Parton Distribution Amplitudes: Revealing diquarks in the Nucleon and the Roper

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In collaboration with:
J. Segovia, C.D. Roberts and L. Chang

based on arXiv 1711.09101

Chapter 1:
Baryon Distribution Amplitudes

Hadrons seen as Fock States

- Lightfront quantization allows to expand hadrons on a Fock basis:

$$|P,\pi\rangle \propto \sum_{\beta} \Psi_{\beta}^{q\bar{q}} |q\bar{q}\rangle + \sum_{\beta} \Psi_{\beta}^{q\bar{q},q\bar{q}} |q\bar{q},q\bar{q}\rangle + \dots$$

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- Non-perturbative physics is contained in the N-particles Lightfront-Wave Functions (LFWF) Ψ^N
- Schematically a distribution amplitude φ is related to the LFWF through:

$$\varphi(x) \propto \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2}\Psi(x,k_\perp)$$

S. Brodsky and G. Lepage, PRD 22, (1980)

Nucleon Distribution Amplitudes

- 3 bodies matrix element:

$$\langle 0|\epsilon^{ijk} u^i_\alpha(z_1) u^j_\beta(z_2) d^k_\gamma(z_3)|P\rangle$$

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- 3 bodies matrix element expanded at leading twist:

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- Isospin symmetry:

$$2T(x_1,x_2,x_3) = \varphi(x_1,x_3,x_2) + \varphi(x_2,x_3,x_1)$$

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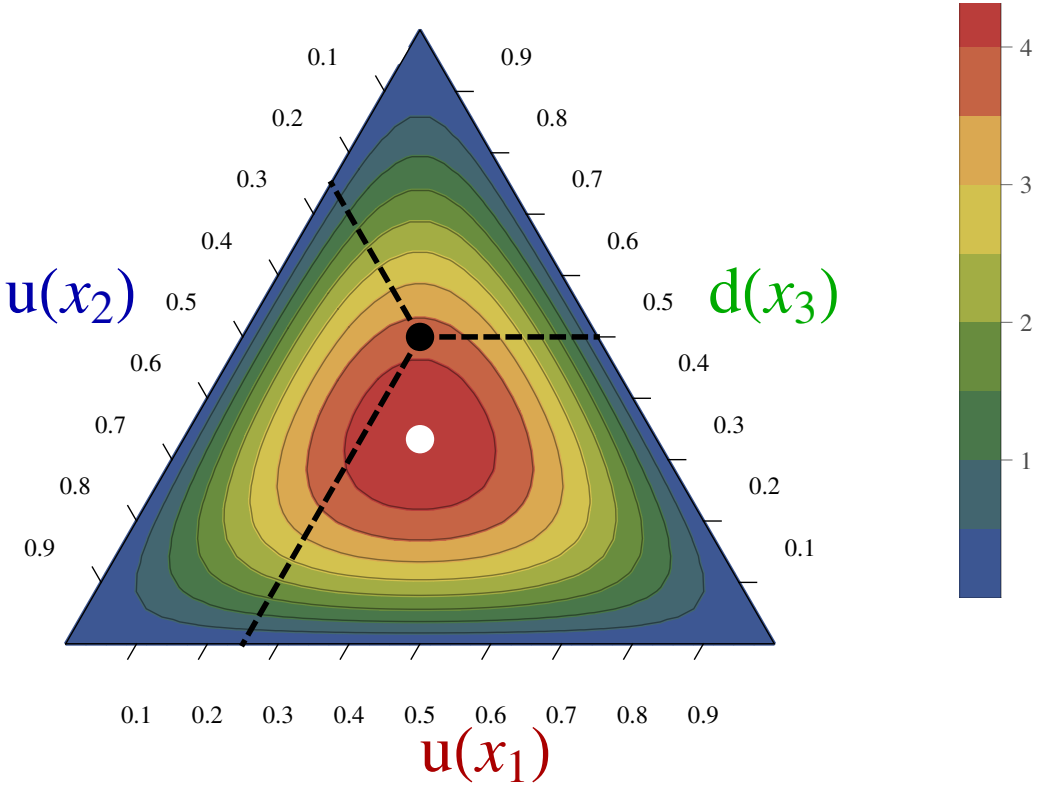
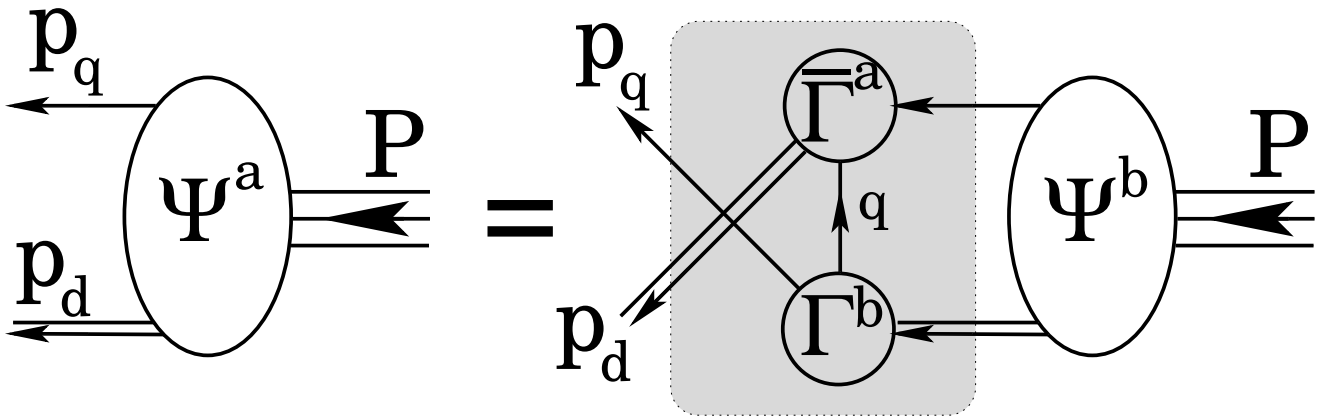


Some previous studies of DA

- QCD Sum Rules
  - V. Chernyak and I. Zhitnitsky, Nucl. Phys. B 246 (1984)
- Relativistic quark model
  - Z. Dziembowski, PRD 37 (1988)
- Scalar diquark clustering
  - Z. Dziembowski and J. Franklin, PRD 42 (1990)
- Phenomenological fit
  - J. Bolz and P. Kroll, Z. Phys. A 356 (1996)
- Lightcone quark model
  - B. Pasquini *et al.*, PRD 80 (2009)
- Lightcone sum rules
  - I. Anikin *et al.*, PRD 88 (2013)
- Lattice Mellin moment computation
  - G. Bali *et al.*, JHEP 2016 02

Our Approach

- Inspired by the results obtained from DSEs and Faddeev equations.
- We do not use numerical solution of the Faddeev equation, but algebraic parametrisations based on the Nakanishi representation.
- This is an exploratory work: we want to know what can be done.
- We also assume the dynamical diquark correlations, both scalar and AV, and compare in the end with Lattice QCD one.



Nucleon DA as a Matrix Element

- Operator point of view for every DA (and at every twist):

$$\langle 0|\epsilon^{ijk}\left(u^i_\uparrow(z_1)C\not{n}u^j_\downarrow(z_2)\right)\not{n}d^k_\uparrow(z_3)|P,\lambda\rangle \to \varphi(x_i) \to O_\varphi,$$

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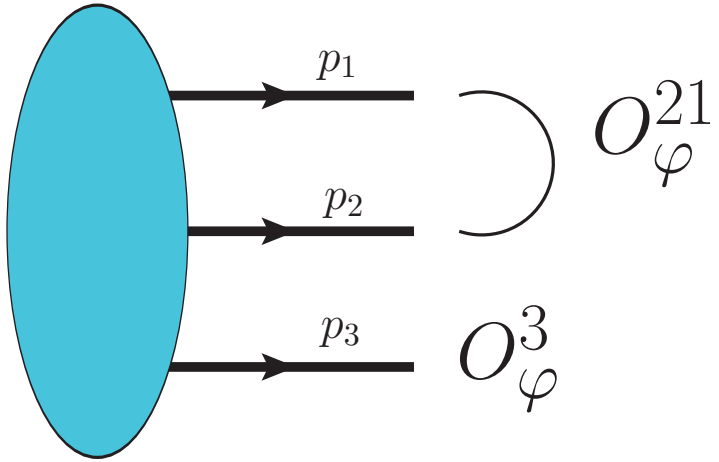

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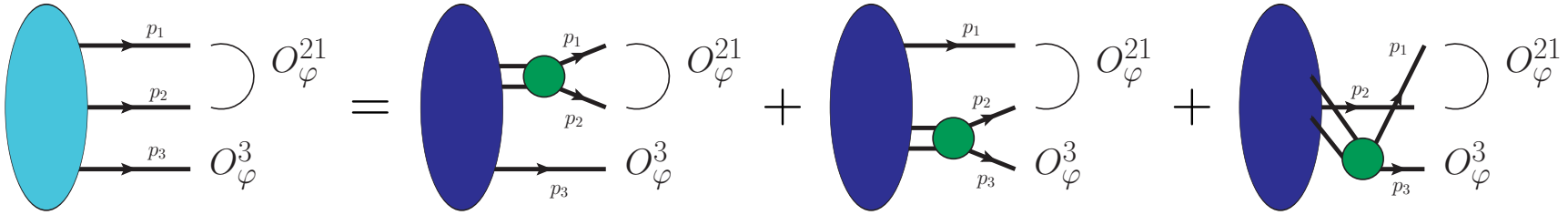

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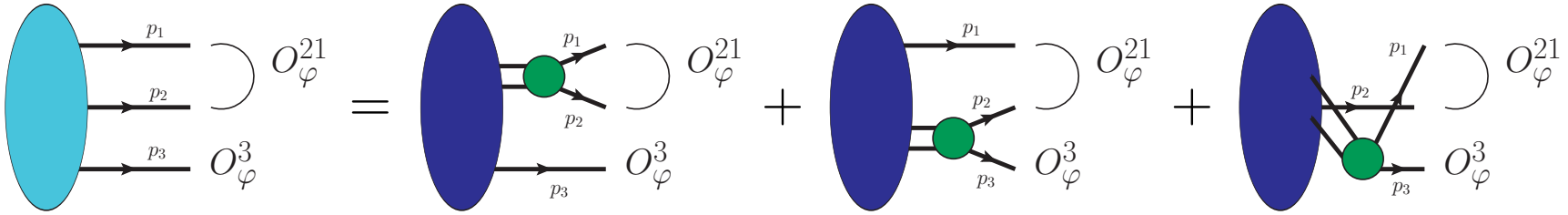
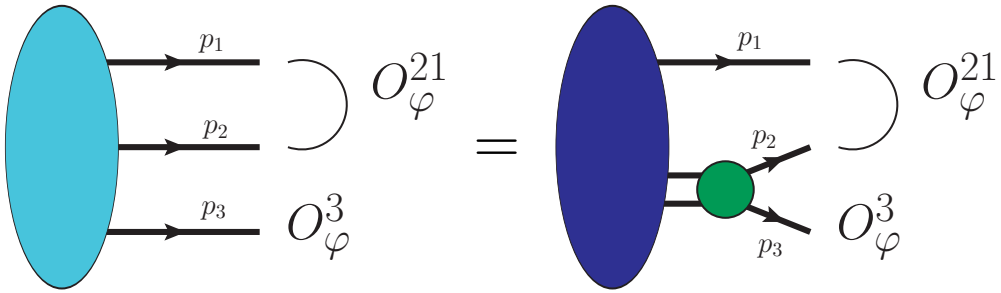


- The operator then selects the relevant component of the wave function.

Scalar diquark contribution

- In the scalar diquark case, only one contribution remains (φ case):



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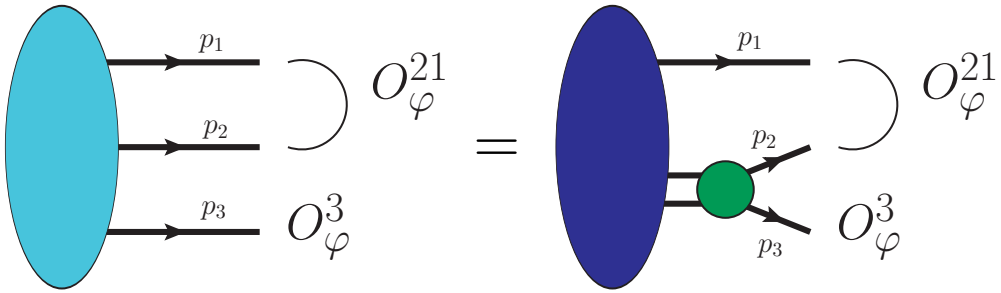

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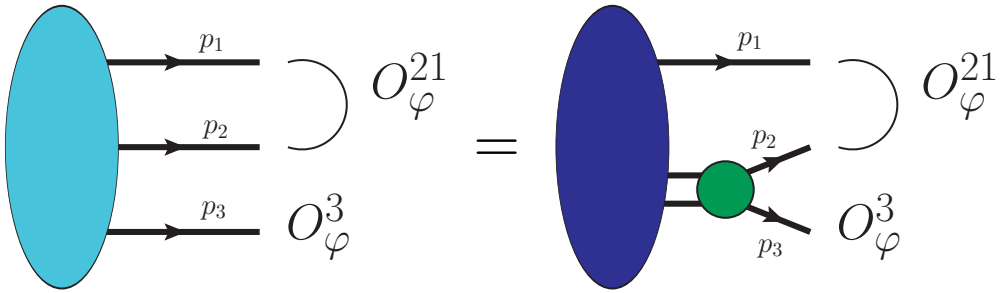

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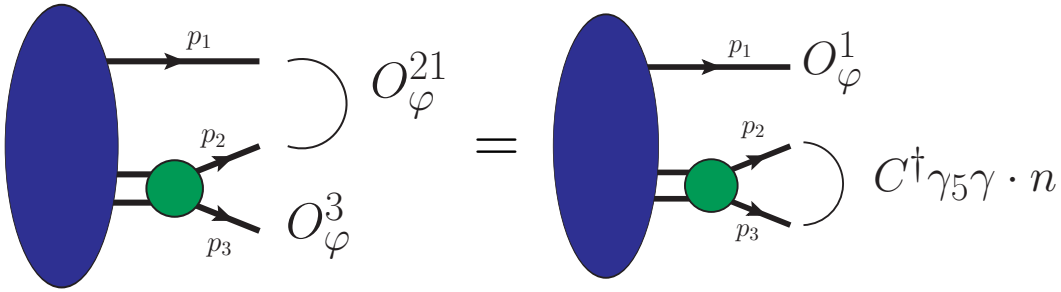

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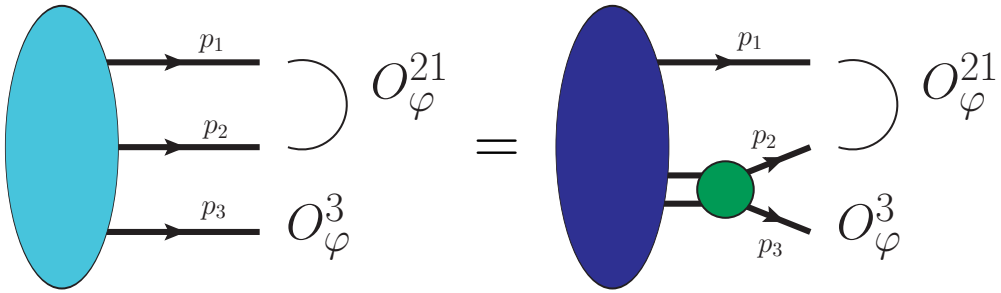

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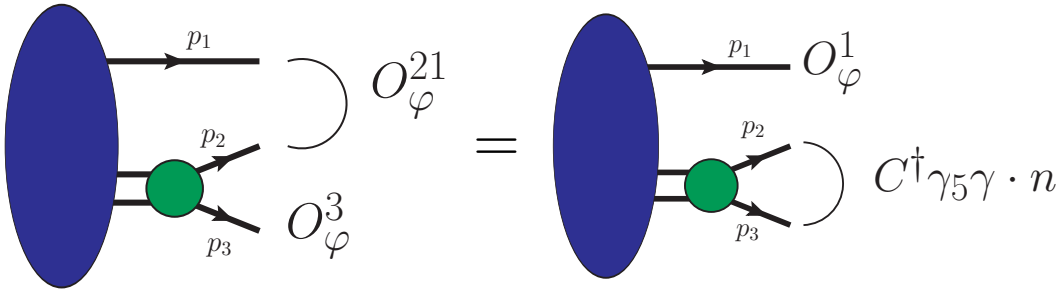

We recognise the leading twist DA of a scalar diquark

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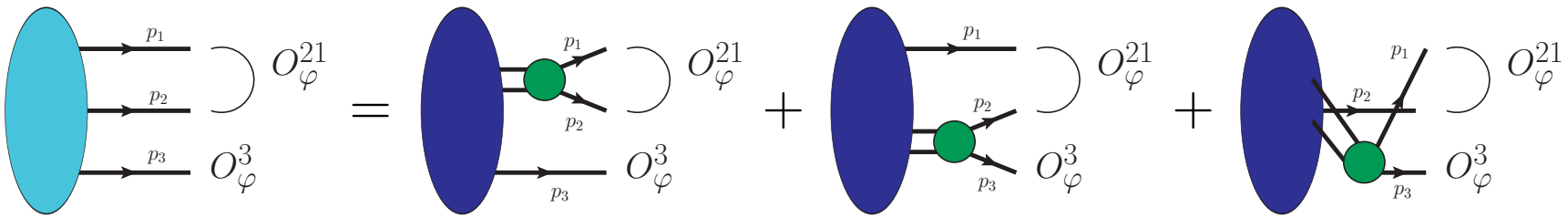

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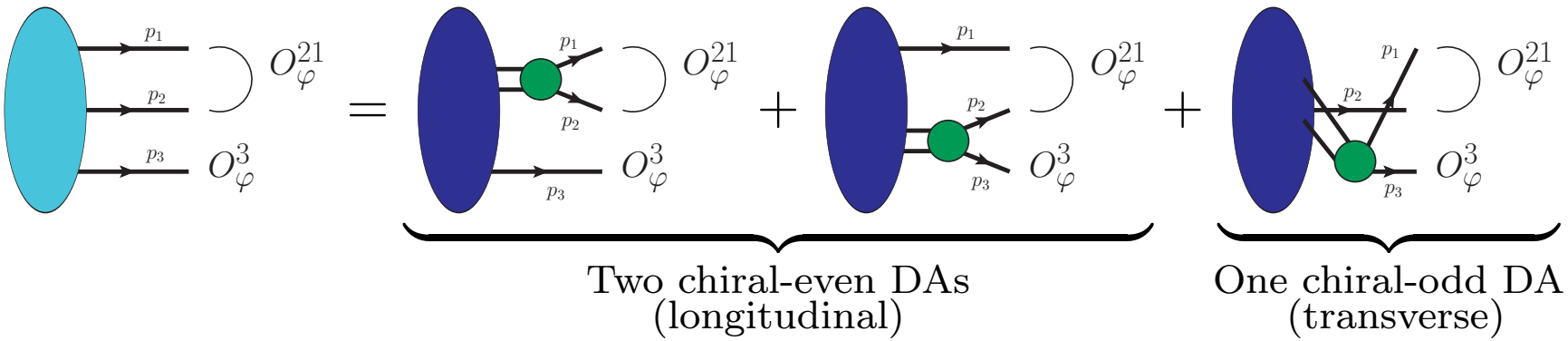

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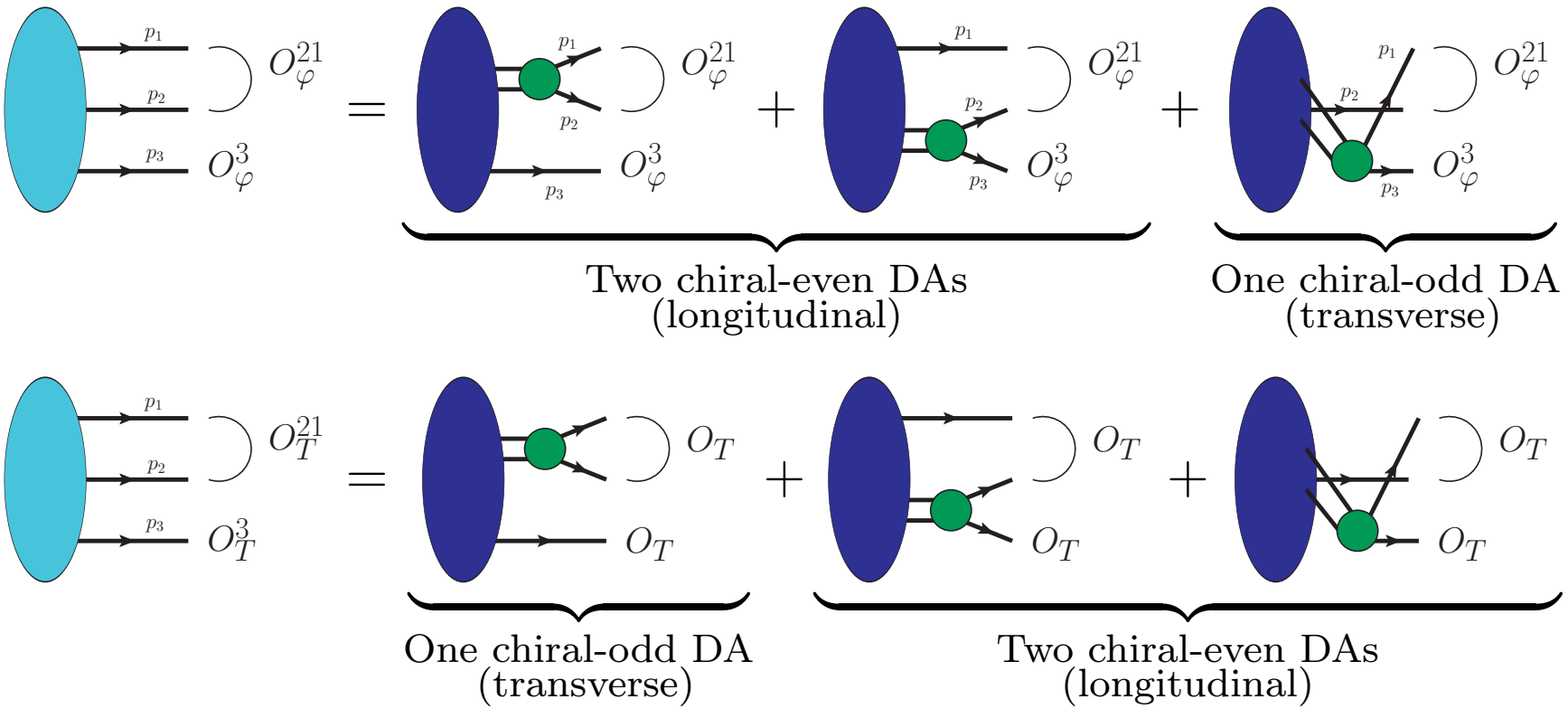

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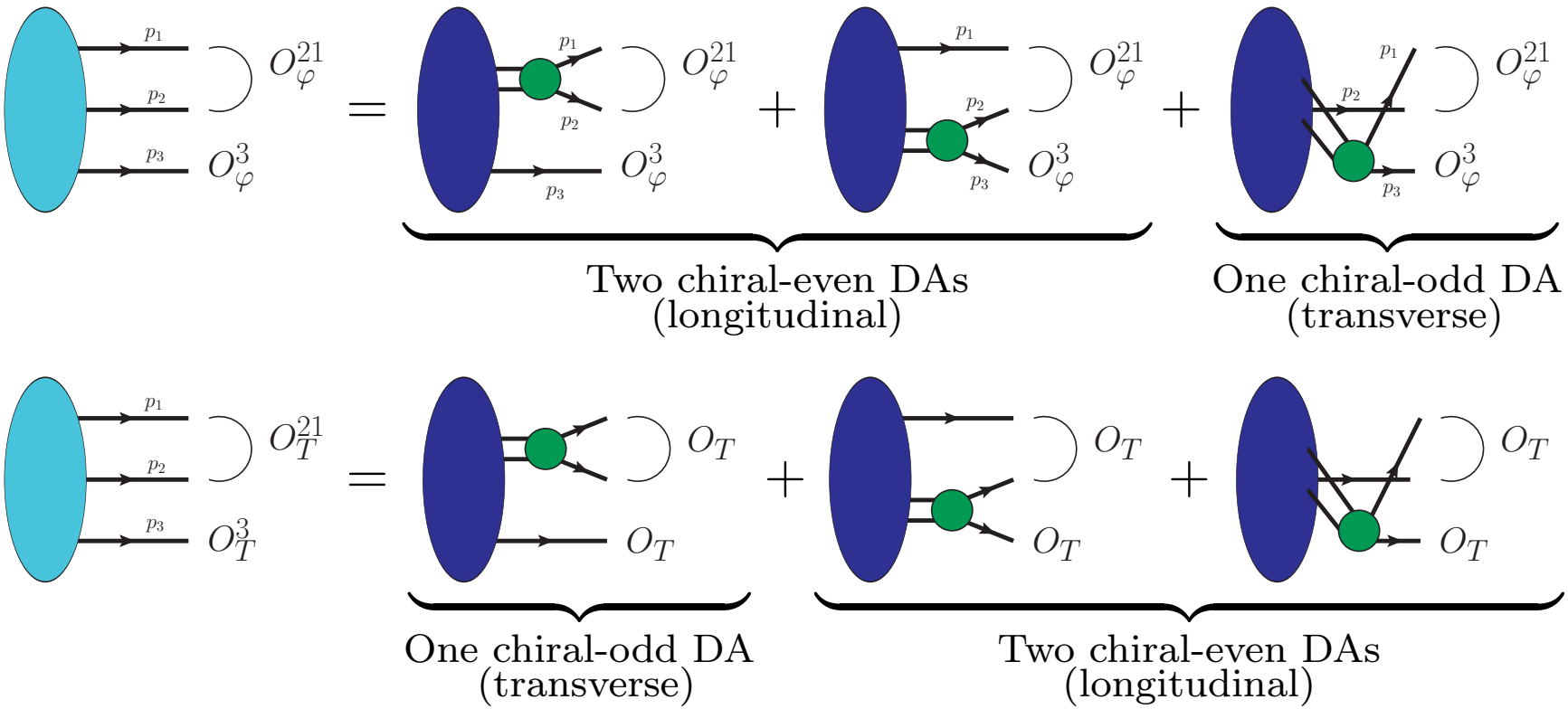

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Modeling the Diquarks

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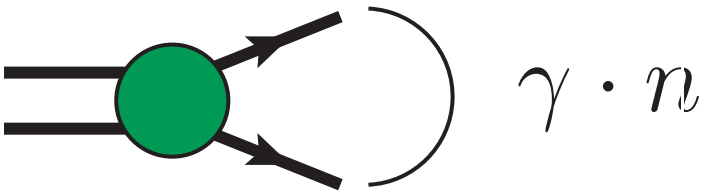

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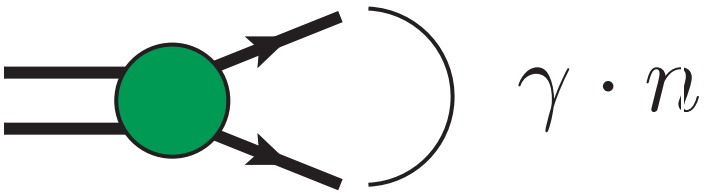

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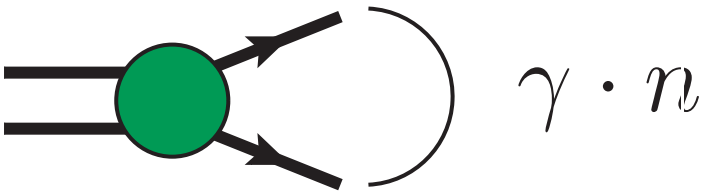

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- This point-like case leads to a flat DA:

$$\phi_{\rm PL}(x) = 1$$

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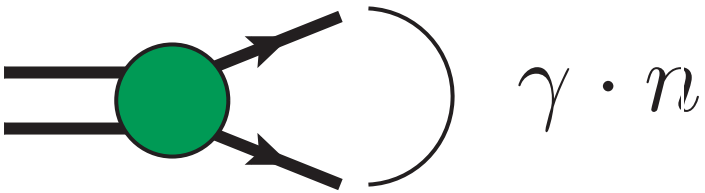

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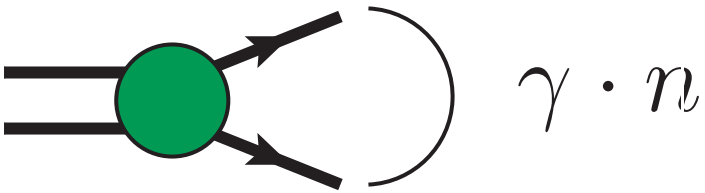

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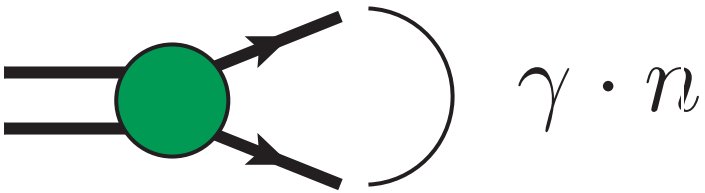

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- The Nakanishi case leads to a non trivial DA:

$$\phi(x) \propto 1 - \frac{M^2}{K^2} \frac{\ln\left[1 + \frac{K^2}{M^2}x(1-x)\right]}{x(1-x)}$$

Scalar DA behaviour

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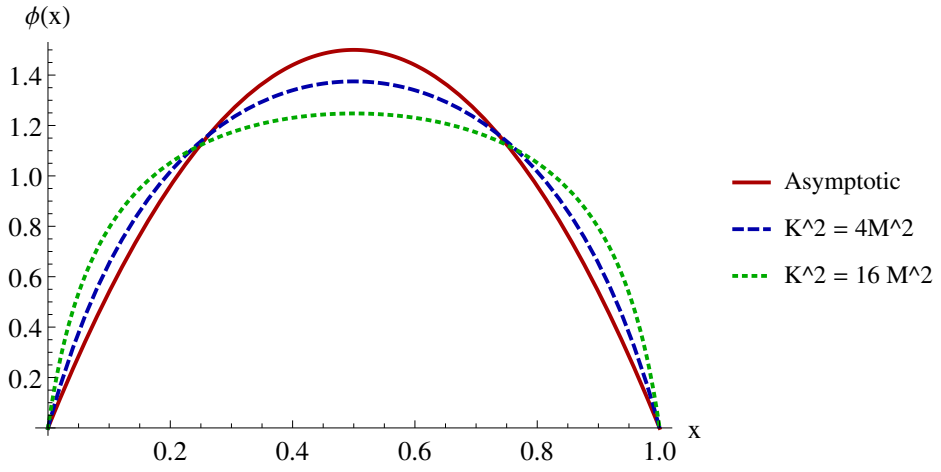

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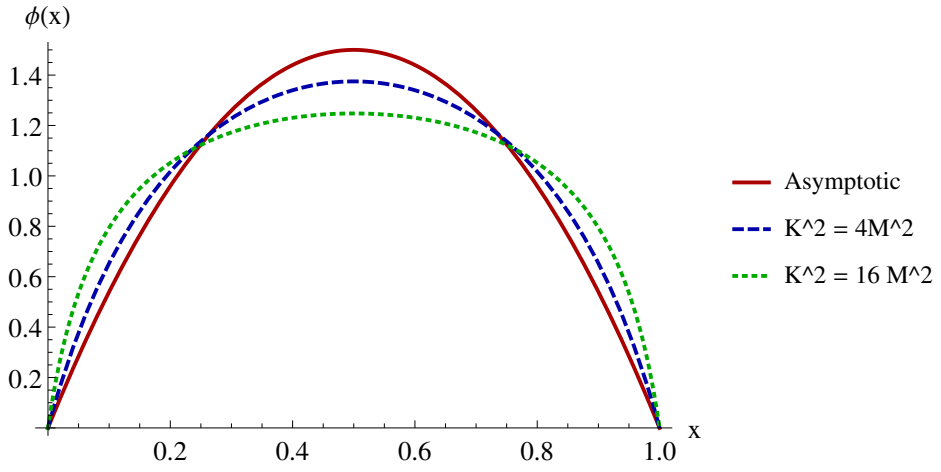
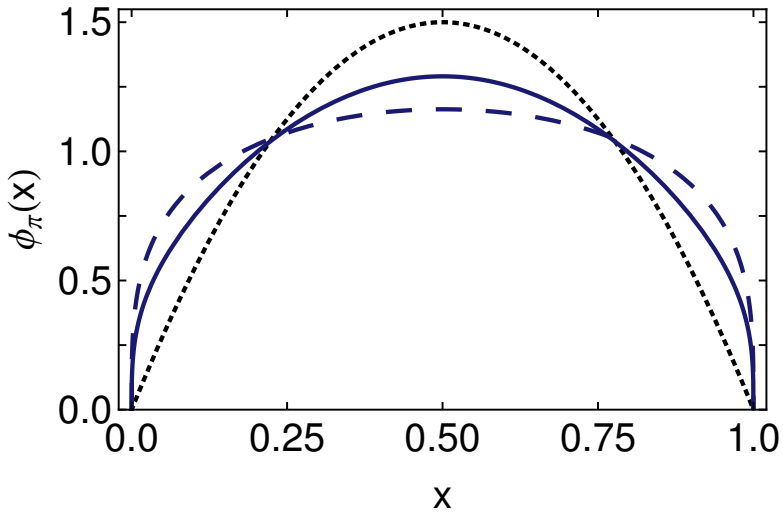


Pion

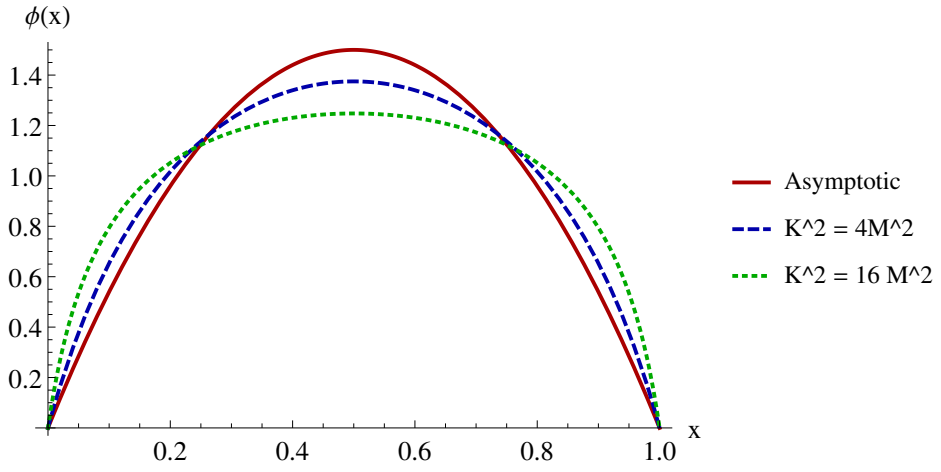


Pion figure from L. Chang *et al.*, PRL 110 (2013)

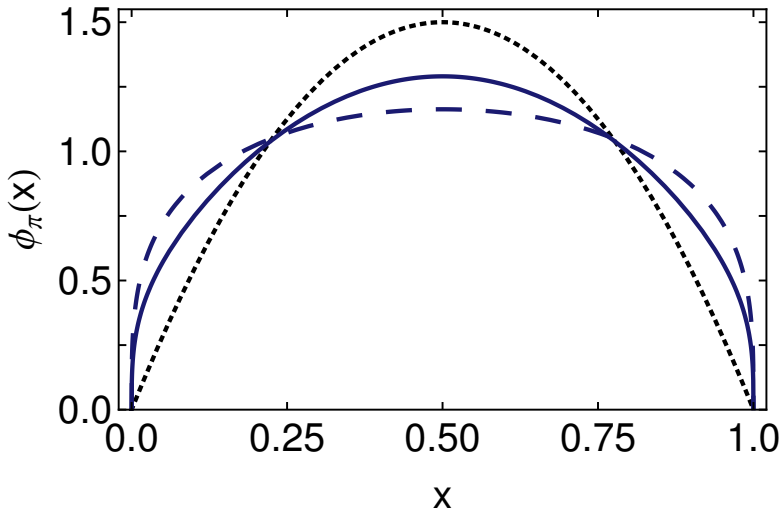
Scalar DA behaviour

$$\phi(x) = \mathcal{N}\left(1 - \frac{M^2}{K^2}\frac{\ln\left[1 + \frac{K^2}{M^2}x(1-x)\right]}{x(1-x)}\right)$$

Scalar diquark



Pion



Pion figure from L. Chang *et al.*, PRL 110 (2013)

This extended version of the DA seems promising!

Scalar DA behaviour

$$\phi(x) = \mathcal{N}\left(1 - \frac{M^2}{K^2}\frac{\ln\left[1 + \frac{K^2}{M^2}x(1-x)\right]}{x(1-x)}\right)$$

Nota Bene

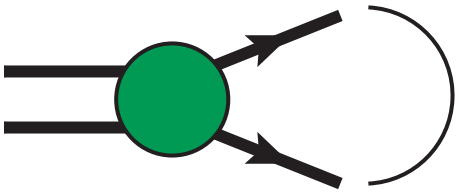
This PDA model can be used for meson easily, replacing $\frac{K^2}{M^2} \to \alpha$, where α become a free parameter $\in [0, +\infty]$ controlling the curvature of the PDA. This model has two main features:

- explore a region where the PDA is broader than the asymptotic one, but remained peaked at $x = 1/2$.
- has linear endpoint behaviour.

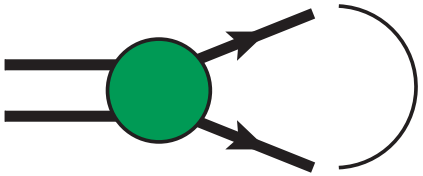
Such a model can for instance be used to explore the effect of the PDA curvature in DVMP, for instance with PARTONS.

See CM *et al.* arXiv 1711.09101

AV diquark DA



$\gamma \cdot n$

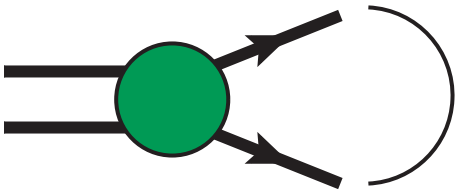


$\sigma^{\mu\nu} n_\nu$

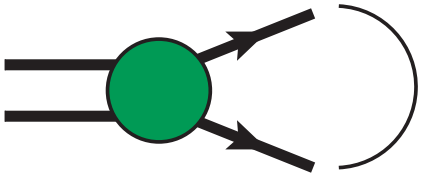
- Quark propagator:

$$S(q) = \frac{-i\not{q} + M}{q^2 + M^2}$$

AV diquark DA



$\gamma \cdot n$



$\sigma^{\mu\nu} n_\nu$

- Quark propagator:

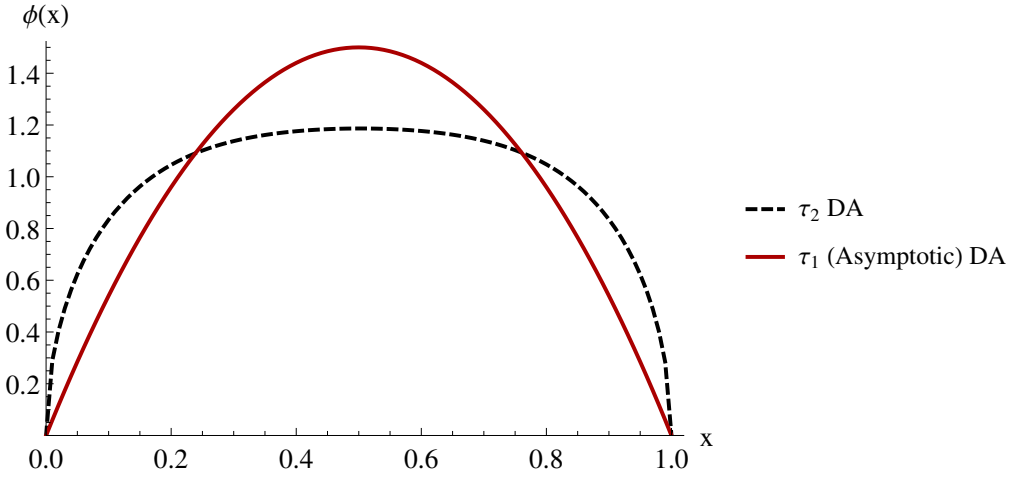
$$S(q) = \frac{-i\not{q} + M}{q^2 + M^2}$$

- Bethe-Salpeter amplitude (2 out of 8 structures):

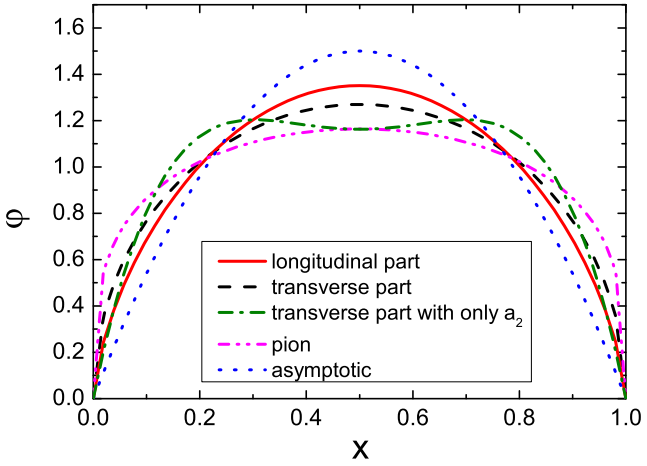
$$\begin{aligned}
\Gamma^{\mu}_{\rm PL}(q,K) &= (\mathcal{N}_1 \tau_1^\mu + \mathcal{N}_2 \tau_2^\mu)\, C \int_{-1}^{1} {\rm d}z \frac{(1-z^2)}{\left[\left(q - \frac{1-z}{2}K\right)^2 + \Lambda_q^2\right]} \\
\tau_1^\mu &= i\left(\gamma^\mu - K^\mu \frac{\not{K}}{K^2}\right) \rightarrow \text{Chiral even} \\
\tau_2^\mu &= \frac{K\cdot q}{\sqrt{q^2(K-q)^2}\sqrt{K^2}} \left(-i\tau_1^\mu \not{q} + i\not{q}\tau_1^\mu\right) \rightarrow \text{Chiral odd}
\end{aligned}$$

Comparison with the ρ meson

AV diquark



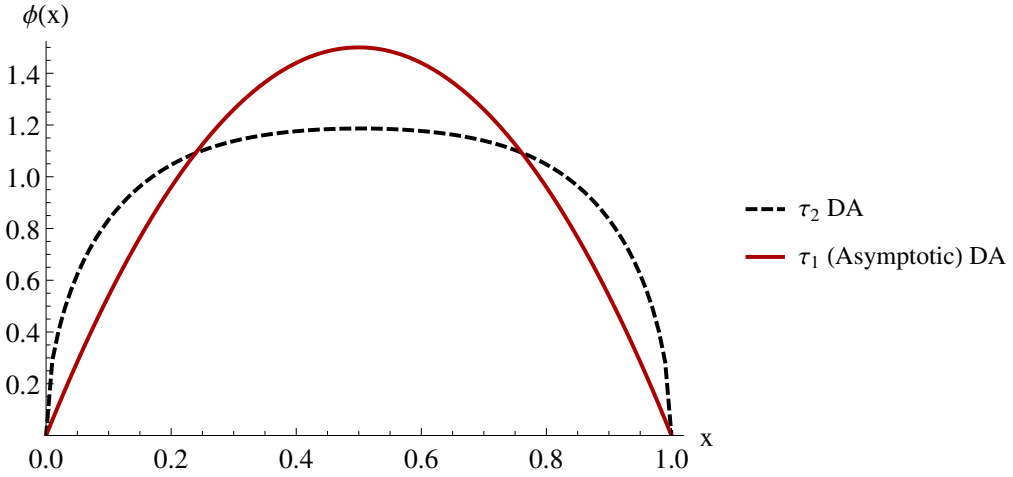
ρ meson



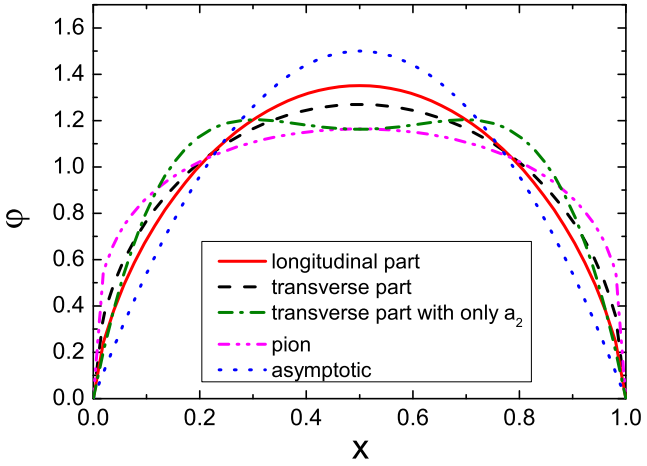
ρ figure from F. Gao *et al.*, PRD 90 (2014)

Comparison with the ρ meson

AV diquark



ρ meson

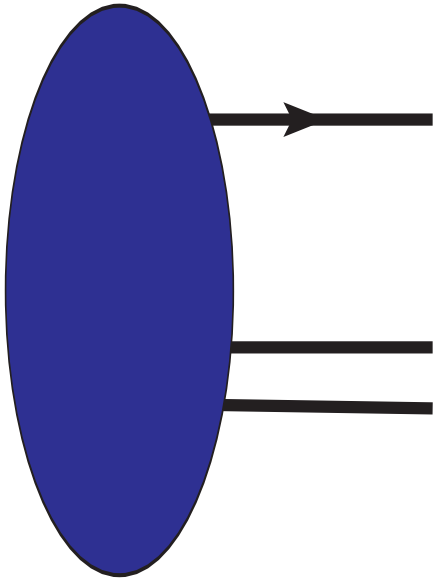


ρ figure from F. Gao *et al.*, PRD 90 (2014)

- Same "shape ordering" $\rightarrow \phi_\perp$ is flatter in both cases.
- Farther apart compared to the ρ meson case.

Modeling the Faddeev Amplitude

Faddeev Amplitude



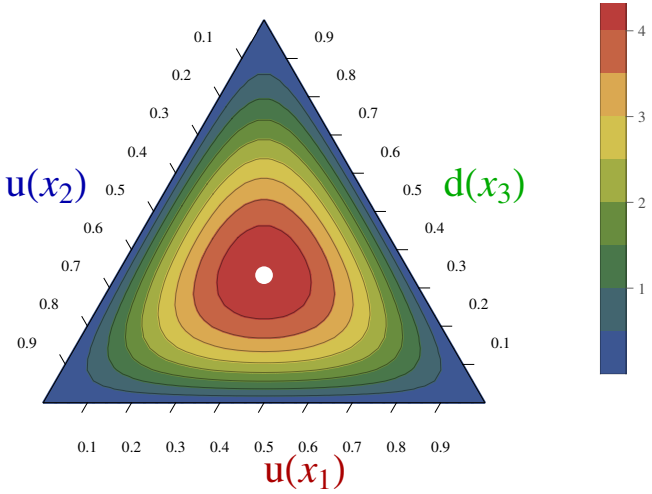
- Scalar case:

$$s_1(K,P) \quad = \mathcal{N}_1 \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{\left[\left(K - \frac{1-z}{2}P\right)^2 + \Lambda_N^2\right]}$$

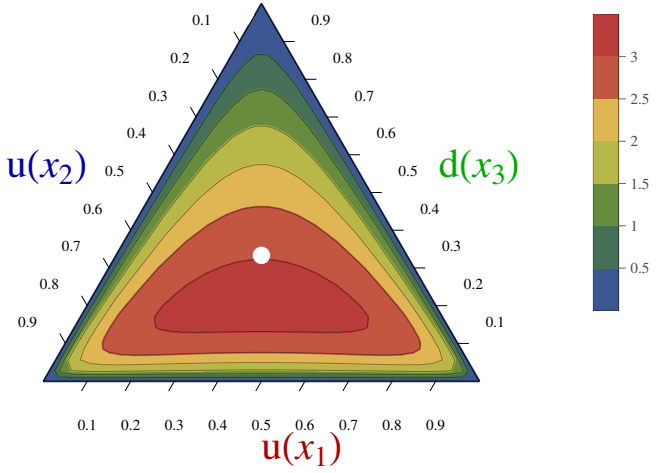
- AV case (2 out of 6 structures):

$$A^\mu(K,P) = \left(\gamma_5\gamma^\mu - i\gamma_5\hat{P}^\mu\right) \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{\left[\left(K - \frac{1-z}{2}P\right)^2 + \Lambda_N^2\right]}$$

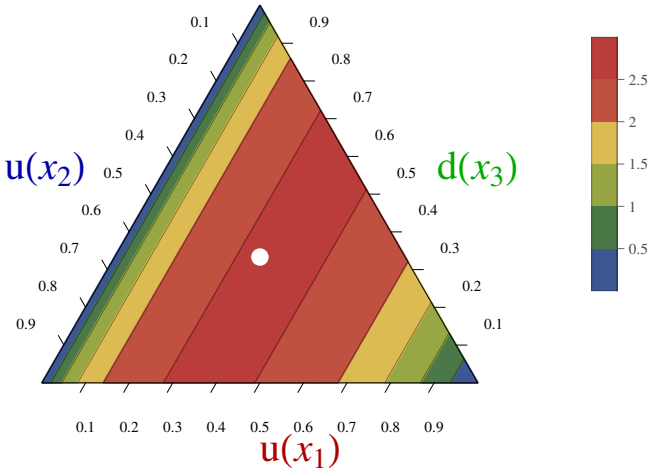
Results in the scalar channel



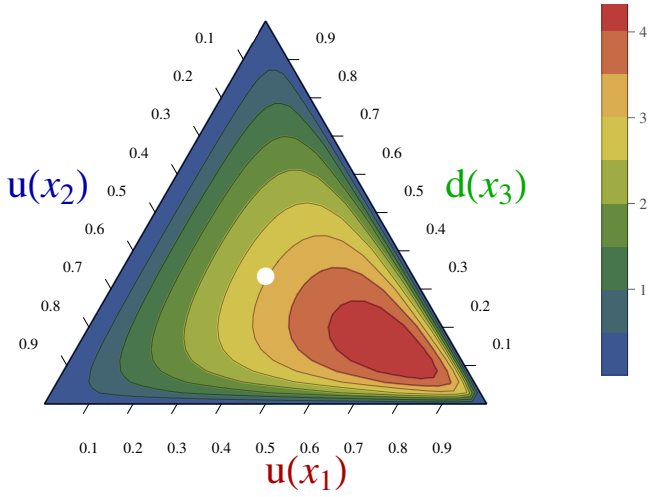
Asymptotic DA



Extended case: T

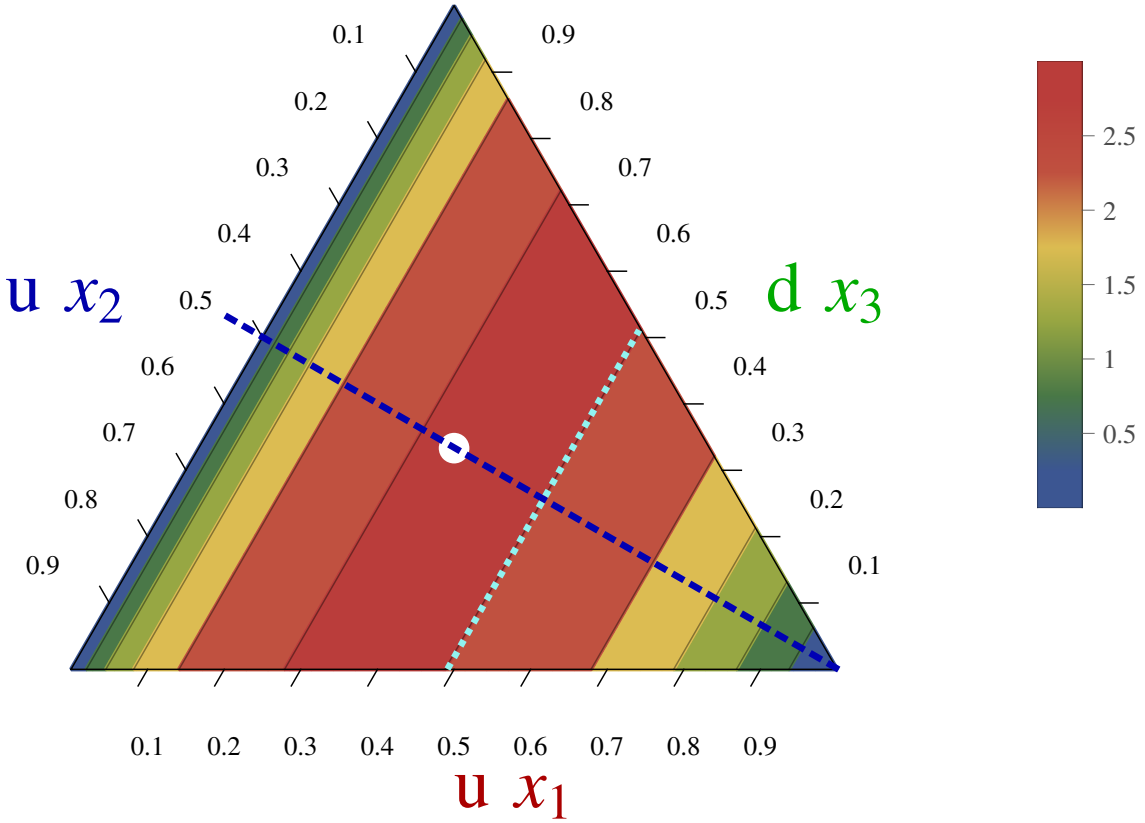


Point-like case: φ



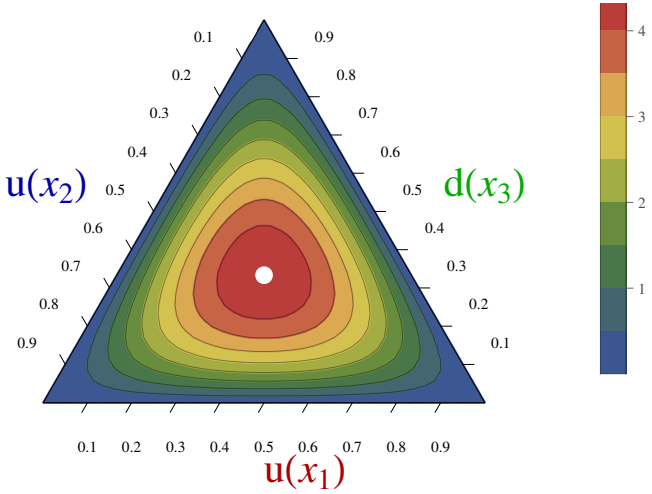
Extended case: φ

Results in the scalar channel

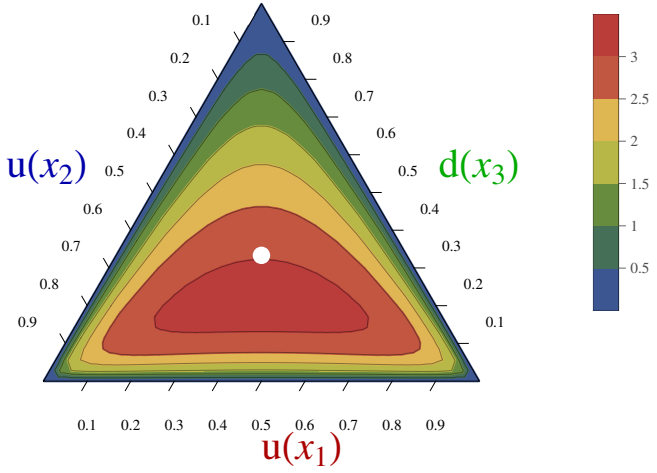


Point-like case: φ

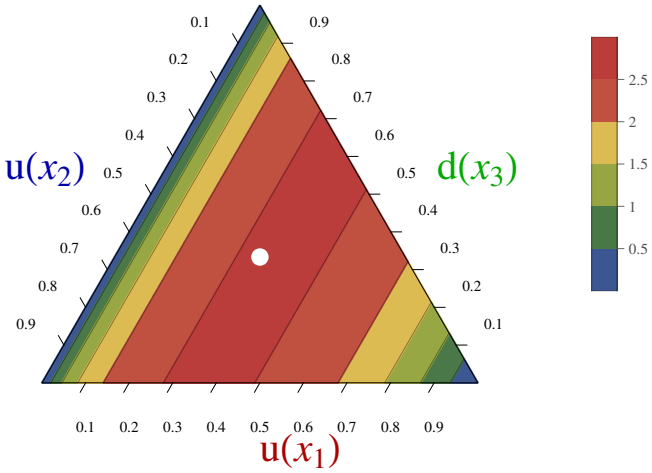
Results in the scalar channel



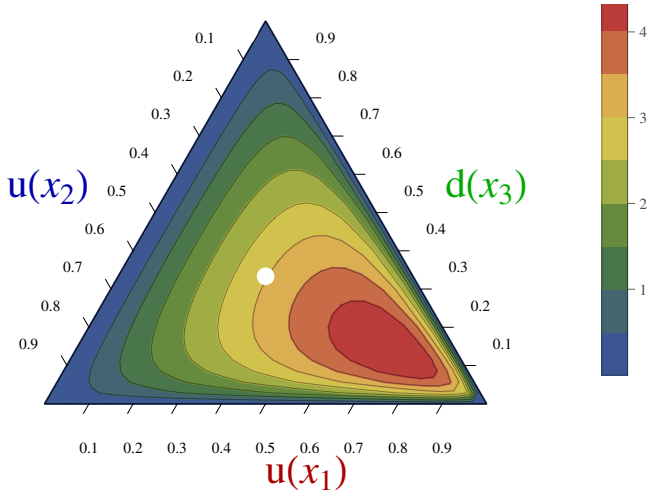
Asymptotic DA



Extended case: T



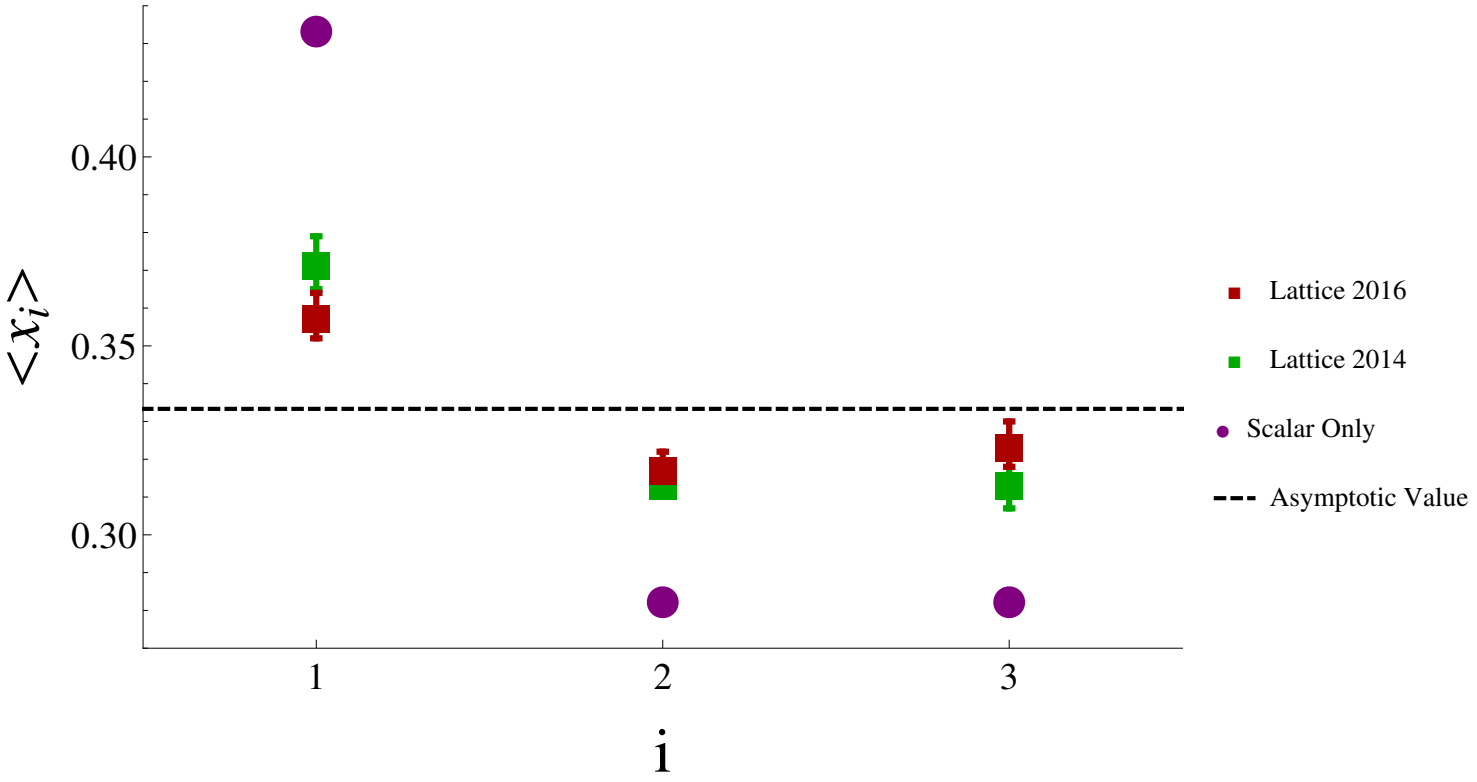
Point-like case: φ



Extended case: φ

Comparison with lattice I

$$< x_i >_\varphi = \int \mathcal{D}x \; x_i \varphi(x_1, x_2, x_3)$$

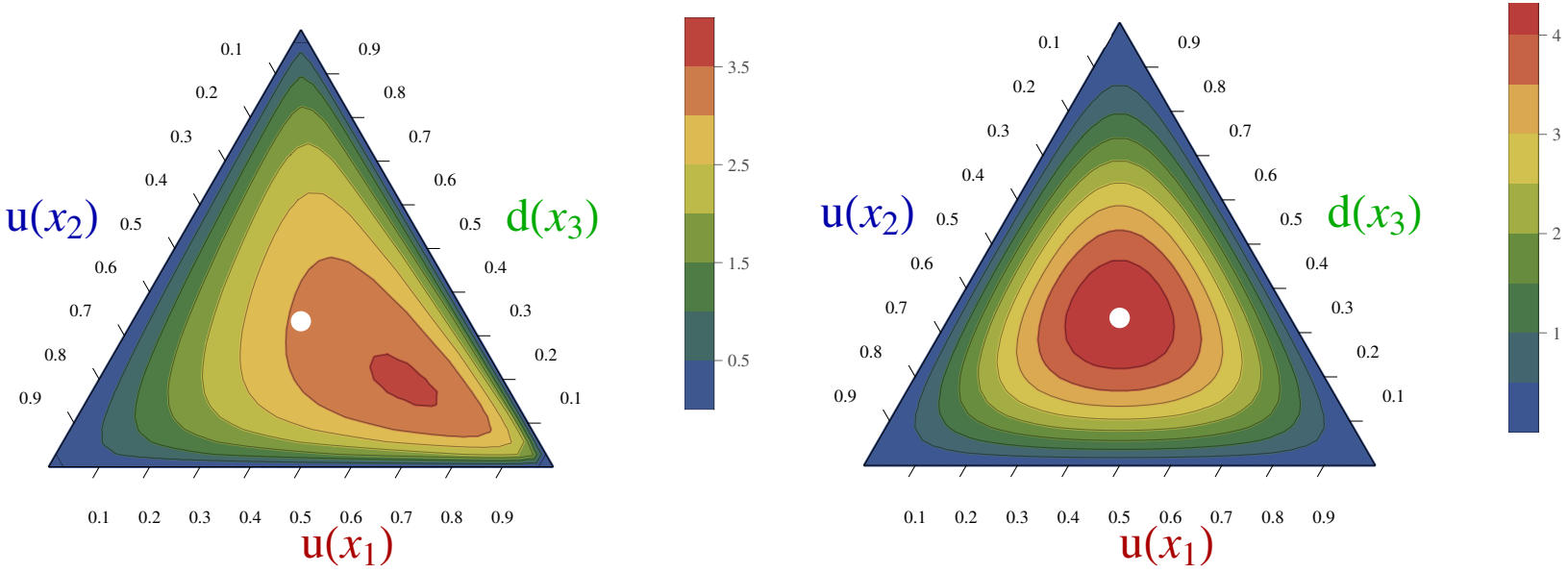


Lattice data from V.Braun *et al*, PRD 89 (2014)

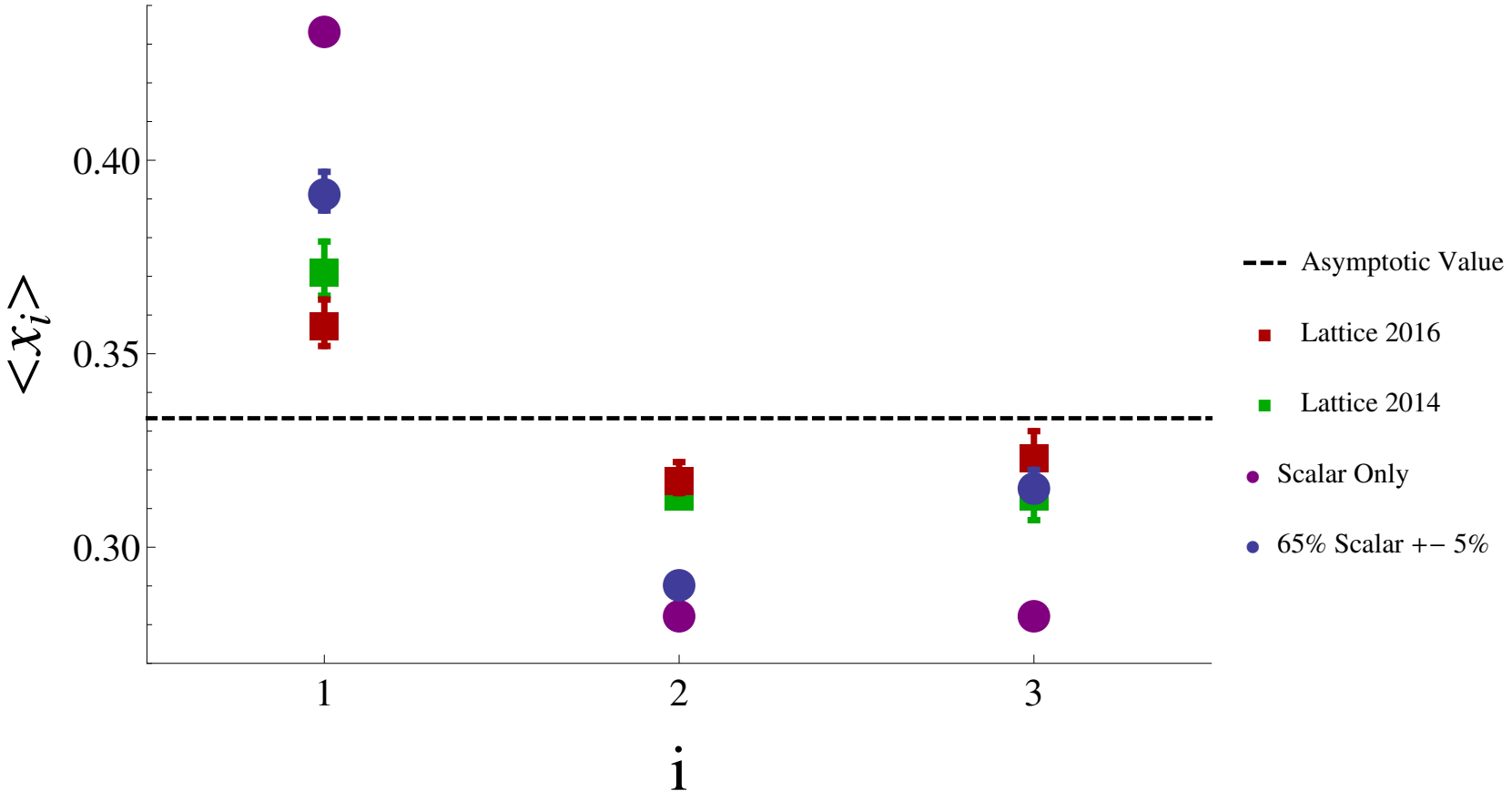
G. Bali *et al.*, JHEP 2016 02

Complete results for φ

- We use the prediction from the Faddeev equation to weight the scalar and AV contributions 65/35:



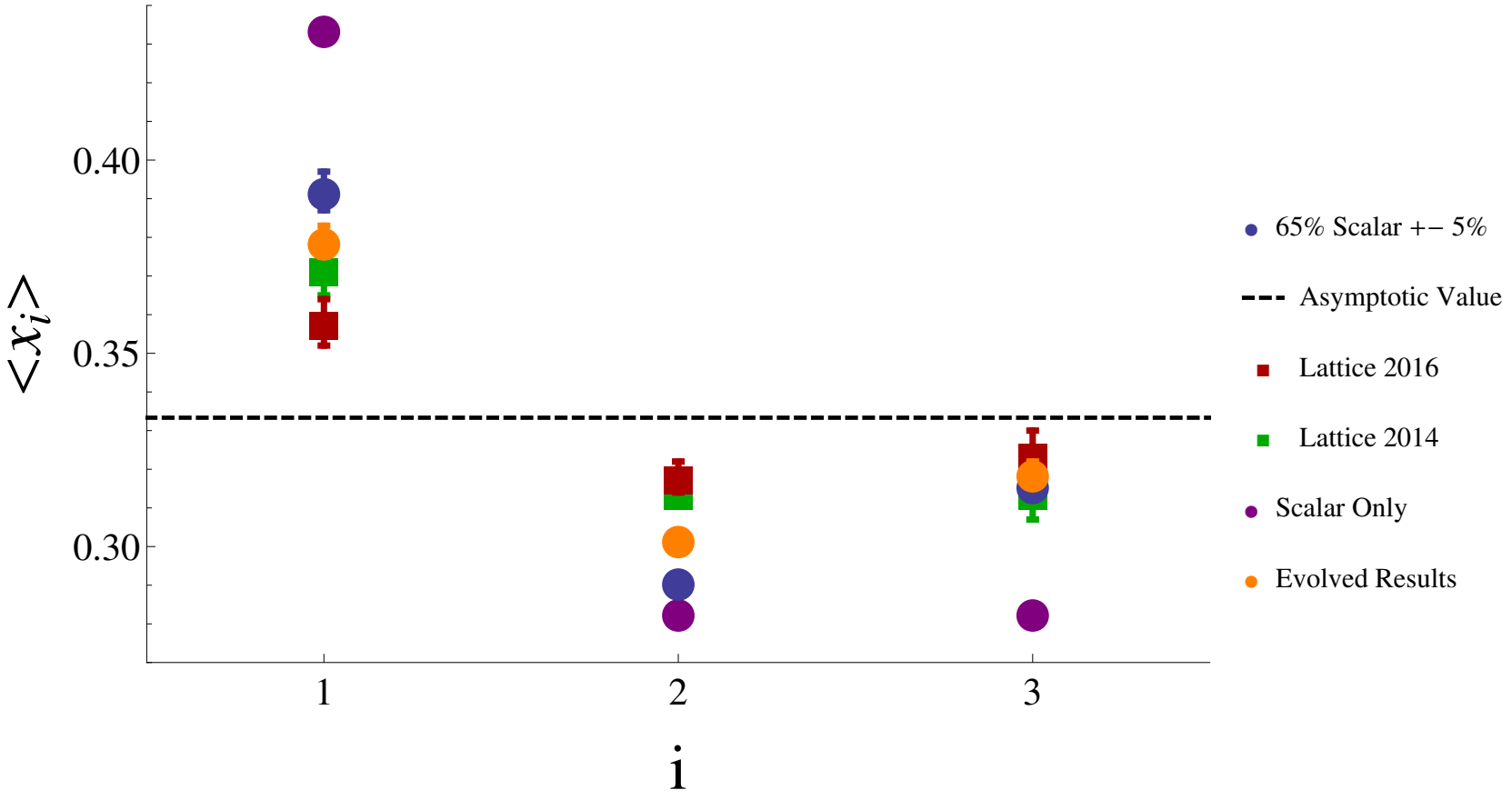
Comparison with lattice II



Lattice data from V.Braun *et al*, PRD 89 (2014)

G. Bali *et al.*, JHEP 2016 02

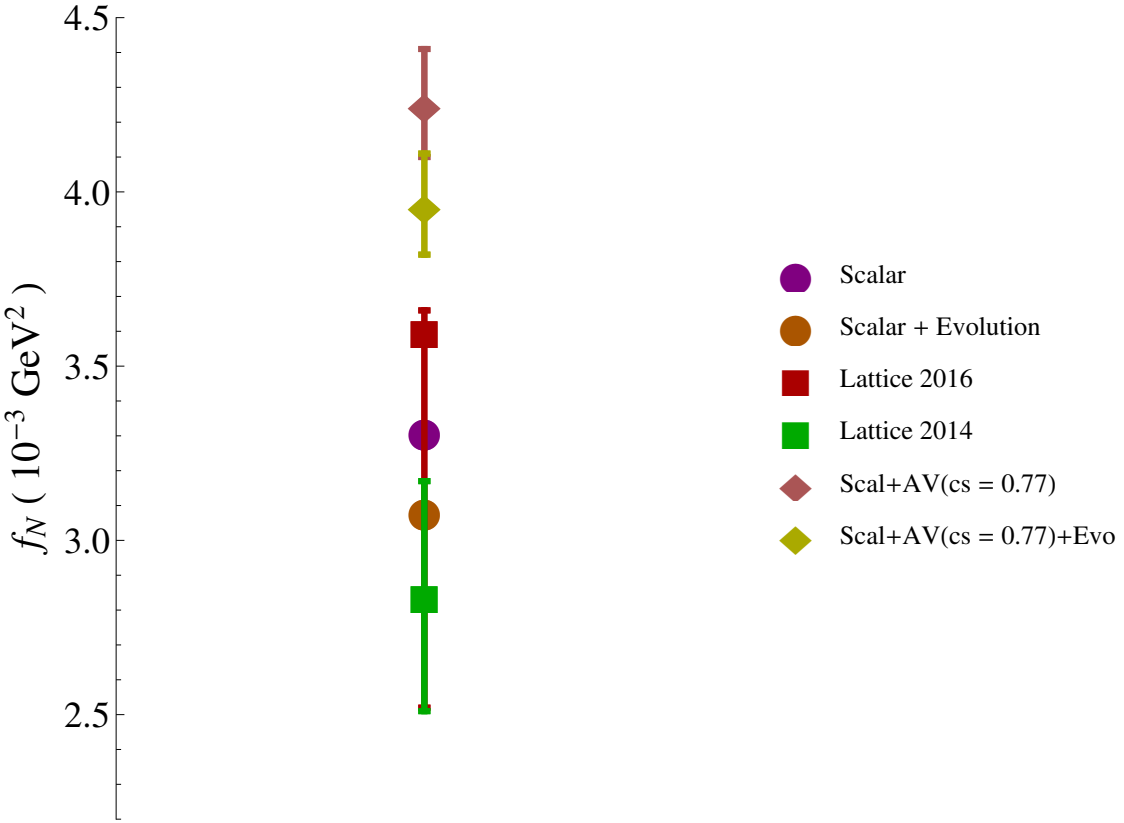
Comparison with lattice II



Lattice data from V.Braun *et al*, PRD 89 (2014)

G. Bali *et al.*, JHEP 2016 02

Comparison with lattice III

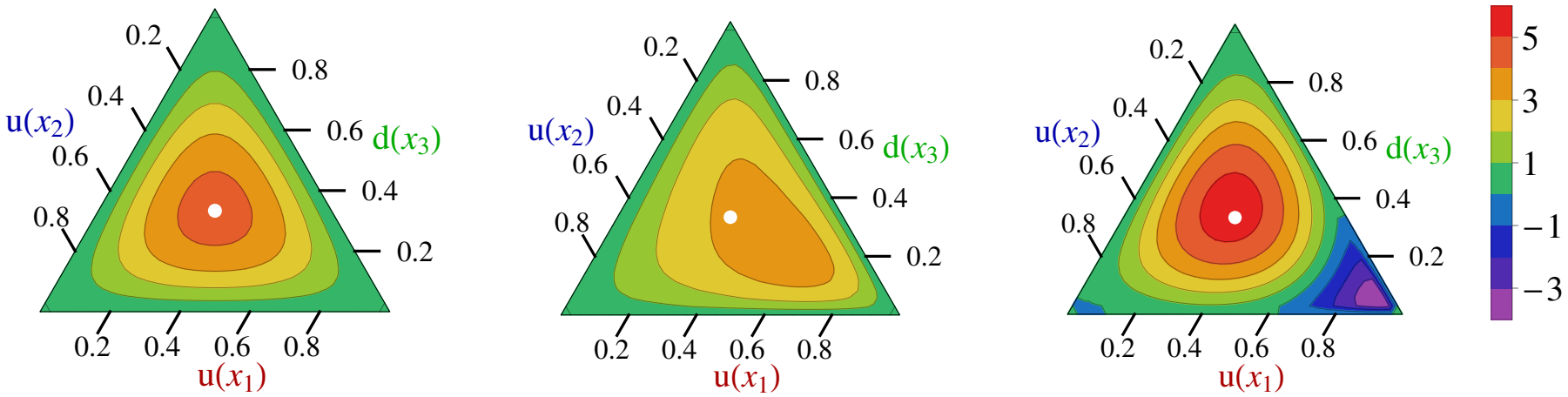


Computations done by J. Segovia

Lattice data from V.Braun *et al*, PRD 89 (2014)

G. Bali *et al.*, JHEP 2016 02

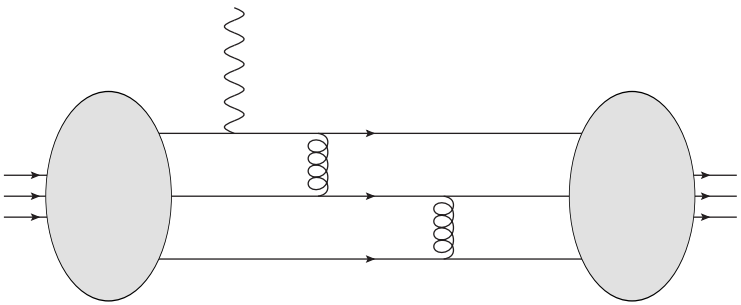
From Nucleon to Roper



- Results for the nucleon and Roper at 2 GeV.
- The nucleon remains broader and the peak shifted toward large x_1
- The roper present a negative area consistently with our understanding of $n = 1$ excited states.
- We provide a parametrisation of our results at 2 GeV.

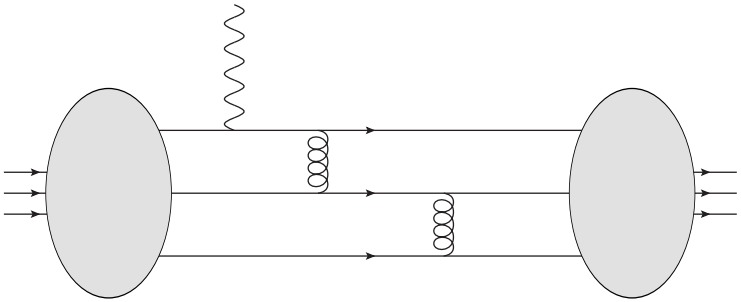
CM *et al.*, arXiv 1711.09101 submitted to PRL

Form Factors



$$F_1(Q^2) = \mathcal{N} \int [\mathrm{d}x_i][\mathrm{d}y_i] \left[\varphi(x_i, \zeta_x^2) H_\varphi(x_i, y_i, Q^2, \zeta_x^2, \zeta_y^2) \varphi(y_i, \zeta_y^2) + T(x_i, \zeta_x^2) H_T(x_i, y_i, Q^2, \zeta_x^2, \zeta_y^2) T(y_i, \zeta_y^2)\right]$$

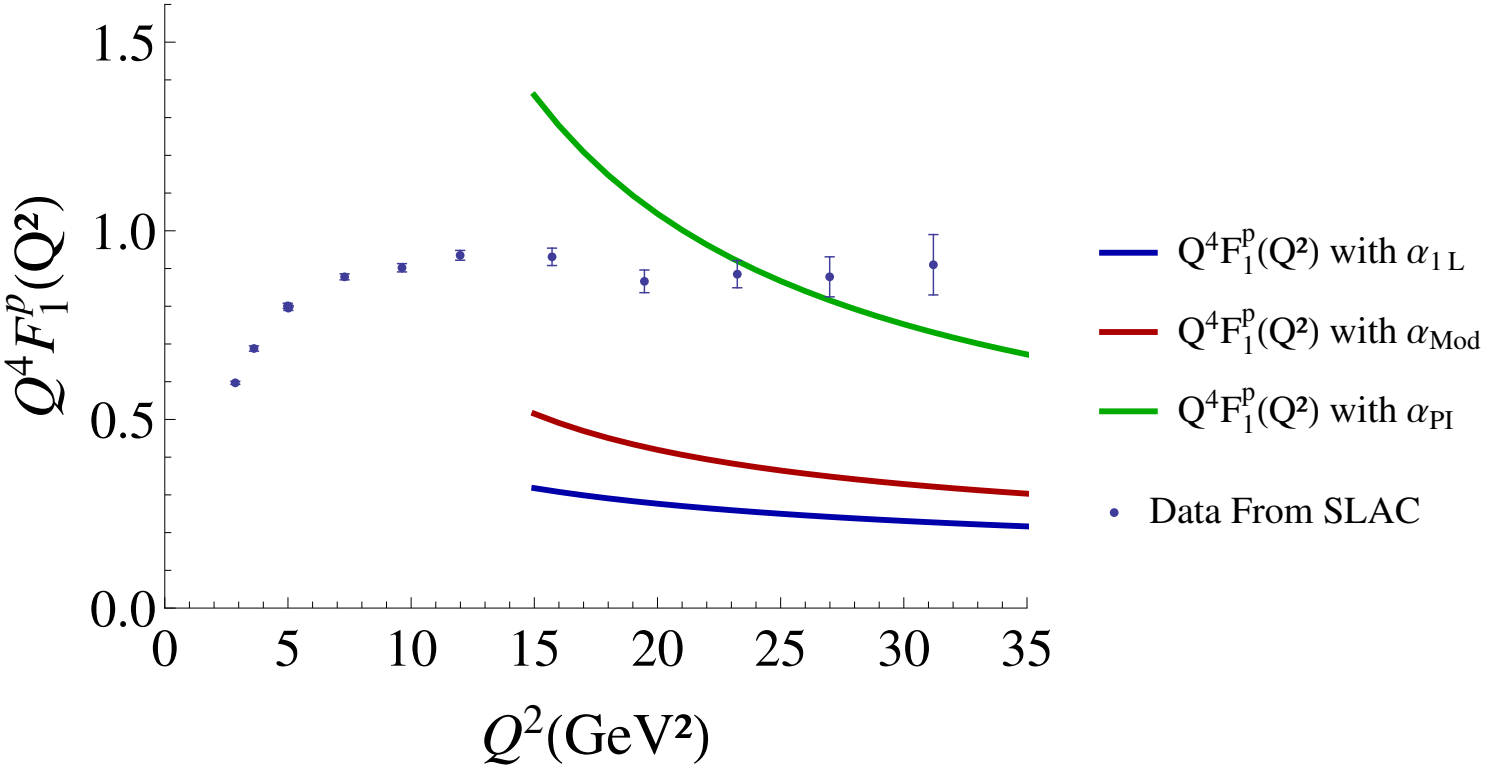
Form Factors



$$F_1(Q^2) = \mathcal{N} \int [\mathrm{d}x_i][\mathrm{d}y_i] \left[\varphi(x_i, \zeta_x^2) H_\varphi(x_i, y_i, Q^2, \zeta_x^2, \zeta_y^2) \varphi(y_i, \zeta_y^2) + T(x_i, \zeta_x^2) H_T(x_i, y_i, Q^2, \zeta_x^2, \zeta_y^2) T(y_i, \zeta_y^2)\right]$$

- LO Kernel well known since more than 30 years...
- ...but different groups have argued different choices for the treatment of scales:
  - for the DA : $\varphi(Q^2), \varphi((min(x_i) \times Q)^2)$...,
  - for the strong coupling constant : $\alpha_S(Q^2), \alpha_s(< x_i >^2 Q^2), \alpha_s^{\mathrm{reg}}(g(x_i, y_j) Q^2)$
- Use of perturbative coupling vs. effective coupling?

Very preliminary results



Data from Arnold et al. PRL 57

More work is required before we can conclude anything.

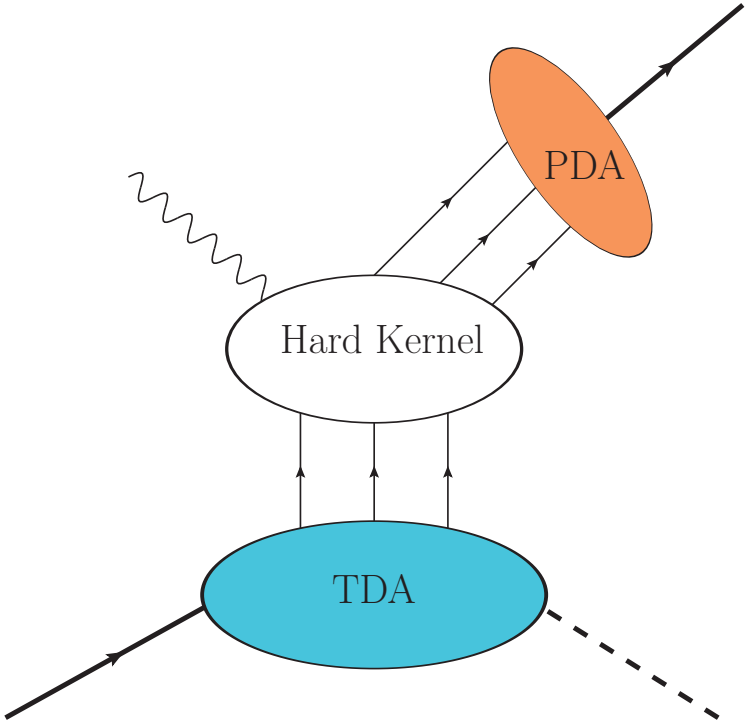
Chapter 2:
Beyond Distribution Amplitudes

TDA : A new understanding of hadron structure

$$\langle 0|\epsilon^{ijk} u^i_\alpha(z_1) u^j_\beta(z_2) d^k_\gamma(z_3)|N,P\rangle$$

$$\downarrow$$

$$\langle \pi, p|\epsilon^{ijk} u^i_\alpha(z_1) u^j_\beta(z_2) d^k_\gamma(z_3)|N,P\rangle$$



- Transition Distribution Amplitude encodes the non-perturbative information contain in an off-diagonal matrix element.
- They can be measured in backward kinematics of DVMP.

L.L. Frankfurt *et al.*, PRD 60 014010 (1999)
B. Pire and L. Szymanowski PRD 71 111501 (2005)
B. Pire and L. Szymanowski PLB 622 83 (2005)
K. Park *et al.*, arXiv 1711.08486

TDA: Entering the measurable era

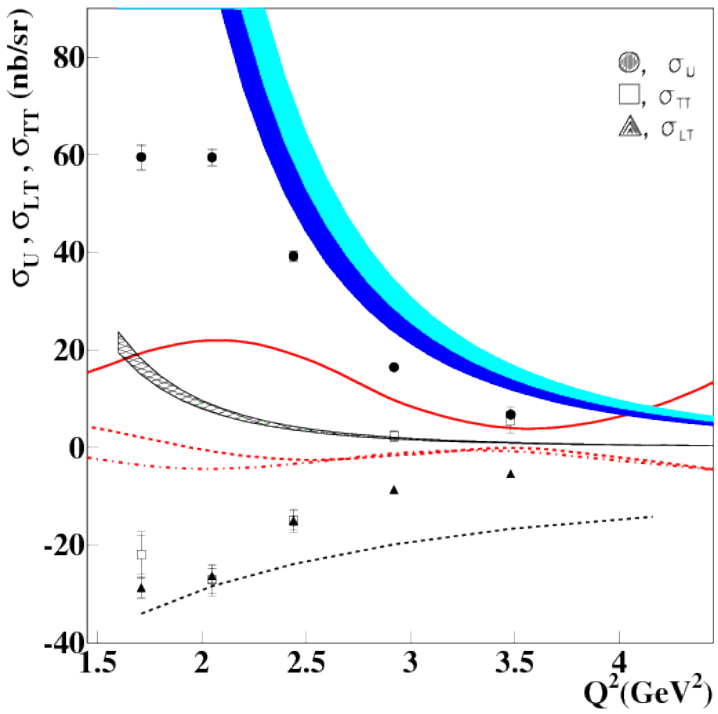


figure from K. Park *et al.*, arXiv 1711.08486

- TDA formalism predict the dominance of σ_T at large Q^2.
- Large σ_{LT} and σ_{TT} suggest a large σ_T contribution.
- σ_U seems not too far from theroretical predictions for σ_T.

TDA: Entering the measurable era

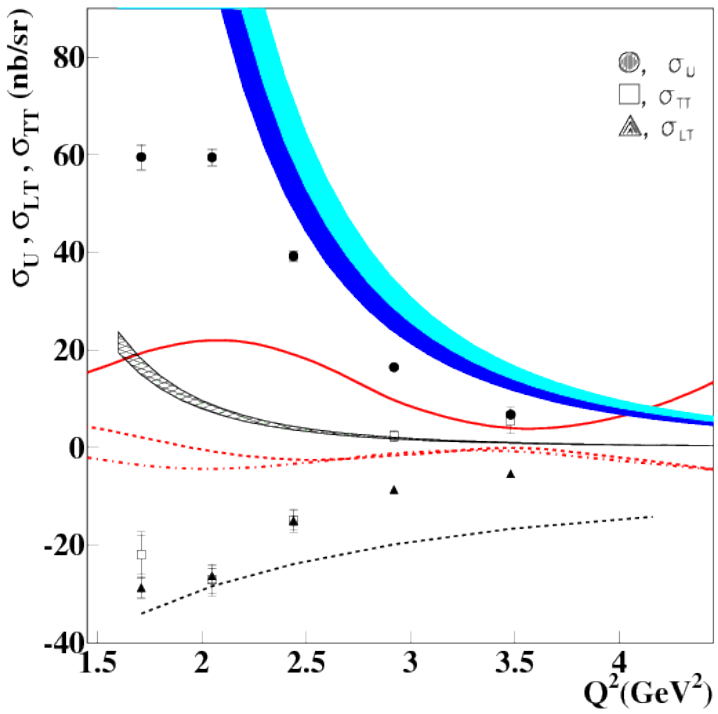


figure from K. Park *et al.*, arXiv 1711.08486

- TDA formalism predict the dominance of σ_T at large Q^2.
- Large σ_{LT} and σ_{TT} suggest a large σ_T contribution.
- σ_U seems not too far from theroretical predictions for σ_T.

This hints toward measurable TDAs in backward kinematics.

TDAs vs. GPDs

TDAs exhibit similar to GPDs:

- They depend on $\{x_i\}, \xi, \Delta^2, \mu_F^2$.
- They also obey polynomiality and can be written in terms of Spectral Functions.
- They reduce to the nucleon PDA when $\xi \to 1$ due to the soft pion theorem, the same way, the pion GPD reduced itself to the pion PDA.

TDAs vs. GPDs

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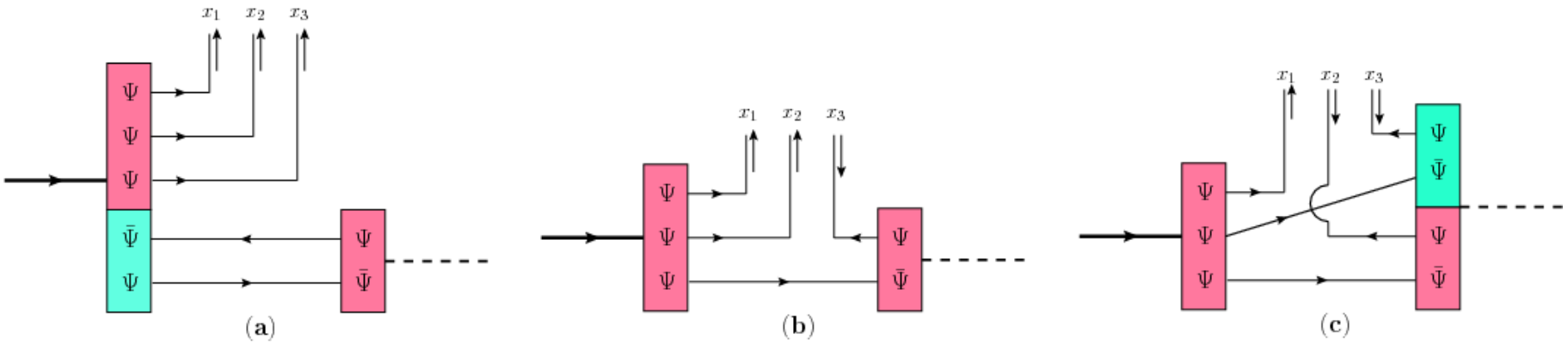
- They reduce to the nucleon PDA when $\xi \to 1$ due to the soft pion theorem, the same way, the pion GPD reduced itself to the pion PDA.

Advantage of TDA

TDA are define through a pion bra $\langle \pi, p|$ while GPDs through a nucleon bra $\langle N, p|$. This simplifies significantly the computations, and TDAs appears to be a natural intermediary steps between nucleon PDA and nucleon GPDs.

Computing TDA



figures from J.-P. Landsberg *et al.*, PRD 85 054021 (2012)

- Three different contribution coming from different kinematics.
- We can compute the (b) involving only valence components.
- The question is to know if we can apply the formalism of the inverse Radon transform to obtain an extension on TDA fulfilling polynomiality, the same way it as recently be developed for GPDs.

N. Chouika, CM *et al.*, arXiv 1711.05108
N. Chouika, CM *et al.*, arXiv 1711.11548

Conclusion

Conclusion

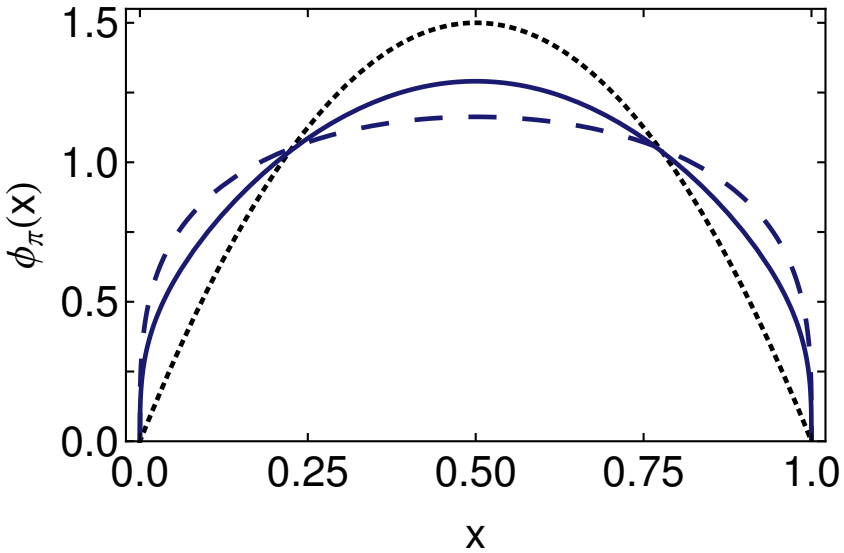
- The nucleon PDA can be described using a quark-diquark approximation.
- As a side-effect, we obtained a meson PDA parametrisation.
- The comparison with lattice computations explains how the different diquarks contribute to the total DAs, and the respective sensitivity of the latter to the AV-diquarks.
- We have obtained the first evaluation of the Roper PDA.
- We have preliminary results on the nucleon Dirac form factors
- Keep working on PDAs but using the numerical solution of DSEs.
- Keep exploring hadron structures with algebraic PTIR model. TDAs and GPDs are our targets.

Thank you for your attention

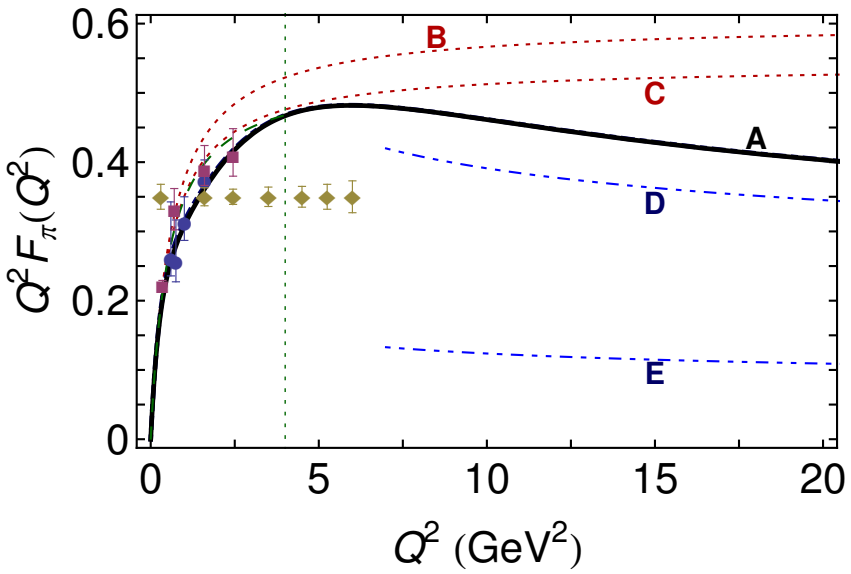
Back up slides

Pion distribution amplitude

$$\phi_{As}(x) = 6x(1-x)$$



L. Chang *et al.* (2013)



L. Chang *et al.* (2013)

- Broad DSE pion DA is much more consistent with the form factor than the asymptotic one.
- The scale when the asymptotic DA become relevant is huge.

N(1535)

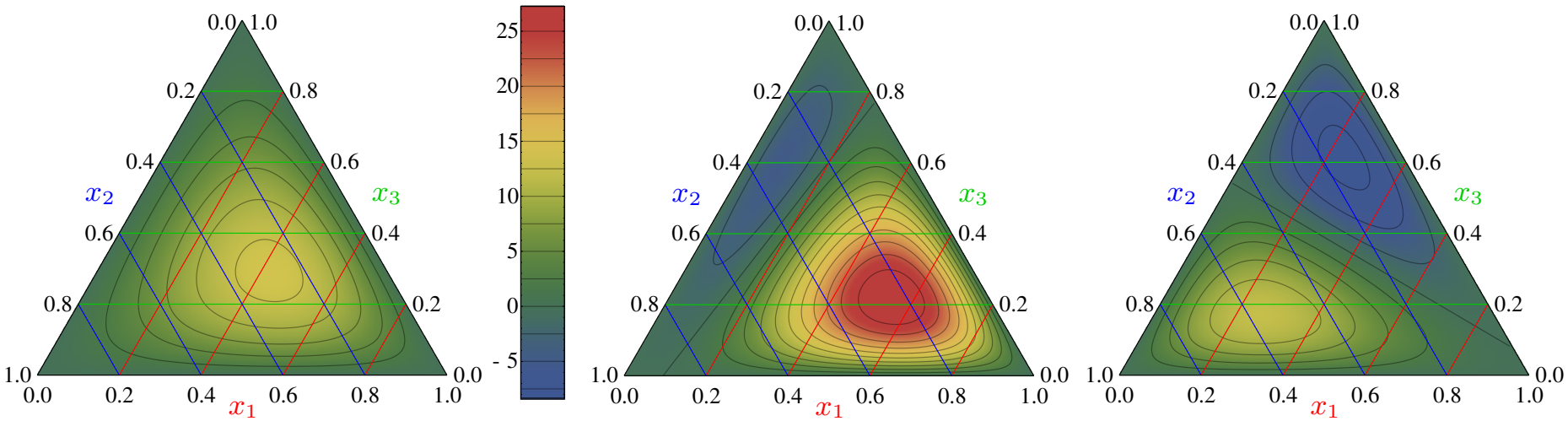


Figure from V. Braun *et la.*,Phys. Rev. **D89**, 094511 (2014)

Beyond the Form Factors

- The form factor is only the first Mellin Moment of GPDs and GDAs.
- The perturbative formula have been generalised to GPDs at large t and GDAs at large s for mesons and baryons.

M. Diehl *et al.*, PRD 61, (2000) 074029
C. Vogt, PRD 64, (2001), 057501
P. Hoodboy *et al.*, PRL 92 (2004) 012003
B. Pire *et al.*, PLB 639, (2006) 642-651

Can we use our DA models to get relevant information on GPDs and GDAs for mesons and baryons?