High-energy resummation in two heavy-quark pairs production in photon-photon collisions

Dmitry Ivanov

Sobolev Institute of Mathematics

in collaboration:

F. Celiberto, B. Murdaca and A. Papa

University of Calabria

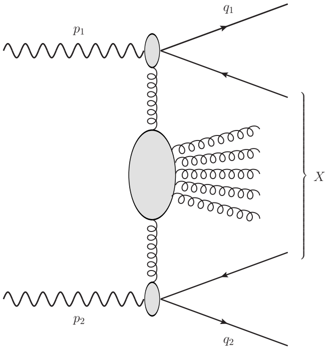
BFKL approach

- BFKL resummation – the basis of our understanding of semihard processes, characterized by a clear hierarchy of scales, $s \gg Q^2 \gg \Lambda^2_{\rm QCD}$.
- [1975] Fadin, Kuraev, Lipatov - LLA resummation ($\sim (\alpha_s \ln(s))^n$) in gauge theories with massive gauge bosons.
- [1978] Balitsky, Lipatov - LLA resummation in QCD. Infrared save predictions for color singlet (Pomeron channel).
  Example: total inclusive cross section of $\gamma\gamma \to Q\bar{Q} + X + Q\bar{Q}$.
- [1998] Fadin, Lipatov - NLA resummation ($\sim \alpha_s(\alpha_s \ln(s))^n$) in QCD
- In most cases - hard scale does not guarantee the dominance of small distances. BFKL is used together with collinear factorization...
  Also I just mention here related with BFKL the dipole approach and its modern developments related with saturation problem.
- my topic: - processes where BFKL can be directly used and confronted with experiment.
  In particular – heavy quark photoproduction $\gamma\gamma \to Q\bar{Q} + X + Q\bar{Q}$

BFKL phenomenology:

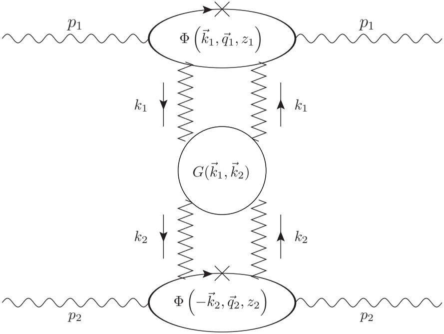
- Total cross section of $\gamma^*(Q_1)\gamma^*(Q_2) \to X$ (NLA BFKL) – e^+e^- experiments at LEP2.
- Mueller-Navelet jets production $pp \to Jet + X + Jet$ (NLA BFKL) – CMS experiment at LHC
- Inclusive di-hadron production $pp \to h_1 + X + h_2$ (NLA BFKL) – proposal for LHC: [2016-2017] our group.
- More jets separated by large rapidity intervals like $pp \to Jet + X + Jet + X + Jet$ – proposal for LHC [2016-2017] Celiberto, Chachamis, Sabio Vera ...
- Why heavy quark photoproduction $\gamma\gamma \to Q\bar{Q} + X + Q\bar{Q}$?
  1. It is exiting to return back to process considered in Balitsky, Lipatov [1978] paper, from which the whole BFKL business was started ...
  2. In order to confront with experiment one needs to consider less inclusive observables

Heavy quark photoproduction



in the case when a heavy quark with transverse momentum q_1 (q_2) from the upper (lower) vertex is tagged (detected).

BFKL cross section



– convolution of the BFKL Green's function and two impact factors. We need impact factors for photoproduction of heavy quark pair when momentum of a quark (an antiquark) is fixed (tagged).

Tagged impact factor

is build from the well known differential amplitude for the pair photoproduction

$$d\phi = \frac{\alpha\alpha_s e_Q^2}{\pi}\left[m^2 R^2 + \vec{P}^2\left(z^2+\bar{z}^2\right)\right] d^2q\ dz\,,$$

where R and $\vec{P}$ read

$$R = \frac{1}{m^2+\vec{q}^{\,2}} - \frac{1}{m^2+(\vec{q}-\vec{k})^2}\,, \qquad \vec{P} = \frac{\vec{q}}{m^2+\vec{q}^{\,2}} + \frac{\vec{k}-\vec{q}}{m^2+(\vec{q}-\vec{k})^2}\,.$$

where $\vec{q}$ ans z are transverse momentum and longitudinal fraction of tagged quark, and $\vec{k}$ – transverse momentum of the Reggeized gluon.

To obtain the tagged quark IF – to make square of this amplitude and to project onto the eigenfunction of LLA BFKL equation, $\sim \left(k^2\right)^{i\nu-3/2} e^{in\vartheta}$. To get its so called (n,ν)-representation.

Tagged impact factor

$$v_{R^2} \equiv \int \frac{d^2k}{\pi\sqrt{2}} \left(k^2\right)^{i\nu-3/2} e^{in\vartheta} R^2$$

$$= \frac{1}{\sqrt{2}} \frac{\Gamma\left(\frac{1}{2}+\frac{n}{2}-i\nu\right)\Gamma\left(\frac{1}{2}+\frac{n}{2}+i\nu\right)\left(\vec{q}^{\,2}\right)^{\frac{n}{2}} e^{in\varphi}\left(\frac{1}{2}+\frac{n}{2}-i\nu\right)}{\Gamma(n+1)\left(m^2+\vec{q}^{\,2}\right)^{\frac{5}{2}+\frac{n}{2}-i\nu}\left(\frac{n}{2}+i\nu-\frac{1}{2}\right)}$$

$$\times\left[\left(\frac{3}{2}+\frac{n}{2}-i\nu\right){}_2F_1\left(\frac{n}{2}-\frac{1}{2}+i\nu,\,\frac{5}{2}+\frac{n}{2}-i\nu,\,1+n,\,\zeta\right)\right.$$

$$\left.-\,2\,{}_2F_1\left(\frac{n}{2}-\frac{1}{2}+i\nu,\,\frac{3}{2}+\frac{n}{2}-i\nu,\,1+n,\,\zeta\right)\right]$$

$$\equiv e^{in\varphi}\, c_{R^2}(n,\nu,\vec{q}^{\,2})$$

and similar equation for the projection of $\vec{P}^2$ structure

Photoproduction differential cross section

$$\frac{d\sigma}{dy_1 dy_2 d|\vec{q}_1| d|\vec{q}_2| d\phi_1 d\phi_2} = \frac{1}{(2\pi)^2} \left[\mathcal{C}_0 + 2 \sum_{n=1}^{\infty} \cos(n\varphi) \mathcal{C}_n \right] ,$$

where $\varphi = \varphi_1 - \varphi_2 - \pi$, while $\mathcal{C}_0$ gives the cross section averaged over the azimuthal angles $\varphi_{1,2}$ of the produced quarks and the other coefficients $\mathcal{C}_n$ determine the distribution of the relative azimuthal angle between the two quarks.

$(q_{1,2} \equiv |\vec{q}_{1,2}|)$:

$$\mathcal{C}_n = \frac{q_1 q_2 \sqrt{m_1^2 + {q_1}^2} \sqrt{m_2^2 + {q_2}^2}}{W^2} e^{\Delta Y}$$

$$\times \int d\nu \left(\frac{W^2}{s_0} \right)^{\bar{\alpha}_s(\mu_R)\chi(n,\nu) + {\bar{\alpha}_s}^2(\mu_R) \left(\bar{\chi}(n,\nu) + \frac{\beta_0}{8N_c} \chi(n,\nu) \left(-\chi(n,\nu) + \frac{10}{3} + 2 \ln \frac{\mu_R^2}{\sqrt{s_1 s_2}} \right) \right)}$$

$$\times \alpha_s^2(\mu_R)\, c_1(n,\nu,{\vec{q}_1}^{\,2},z_1) c_2(n,\nu,{\vec{q}_2}^{\,2},z_2)$$

$$\times \left\{ 1 + \bar{\alpha}_s(\mu_R) \left(\frac{\bar{c}_1^{(1)}}{c_1} + \frac{\bar{c}_2^{(1)}}{c_2} \right) + \bar{\alpha}_s(\mu_R) \frac{\beta_0}{2N_c} \left(\frac{5}{3} + \ln \frac{\mu_R^2}{s_1 s_2} + f(\nu) \right) \right.$$

$$\left. + {\bar{\alpha}_s}^2(\mu_R) \ln \left(\frac{W^2}{s_0} \right) \frac{\beta_0}{4N_c} \chi(n,\nu) f(\nu) \right\} ,$$

e^+e^- cross section

convolution with the WW photon spectrum:

$$d\sigma_{e^+e^-} = dn_1 dn_2 d\sigma_{\gamma\gamma} \ ,$$

with

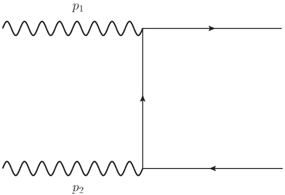
$$dn = \frac{\alpha}{\pi}\frac{dx}{x}\left[\left(1 - x + \frac{x^2}{2}\right)\ln\left(\frac{E_e^2\theta_0^2(1-x)^2 + m_e^2 x^2}{m_e^2 x^2}\right) - (1-x)\right] \ ,$$

θ_0- is antitag electron angle parameter.

$$\frac{d\sigma_{e^+e^-}}{d(\Delta Y)} = \int dq_1 \int dq_2 \int_{-y_{\max}^{(1)}}^{y_{\max}^{(1)}} dy_1 \int_{-y_{\max}^{(2)}}^{y_{\max}^{(2)}} dy_2\, \delta(y_1 - y_2 - \Delta Y)$$

$$\times \int_{e^{-\left(y_{\max}^{(1)} - y_1\right)}}^{1} \frac{dn_1}{dx_1} dx_1 \int_{e^{-\left(y_{\max}^{(2)} + y_2\right)}}^{1} \frac{dn_2}{dx_2} dx_2\, d\sigma_{\gamma\gamma} \ ,$$

The "box" $Q\bar{Q}$ cross section



$$\frac{d\sigma_{ee}}{d(\Delta Y)} = \int_0^{\frac{s_{ee}}{2(1+\cosh(\Delta Y))}-m^2} \frac{dq^2}{(m^2+q^2)^2} \frac{2\pi\alpha^2 e_q^4 N_c}{(1+\cosh(\Delta Y))^2}$$

$$\left[\frac{\cosh(\Delta Y)}{2} + \frac{m^2}{m^2+q^2} - \left(\frac{m^2}{m^2+q^2}\right)^2\right]$$

$$\times\left(\frac{\alpha}{\pi}\right)^2 \left[f(y)\left(\ln\left(\frac{\Lambda^2}{m_e^2 y}\right)-1\right)^2 - \frac{1}{3}\left(\ln\frac{1}{y}\right)^3\right] ,$$

where

$$y = \frac{w^2}{s_{ee}} = \frac{2\left(1+\cosh(\Delta Y)\right)\left(m^2+q^2\right)}{s_{ee}} ,$$

with

$$f(y) = \left(1+\frac{y}{2}\right)^2 \ln\frac{1}{y} - \frac{1}{2}(1-y)(3+y)$$

and $\Lambda \simeq m^2$.

Box vs BFKL

ΔY-dependence of the φ-averaged cross section C_0 [pb] for $q_{\min} = 0$ GeV.

$\sqrt{s} = 200$ GeV.

ΔY	Box $q\bar{q}$	$\mathrm{NLA, C} = 1/2$	$\mathrm{NLA, C} = 1$	$\mathrm{NLA, C} = 2$
1.5	98.26	2.120(13)	1.4046(91)	1.2861(93)
2.5	42.73	2.197(11)	1.1976(71)	1.067(7)
3.5	14.077	2.315(12)	0.9986(54)	0.8296(45)
4.5	3.9497	2.3015(23)	0.7763(39)	0.6116(32)
5.5	0.9862	2.12(1)	0.5411(27)	0.3922(19)

$\sqrt{s} = 3$ TeV.

ΔY	Box $q\bar{q}$	$\mathrm{NLA, C} = 1/2$	$\mathrm{NLAC} = 1$	$\mathrm{NLAC} = 2$
1.5	280.98	12.45(11)	7.292(72)	6.521(73)
3.5	48.93	23.07(14)	8.153(62)	6.798(59)
5.5	4.9819	47.53(23)	9.479(67)	6.903(45)
7.5	0.4318	94.54(44)	10.243(56)	6.435(33)
9.5	0.0323	158.38(76)	9.092(45)	4.858(24)
10.5	0.0081	180.4(9)	7.497(37)	3.651(18)

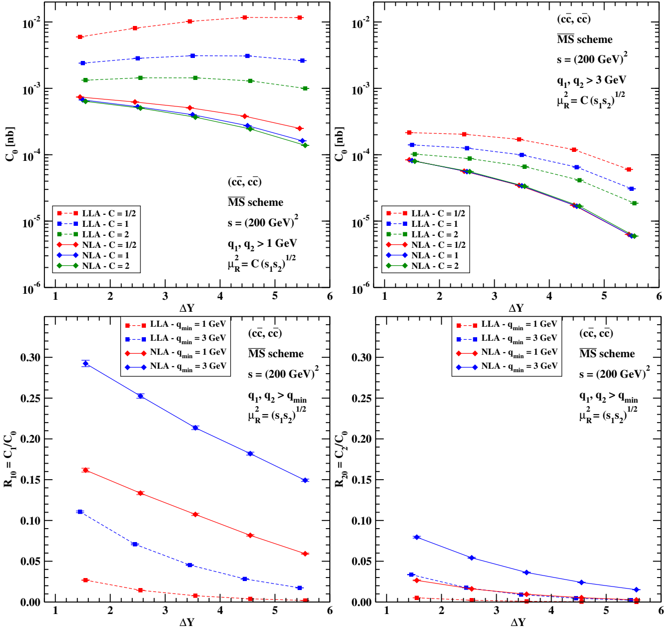


Figure: ΔY-dependence of C_0, R_{10}, and R_{20} for $q_{\min} = 1$, 3 GeV, $\sqrt{s} = 200$ GeV, and for different values of $C = \mu_R^2/\sqrt{s_1 s_2}$, with $s_{1,2} = m_{1,2}^2 + q_{1,2}^2$.

Future e^+e^- collider

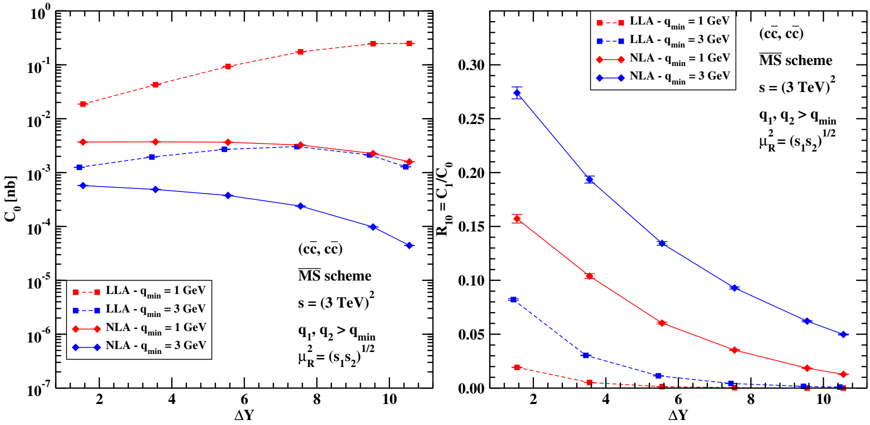


Figure: ΔY-dependence of C_0 and R_{10} for $q_{\min} = 1,\ 3$ GeV, $\sqrt{s} = 3$ TeV, and for $\mu_R^2 = \sqrt{s_1 s_2}$, with $s_{1,2} = m_{1,2}^2 + q_{1,2}^2$.

Summary and outlook

- We performed analysis of inclusive heavy quark photoproduction process where two heavy quarks are detected separated by large rapidity interval.
- This process extends the list of semihard processes by which strong interactions in the high-energy limit, and in particular the BFKL resummation procedure, can be probed at e^+e^- colliders.

possible developments:

- NLA impact factors – Complete NLA predictions
- Treatment of heavy quark fragmentation.
- LHC phenomenology:
  From photoproduction to the processes initiated by the gluons. It opens the direct way to study similar process in proton-proton collision at LHC:
  $pp \to Q\bar{Q} + X + Q\bar{Q}$
- see recent study of
  $pp \to \bar{J}/\Psi + X + Jet$
  by R. Boussarie, B. Ducloué, L. Szymanowski and S. Wallon