Moving to NLO accuracy in the saturation formalism

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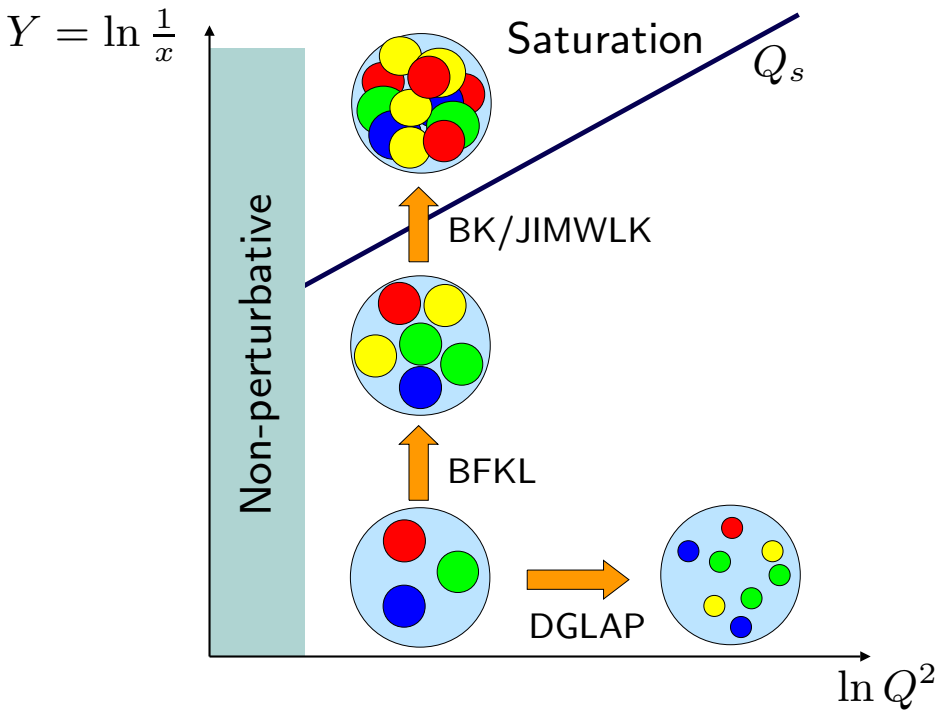
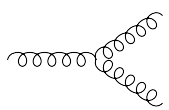
B. D., T. Lappi, Y. Zhu, PRD 93 (2016) 114016 [arXiv:1604.00225]
B. D., T. Lappi, Y. Zhu, PRD 95 (2017) 114007 [arXiv:1703.04962]
B. D., H. Hänninen, T. Lappi, Y. Zhu, PRD 96 (2017) 094017 [arXiv:1708.07328]
B. D., E. Iancu, T. Lappi, A.H. Mueller, G. Soyez, D.N. Triantafyllopoulos, Y. Zhu, in preparation

Our goal is to study QCD in the saturation regime

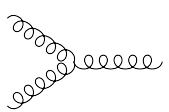


We want to stay in the perturbative regime: requires a hard scale in the process

Linear small x evolution (BFKL): dominated by $1 \to 2$ scatterings: rapid growth of parton densities when going to smaller x values



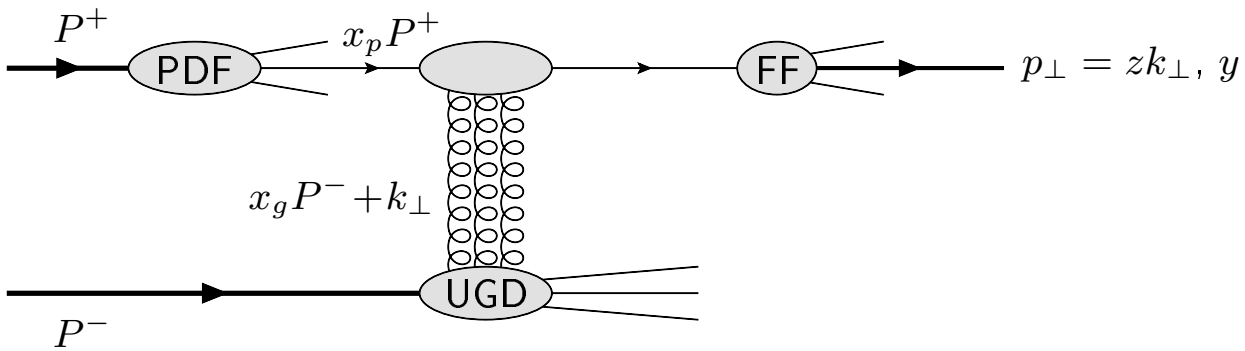
At large densities, recombination effects become important: Non-linear evolution (BK, JIMWLK)



Probing saturation: reach small x values, preferably in a nucleus ($Q_s^2 \sim A^{1/3}$)

Forward hadron production at LO

Single inclusive forward hadron production at LO in the $q \to q$ channel:



Dilute projectile: $x_p = \dfrac{k_\perp}{\sqrt{s}} e^{y}$, described by a collinear PDF

Dense target: $x_g = \dfrac{k_\perp}{\sqrt{s}} e^{-y} \ll 1$, described by unintegrated gluon distribution $\mathcal{S}$

LO quark multiplicity: $\dfrac{\mathrm{d}N}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y} \propto \mathrm{PDF} \otimes \mathcal{S} \otimes \mathrm{FF}$ $\qquad \left(\dfrac{\mathrm{d}\sigma}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y} = \int \mathrm{d}^2\mathbf{b} \dfrac{\mathrm{d}N}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y} \right)$

$\mathcal{S}$ is the Fourier transform of the correlator $S(\mathbf{r})$:

$$\mathcal{S}(k_\perp) = \int \mathrm{d}^2\mathbf{r} e^{-i\mathbf{k}\cdot\mathbf{r}} S(\mathbf{r}) \ , \ S(\mathbf{r} = \mathbf{x} - \mathbf{y}) = \left\langle \frac{1}{N_c} \operatorname{Tr} V(\mathbf{x}) V^\dagger(\mathbf{y}) \right\rangle$$

Where $V(\mathbf{x})$ is a fundamental representation Wilson line in the target color field

The rapidity (or x) evolution of S is governed by the Balitsky-Kovchegov (BK) equation:

$$\frac{\partial S(\mathbf{x}-\mathbf{y},x)}{\partial \ln x} = \frac{\bar{\alpha}_s}{2\pi} \int \mathrm{d}^2\mathbf{z}\, \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2} \left[S(\mathbf{x}-\mathbf{y},x) - S(\mathbf{x}-\mathbf{z},x)S(\mathbf{z}-\mathbf{y},x)\right]$$

Solving numerically this equation, we can evolve perturbatively S down to lower values of x

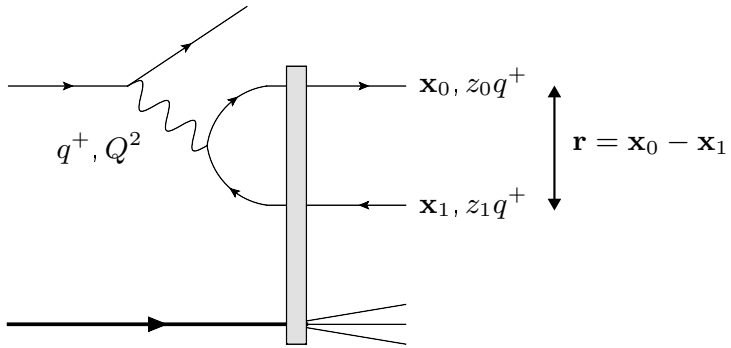
For this we need an initial condition $S(\mathbf{r},x_0)$ to start the evolution

The initial condition involves non-perturbative dynamics. It can be extracted by a fit to data

Initial condition for BK evolution

Process generally used to fit the initial condition: deep inelastic scattering

DIS in the dipole picture:



Structure functions F_L and F_T:

$$F_{L,T}(x, Q^2) = \frac{Q^2}{4\pi^2 \alpha_{em}} \sigma_{L,T}(x, Q^2)$$

with e.g.

$$\sigma_L(x, Q^2) = \frac{4 N_c \alpha_{em}}{\pi^2} \frac{\sigma_0}{2} \sum_f e_f^2 \int \mathrm{d}z_1 \mathrm{d}^2\mathbf{r}\, Q^2 z_1^2 (1-z_1)^2 K_0^2 \left(Q \sqrt{z_1(1-z_1)\mathbf{r}^2} \right) (1 - S(\mathbf{r}, x))$$

$\frac{\sigma_0}{2}$: normalization parameter (transverse area of the proton)

Initial condition for BK evolution

Possible parametrization for the initial condition of a proton target:

$$S(\mathbf{r}, x_0) = \exp\left[-\frac{(\mathbf{r}^2 Q_{\mathbf{s},\mathbf{0}}^2)^\gamma}{4} \ln\left(\frac{1}{|\mathbf{r}|\Lambda_{\mathbf{QCD}}} + e_c \cdot e\right)\right]$$

($\gamma = 1$, $e_c = 1$: original McLerran-Venugopalan model)

Example of fit to HERA DIS data in the region $Q^2 < 50$ GeV^2 and $x < 0.01$ (Lappi, Mäntysaari):

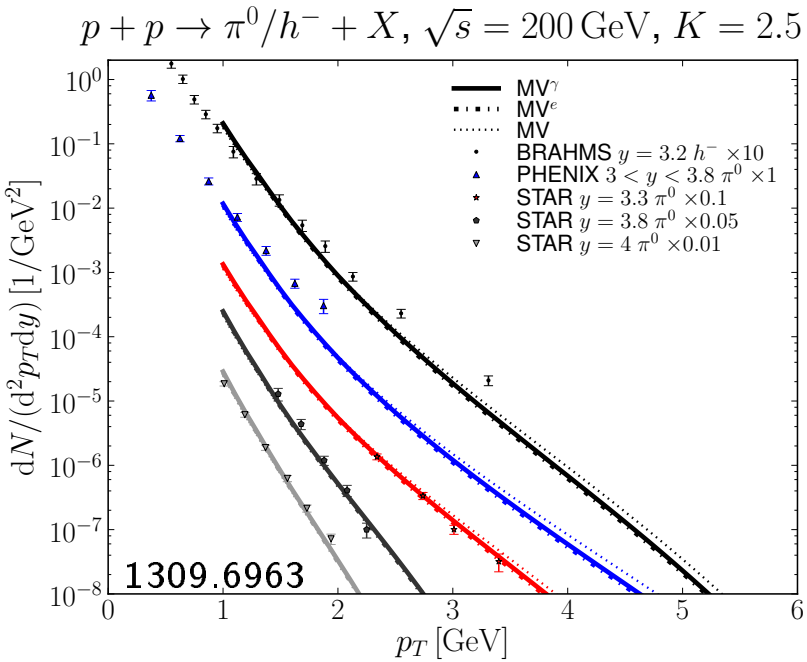
Model	χ^2/d.o.f	$Q_{\mathrm{s},0}^2$ [GeV^2]	γ	e_c	$\sigma_0/2$ [mb]
MV	2.76	0.104	1	1	18.81
MV$^\gamma$	1.17	0.165	1.135	1	16.45
MV^e	1.15	0.060	1	18.9	16.36

Quite good description of the data

There is no such precise data for nuclear targets. Initial condition for a nucleus: need some modeling (e.g. Glauber model)

Forward hadron production at LO

Using the obtained dipole correlators to compute forward hadron production (Lappi, Mäntysaari):



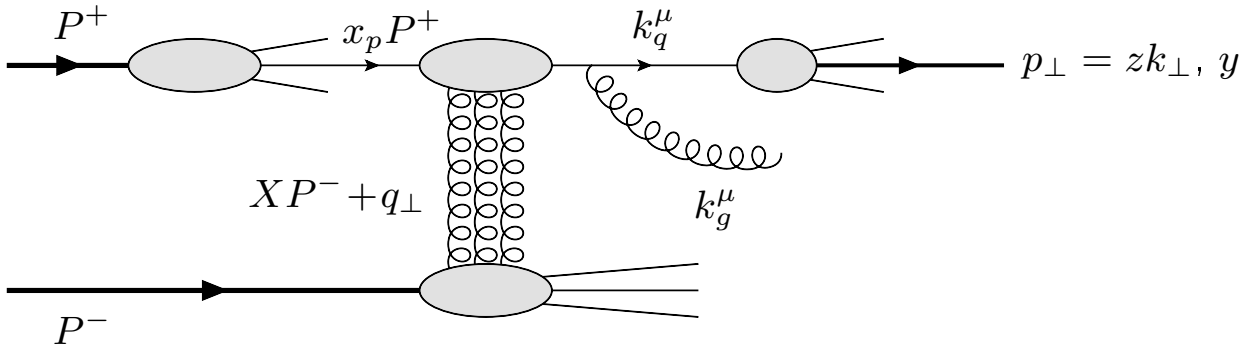
Reasonable description of the trend of the data (but large K factor needed)

This is only leading order. What about NLO corrections?

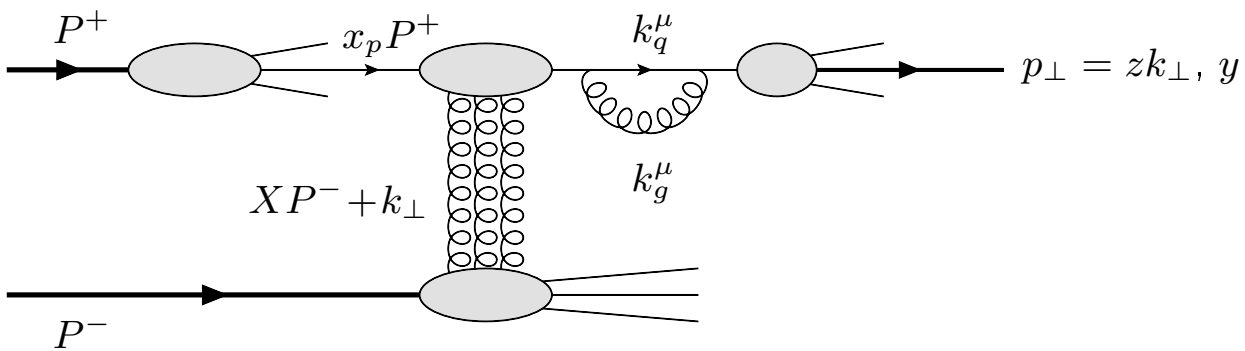
Forward hadron production at NLO

NLO corrections to the impact factor for this process: Chirilli, Xiao, Yuan

Example of real $q \to q$ contribution:

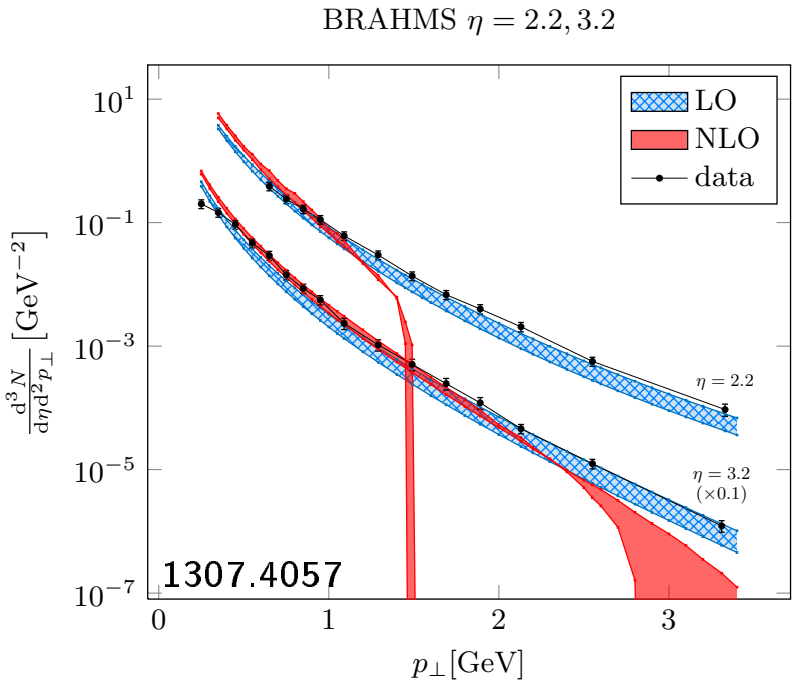


Example of virtual $q \to q$ contribution:



$1 - \xi = \frac{k_g^+}{x_p P^+}$ is the momentum fraction of the incoming quark carried by the gluon

First numerical implementation of these expressions (Staśto, Xiao, Zaslavsky):



Negative cross section above some $p_\perp \sim Q_s$

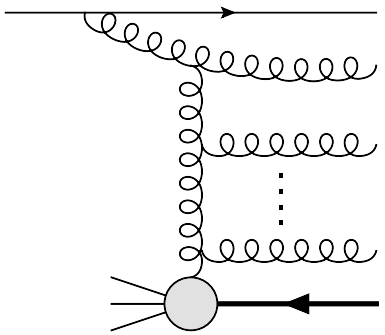
Many works devoted to solving this issue, using for example the kinematical constraint / Ioffe time cutoff (Altinoluk, Armesto, Beuf, Kovner, Lublinsky). Numerical implementation: Watanabe, Xiao, Yuan, Zaslavsky. Can extend the positivity range but doesn't solve the problem completely.

It turned out that the cause of the negativity is the subtraction of the LO contribution from the NLO corrections

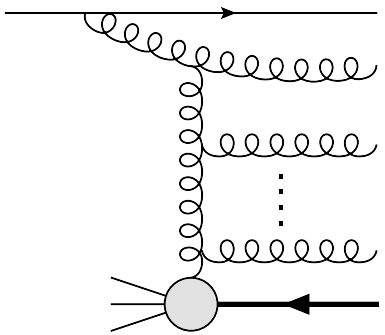
LO vs. NLO

Balitsky-Kovchegov (BK) evolution: resummation of $(\alpha_s \ln 1/x)^n$, corresponding to any number of soft gluons already at LO

LO: all gluons are soft:



NLO impact factor: the first gluon can be hard:

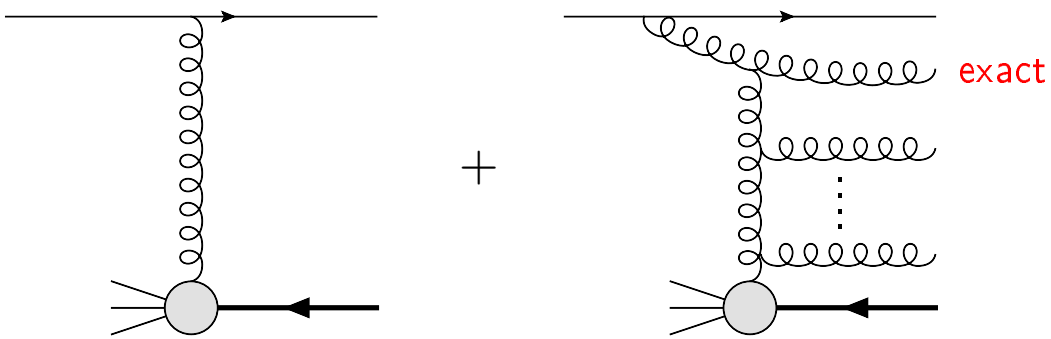


The case where the first gluon is soft is already included in the leading order
$\Rightarrow$ Need to avoid double counting between LO and NLO

Two possible solutions to avoid double counting:

1) Subtract the case where the gluon in the NLO impact factor is soft
Chirilli, Xiao, Yuan ('CXY')

2) Rearrange the terms to avoid doing a subtraction. The expression for the cross section is explicitly positive
Iancu, Mueller, Triantafyllopoulos



These two choices should be equivalent

The expression for the (quark production) multiplicity at NLO reads

$$\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{k}\mathrm{d}y} = x_p q(x_p)\frac{\mathcal{S}(k_\perp, x_0)}{(2\pi)^2} \quad \leftarrow \text{No emission}$$

$$+\frac{\alpha_s}{2\pi^2}\int_{x_p}^{\xi_{\max}} \mathrm{d}\xi \frac{1+\xi^2}{1-\xi}\frac{x_p}{\xi} q\left(\frac{x_p}{\xi}\right)\left\{C_\mathrm{F}\mathcal{I}(k_\perp, \xi, X(\xi)) + \frac{N_\mathrm{c}}{2}\mathcal{J}(k_\perp, \xi, X(\xi))\right\} \quad \leftarrow \text{real}$$

$$-\frac{\alpha_s}{2\pi^2}\int_{0}^{\xi_{\max}} \mathrm{d}\xi \frac{1+\xi^2}{1-\xi} x_p q\left(x_p\right)\left\{C_\mathrm{F}\mathcal{I}_v(k_\perp, \xi, X(\xi)) + \frac{N_\mathrm{c}}{2}\mathcal{J}_v(k_\perp, \xi, X(\xi))\right\} \quad \leftarrow \text{virtual}$$

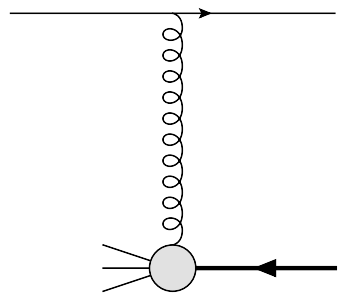
with e.g.

$$\mathcal{J}(k_\perp, \xi, X(\xi)) = \int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{2(\mathbf{k}-\xi\mathbf{q})\cdot(\mathbf{k}-\mathbf{q})}{(\mathbf{k}-\xi\mathbf{q})^2(\mathbf{k}-\mathbf{q})^2}\mathcal{S}(q_\perp, X(\xi))$$
$$-\int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{l}}{(2\pi)^2}\frac{2(\mathbf{k}-\xi\mathbf{q})\cdot(\mathbf{k}-\mathbf{l})}{(\mathbf{k}-\xi\mathbf{q})^2(\mathbf{k}-\mathbf{l})^2}\mathcal{S}(q_\perp, X(\xi))\mathcal{S}(l_\perp, X(\xi))$$

$$\mathcal{J}_v(k_\perp, \xi, X(\xi)) = \int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{2(\xi\mathbf{k}-\mathbf{q})\cdot(\mathbf{k}-\mathbf{q})}{(\xi\mathbf{k}-\mathbf{q})^2(\mathbf{k}-\mathbf{q})^2}\mathcal{S}(k_\perp, X(\xi))$$
$$-\int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{l}}{(2\pi)^2}\frac{2(\xi\mathbf{k}-\mathbf{q})\cdot(\mathbf{l}-\mathbf{q})}{(\xi\mathbf{k}-\mathbf{q})^2(\mathbf{l}-\mathbf{q})^2}\mathcal{S}(k_\perp, X(\xi))\mathcal{S}(l_\perp, X(\xi))$$

The NLO cross section

In the previous expressions:



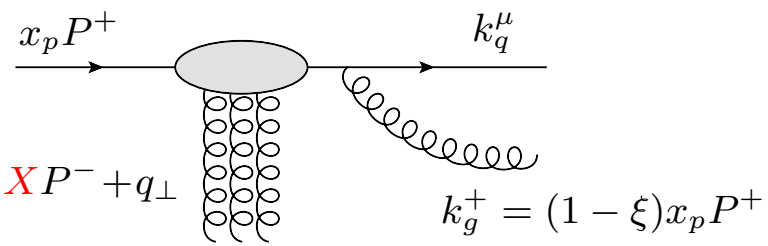
- $x_p q(x_p) \dfrac{\mathcal{S}(k_\perp, x_0)}{(2\pi)^2}$ represents the lowest order contribution

  (no phase space for BK evolution. x_0: initial condition)

- $X(\xi)$ is the rapidity scale at which the dipole correlators are evaluated

  At LO: the P^- fraction needed from the target is $\dfrac{k_\perp}{\sqrt{s}} e^{-y} \equiv x_g$

  At NLO:



$$X = \frac{k_\perp}{\sqrt{s}} e^{-y} \left(1 + \frac{\xi}{1-\xi} \frac{(q_\perp - k_\perp)^2}{k_\perp^2}\right)$$
$$\approx \frac{x_g}{1-\xi} \equiv X(\xi) \text{ when } k_\perp \gtrsim Q_s$$

The limit $\xi < 1 - \frac{x_g}{x_0} \equiv \xi_{\text{max}}$ enforces $X(\xi) < x_0$

The 'CXY' approximation corresponds to making the replacements $X(\xi) \to x_g$ and $\xi_{\text{max}} \to 1$

The terms proportional to C_F are divergent when the additional gluon at NLO is collinear to the initial or final state quark

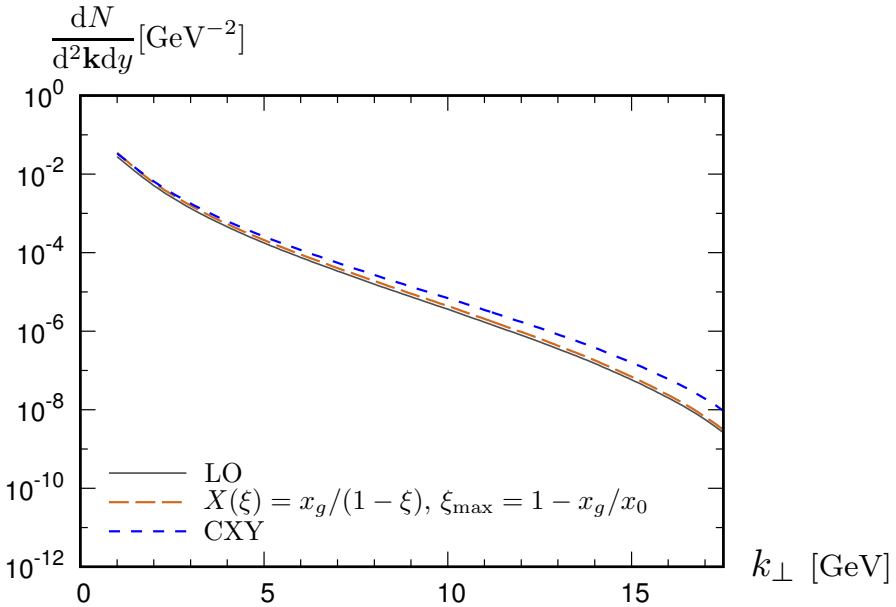
These divergences are absorbed in the DGLAP evolution of the PDFs and fragmentation functions

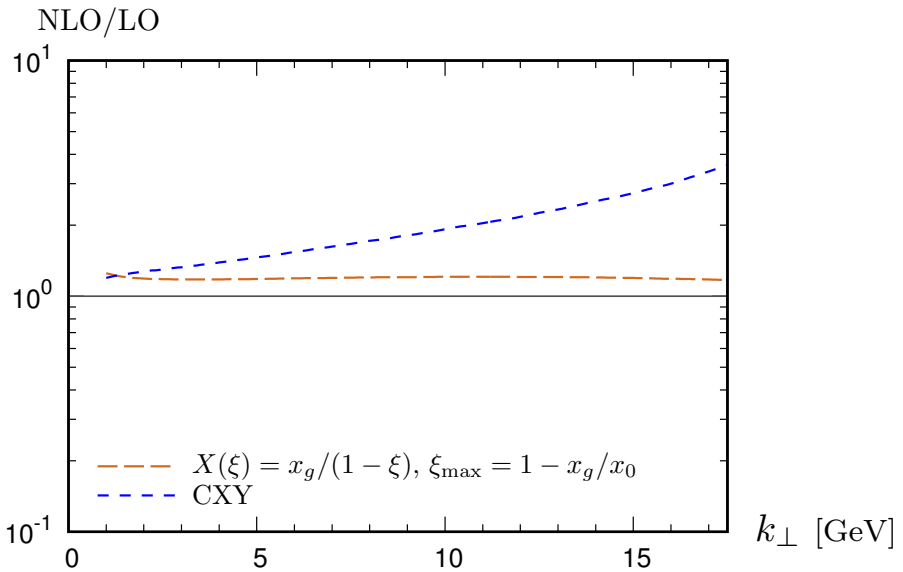
After subtracting the corresponding $1/\varepsilon$ poles, we should replace $\mathcal{I}$ and $\mathcal{I}_v$ by

$$\mathcal{I}^{\text{finite}}(k_\perp,\xi,X(\xi)) = \int \frac{\mathrm{d}^2\mathbf{r}}{4\pi} S(\mathbf{r},X(\xi)) \ln\frac{c_0^2}{\mathbf{r}^2\mu^2}\left(e^{-i\mathbf{k}\cdot\mathbf{r}} + \frac{1}{\xi^2}e^{-i\frac{\mathbf{k}}{\xi}\cdot\mathbf{r}}\right)$$
$$-2\int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{(\mathbf{k}-\xi\mathbf{q})\cdot(\mathbf{k}-\mathbf{q})}{(\mathbf{k}-\xi\mathbf{q})^2(\mathbf{k}-\mathbf{q})^2}\mathcal{S}(q_\perp,X(\xi))$$

$$\mathcal{I}_v^{\text{finite}}(k_\perp,\xi,X(\xi)) = \frac{\mathcal{S}(k_\perp,X(\xi))}{2\pi}\left(\ln\frac{k_\perp^2}{\mu^2} + \ln(1-\xi)^2\right)$$

Results for the LO+C_F NLO corrections at fixed coupling ($\alpha_s = 0.2$):





'CXY' approximation: $X(\xi) \to x_g$ and $\xi_{\max} \to 1$

In both cases the NLO corrections proportional to C_F are positive $\to$ not the cause of the negativity

(Initial condition for the BK evolution at $x_0 = 0.01$: MV model

$S(\mathbf{r}, x_0) = \exp\left[-\frac{\mathbf{r}^2 Q_{s,0}^2}{4} \ln\left(\frac{1}{|\mathbf{r}|\Lambda_{QCD}} + e\right)\right]$, $Q_{s,0}^2 = 0.2$ GeV^2 and $\Lambda_{QCD} = 0.241$ GeV)

We can write the sum of the LO and N_c terms as

$$\frac{\mathrm{d}N^{\mathrm{LO}+N_c}}{\mathrm{d}^2\mathbf{k}\mathrm{d}y} = x_p q(x_p)\frac{\mathcal{S}(k_\perp, x_0)}{(2\pi)^2} + \alpha_s \int_0^{1-x_g/x_0} \frac{\mathrm{d}\xi}{1-\xi}\mathcal{K}(k_\perp, \xi, X(\xi)) \equiv \frac{\mathrm{d}N^{\mathrm{LO}+N_c, unsub}}{\mathrm{d}^2\mathbf{k}\mathrm{d}y},$$

$$\mathcal{K}(k_\perp, \xi, X) = \frac{N_c}{(2\pi)^2}(1+\xi^2)\left[\theta(\xi - x_p)\frac{x_p}{\xi} q\left(\frac{x_p}{\xi}\right)\mathcal{J}(k_\perp, \xi, X) - x_p q\left(x_p\right)\mathcal{J}_v(k_\perp, \xi, X)\right].$$

At large $k_\perp$ the function $\mathcal{K}(k_\perp, \xi, X)$ is positive and so is the cross section.

Using the integral BK equation,

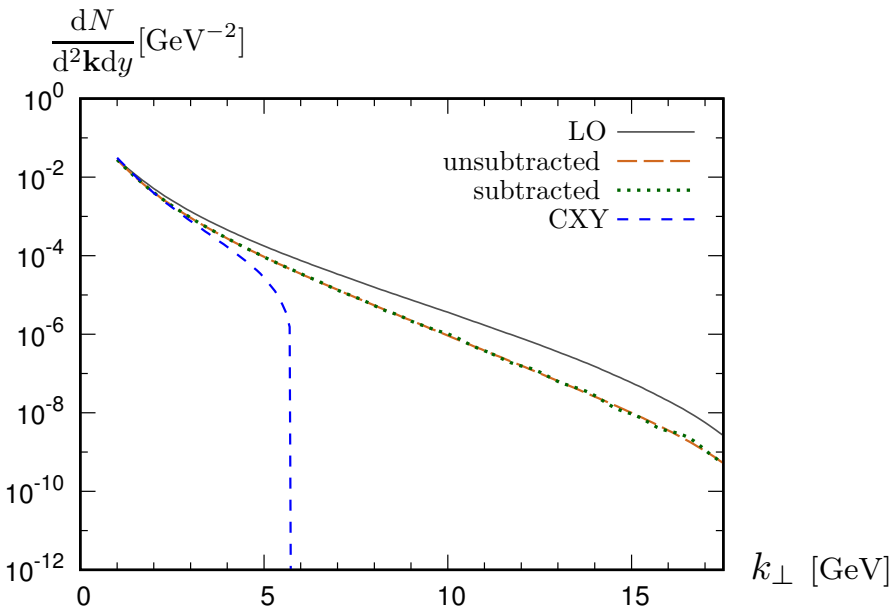
$$\mathcal{S}(k_\perp, x_g) = \mathcal{S}(k_\perp, x_0) + 2\alpha_s N_c \int_0^{1-x_g/x_0} \frac{\mathrm{d}\xi}{1-\xi}\left[\mathcal{J}(k_\perp, 1, X(\xi)) - \mathcal{J}_v(k_\perp, 1, X(\xi))\right],$$

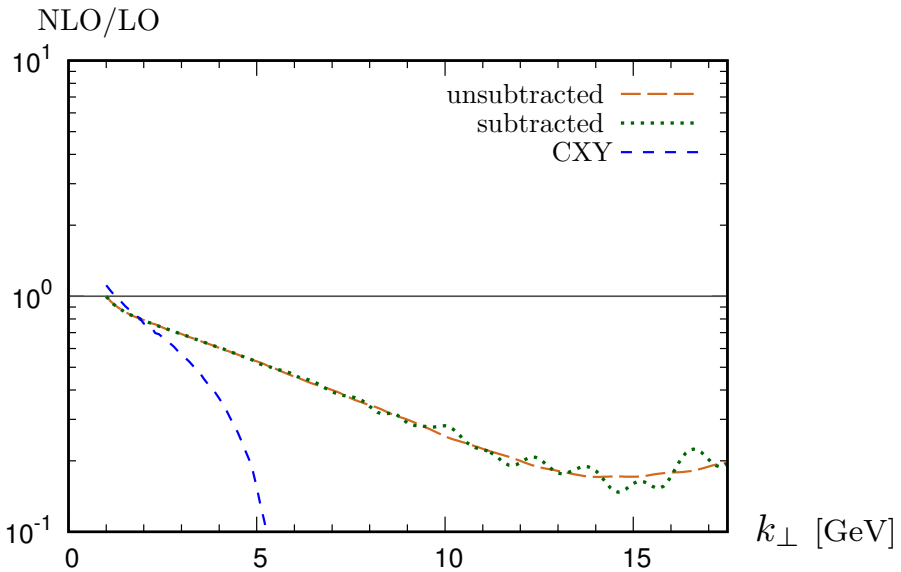
the LO+N_c terms can be rewritten as

$$\frac{\mathrm{d}N^{\mathrm{LO}+N_c, sub}}{\mathrm{d}^2\mathbf{k}\mathrm{d}y} = x_p q(x_p)\frac{\mathcal{S}(k_\perp, x_g)}{(2\pi)^2} + \alpha_s \int_0^{1-x_g/x_0} \frac{\mathrm{d}\xi}{1-\xi}\left[\mathcal{K}(k_\perp, \xi, X(\xi)) - \mathcal{K}(k_\perp, 1, X(\xi))\right].$$

'CXY' approximation: $X(\xi) \to x_g$ and $\xi_{\mathrm{max}} \to 1$ in this subtracted version (not explicitly positive)

Results for the LO+N_c NLO corrections at fixed coupling ($\alpha_s = 0.2$):





The 'subtracted' and 'unsubtracted' expressions give the same positive results

The 'CXY' approximation leads to negative results for $k_\perp \gtrsim 5$ GeV.

Conclusions at fixed coupling

$\Rightarrow$ The negativity issue observed in the first implementation of the NLO impact factor can be attributed to approximations made in the LO subtraction

In the 'subtracted' formulation, we add and subtract a large contribution. If we use the CXY approximation what we add and subtract is no longer the same which can make the final result negative

Some other points still need to be understood. So far we considered only fixed coupling. Additional complications appear when introducing running coupling

The equivalence between the 'subtracted' and 'unsubtracted' formulations holds only if one uses the same coupling α_s when computing the cross section and when solving the BK equation

In practice the BK equation is usually solved in coordinate space, with some prescription for the running coupling

Fixed coupling BK equation:

$$\frac{\partial S(\mathbf{r},X)}{\partial \ln X} = 2\alpha_s N_c \int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2} \frac{\mathbf{r}^2}{\mathbf{x}^2(\mathbf{r}-\mathbf{x})^2} \left[S(\mathbf{r},X) - S(\mathbf{x},X)S(\mathbf{r}-\mathbf{x},X)\right]$$

BK equation with Balitsky's prescription for the running coupling:

$$\begin{aligned}\frac{\partial S(\mathbf{r},X)}{\partial \ln X} = 2\alpha_s(\mathbf{r}^2) N_c \int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2} \left[S(\mathbf{r},X) - S(\mathbf{x},X)S(\mathbf{r}-\mathbf{x},X)\right] \\ \times \left[\frac{\mathbf{r}^2}{\mathbf{x}^2(\mathbf{r}-\mathbf{x})^2} + \frac{1}{\mathbf{x}^2}\left(\frac{\alpha_s(\mathbf{x}^2)}{\alpha_s((\mathbf{r}-\mathbf{x})^2)} - 1\right)\right. \\ \left. + \frac{1}{(\mathbf{r}-\mathbf{x})^2}\left(\frac{\alpha_s((\mathbf{r}-\mathbf{x})^2)}{\alpha_s(\mathbf{x}^2)} - 1\right)\right]\end{aligned}$$

Running coupling

We use dipole correlators obtained by solving the BK equation with the Balitsky prescription for the running coupling. The initial condition is the 'MV^e' parametrization shown previously:

$$S(\mathbf{r}, x_0 = 0.01) = \exp\left[-\frac{\mathbf{r}^2 Q_{\mathbf{s},0}^2}{4} \ln\left(\frac{1}{|\mathbf{r}|\Lambda_{\mathbf{QCD}}} + e_c \cdot e\right)\right],$$

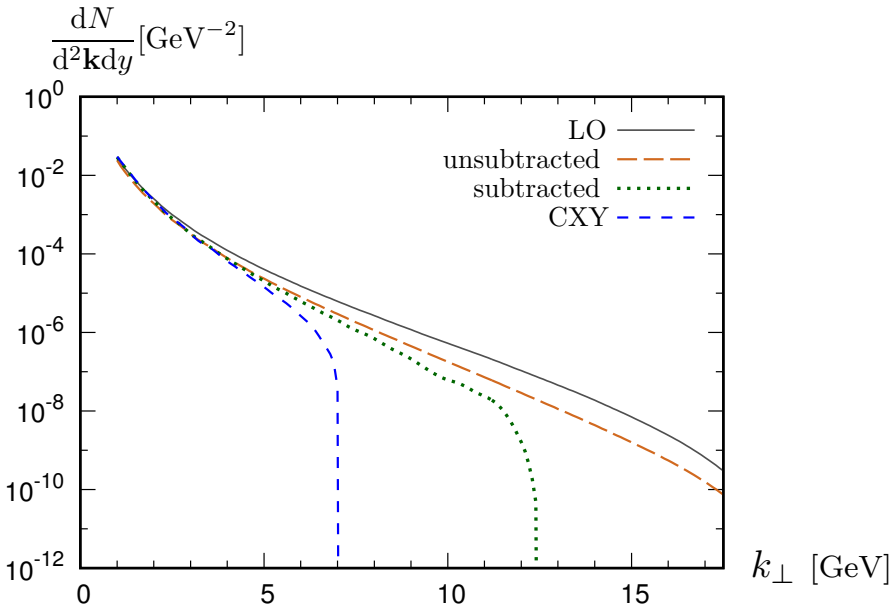
and the running coupling is taken as $\alpha_s(\mathbf{r}^2) = \dfrac{4\pi}{\beta_0 \ln\left(\frac{4C^2}{\mathbf{r}^2\Lambda_{\mathbf{QCD}}^2}\right)}$.

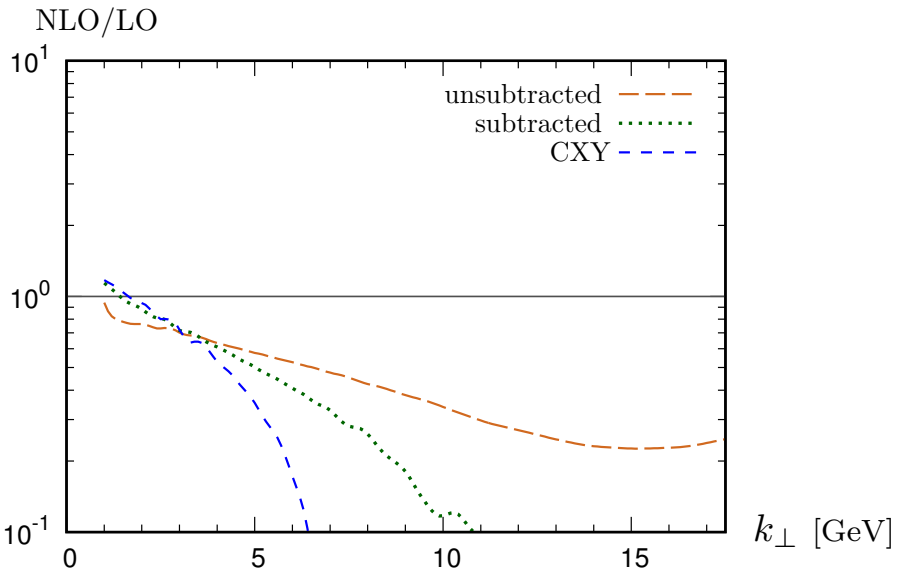
The values $Q_{\mathbf{s},0}^2 = 0.06$ GeV^2, $C^2 = 7.2$ and $e_c = 18.9$ are obtained by a fit to HERA DIS data

We can't use the (coordinate-space) Balitsky prescription using the previously shown momentum-space expressions for the cross section. Here we use the natural choice $\alpha_s(k_\perp^2)$ for the explicit α_s appearing in the cross section

Running coupling

Results with running coupling:





The 'subtracted' and 'unsubtracted' expressions are no longer equivalent

'Subtracted' expression: closer to the 'CXY' result at small $k_\perp$, negative results at large $k_\perp$

'Unubtracted' expression: positive results at all $k_\perp$, but does not reduce to the correct LO result in the eikonal limit

Possible way to use consistently a coordinate-space running coupling: rewrite the cross section expression in coordinate space

We write $\mathcal{J} = \int \mathrm{d}^2\mathbf{r} e^{-i\mathbf{k}\cdot\mathbf{r}} \widetilde{\mathcal{J}}$ and $\mathcal{J}_v = \int \mathrm{d}^2\mathbf{r} e^{-i\mathbf{k}\cdot\mathbf{r}} \widetilde{\mathcal{J}}_v$, with

$$\widetilde{\mathcal{J}}(\mathbf{r},\xi,X) = 2\int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2} \frac{\mathbf{x}\cdot(\mathbf{x}-\mathbf{r})}{\mathbf{x}^2(\mathbf{r}-\mathbf{x})^2} \left[S(\mathbf{r}-(1-\xi)\mathbf{x},X) - S(\xi\mathbf{x},X)S(\mathbf{r}-\mathbf{x},X)\right],$$

$$\widetilde{\mathcal{J}}_v(\mathbf{r},\xi,X) = 2\int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2} \frac{1}{\mathbf{x}^2} \left[S(\mathbf{r}+(1-\xi)\mathbf{x},X) - S(\mathbf{x},X)S(\mathbf{r}-\xi\mathbf{x},X)\right].$$

(and similarly for the C_F terms)

In these notations the BK equation reads

$$\frac{\partial S(\mathbf{r},X)}{\partial \ln X} = -2\alpha_s N_\mathsf{c} \left[\widetilde{\mathcal{J}}(\mathbf{r},1,X) - \widetilde{\mathcal{J}}_v(\mathbf{r},1,X)\right]$$

BK equation with Balitsky's prescription for the running coupling:

$$\frac{\partial S(\mathbf{r},X)}{\partial \ln X} = 2\alpha_s(\mathbf{r}^2)N_c \int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2}\left[S(\mathbf{r},X) - S(\mathbf{x},X)S(\mathbf{r}-\mathbf{x},X)\right] \times \left[\frac{\mathbf{r}^2}{\mathbf{x}^2(\mathbf{r}-\mathbf{x})^2} + \frac{1}{\mathbf{x}^2}\left(\frac{\alpha_s(\mathbf{x}^2)}{\alpha_s((\mathbf{r}-\mathbf{x})^2)} - 1\right) + \frac{1}{(\mathbf{r}-\mathbf{x})^2}\left(\frac{\alpha_s((\mathbf{r}-\mathbf{x})^2)}{\alpha_s(\mathbf{x}^2)} - 1\right)\right]$$

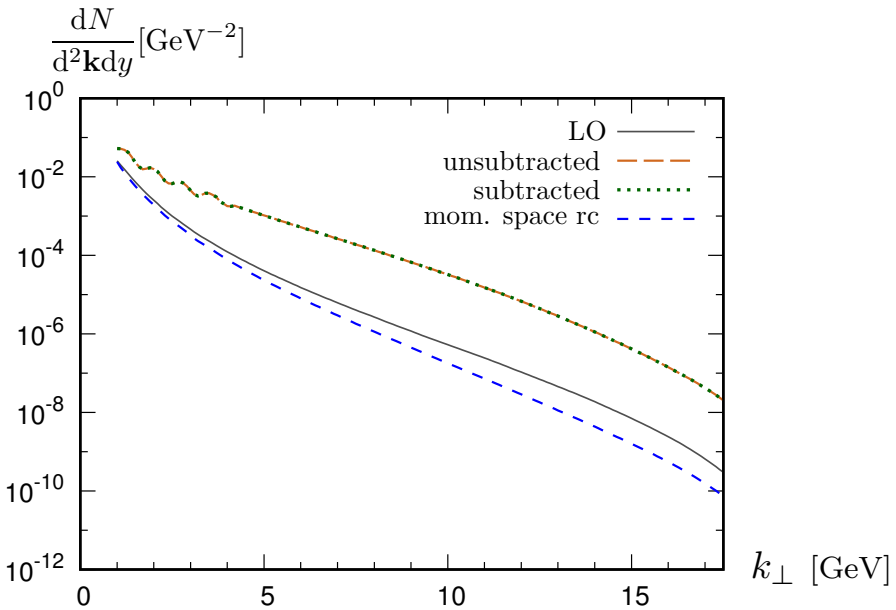
This can be generalized to $\xi \neq 1$ by replacing $\widetilde{\mathcal{J}}_v$ with

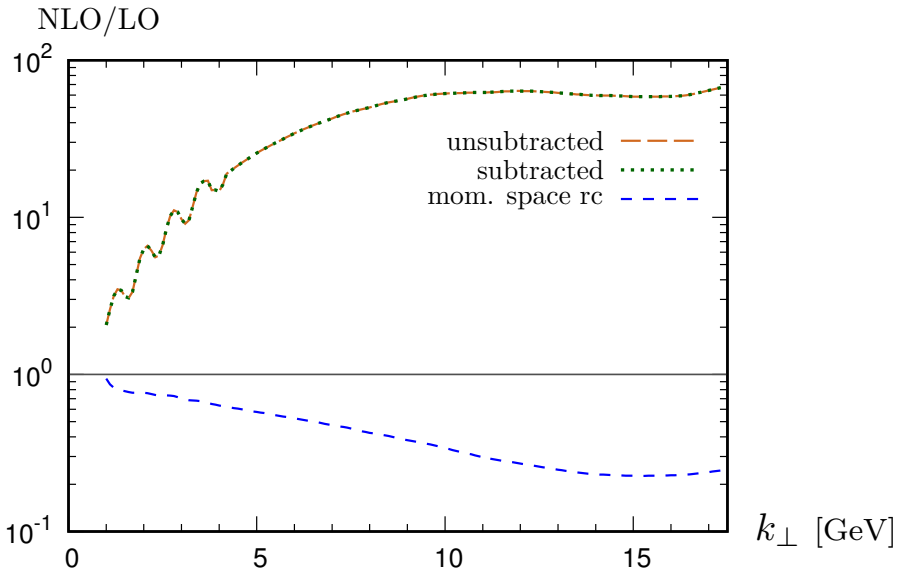
$$\widetilde{\mathcal{J}}_v^{\mathrm{rc}}(\mathbf{r},\xi,X) = 2\int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2}\frac{1}{\mathbf{x}^2}\frac{\alpha_s(\mathbf{x}^2)}{\alpha_s((\mathbf{r}-\xi\mathbf{x})^2)}\left[S(\mathbf{r}+(1-\xi)\mathbf{x},X) - S(\mathbf{x},X)S(\mathbf{r}-\xi\mathbf{x},X)\right],$$

and by replacing the explicit α_s factors by $\alpha_s(\mathbf{r}^2)$. Not a unique choice but:

- $\xi = 1$: recovers Balitsky's prescription
- Fixed coupling results unchanged

Results with this formulation:





The 'subtracted' expression gives the same results as the 'unsubtracted' one

Completely different results compared to fixed coupling or $\alpha_s(k_\perp)$

Similar situation with a simpler parent dipole running coupling $\alpha_s(\mathbf{r}^2)$
$\rightarrow$ the problem is more general than just the Balitsky prescription

Coordinate space formulation

To illustrate the problem, let's look at the following quantities:

$$\mathcal{N}_k \equiv \bar{\alpha}_s(k_\perp)\,\mathcal{S}(k_\perp) = \bar{\alpha}_s(k_\perp)\int \mathrm{d}^2\mathbf{r}\, e^{-i\mathbf{k}\cdot\mathbf{r}} S(\mathbf{r})$$

$$\mathcal{N}_r \equiv \int \mathrm{d}^2\mathbf{r}\, \bar{\alpha}_s(r_\perp) e^{-i\mathbf{k}\cdot\mathbf{r}} S(\mathbf{r})$$

These two quantities do not differ by only a small factor. Indeed, one finds at large $k_\perp$:

$$\mathcal{N}_k \sim \frac{4\pi\bar{\alpha}_s(k_\perp)Q_s^2}{k_\perp^4}$$

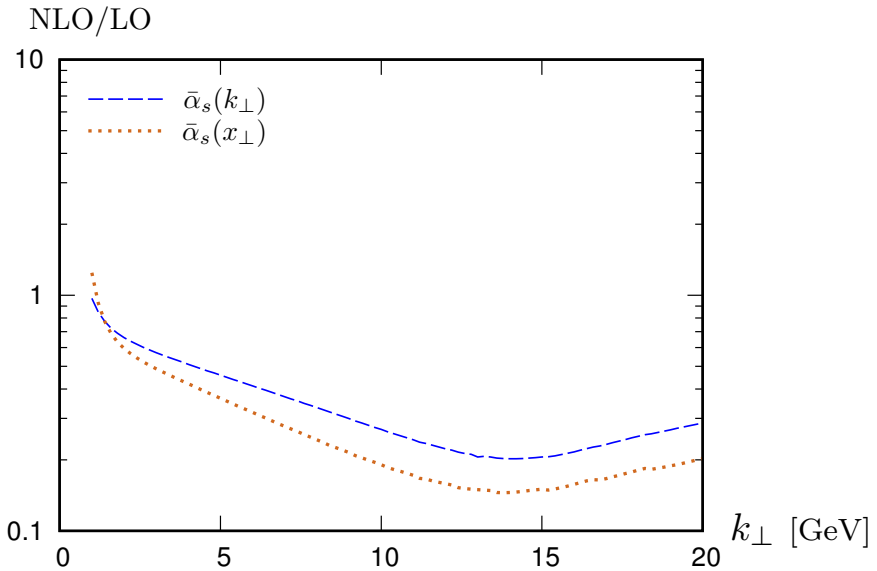
$$\mathcal{N}_r \sim -\frac{4\pi}{\bar{b}[\ln(k_\perp^2/\Lambda^2)]^2}\,\frac{1}{k_\perp^2}$$

Which are opposite in sign and have different tails

The choice of the running coupling prescription and the Fourier transform do not 'commute'

Coordinate space formulation

Possible way to alleviate the issue (at least for the N_c terms): use daughter dipole prescription $\alpha_s(\mathbf{x}^2)$ both when solving BK and in the cross section



Results similar to using $\alpha_s(k_\perp)$

But not natural to use this prescription when solving BK

And can't be used for the C_F terms: collinear divergence subtracted in momentum space $\rightarrow$ impossible to use consistently a coordinate space α_s

A fully consistent treatment would probably require to perform the whole calculation in momentum space

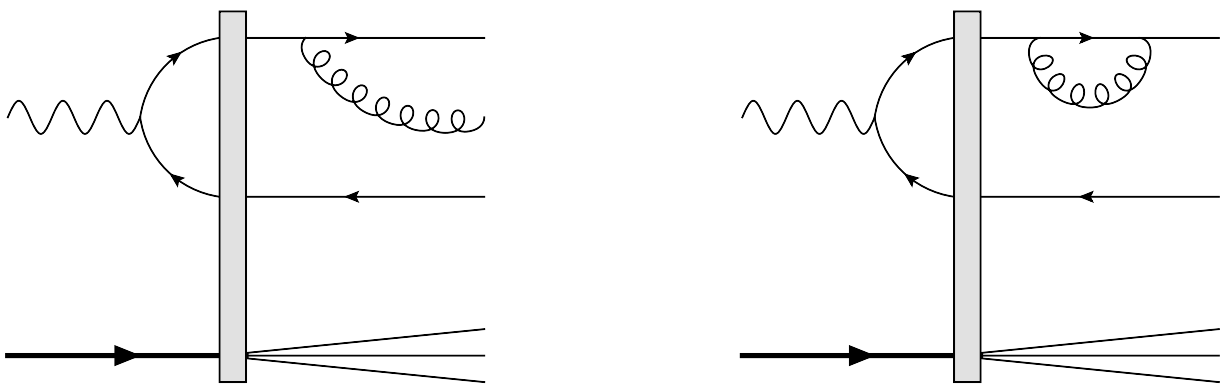
DIS at NLO

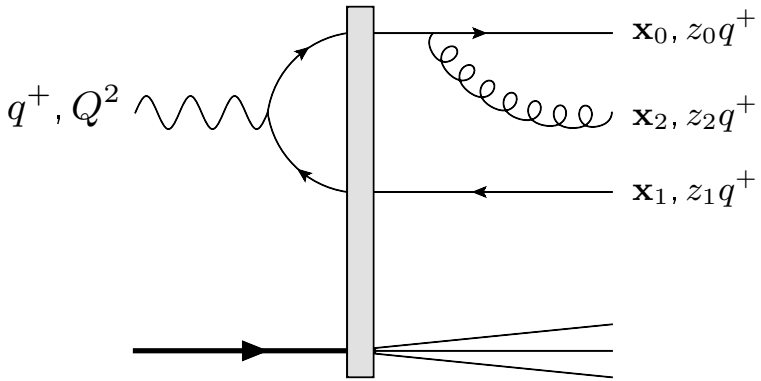
As explained previously the initial condition for the BK evolution can be obtained by a fit to DIS data

We observed rather large NLO corrections to forward hadron production $\rightarrow$ no reason to expect that they are small for DIS

For consistency, DIS fits should also include these corrections

We use the NLO expressions for $F_{L,T}$ derived recently (Beuf)





we can write $\sigma_L^{\mathsf{NLO}} = \sigma_L^{\mathsf{IC}} + \sigma_L^{qg} + \sigma_L^{\mathsf{dip}}$, with

$$\sigma_L^{\mathsf{IC}} = 4N_{\mathsf{c}}\alpha_{em}\sum_f e_f^2 \int \mathrm{d}z_1 \int_{\mathbf{x}_0,\mathbf{x}_1} \mathcal{K}_L^{\mathsf{LO}}(z_1, \mathbf{x}_0, \mathbf{x}_1, x_0)$$

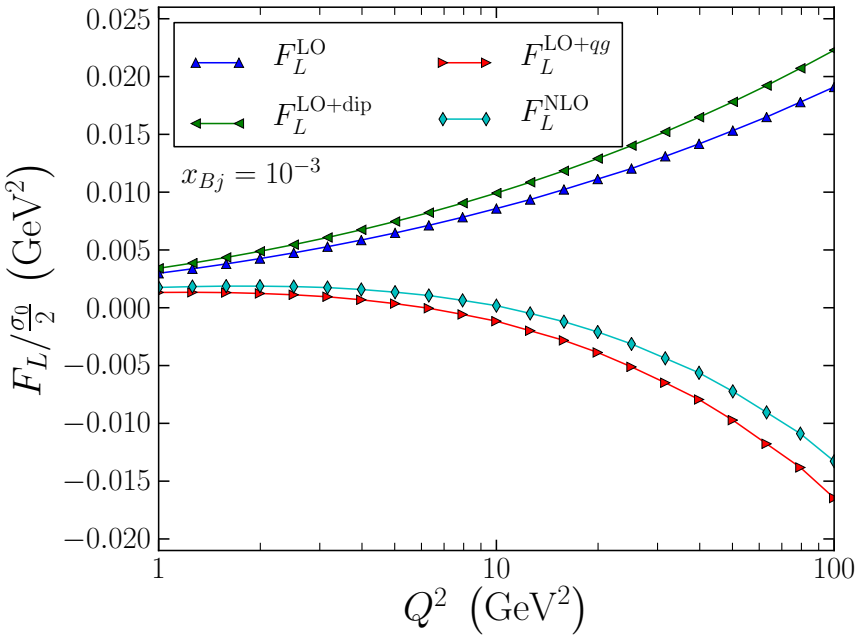
$$\sigma_L^{qg} = 8N_{\mathsf{c}}\alpha_{em}\frac{\alpha_s C_{\mathsf{F}}}{\pi}\sum_f e_f^2 \int \mathrm{d}z_1 \frac{\mathrm{d}z_2}{z_2} \int_{\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \mathcal{K}_L^{\mathsf{NLO}}(z_1, z_2, \mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, X(z_2))$$

$$\sigma_L^{\mathsf{dip}} = 4N_{\mathsf{c}}\alpha_{em}\frac{\alpha_s C_{\mathsf{F}}}{\pi}\sum_f e_f^2 \int \mathrm{d}z_1 \int_{\mathbf{x}_0,\mathbf{x}_1} \mathcal{K}_L^{\mathsf{LO}}(z_1, \mathbf{x}_0, \mathbf{x}_1, X^{\mathsf{dip}}) \left[\frac{1}{2}\ln^2\left(\frac{z_1}{1-z_1}\right) - \frac{\pi^2}{6} + \frac{5}{2}\right]$$

Variables in red: rapidity scale of the dipole correlators

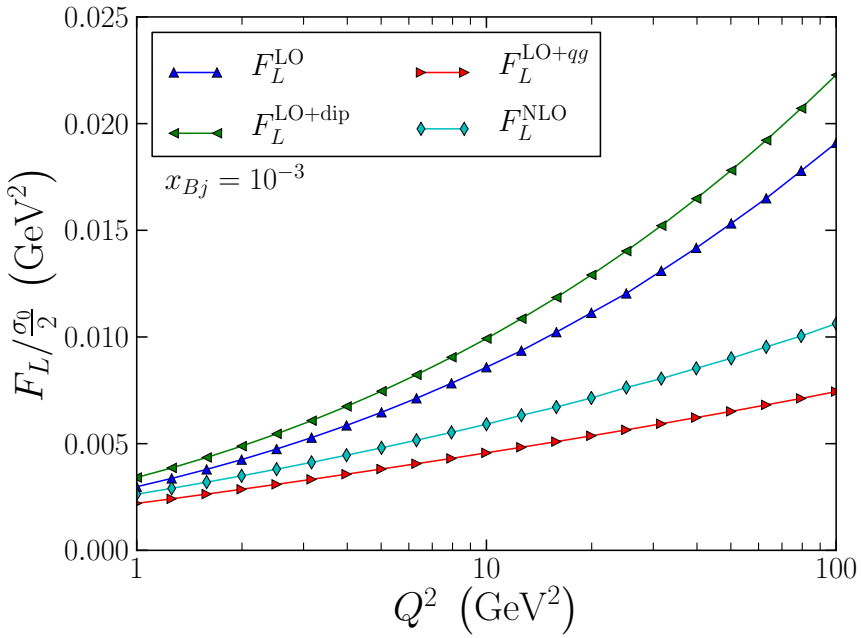
z_2: momentum fraction of the gluon: similar role as $1-\xi$ in hadron production

Using $X(z_2) = x_{Bj}$ (similar to the CXY approximation in forward hadron production):



Negative results at large Q^2

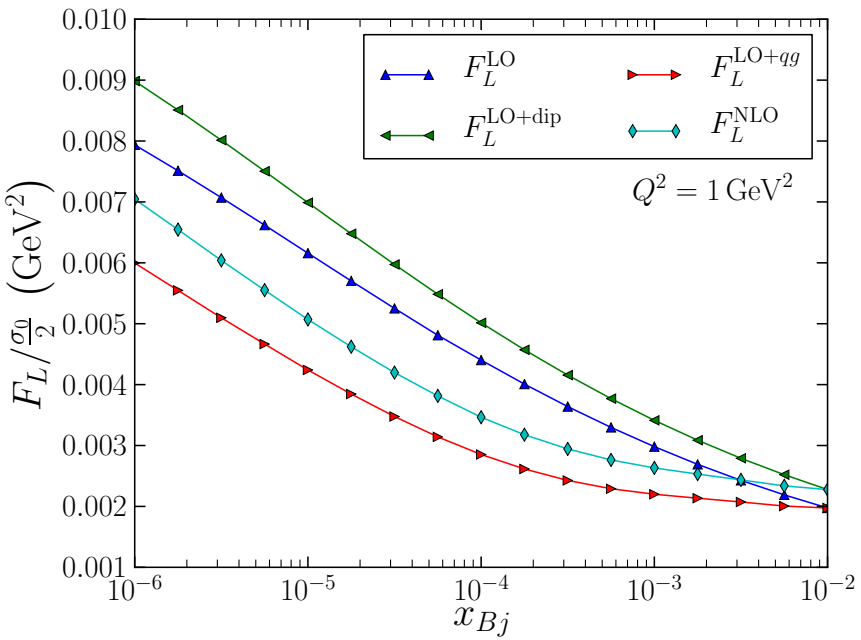
Taking $X(z_2) = x_{Bj}/z_2$:



Positive results at all Q^2, $\sigma^{\rm NLO} < \sigma^{\rm LO}$: very similar situation as in forward hadron production

Technical difficulty: in the expressions we use, σ^{dip} is already integrated over z_2 from 0 to 1 assuming X does not depend on $z_2 \rightarrow$ cannot use $X(z_2) = x_{Bj}/z_2$

Because of that, NLO does not go back to leading order when $x_{Bj} \rightarrow x_0$ (no phase space for evolution):



Not a fully consistent scheme for now

The next step: BK evolution at NLO

Here we discussed only the NLO corrections to the impact factor

A complete NLO calculation must also include the NLO corrections to the BK evolution

NLO BK equation known (Balitsky, Chirilli) and solved numerically (Lappi, Mäntysaari). Large collinear contributions lead to unphysical results

The resummation of these contributions leads to physical results (Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos)

$\Rightarrow$ Almost all the necessary ingredients for complete NLO calculations are here

Conclusions

- Phenomenological studies in saturation so far: LO impact factor, LO BK evolution with running coupling corrections
- First study with NLO impact factor for forward hadron production: unphysical results due to some approximations made. Similar features observed in DIS

  Now understood but some difficulties remain:
  - Forward hadron production: treatment of the running coupling
  - DIS: need to 'undo' the z_2 integral of some terms
- Next steps for phenomenology:
  - Implement the $q \to g$, $g \to q$ and $g \to g$ channels + fragmentation functions in forward hadron production
  - Use collinearly-improved NLO BK for the evolution of dipole correlators
  - NLO fit to DIS data for the initial condition of the BK evolution