assumptions:

- $\mathcal{M}_{\pi A \to J_q J_{\bar{q}} X} \propto \mathcal{M}_{\pi g \to q \bar{q} g'}$,
- gluons in the nucleus have small traverse momentum,
- gluons from the nucleus are not absorbed by pion.

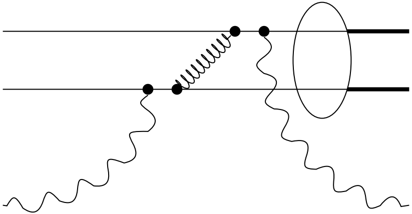
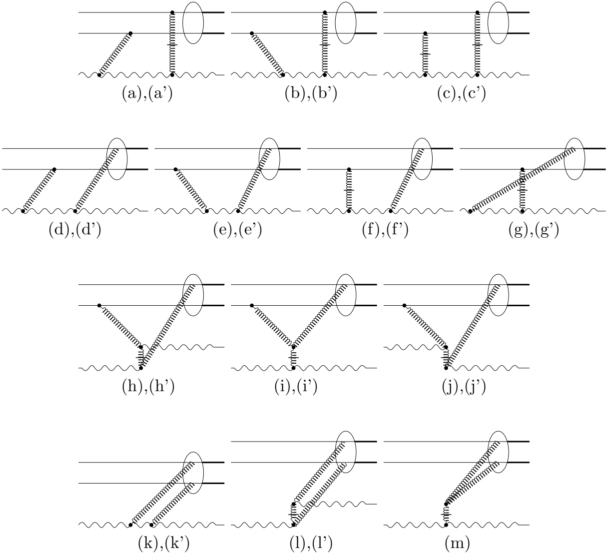


Figure: The figure exemplifies a time-ordered diagram in which gluon from the nucleus is absorbed by the pion.

Diagrams



The expansion of $\mathcal{H}_I$ for any λ in the bare coupling constant g is:

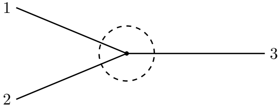
$$\mathcal{H}_I(\hat{a}_\lambda;\lambda) = g\,\mathcal{H}_1(\hat{a}_\lambda;\lambda) + g^2\,\mathcal{H}_2(\hat{a}_\lambda;\lambda) + \ldots\,,$$

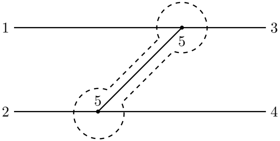
where

$$\mathcal{H}_1(\hat{a}_\lambda;\lambda) = \mathcal{H}_1(\hat{a}_\lambda;\infty)\,f_{12,3}(\lambda)\,,$$

$$\mathcal{H}_2(\hat{a}_\lambda;\lambda) = \mathcal{H}_2(\hat{a}_\lambda;\infty)\,f_{12,34}(\lambda) + \mathcal{H}_1(\hat{a}_\lambda;\infty)\,\frac{1}{D}\,\mathcal{H}_1(\hat{a}_\lambda;\infty)\Big[f_{12,34}(\lambda) - f_{45,2}(\lambda)\,f_{15,3}(\lambda)\Big]\,,$$

and $f_{ij,k}(\lambda)$, $f_{ij,kl}(\lambda)$ are Gaussian shape vertex form-factors, and **the denominator D is symmetric around the pole location.**





S. Glazek, Acta Phys.Polon. B43 (2012) 1843-1862

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G. de Téramond, S. Brodsky, Phys.Rev.Lett. 102 (2009) 081601
APT, Few Body Syst. 57 (2016) 449-453
APT *et al.*, Phys.Rev. D90 (2014) 7,074017