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Narrow-width tetraquarks in large-N_c QCD

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Tetraquark problem in QCD

Hadrons are color-singlet bound states of quarks and gluons.

Mesons are essentially made of $\overline{q}q$.

Baryons are essentially made of qqq.

Are there other types of structure for bound states (exotics)?

Tetraquarks would be made of $\overline{q}\overline{q}qq$.

Pentaquarks would be made of $\overline{q}qqqq$.

Possibility considered long ago by many authors (Jaffe, 1977).

However, theoretical difficulties arise in QCD.

We concentrate in the following on the tetraquark problem; pentaquarks can be treated in the same way.

The difficulty is related to the fact that a tetraquark field, local or nonlocal, made of a pair of quark and a pair of antiquark fields, which would be color-gauge invariant, could be decomposed, by Fierz transformations, into a combination of products of color-singlet bilinear operators of quark-antiquark pairs.

For instance, in local form

$$T(x) = (\overline{q}\overline{q}qq)(x) \sim \sum (\overline{q}q)(x)(\overline{q}q)(x),$$

where $(\overline{q}q)(x)$ are themselves color-singlet.

However, color-singlet $(\overline{q}q)(x)$'s essentially describe ordinary meson fields or states. The above decomposition is suggestive of a property that tetraquarks would be factorizable into independent mesons and could at best be bound states or resonances of mesons, called also molecular tetraquarks, and not genuine bound states of two quarks and two antiquarks, resulting from the direct confinement of the four constituents.

What would be, on phenomenological grounds, the difference of the two types of bound state, since both of them would be represented by poles in the hadronic sector?

Meson-meson interaction forces are short-range and weak, as compared to the strong long-distance confining forces.

Therefore, molecular type tetraquarks, would be loosely bound states, with relatively large space extensions, while tetraquarks, which would be formed directly by confining forces, would be more tightly bound. The latter are called compact tetraquarks.

Compact tetraquarks would also exist in multiplicities, since confinement is independent of flavor.

The above qualitative differences have their influence on phenomenological quantities, like the number of states, decay modes, decay widths and transition amplitudes.

For more than ten years, many tetraquark candidate states have been signalled by several experiments: Belle, BaBar, BESIII, LHCb, CDF, D0, CMS. Ordinary meson structures could not fit their properties.

Some of the candidates have disappeared or were invalidated, but there is still a certain number of states which might be interpreted as tetraquarks. Intense theoretical activity around the extraction of their physical properties and their interpretation.

We will be interested here by the qualitative properties of compact tetraquarks.

To this end we will have recourse to the large-N_c limit of QCD.

QCD at large N_c

Framework: $SU(N_c)$ gauge theory, with quarks in the fundamental representation, considered in the limit $N_c \to \infty$ with $g \sim 1/N_c^{1/2}$. ('t Hooft, 1974.)

In this limit, QCD catches the main properties of confinement, while being simplified with respect to secondary complications. $1/N_c$ plays the role of a perturbative parameter.

Properties of the theory analyzed by Witten (1979).

Consider a quark color-singlet bilinear operator and its two-point function. Intermediate states are color-singlet mesons.

$$B(x) = (\overline{q}q)(x).$$

$$\int d^4x e^{ip.x} \langle B(x) B^\dagger(0) \rangle \underset{N_c \to \infty}{=} \sum_{n=1}^{\infty} \frac{f_n^2}{p^2 - M_n^2}.$$

The spectrum is saturated by an infinite number of free stable mesons.

Many-meson states contribute only to subleading orders.

Interaction forces are also classified with respect to N_c.

- Three-meson interaction $\sim N_c^{-1/2}$.
- Four-meson interaction $\sim N_c^{-1}$.
- Meson decay widths $\sim N_c^{-1}$.

Can we have similar predictions with tetraquarks?

$$T(x) = (\overline{q}\overline{q}qq)(x).$$

$$\langle T(x)T^{\dagger}(0)\rangle \underset{N_c\to\infty}{=} \langle B(x)B^{\dagger}(0)\rangle\langle B(x)B^{\dagger}(0)\rangle.$$

Equivalent to the propagation of two free mesons. (Coleman, 1980.)

No tetraquark poles can appear at this order.

For a long time, this fact has been considered as a theoretical proof of the non-existence of tetraquarks as elementary stable particles, surviving in the large-N_c limit, like the ordinary mesons.

Recently, Weinberg (2013) observed that if tetraquarks exist as bound states in the large-N_c limit with finite masses, the crucial point is, even if they contribute to subleading diagrams, the qualitative property of their decay widths: are they broad or narrow? In the latter case, they might be observable. He showed that, generally, they should be narrow, with decay widths of the order of $1/N_c$, which is compatible with the stability assumption in the large-N_c limit.

Knecht and Peris (2013) showed that in a particular exotic channel, tetraquarks should even be narrower, with decay widths of the order of $1/N_c^2$.

Cohen and Lebed (2014) showed, in more general exotic channels, with an analysis based on the analyticity properties of two-meson scattering amplitudes, that the decay widths should indeed be of the order of $1/N_c^2$.

Line of approach

Study of exotic and cryptoexotic tetraquark properties, through the analysis of meson-meson scattering amplitudes.

Exotics: contain four different quark flavors.

Cryptoexotics: contain three different quark flavors.

Four-point correlation functions of color-singlet quark bilinears,

$$j_{ab} = \overline{q}_a q_b,$$

having coupling with a meson M_{ab}:

$$\langle 0|j_{ab}|M_{ab}\rangle = f_{M_{ab}}; \qquad f_M \sim N_c^{1/2}.$$

Spin and parity ignored; not relevant for the qualitative aspects.

Consider all possible channels where a tetraquark may be present.

To be sure that a QCD diagram may contain a tetraquark contribution, through a pole term, one has to check that it receives a four-quark contribution in its s-channel singularities, plus additional gluon singularities that do not modify the N_c-behavior of the diagram.

If the tetraquark contains quarks and antiquarks with masses $m_j,\ j = a, b, c, d$, then the diagram should have a four-particle cut starting at $s = (m_a + m_b + m_c + m_d)^2$.
Its existence is checked with the use of the Landau equations.

Diagrams that do not have s-channel singularities, or have only two-particle singularities (quark-antiquark), cannot contribute to the formation of tetraquarks at their N_c-leading order. They should not be taken into account for the N_c-behavior analysis of the tetraquark properties.

Exotic tetraquarks

Four distinct quark flavors, denoted 1,2,3,4, with meson currents

$$j_{12} = \overline{q}_1 q_2, \quad j_{34} = \overline{q}_3 q_4, \quad j_{14} = \overline{q}_1 q_4, \quad j_{32} = \overline{q}_3 q_2.$$

The following scattering processes are considered:

$$M_{12} + M_{34} \;\to\; M_{12} + M_{34}; \qquad \text{Direct channel I};$$

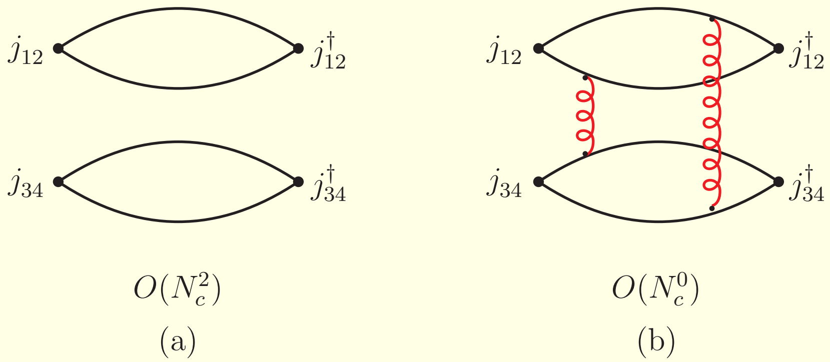
$$M_{14} + M_{32} \;\to\; M_{14} + M_{32}; \qquad \text{Direct channel II};$$

$$M_{12} + M_{34} \;\to\; M_{14} + M_{32}; \qquad \text{Recombination channel}.$$

‘Direct’ 4-point functions

$$\boldsymbol{\Gamma_{I}^{(\mathrm{dir})} = \langle j_{12} j_{34} j_{34}^{\dagger} j_{12}^{\dagger} \rangle} \ , \qquad \boldsymbol{\Gamma_{II}^{(\mathrm{dir})} = \langle j_{14} j_{32} j_{32}^{\dagger} j_{14}^{\dagger} \rangle} \ .$$

Leading and subleading diagrams for $\boldsymbol{\Gamma_{I}^{(dir)}}$:



Similar diagrams for $\boldsymbol{\Gamma_{II}^{(dir)}}$.

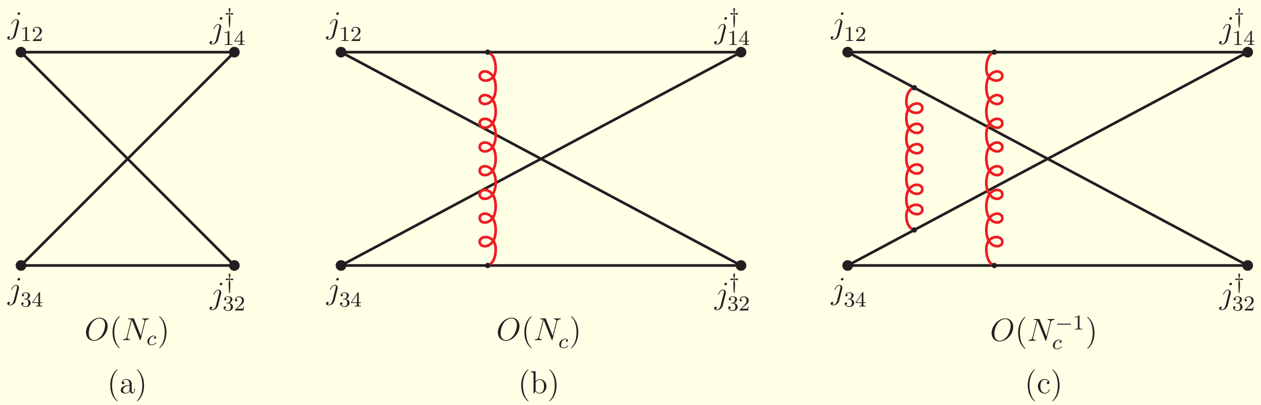
Only diagram (b) may receive contributions from tetraquark states.

$$\boldsymbol{\Gamma_{I,T}^{(\mathrm{dir})} = O(N_c^0)}, \qquad \boldsymbol{\Gamma_{II,T}^{(\mathrm{dir})} = O(N_c^0)}.$$

‘Recombination’ 4-point function

$$\Gamma^{(\mathrm{recomb})} = \langle j_{12} j_{34} j_{32}^{\dagger} j_{14}^{\dagger} \rangle \, .$$

Leading and subleading diagrams:



Only diagram (c) may receive contributions from tetraquark states.

$$\Gamma_T^{(\mathrm{recomb})} = O(N_c^{-1}).$$

The fact that the direct and recombination amplitudes have different behaviors in N_c, requires the contribution of two different tetraquarks, T_A and T_B, each having different couplings to the meson pairs.

Factorizing in the correlation functions the external current couplings to the mesons ($\sim f_M \sim N_c^{1/2}$), one obtains

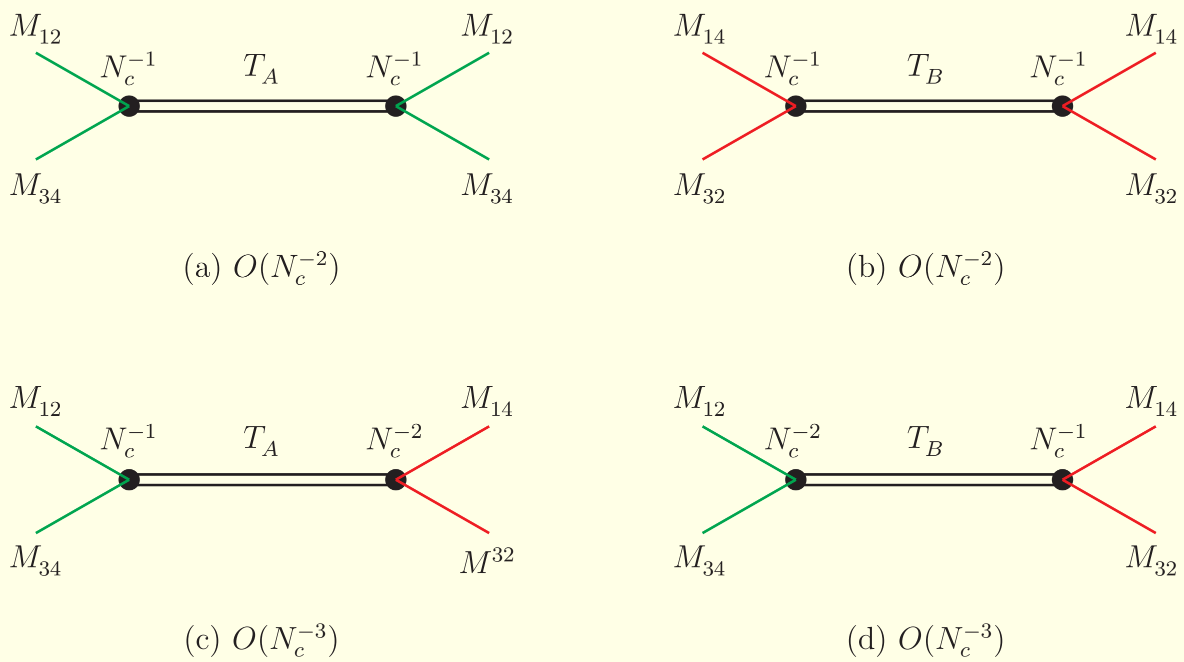
$$A(T_A \to M_{12}M_{34}) = O(N_c^{-1}), \qquad A(T_A \to M_{14}M_{32}) = O(N_c^{-2}),$$

$$A(T_B \to M_{12}M_{34}) = O(N_c^{-2}), \qquad A(T_B \to M_{14}M_{32}) = O(N_c^{-1}).$$

Total widths:

$$\Gamma(T_A) = O(N_c^{-2}), \qquad \Gamma(T_B) = O(N_c^{-2}).$$

The meson-meson scattering amplitudes at the tetraquark poles (leading contributions):



Cryptoexotic tetraquarks

Three distinct quark flavors, denoted 1,2,3, with meson currents

$$j_{12} = \overline{q}_1 q_2, \quad j_{23} = \overline{q}_2 q_3, \quad j_{22} = \overline{q}_2 q_2.$$

The calculations are similar as before, with the existence of a larger number of diagrams.

The net result is that one single tetraquark T can saturate all the consistency equations. Its width is again of the order of N_c^{-2}:

$$\Gamma(T) = O(N_c^{-2}).$$

Conclusion

Analysis of the s-channel singularities of Feynman diagrams crucial for the detection of the possible presence of tetraquark intermediate states in correlation functions of meson currents.

If tetraquarks exist as stable bound states of two quarks and two antiquarks in the large-N_c limit, with finite masses, due to the operating confining forces, then they should have narrow decay widths, of the order of N_c^{-2}, much smaller than those of ordinary mesons ($\sim N_c^{-1}$).

For the fully exotic channel, with four different quark flavors, two different tetraquarks are needed to accommodate the theoretical constraints of the large-N_c limit. In this case, each tetraquark has one predominant decay channel.