



Groupement de Recherche
Chromodynamique Quantique



Hadron energy-momentum tensor and mass decomposition

Based on [\[C.L., arXiv:1706.05853\]](#)

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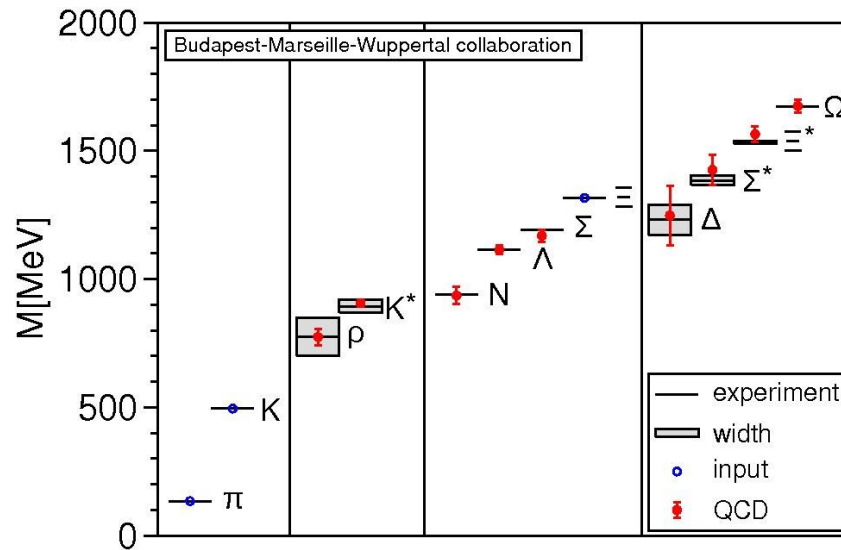
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Outline

- 1. Hadron mass and energy-momentum tensor**
- 2. Trace decomposition**
- 3. Ji's decomposition**
- 4. New decomposition**
- 5. Summary**

Lattice QCD

Ab initio mass calculation based on Euclidean space-time correlators $\sim \sum_n e^{-E_n \tau}$



[Dürr *et al.* (2008)]

Unfortunately, little insight on where mass comes from ...

Energy-momentum tensor

Classical QCD energy-momentum tensor

$$T^{\mu\nu} = \bar{\psi} \gamma^\mu \frac{i}{2} \overleftrightarrow{D}^\nu \psi - G^{a\mu\alpha} G^{a\nu}{}_\alpha + \frac{1}{4} \eta^{\mu\nu} G^2$$

Translation invariance  energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

Renormalized trace of the QCD EMT

$$T^\mu{}_\mu = \underbrace{\frac{\beta(g)}{2g} G^2}_{\text{Trace anomaly}} + (1 + \gamma_m) \bar{\psi} \overset{\uparrow}{m} \psi$$

Quark mass matrix

[Crewther (1972)]
[Chanowitz, Ellis (1972)]
[Nielsen (1975)]
[Adler, Collins, Duncan (1977)]
[Collins, Duncan, Joglekar (1977)]
[Nielsen (1977)]

Textbook decomposition

Forward matrix element

$$\langle P|T^{\mu\nu}(0)|P\rangle = 2P^\mu P^\nu$$

$$\langle P'|P\rangle = 2P^0 (2\pi)^3 \delta^{(3)}(\vec{P}' - \vec{P})$$

Trace decomposition

$$\begin{aligned} 2M^2 &= \langle P|T^\mu{}_\mu(0)|P\rangle \\ &= \underbrace{\langle P|\frac{\beta(g)}{2g} G^2|P\rangle}_{\sim 89\%} + \underbrace{\langle P|(1 + \gamma_m)\bar{\psi}m\psi|P\rangle}_{\sim 11\%} \end{aligned}$$

[Shifman, Vainshtein, Zakharov (1978)]
[Luke, Manohar, Savage (1992)]
[Donoghue, Golowich, Holstein (1992)]
[Kharzeev (1996)]
[Bressani, Wiedner, Filippi (2005)]
[Roberts (2017)]
[Krein, Thomas, Tsushima (2017)]



Manifestly covariant



Compatible with Gell-Mann–Oakes–Renner formula for pion



Depends on state normalization



No spatial extension



No clear relation to energy

Ji's decomposition

[Ji (1995)]
[Gaiete (2013)]

Separation of trace and traceless parts

$$T^{\mu\nu} = \bar{T}^{\mu\nu} + \hat{T}^{\mu\nu} \qquad \hat{T}^{\mu\nu} = \frac{1}{4} \eta^{\mu\nu} T^\alpha{}_\alpha$$

Forward matrix elements

$$\begin{aligned} \langle P | \bar{T}^{\mu\nu}(0) | P \rangle &= 2 \left(P^\mu P^\nu - \frac{1}{4} \eta^{\mu\nu} M^2 \right) \\ \langle P | \hat{T}^{\mu\nu}(0) | P \rangle &= \frac{1}{2} \eta^{\mu\nu} M^2 \end{aligned}$$

Virial decomposition

$$\begin{aligned} M &= \langle T^{00} \rangle |_{\vec{P}=\vec{0}} \\ &= \underbrace{\langle \bar{T}^{00} \rangle |_{\vec{P}=\vec{0}}}_{75\%} + \underbrace{\langle \hat{T}^{00} \rangle |_{\vec{P}=\vec{0}}}_{25\%} \end{aligned}$$

$$\langle O \rangle = \frac{\langle P | \int d^3r O(r) | P \rangle}{\langle P | P \rangle}$$

Ji's decomposition

[Ji (1995)]

Separation of quark and gluon contributions

$$\bar{T}^{\mu\nu} = \bar{T}_q^{\mu\nu} + \bar{T}_g^{\mu\nu}$$

$$\hat{T}^{\mu\nu} = \hat{T}_m^{\mu\nu} + \hat{T}_a^{\mu\nu}$$

Forward matrix elements

$$\langle P | \bar{T}_i^{\mu\nu}(0) | P \rangle = 2 a_i(\mu^2) \left(P^\mu P^\nu - \frac{1}{4} \eta^{\mu\nu} M^2 \right)$$

$$\langle P | \hat{T}_i^{\mu\nu}(0) | P \rangle = \frac{1}{2} b_i(\mu^2) \eta^{\mu\nu} M^2$$

$$\sum_i a_i(\mu^2) = \sum_i b_i(\mu^2) = 1$$

Ji's decomposition

[Gao *et al.* (2015)]

$$M = \underbrace{M_q}_{\sim 33\%} + \underbrace{M_g}_{\sim 34\%} + \underbrace{M_m}_{\sim 11\%} + \underbrace{M_a}_{\sim 22\%}$$

$\mu = 2 \text{ GeV}$

$$M_q = \langle \bar{T}_q^{00} \rangle |_{\vec{P}=\vec{0}} - \frac{3}{1+\gamma_m} \langle \hat{T}_m^{00} \rangle |_{\vec{P}=\vec{0}}$$

$$M_g = \langle \bar{T}_g^{00} \rangle |_{\vec{P}=\vec{0}}$$

$$M_m = \frac{4+\gamma_m}{1+\gamma_m} \langle \hat{T}_m^{00} \rangle |_{\vec{P}=\vec{0}}$$

$$M_a = \langle \hat{T}_a^{00} \rangle |_{\vec{P}=\vec{0}}$$

✓ Proper normalization

✓ Clear relation to energy distribution

! Scale-dependent interpretation in the rest frame

✗ Pressure effects not taken into account

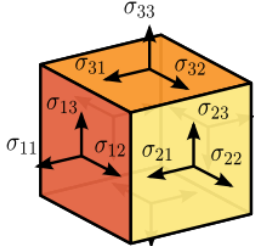
Continuum mechanics

[C.L. (2017)]

Interpretation of the EMT components

$$T^{\mu\nu} = \begin{bmatrix} \text{Energy density} & \text{Momentum density} & & \\ T^{00} & T^{01} & T^{02} & T^{03} \\ \text{Energy flux} & \text{Momentum flux} & & \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

Shear stress
Normal stress (pressure)



Analogy with relativistic hydrodynamics

Hadron $\langle P | T_i^{\mu\nu}(0) | P \rangle = 2P^\mu P^\nu A_i(0) + 2M^2 \eta^{\mu\nu} \bar{C}_i(0)$

Perfect fluid element

$$\Theta_i^{\mu\nu} = (\varepsilon_i + p_i) u^\mu u^\nu - p_i \eta^{\mu\nu}$$



Four-velocity

$$u^\mu = P^\mu / M$$

Energy density

$$\varepsilon_i = [A_i(0) + \bar{C}_i(0)] \frac{M}{V}$$

Isotropic pressure

$$p_i = -\bar{C}_i(0) \frac{M}{V}$$

New decomposition

[C.L. (2017)]

Mass as the total internal energy

$$M = \sum_i U_i \qquad U_i = \varepsilon_i V$$

Stability condition \longrightarrow energy-momentum conservation

$$\sum_i p_i = 0 \qquad \longrightarrow \qquad \sum_i \bar{C}_i(0) = 0 \qquad \longrightarrow \qquad \sum_i A_i(0) = 1$$

Nucleon mass decomposition

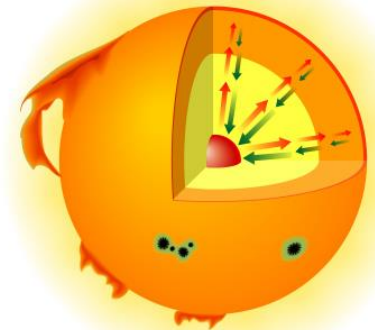
$$M = \underbrace{U_q}_{\sim 44\%} + \underbrace{U_g}_{\sim 56\%}$$

$\mu = 2 \text{ GeV}$

$$p_q = -p_g$$

$\sim 11\%$

pressure \longrightarrow quark pressure
gravity \longrightarrow gluon pressure



Some numerology

At the center of the Sun

$$\rho_{\odot} \approx 1.6 \times 10^5 \text{ kg/m}^3$$

$$p_{\odot} \approx 2.5 \times 10^{16} \text{ N/m}^2$$

$$\longrightarrow p_{\odot}/\rho_{\odot}c^2 \approx 1.7 \times 10^{-6}$$

Inside a nucleon

$$\rho_N \approx 2 \times 10^{17} \text{ kg/m}^3$$

$$p_N \approx 2 \times 10^{33} \text{ N/m}^2$$

$$\longrightarrow p_N/\rho_Nc^2 \approx 0.11$$

Inside a black hole

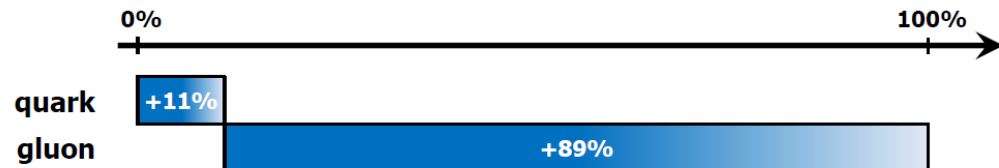
$$\rho_{\text{BH}} = \frac{3c^6}{32\pi G^3 M^2}$$

$$\rho_{\text{BH}}^{\odot} \approx 1.8 \times 10^{19} \text{ kg/m}^3$$

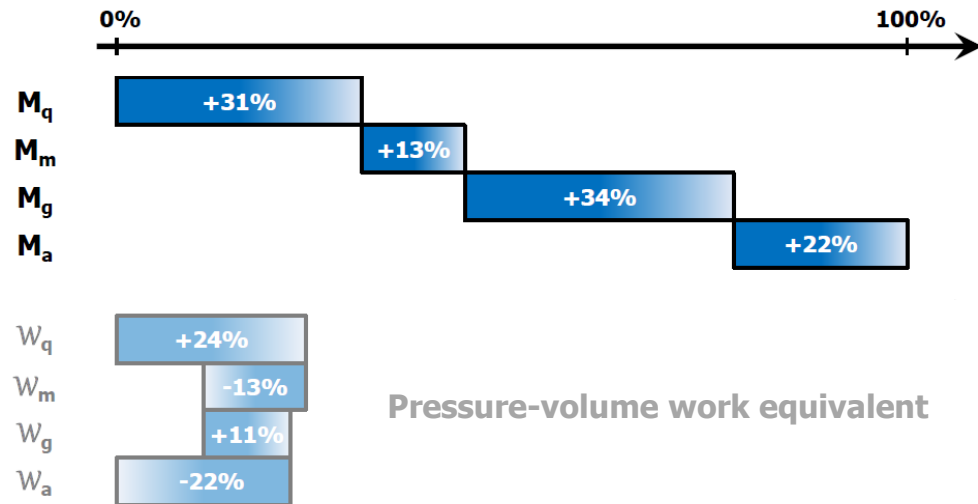
$$\rho_{\text{BH}}^N \approx 2.6 \times 10^{133} \text{ kg/m}^3$$

Summary

Trace decomposition



Ji's decomposition



New decomposition

