HQ mass effects in Monte Carlo generation

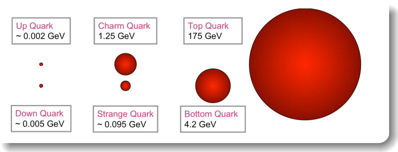
Davide Napoletano, GDR QCD 2017

04/12/2017

Outline

Outline

Introduction



$\Lambda_{QCD} \sim 250$ MeV,
A quark Q is **heavy** $\Leftrightarrow m_Q \gg \Lambda_{QCD}$.

$m_u, m_d, m_s \ll \Lambda_{QCD} \Rightarrow$ light quarks

$m_c > \Lambda_{QCD}$ but not by much!

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$$\Lambda_{QCD} \ll m \ll M(m_W, m_Z, m_H, m_t)$$

- b phenomenology crucially important at the LCH, from flavour physics, to Higgs characterisation and measurements and as window to New Physics.
- From a theoretical viewpoint we need better control on this kind of processes which appear as both BSM signals and SM irreducible backgrounds.
- Important examples: H and Z associated production.

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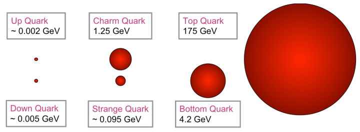

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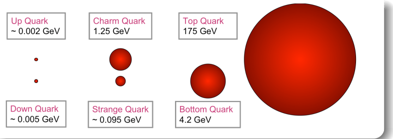

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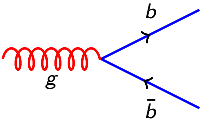

The gluon splitting is the dominant mode of production of b quarks at the LHC

Contribution to σ:

$$\sigma \propto \alpha_S \int_0^{\eta^2} \frac{\mathrm{d}\mu^2}{\mu^2 + m_b^2} \sim \alpha_S \log \frac{\eta^2}{m_b^2}$$

Can take place either in the initial state or in the final state:

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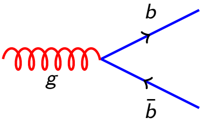

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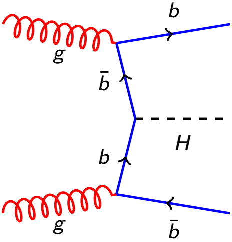

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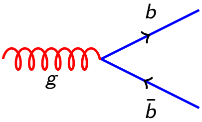

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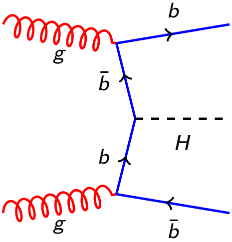

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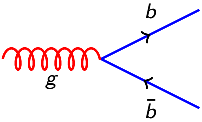

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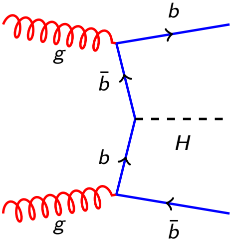

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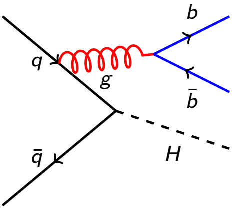

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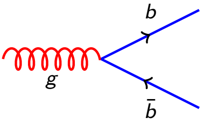

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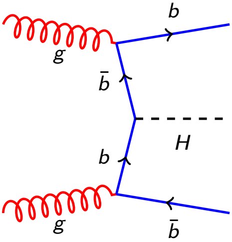

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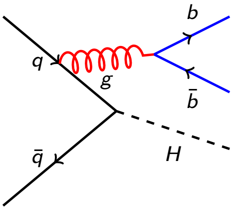

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In the kinematic region $\hat{s} \gg m_b^2$

$$\alpha_S(\hat{s}) \log \frac{\hat{s}}{m_b^2} \sim \mathcal{O}(1)$$

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Solution

5 flavour scheme, re-sum such logs via DGLAP eqs in b-PDF.

$$m_b = 0$$

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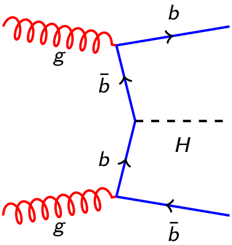

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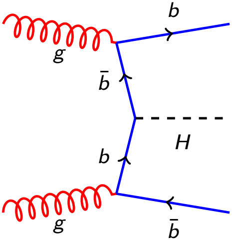

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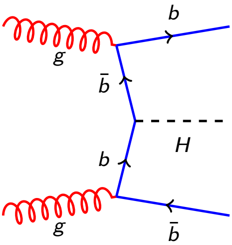

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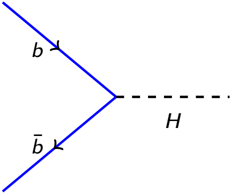

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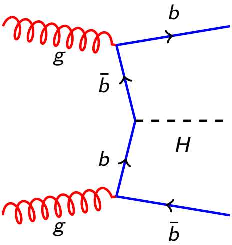

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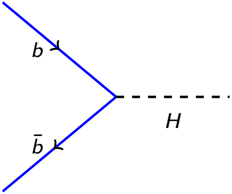

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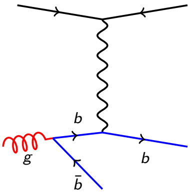
What happened?

When logs are dominant over mass effects we have that:



$\Longrightarrow \quad \alpha_S \log \frac{\eta^2}{m_b^2} \quad \times$

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$$\lim_{m_b^2/\eta^2 \to 0} f_g \otimes \hat{\sigma}_{Xg \to b\bar{b}Y} = \underbrace{\alpha_S \log \frac{\eta^2}{m_b^2} P_{qg} \otimes f_g}_{=\widetilde{b}(x,\mu^2)} \sigma_{Xb \to Y}$$

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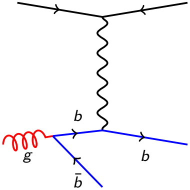

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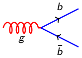

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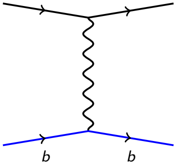

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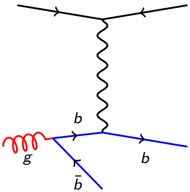

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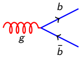

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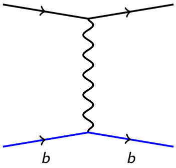

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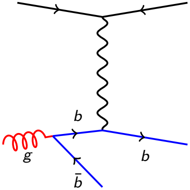

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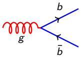

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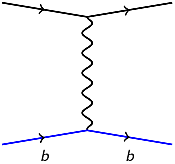

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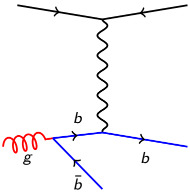

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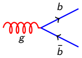

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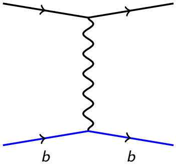

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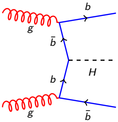
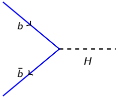
4F versus 5F scheme

4F scheme



- × Doesn't re-sum possibly large logs, but it does have them explicitly
- × Higher orders are computationally more difficult
- ✓ Mass effects present at any order
- ✓ MC@NLO no problem

5F scheme



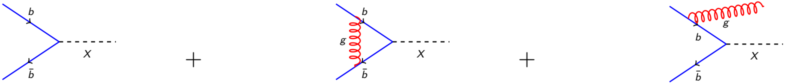
- ✓ Stabler predictions, re-summation of IS large logs into b-PDF
- ✓ Higher order easily accessible
- × p_T of b and mass effects are pushed to higher orders
- × Implementation in MC depends on the $g \to b\bar{b}$ splitting implemented

Outline

Computing NLO observables

To compute a NLO observable we need:

$$\mathrm{d}\sigma = \mathrm{d}\Phi_{\mathcal{B}}\left[\mathcal{B}(\Phi_{\mathcal{B}}) + \mathcal{V}(\Phi_{\mathcal{B}})\right] + \mathrm{d}\Phi_{\mathcal{B}+1}\mathcal{R}(\Phi_{\mathcal{B}+1})$$



- $\mathcal{V}(\Phi_{\mathcal{B}})$ and $\int \mathrm{d}\Phi_{\mathcal{B}+1}\,\mathcal{R}(\Phi_{\mathcal{B}+1})$ are separately soft (and collinear) divergent in $4d$
- $\int \mathrm{d}\Phi_{\mathcal{B}}\,\mathcal{V}(\Phi_{\mathcal{B}}) + \int \mathrm{d}\Phi_{\mathcal{B}+1}\,\mathcal{R}(\Phi_{\mathcal{B}+1})$ is finite!
- Need method to render the integrand finite for MC integration!
  $\Longrightarrow$ **Catani-Seymour Dipole formalism**.

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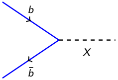

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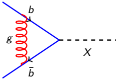

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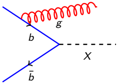
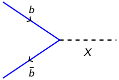


- $\mathcal{V}(\Phi_{\mathcal{B}})$ and $\int d\Phi_{\mathcal{B}+1}\, \mathcal{R}(\Phi_{\mathcal{B}+1})$ are separately soft (and collinear) divergent in $4d$
- $\int d\Phi_{\mathcal{B}}\, \mathcal{V}(\Phi_{\mathcal{B}}) + \int d\Phi_{\mathcal{B}+1}\, \mathcal{R}(\Phi_{\mathcal{B}+1})$ is finite!
- Need method to render the integrand finite for MC integration!
  $\Longrightarrow$ **Catani-Seymour Dipole formalism**.

Computing NLO observables

To compute a NLO observable we need:

$$\mathrm{d}\sigma = \mathrm{d}\Phi_{\mathcal{B}} \left[\mathcal{B}(\Phi_{\mathcal{B}}) + \mathcal{V}(\Phi_{\mathcal{B}}) \right] + \mathrm{d}\Phi_{\mathcal{B}+1} \mathcal{R}(\Phi_{\mathcal{B}+1})$$



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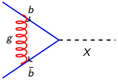

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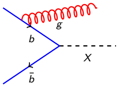

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Catani-Seymour Dipoles

CS-Dipoles

Exploit universal structure of **soft-** and **collinear- singularities** $\Rightarrow$ in these limits:

$$|\mathcal{M}(\{p_n\}, p_k)|^2 \sim \sum_{ijk} \mathcal{D}_{ijk} = \mathcal{S}$$

$$\mathcal{D}_{ijk} \propto V_{ij,k}(\{p_n\}, p_k) \otimes |\mathcal{M}(\{\widetilde{p}_n\})|^2$$

If we also use this to factorise the PS $\Rightarrow$ $\mathrm{d}\Phi_{\mathcal{B}+1} = \mathrm{d}\widetilde{\Phi}_{\mathcal{B}} \otimes \mathrm{d}\Phi_1$ we can write:

$$\mathrm{d}\sigma = \mathrm{d}\Phi_{\mathcal{B}}\left[\mathcal{B}(\Phi_{\mathcal{B}}) + \mathcal{V}(\Phi_{\mathcal{B}}) + \mathcal{I}(\Phi_{\mathcal{B}})\right] + \mathrm{d}\Phi_{\mathcal{B}+1}\left[\mathcal{R}(\Phi_{\mathcal{B}+1}) - \mathcal{S}(\Phi_{\mathcal{B}} \otimes \Phi_1)\right]$$

$$\mathcal{I}(\Phi_{\mathcal{B}}) = \int \mathrm{d}\Phi_1 \mathcal{S}(\Phi_{\mathcal{B}} \otimes \Phi_1)$$

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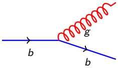
Dressing partons



$$\Rightarrow \quad d\sigma \propto \sigma_0 \int_{Q_0^2}^{Q^2} \frac{dt}{t} \propto \alpha_S \log \frac{Q^2}{Q_0^2}$$

$\Rightarrow$ One additional emission

$$\Rightarrow \quad d\sigma \propto \sigma_0 \int_{Q_0^2}^{Q^2} \frac{dt_1}{t_1} \cdots \int_{Q_0^2}^{t_{n-1}} \frac{dt_n}{t_n} \propto \alpha_S^n \log^n \frac{Q^2}{Q_0^2}$$

$\Rightarrow$ Many sub-sequential emissions, with $t_1 > t_2 > \cdots > t_n$

Sudakov Form-Factor exponentiate these logs (DGLAP equations):

$$\Delta(Q_0^2, Q^2) = \exp\left[-\int_{Q_0^2}^{Q^2} \frac{dt}{t} \int dz\, \alpha_S(t(z))\, P_{ab}(z)\right] \sim \exp\left[-C_F \alpha_S \log^2 \frac{Q^2}{Q_0^2}\right]$$

$\Rightarrow$ No emission probability!

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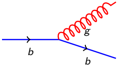

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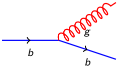

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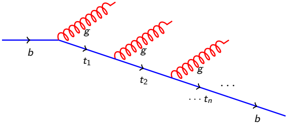

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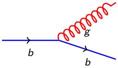

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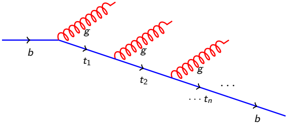

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$\Rightarrow$ **No emission probability!**

Matching to the Fixed Order

Leading-Order

At LO, we start with the $\mathcal{B}$ cross section:

$$\mathrm{d}\sigma^{(\mathrm{Born})} = \mathrm{d}\Phi_{\mathcal{B}}\mathcal{B}(\Phi_{\mathcal{B}}) \underbrace{\left\{ \Delta(Q_0^2, Q^2) + \int_{Q^2}^{Q_0^2} \mathrm{d}\Phi_1 \left[\mathcal{K}(\Phi_1)\Delta\left(Q_0^2, t(\Phi_1)\right)\right] \right\}}_{\text{Unitarity of the PS}}$$

$\mathcal{K}(\Phi_1) = \int \mathrm{d}z \alpha_S\left(t(z)\right) P_{ab}(z)$

- Note that $\mathcal{R}(\Phi_{\mathcal{B}} \otimes \Phi_1) \leq \mathcal{B}(\Phi_{\mathcal{B}}) \otimes \mathcal{K}(\Phi_1)$
- introduce $\widetilde{\mathcal{K}}(\Phi_1) = \mathcal{R}(\Phi_{\mathcal{B}} \otimes \Phi_1)/\mathcal{B}(\Phi_{\mathcal{B}})$ thus:

$$\mathrm{d}\sigma^{(\mathrm{Born})} = \mathrm{d}\Phi_{\mathcal{B}}\mathcal{B} \left\{ \widetilde{\Delta}(Q_0^2, Q^2) + \int_{Q^2}^{Q_0^2} \mathrm{d}\Phi_1 \left[\widetilde{\mathcal{K}}(\Phi_1)\widetilde{\Delta}\left(Q_0^2, t(\Phi_1)\right)\right] \right\}$$

$$\widetilde{\Delta}(Q_0^2, Q^2) = \exp\left[-\int_{Q_0^2}^{Q^2} \mathrm{d}\Phi_1 \widetilde{\mathcal{K}}(\Phi_1) \right]$$

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Matching to the FO (NLO)

Going MC@NLO

Can we do even better? First recall Catani-Seymour:

$$d\sigma^{(\mathrm{NLO})} = d\Phi_{\mathcal{B}}\widetilde{\mathcal{B}}(\Phi_{\mathcal{B}}) + d\Phi_{\mathcal{B}+1}\left[\mathcal{R}(\Phi_{\mathcal{B}+1}) - \mathcal{S}(\Phi_{\mathcal{B}}\otimes\Phi_1)\right]$$

$$\widetilde{\mathcal{B}}(\Phi_{\mathcal{B}}) = \mathcal{B}(\Phi_{\mathcal{B}}) + \mathcal{V}(\Phi_{\mathcal{B}}) + \mathcal{I}(\Phi_{\mathcal{B}})$$

- Expand $\mathcal{R}$, recalling that $\mathcal{R}(\Phi_{\mathcal{B}+1}) = \mathcal{R}^{(S)}(\Phi_{\mathcal{B}}\otimes\Phi_1) + \mathcal{R}^{(H)}(\Phi_{\mathcal{B}+1}) = \mathcal{S}(\Phi_{\mathcal{B}}\otimes\Phi_1) + \mathcal{H}(\Phi_{\mathcal{B}+1})$
- (Symbolically): $\mathcal{S}(\Phi_{\mathcal{B}}\otimes\Phi_1) = \sum_{ijk}\mathcal{B}(\Phi_{\mathcal{B}})\otimes V_{ijk}(\Phi_1)$

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Going MC@NLO

- $\Rightarrow$ Identify the shower kernels with the CS dipoles:

$$\mathcal{S}(\Phi_\mathcal{B} \otimes \Phi_1) = \sum_{ijk} \mathcal{B}(\Phi_\mathcal{B}) \otimes V_{ijk}(\Phi_1) = \mathcal{B}(\Phi_\mathcal{B}) \otimes \mathcal{K}(\Phi_1)$$

- In this way we get

$$\mathrm{d}\sigma^{\mathrm{MC@NLO}} = \mathrm{d}\,\Phi_\mathcal{B}\widetilde{\mathcal{B}} \left\{ \Delta(Q_0^2, Q^2) + \int_{Q^2}^{Q_0^2} \mathrm{d}\Phi_1 \mathcal{K}(\Phi_1) \Delta\left(Q_0^2, t(\Phi_1)\right) \right\} + \mathrm{d}\Phi_{\mathcal{B}+1}\mathcal{H}(\Phi_{\mathcal{B}+1})$$

where

$$\Delta(Q_0^2, Q^2) = \exp\left[-\int_{Q_0^2}^{Q^2} d\Phi_1 \mathcal{K}(\Phi_1) \right]$$

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where

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LO-Merging

Basic idea

- PS $\Rightarrow$ re-sums logs in **soft- collinear-region** $\rightarrow$ jet evolution
- ME exact at any give order and description of **hard region** $\rightarrow$ jet production
- Separate jet production from jet evolution with jet measure Q_J
- ME populate **hard region**
- PS populate **soft- collinear-region**

$$\mathrm{d}\sigma = \mathrm{d}\Phi_N \mathcal{B}_N \left\{ \Delta_N(\mu_N^2, t_0) + \int_{t_0}^{\mu_N^2} \mathrm{d}\Phi_{N+1} \left[\mathcal{K}_N \Delta_N \left(\mu_N^2, t_{N+1}\right) \right] \Theta(Q_J - Q_{N+1}) \right\}$$

$$+ \quad \mathrm{d}\Phi_{N+1} \mathcal{B}_{N+1} \Delta_N \left(\mu_{N+1}, t_{N+1}\right) \Theta(Q_J - Q_{N+1})$$

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- Separate jet production from jet evolution with jet measure Q_J
- ME populate **hard region**
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$$d\sigma = d\Phi_N \mathcal{B}_N \left\{ \Delta_N(\mu_N^2, t_0) + \int_{t_0}^{\mu_N^2} d\Phi_{N+1} \left[\mathcal{K}_N \Delta_N \left(\mu_N^2, t_{N+1}\right)\right] \Theta(Q_J - Q_{N+1}) \right\}$$
$$+ \quad d\Phi_{N+1} \mathcal{B}_{N+1} \Delta_N \left(\mu_{N+1}, t_{N+1}\right) \Theta(Q_J - Q_{N+1})$$

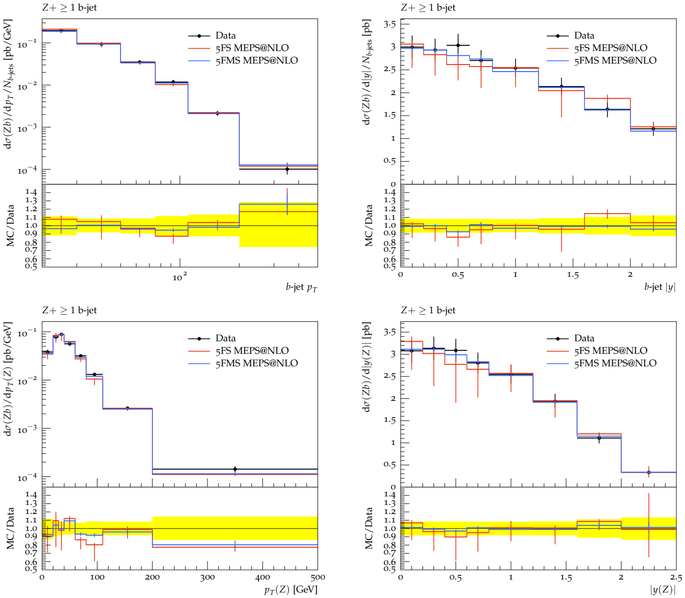
NLO-Merging

Same idea for NLO

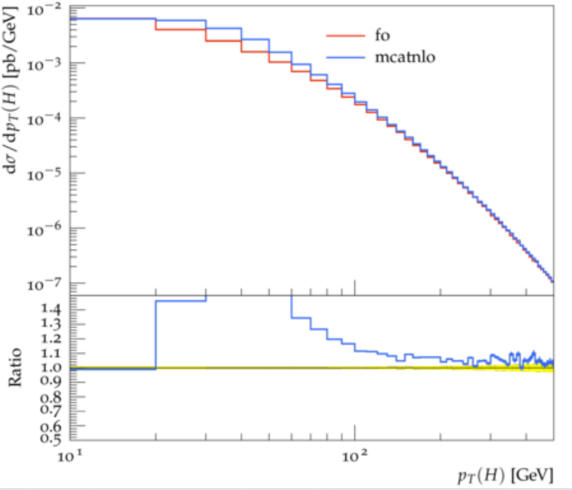
$$
\begin{aligned}
d\sigma = d\Phi_N\, \widetilde{\mathcal{B}}_N & \left[\Delta_N(\mu_N^2, t_0) + \int_{t_0}^{\mu_N^2} d\Phi_1\, \mathcal{K}_N \Delta_N(\mu_N^2, t_{N+1}) \Theta(Q_J - Q_{N+1})\right] \\
& + d\Phi_{N+1}\, \mathcal{H}_N \Delta_N(\mu_N^2, t_{N+1}) \Theta(Q_J - Q_{N+1}) \\
& + d\Phi_{N+1}\, \tilde{\mathcal{B}}_{N+1} \left(1 + \frac{\mathcal{B}_{N+1}}{\tilde{\mathcal{B}}_{N+1}} \int_{t_{N+1}}^{\mu_N^2} d\Phi_1\, \mathcal{K}_N \right) \Theta(Q_{N+1} - Q_J) \\
& \cdot \Delta_N(\mu_N^2, t_{N+1}) \cdot \left[\Delta_{N+1}(t_{N+1}, t_0) + \int_{t_0}^{t_{N+1}} d\Phi_1\, \mathcal{K}_{N+1} \Delta_{N+1}(t_{N+1}, t_{N+2})\right] \\
& + d\Phi_{N+2}\, \mathcal{H}_{N+1} \Delta_N(\mu_N^2, t_{N+1}) \Delta_{N+1}(t_{N+1}, t_{N+2}) \Theta(Q_{N+1} - Q_J) + \ldots
\end{aligned}
$$

Outline

$pp \to Z+ \geq 1b$ jet, @7 TeV



$pp \to H+ \geq 1b$ **jets, @13 TeV**



Conclusions

- Historical treatment of heavy quarks, 4F vs 5F scheme is starting to change... for many reasons
- Correct inclusion of mass effects might play an important role in some precision physics measurements
- Still not a 100% there yet though... A better understanding of factorisazion is needed...