Recent results from EW fits

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Higgs Hunting 2018

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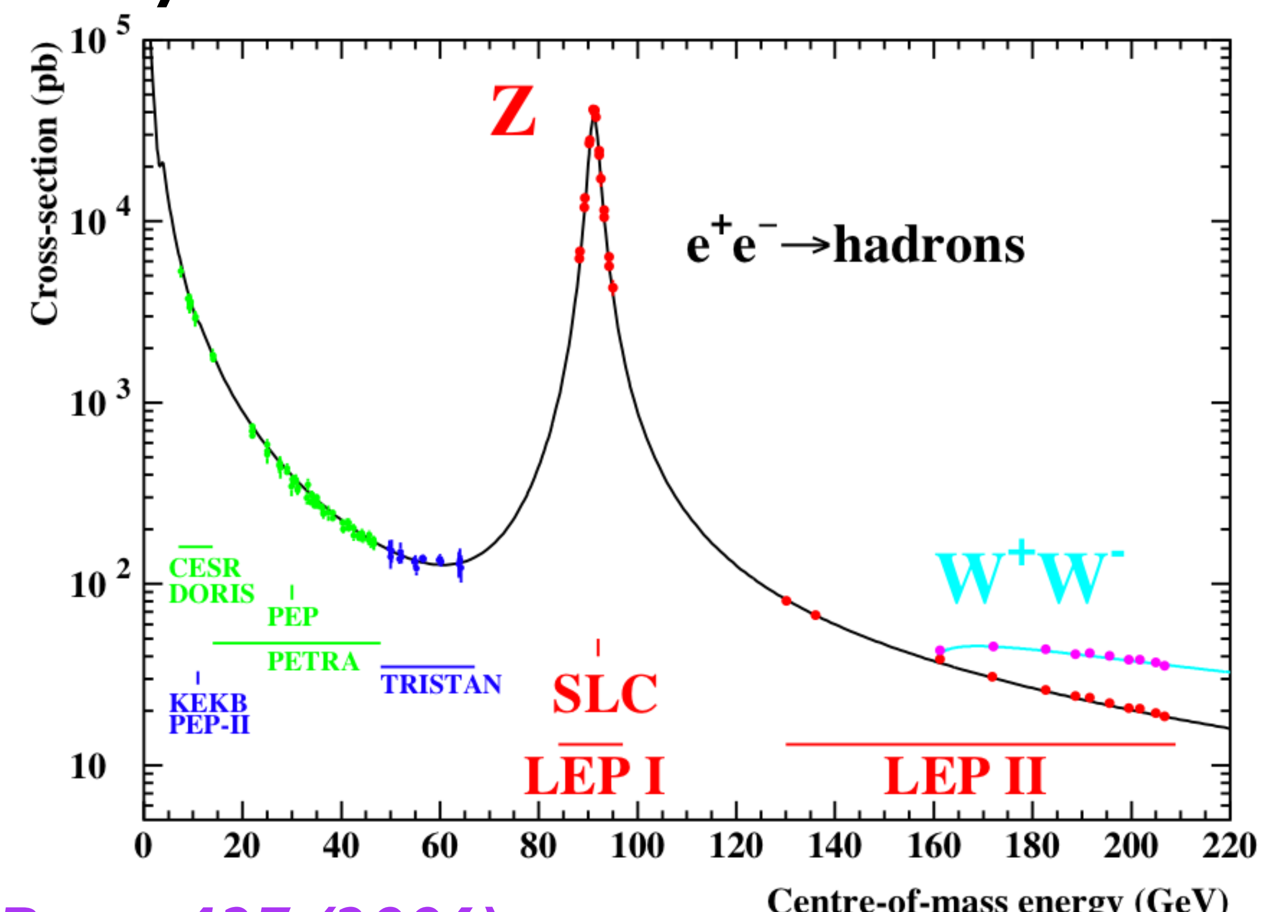
Outline

- Key Observables and Inputs
- Gauge Couplings at Lower Energies
- Electroweak Fits
- Conclusions

Key observables and inputs

Z pole

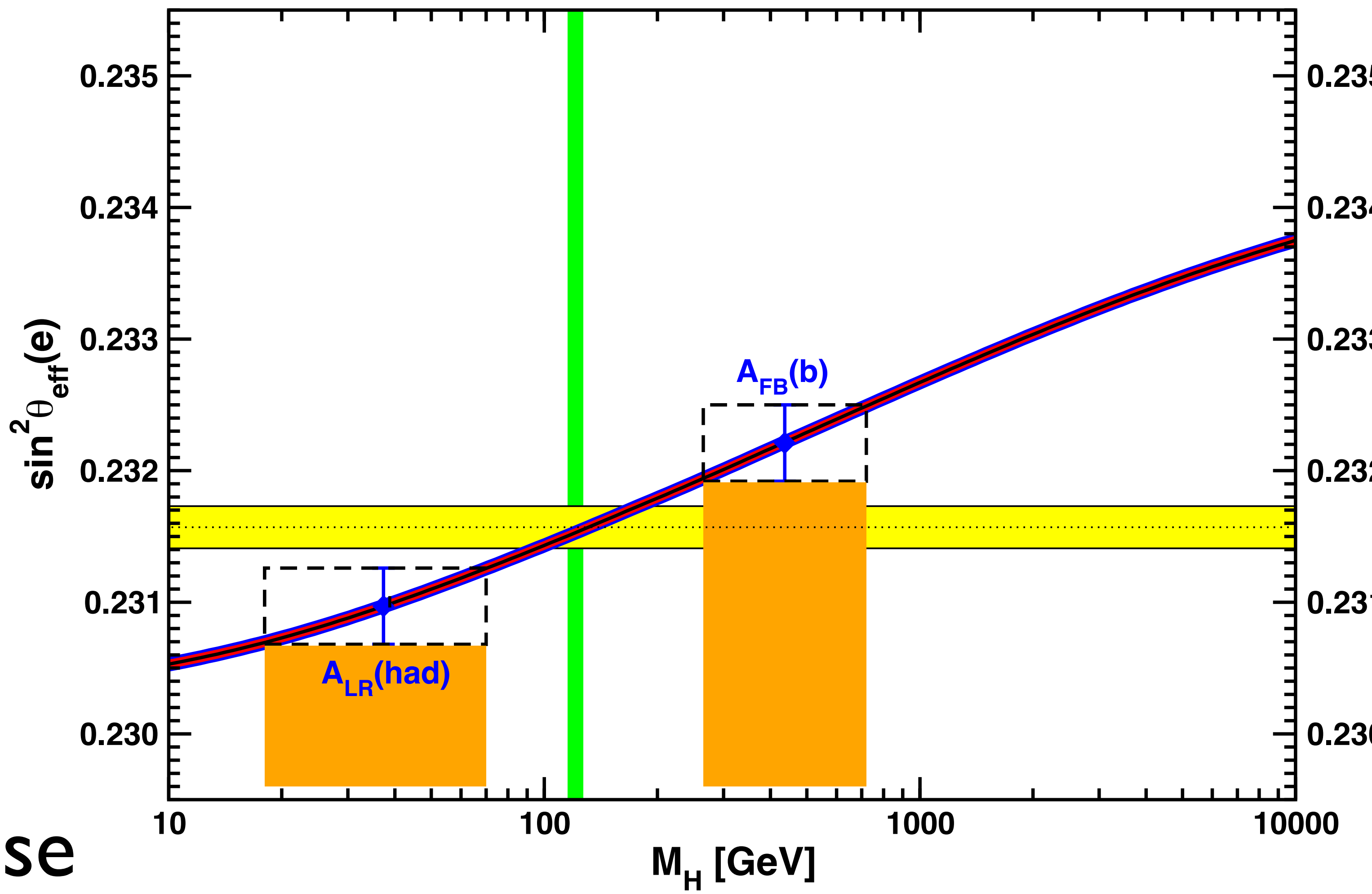
- M_Z = 91.1876 ± 0.0021 GeV (error no longer negligible)
- Γ_Z, σ_{had} and hadronic-to-leptonic BRs provide only α_s constraints not limited by theory
- forward-backward and left-right asymmetries $\propto A_e \sim 1 - 4\sin^2\theta_W(M_Z)$ have strong sensitivity to $\sin^2\theta_W = g'^2/(g^2 + g'^2)$



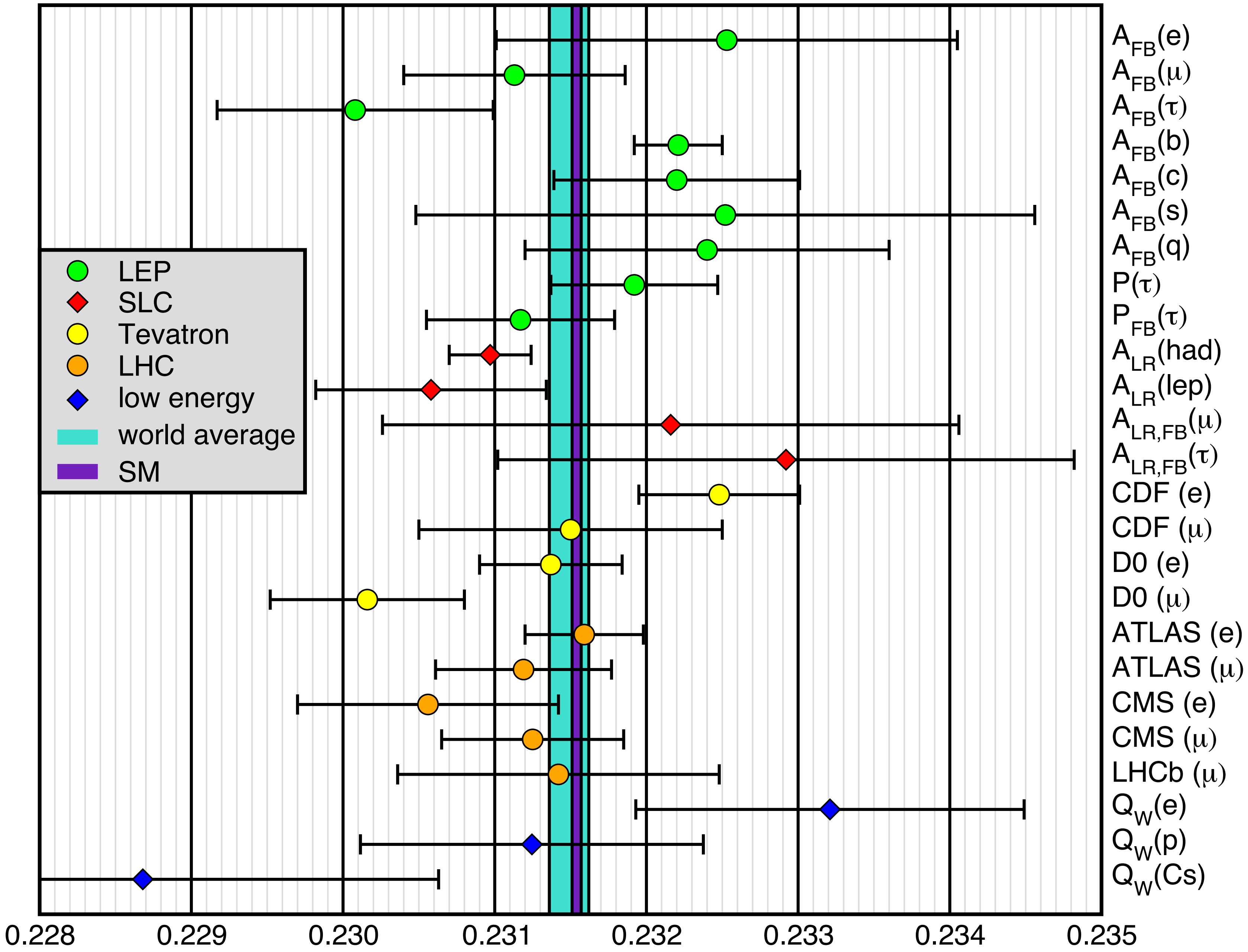
ALEPH, DELPHI, L3 & OPAL, Phys. Rept. 427 (2006)

$\sin^2\theta_W$ within the SM

- $\sin^2\theta_W$ & M_W most precise derived quantities in EW sector:
  - Standard Model: key test of EW symmetry breaking
  - Higgs sector: predict M_H and compare with LHC
  - 3 σ conflict: between most precise LEP and SLC results



$\sin^2\theta_W$ measurements



LEP & SLC:

0.23153 ± 0.00016

Tevatron:

0.23148 ± 0.00033

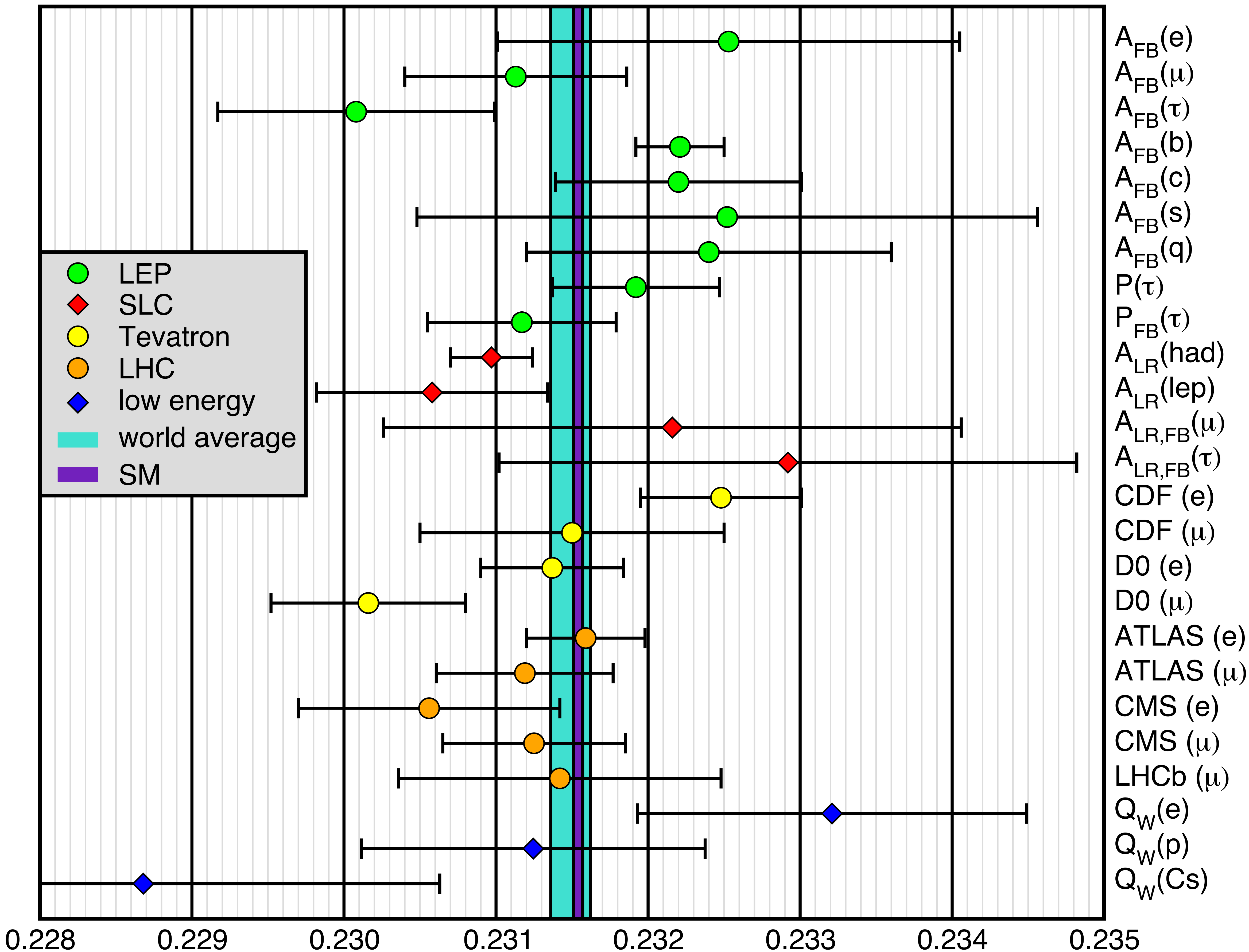
LHC:

0.23131 ± 0.00033

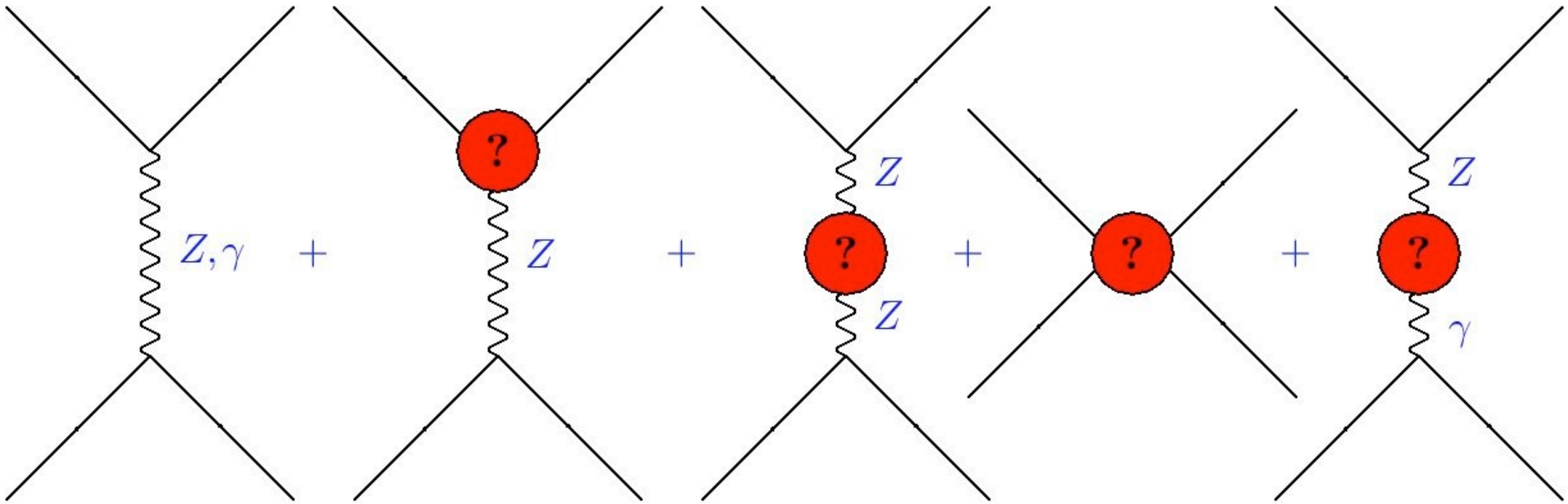
post ICHEP 2018:

0.23154 ± 0.00003

0.23149 ± 0.00013

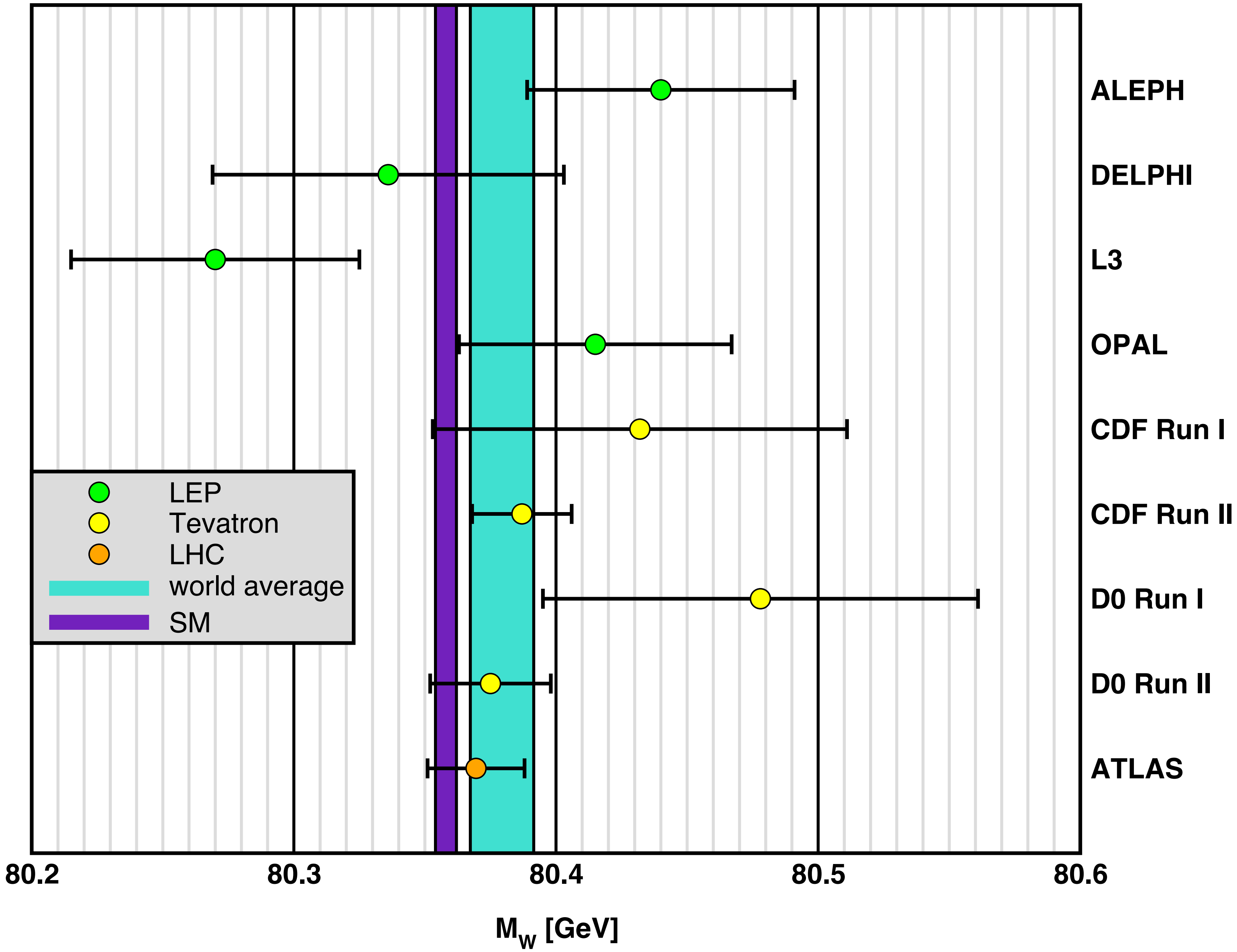

LEP
SLC
Tevatron
LHC
low energy
world average
SM
$A_{FB}(e)$
$A_{FB}(\mu)$
$A_{FB}(\tau)$
$A_{FB}(b)$
$A_{FB}(c)$
$A_{FB}(s)$
$A_{FB}(q)$
$P(\tau)$
$P_{FB}(\tau)$
$A_{LR}(had)$
$A_{LR}(lep)$
$A_{LR,FB}(\mu)$
$A_{LR,FB}(\tau)$
CDF (e)
CDF (μ)
D0 (e)
D0 (μ)
ATLAS (e)
ATLAS (μ)
CMS (e)
CMS (μ)
LHCb (μ)
$Q_W(e)$
$Q_W(p)$
$Q_W(Cs)$
0.228
0.229
0.230
0.231
0.232
0.233
0.234
0.235

$\sin^2\theta_W$ beyond the SM



- Z-Z′ mixing: modification of Z vector coupling
- oblique parameters: STU (also need M_W and Γ_Z)
- new amplitudes: off- versus on-Z pole measurements (e.g. Z′)
- dark Z: renormalization group evolution (running)

M_W measurements



80.358 ± 0.004 GeV

80.379 ± 0.012 GeV

m_t measurements

	central	statistical	systematic	total
Tevatron	174.30	0.35	0.54	0.64
ATLAS	172.51	0.27	0.42	0.50
CMS	172.43	0.13	0.46	0.48
CMS Run 2	172.25	0.08	0.62	0.63
grand	172.74	0.11	0.31	0.33

JE, EPJC 75 (2015)

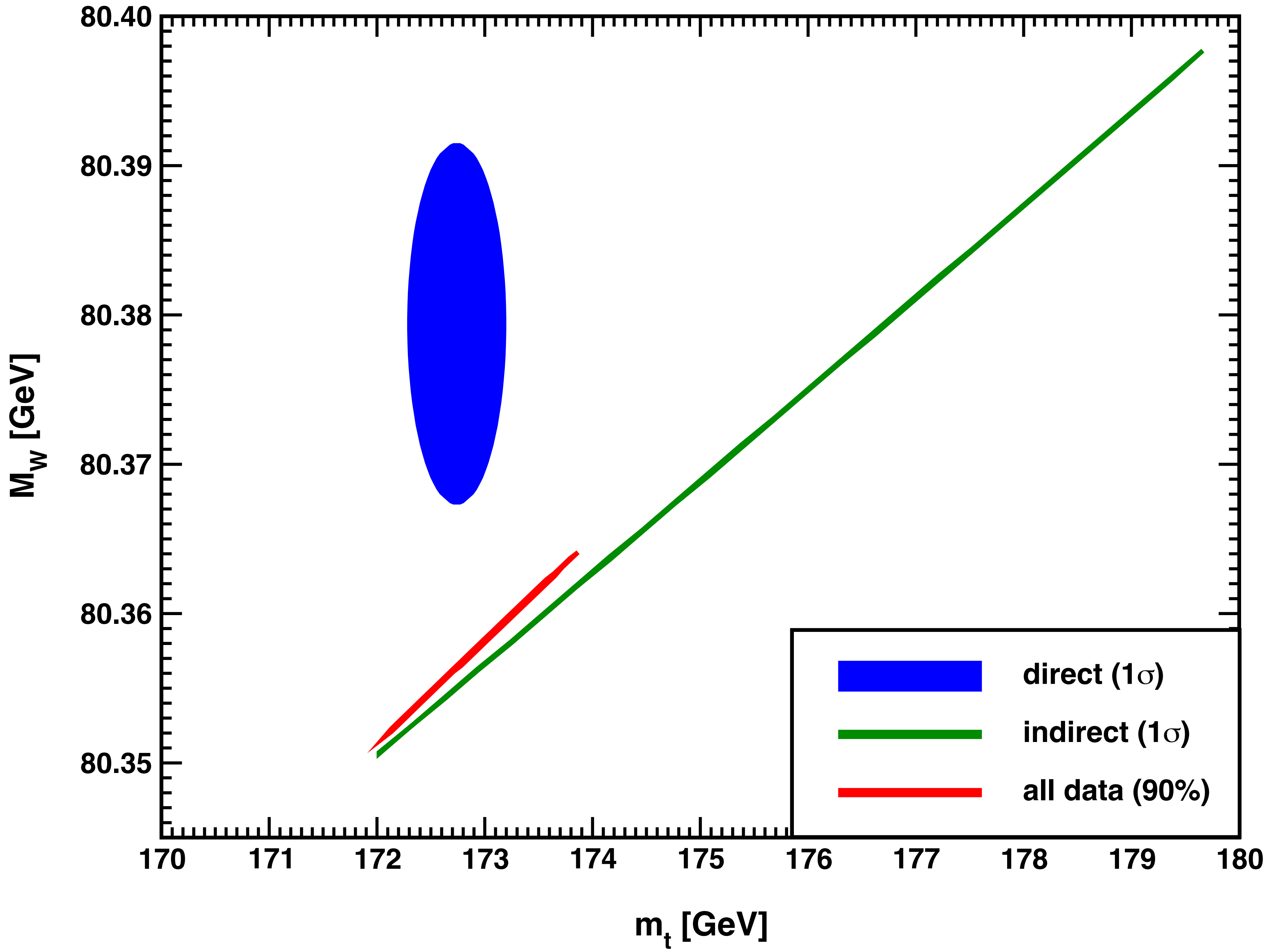
- $m_t = 172.74 \pm 0.25_{uncorr.} \pm 0.21_{corr.} \pm 0.32_{QCD}$ GeV = 172.74 ± 0.46 GeV
- somewhat larger shifts and smaller errors conceivable in the future
  Butenschoen et al., PRL 117 (2016); Andreassen & Schwartz, JHEP 10 (2017)
- 2.8 σ discrepancy between lepton + jet channels from DØ and CMS Run 2
- indirectly from EW fit: $m_t = 176.4 \pm 1.8$ GeV (2 σ) *Freitas & JE (PDG 2018)*

top “pole mass measurements”

	E_{CM}	analysis	value	uncertainty
DØ	1.96 TeV	inclusive $\sigma(t\bar{t})$	172.8	3.3
ATLAS	7+8 TeV	inclusive $\sigma(t\bar{t})$	172.9	2.6
CMS	7+8 TeV	inclusive $\sigma(t\bar{t})$	173.8	1.8
CMS	13 TeV	inclusive $\sigma(t\bar{t})$	170.6	2.7
DØ	1.96 TeV	differential p_t	169.1	2.5
ATLAS	7 TeV	differential $\sigma(t\bar{t}+1\text{jet})$	173.7	2.2
CMS	8 TeV	differential $\sigma(t\bar{t}+1\text{jet})$	169.9	4.1
ATLAS	8 TeV	$e^{\pm}\mu^{\mp}\ \sigma(t\bar{t})$	173.2	1.6
average			172.9	1.0

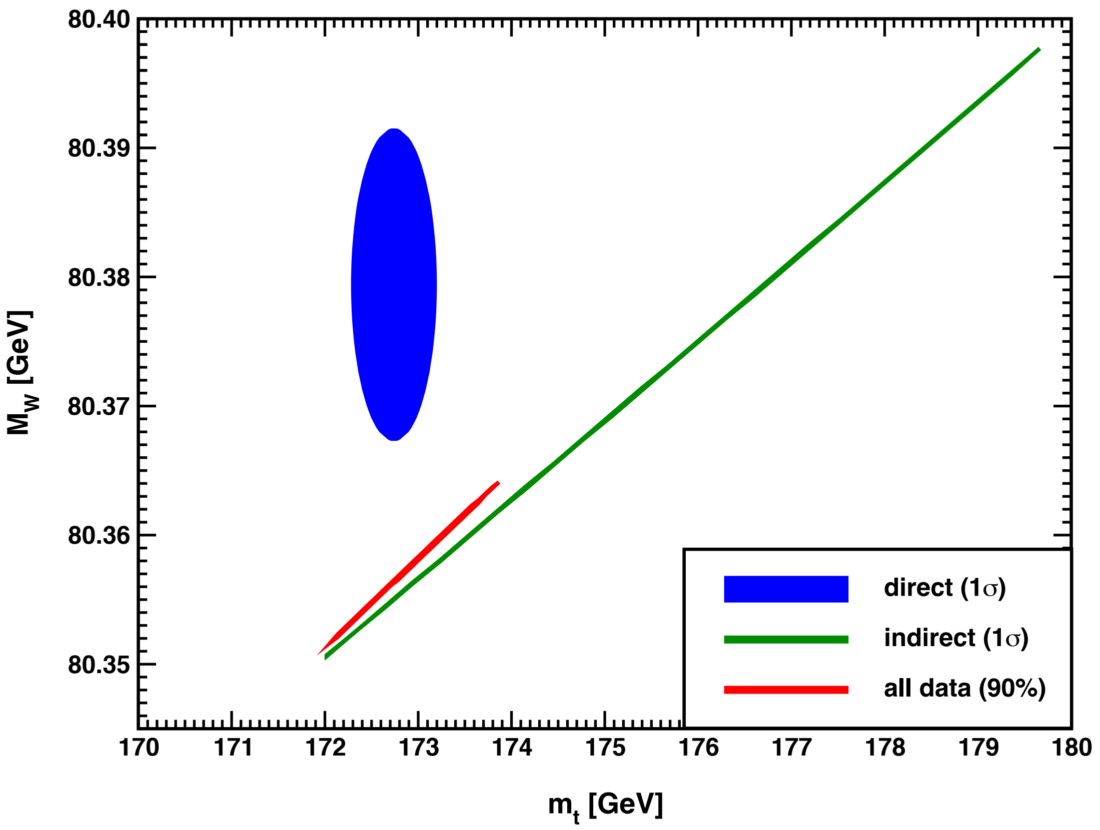
2 σ difference to EW fit $m_t = 176.4 \pm 1.8$ GeV reduces to 1.7 σ

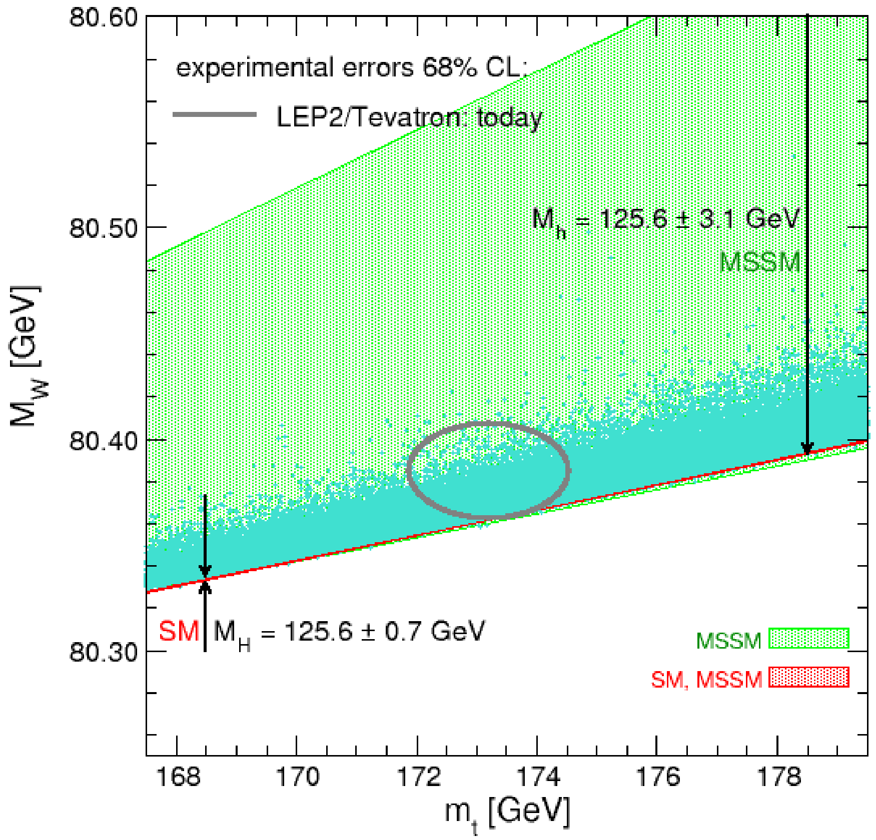
$M_W - m_t$

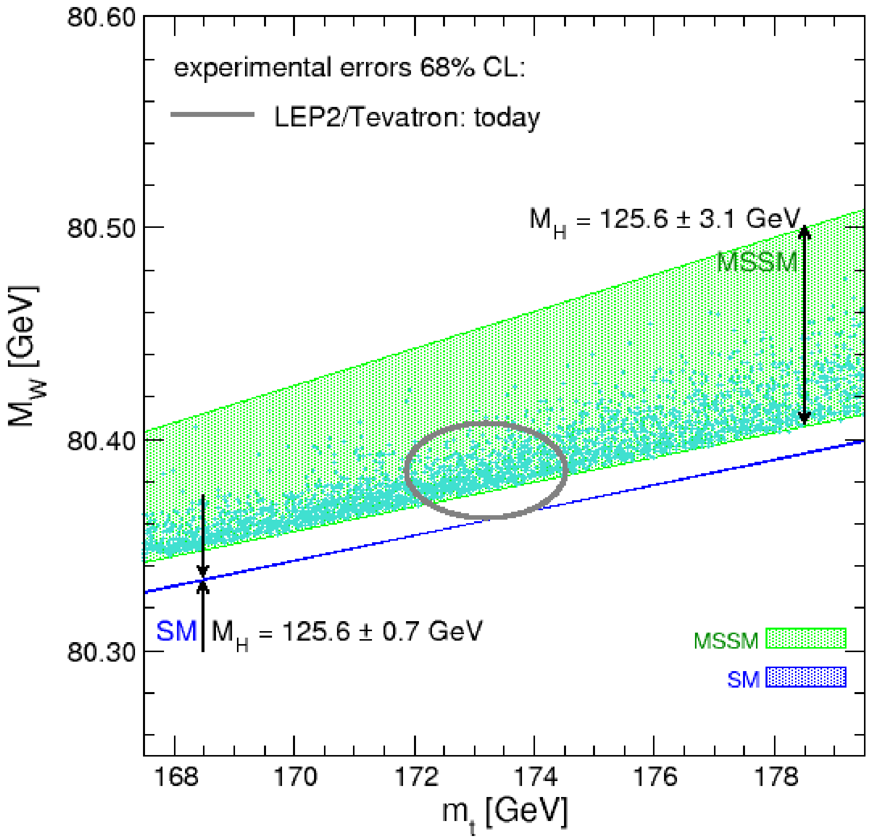


Freitas & JE (PDG 2018)

M_W in the MSSM

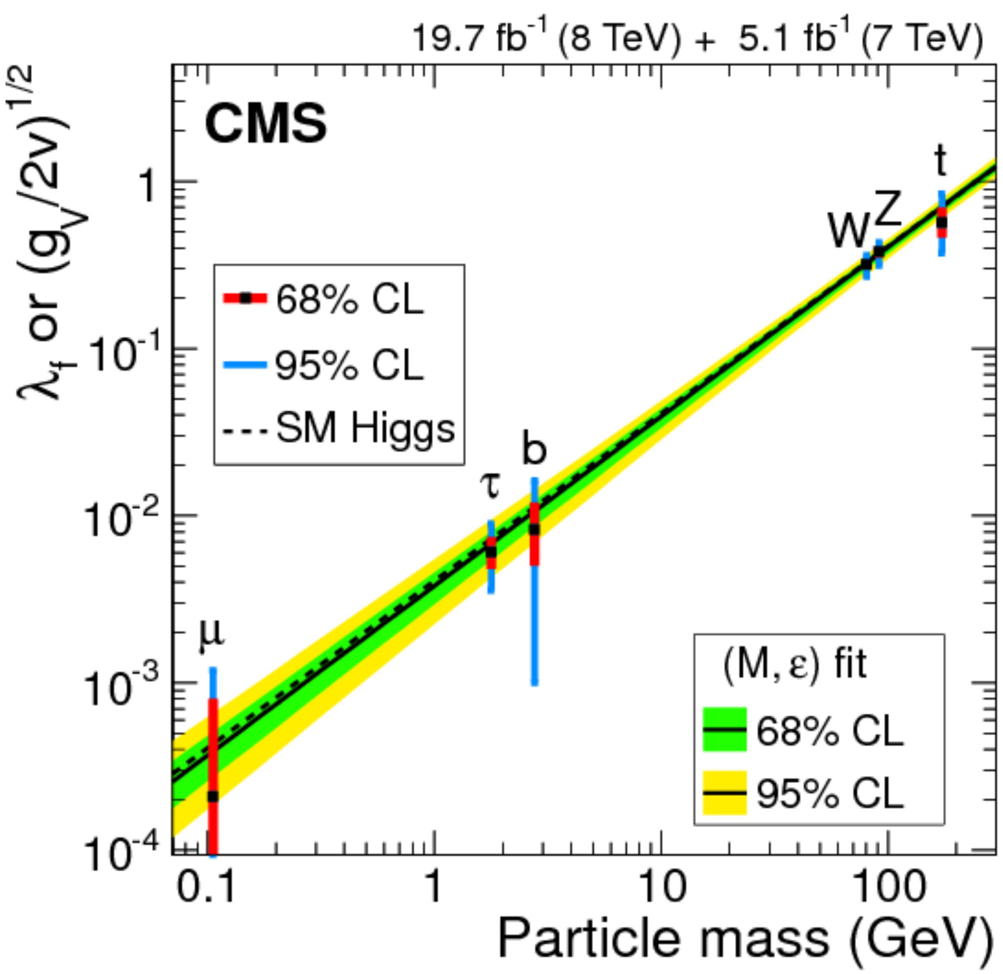






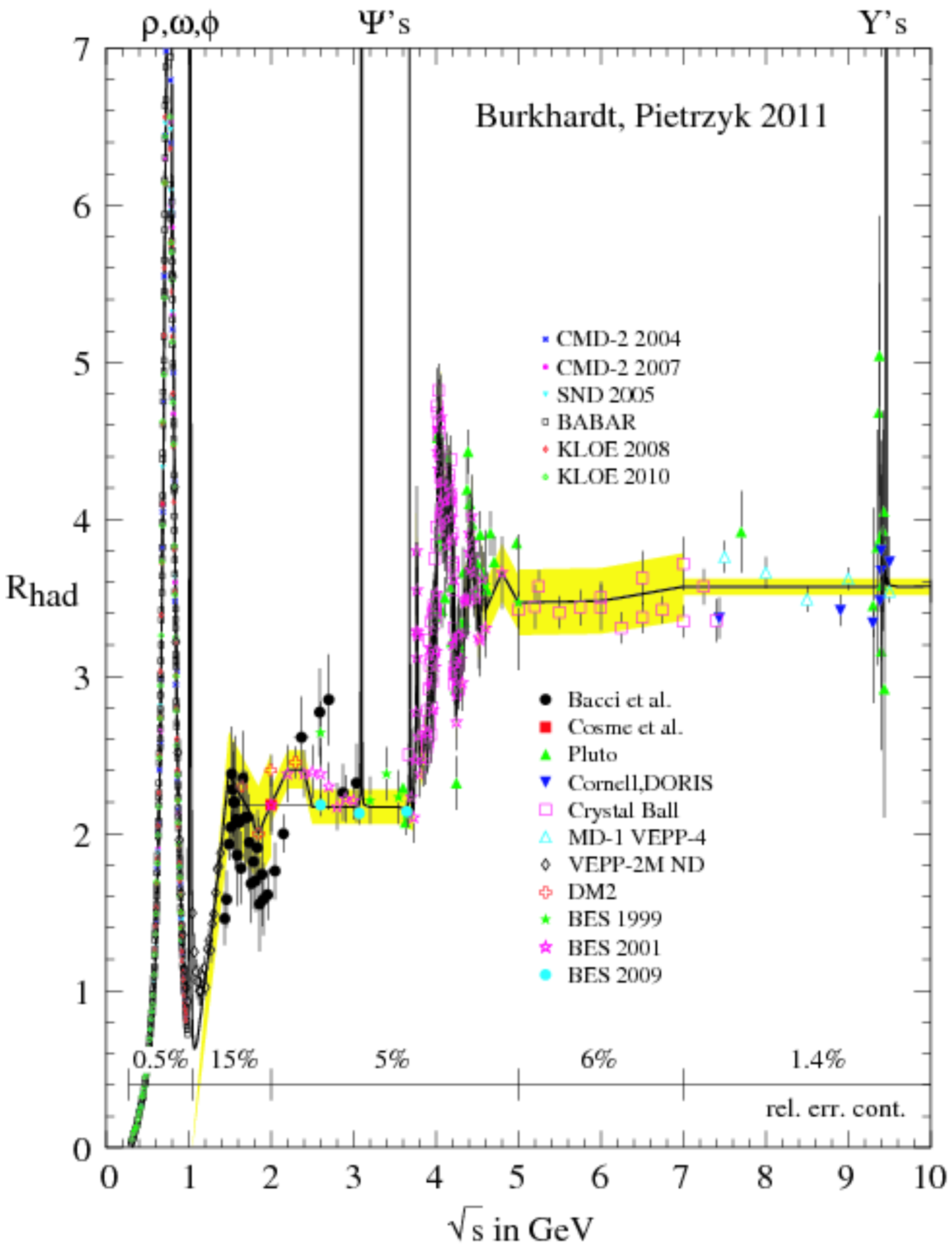
Heinemeyer, Hollik, Weiglein, Zeune 2013

m_c



- α(M_Z) and $\sin^2\theta_W(0)$: can use PQCD for heavy quark contribution if masses are known.
- g–2: c quark contribution to muon g–2 similar to γ×γ; ± 70 MeV uncertainty in m_c induces an error of $\pm 1.6 \times 10^{-10}$ comparable to the projected errors for the FNAL and J-PARC experiments.
- Yukawa coupling – mass relation (in single Higgs doublet SM): $\Delta m_b = \pm 9$ MeV and $\Delta m_c = \pm 8$ MeV to match precision from HiggsBRs @ FCC-ee
- QCD sum rule: m_c = 1272 ± 8 MeV
  Masjuan, Spiesberger & JE, EPJC 77 (2017)
  (expect about twice the error for m_b)

m_c



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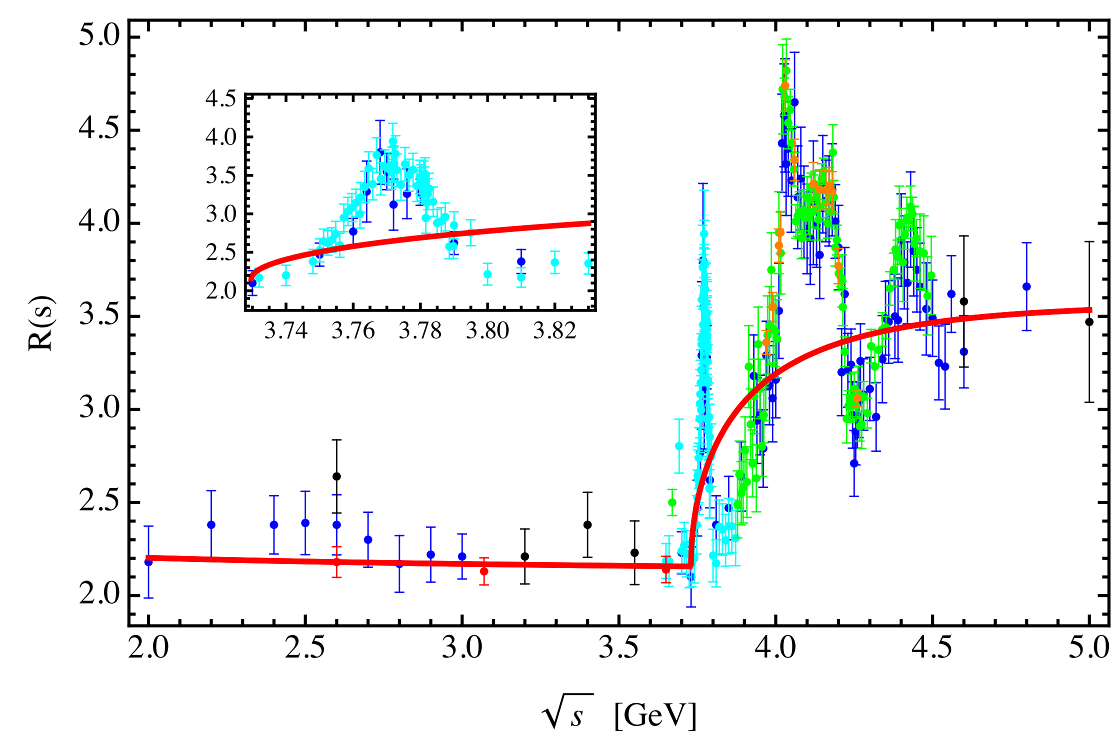
Features of our approach

- only experimental input: electronic widths of J/ψ and ψ(2S)

- continuum contribution from self-consistency between sum rules

- include $\mathcal{M}_0$ → stronger (milder) sensitivity to continuum (m_c)

- quark-hadron duality needed only in finite region (not locally)



- $\overline{m}_c(\overline{m}_c) = 1272 \pm 8 + 2616\,[\overline{\alpha}_s(M_Z) - 0.1182]$ MeV

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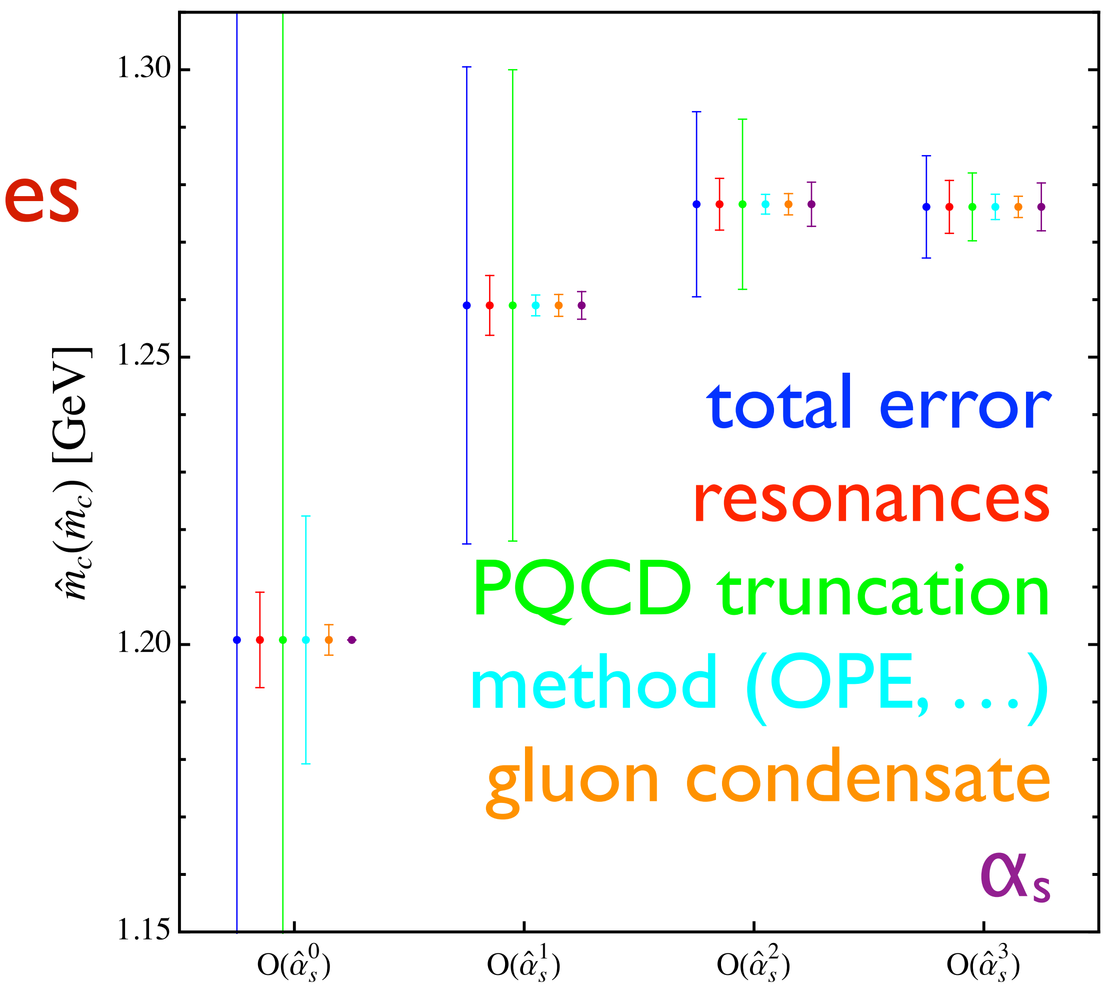

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Gauge Couplings
at Lower Energies

α_s from τ decays

$$\tau_\tau = \hbar \frac{1 - \mathcal{B}_\tau^s}{\Gamma_\tau^e + \Gamma_\tau^\mu + \Gamma_\tau^{ud}} = 290.75 \pm 0.36 \text{ fs},$$

$$\Gamma_\tau^{ud} = \frac{G_F^2 m_\tau^5 |V_{ud}|^2}{64\pi^3} S\left(m_\tau, M_Z\right) \left(1 + \frac{3}{5} \frac{m_\tau^2 - m_\mu^2}{M_W^2}\right) \times$$

$$\left[1 + \frac{\alpha_s\left(m_\tau\right)}{\pi} + 5.202 \frac{\alpha_s^2}{\pi^2} + 26.37 \frac{\alpha_s^3}{\pi^3} + 127.1 \frac{\alpha_s^4}{\pi^4} + \frac{\widehat{\alpha}}{\pi}\left(\frac{85}{24} - \frac{\pi^2}{2}\right) + \delta_{\rm NP}\right]$$

- τ_τ result includes leptonic branching ratios
- $\mathcal{B}_\tau{}^s$ = 0.0292 ± 0.0004 ($\Delta S = -1$) ***PDG 2018***
- $S(m_\tau, M_Z)$ = 1.01907 ± 0.0003 ***JE, Rev. Mex. Fis. 50 (2004)***
- δ_{NP} = 0.003 ± 0.009 (within OPE & OPE breaking) based on (controversial)
  Boito et al., PRD 85 (2012) & PRD 91 (2015); Davier et al., EPJC 74 (2014); Pich & Rodríguez-Sánchez, PRD 94 (2016)

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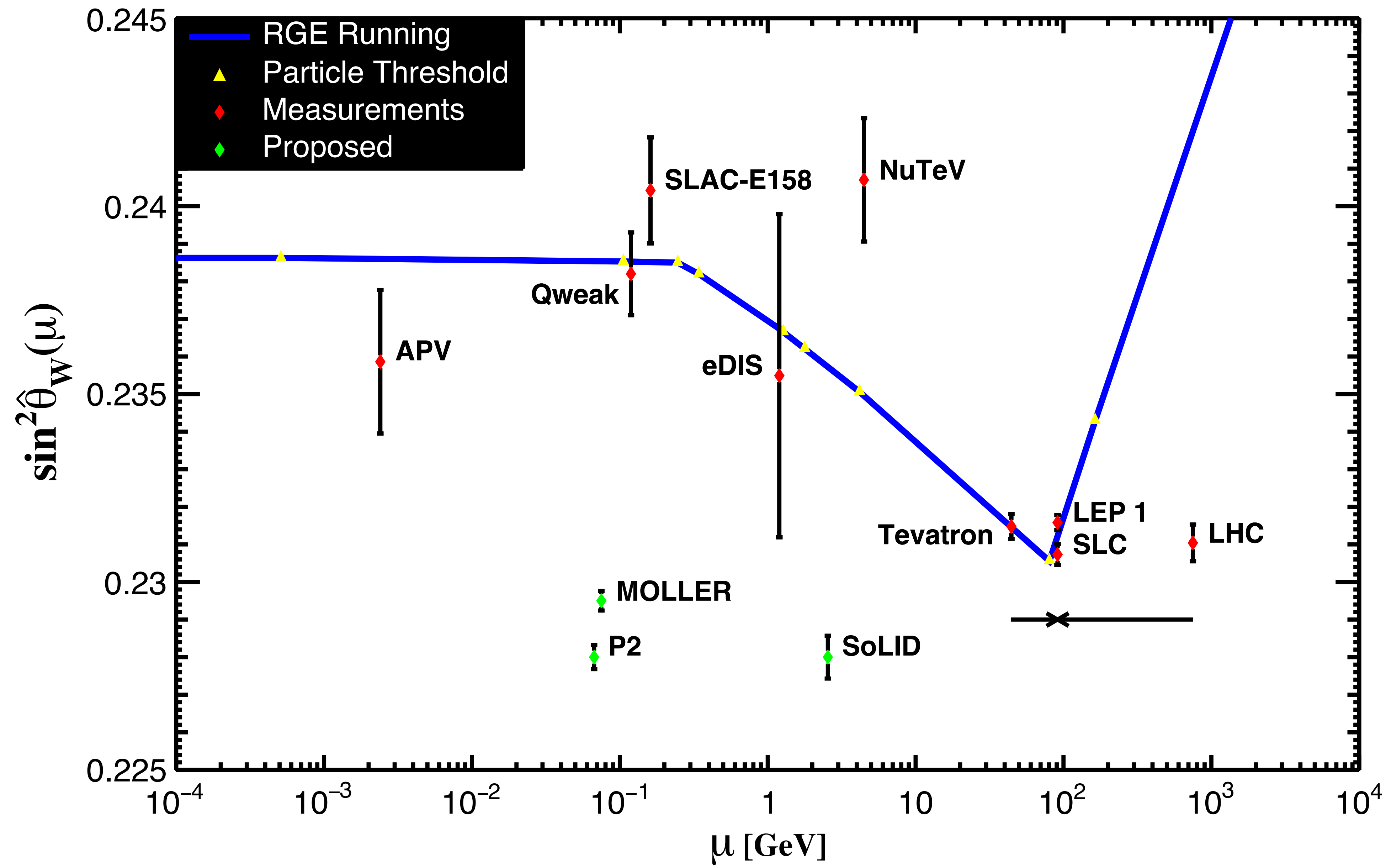
- dominant uncertainty from PQCD truncation (FOPT vs. CIPT vs. geometric continuation)
- $\alpha_S^{(4)}(m_\tau) = 0.323^{+0.018}_{-0.014}$
- $\alpha_S^{(5)}(M_Z) = 0.1184^{+0.0020}_{-0.0018}$
- updated from ***Luo & JE, PLB 558 (2003)*** in ***Freitas & JE (PDG 2018)***

$\alpha(M_Z)$

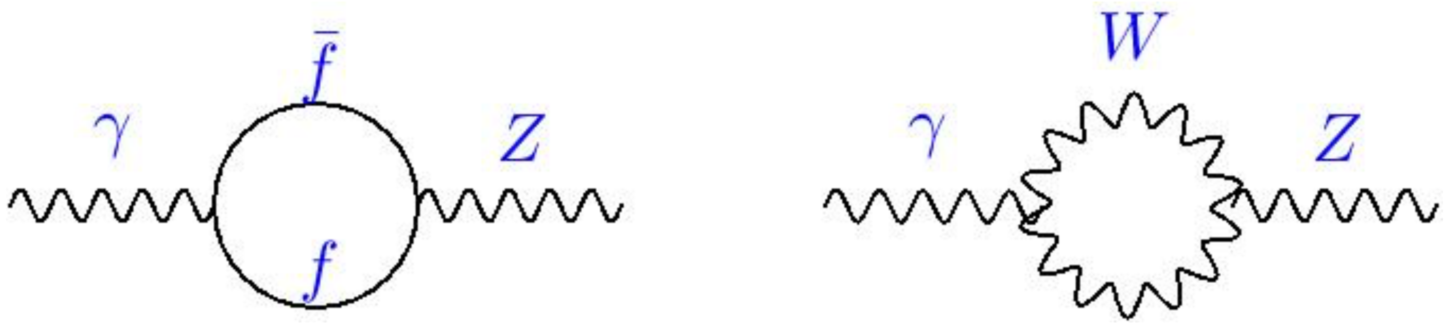
- Dispersive approach:
  - $\alpha^{-1}(M_Z) = 128.947 \pm 0.012$ *Davier et al., EPJC 77 (2017)*
  - $\alpha^{-1}(M_Z) = 128.958 \pm 0.016$ *Jegerlehner, arXiv:1711.06089*
  - $\alpha^{-1}(M_Z) = 128.946 \pm 0.015$ *Keshavarzi et al., arXiv:1802.02995*
- $\alpha^{-1}(M_Z) = 128.949 \pm 0.010$ *Ferro-Hernández & JE, JHEP 03 (2018)*
  - This value is converted from the $\overline{\text{MS}}$ scheme and uses both e^+e^- annihilation and τ decay spectral functions *Davier et al., EPJC 77 (2017)*
  - τ data corrected for γ-ρ mixing *Jegerlehner & Szafron, EPJC 71 (2011)*
  - PQCD for $\sqrt{s} > 2$ GeV (using $\overline{m}_c$ & $\overline{m}_b$) *Ferro-Hernández & JE, in preparation*

$\sin^2\theta_W(\mu)$



Ferro-Hernández & JE, JHEP 03 (2018)

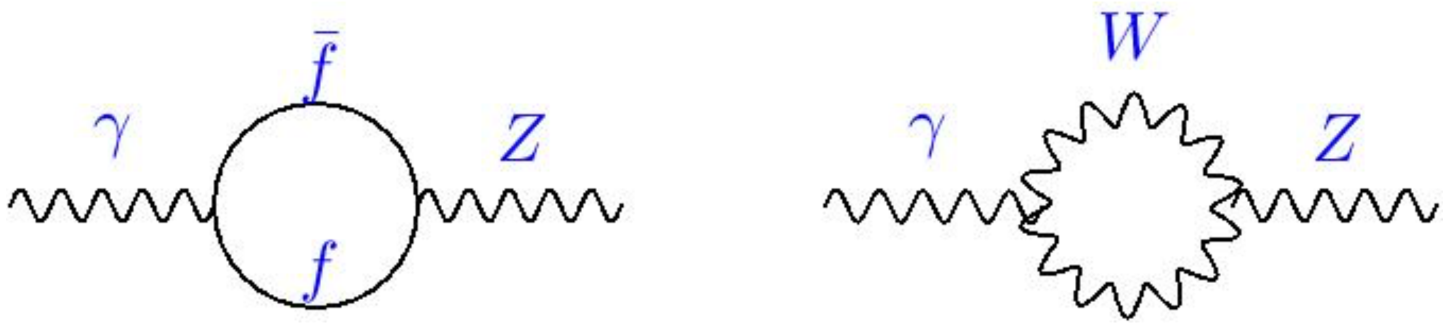
$\sin^2\theta_W(0)$: RGE



$$\mu^2 \frac{d\hat{v}_f}{d\mu^2} = \frac{\hat{\alpha} Q_f}{24\pi} \left[\sum_i K_i \gamma_i \hat{v}_i Q_i + 12\sigma \left(\sum_q Q_q \right) \left(\sum_q \hat{v}_q \right) \right]$$

- v_f : Z vector coupling to fermion f
- K_i : QCD factor known to $O(\alpha_S^4)$ *Baikov et al., JHEP 07 (2012)*
- σ: singlet piece at $O(\alpha_S^3)$ and $O(\alpha_S^4)$ *Baikov et al., JHEP 07 (2012)*
- γ_i : field type dependent constants *Ramsey-Musolf & JE, PRD 72 (2005)*

$\sin^2\theta_W(0)$ and $\Delta\alpha(M_Z)$



$$\mu^2 \frac{d\hat{v}_f}{d\mu^2} = \frac{\hat{\alpha} Q_f}{24\pi} \left[\sum_i K_i \gamma_i \hat{v}_i Q_i + 12\sigma \left(\sum_q Q_q \right) \left(\sum_q \hat{v}_q \right) \right]$$

compare with

$$\mu^2 \frac{d\hat{\alpha}}{d\mu^2} = \frac{\hat{\alpha}^2}{\pi} \left[\frac{1}{24} \sum_i K_i \gamma_i Q_i^2 + \sigma \left(\sum_q Q_q \right)^2 \right]$$

➡ coupled system of differential equations

Ramsey-Musolf & JE, PRD 72 (2005)

$\sin^2\theta_W(0)$: result

source	uncertainty in $\sin^2\theta_W(0)$
$\Delta\alpha^{(3)}(2\text{ GeV})$	1.2×10^{-5}
flavor separation	1.0×10^{-5}
isospin breaking	0.7×10^{-5}
singlet contribution	0.3×10^{-5}
PQCD	0.6×10^{-5}
Total	1.8×10^{-5}

➡ $\sin^2\theta_W(0) = 0.23861 \pm 0.00005_{\text{Z-pole}} \pm 0.00002_{\text{theory}} \pm 0.00001_{\alpha_s}$

Ferro-Hernández & JE, JHEP 03 (2018); Freitas & JE, PDG 2018

Electroweak Fits

Performed with package
Global Analysis of Particle Properties
(GAPP)

Inputs

- 5 inputs needed to fix the bosonic sector of the SM:
  SU(3) × SU(2) × U(1) gauge couplings and 2 Higgs parameters
- fine structure constant: α e.g. from the Rydberg constant (leaves g_e–2 as derived quantity and extra SM test)
- Fermi constant: G_F from PSI (muon lifetime)
- Z mass: M_Z from LEP
- Higgs mass: M_H from the LHC
- strong coupling constant: $\alpha_s(M_Z)$ is fit output ☛

Standard global fit

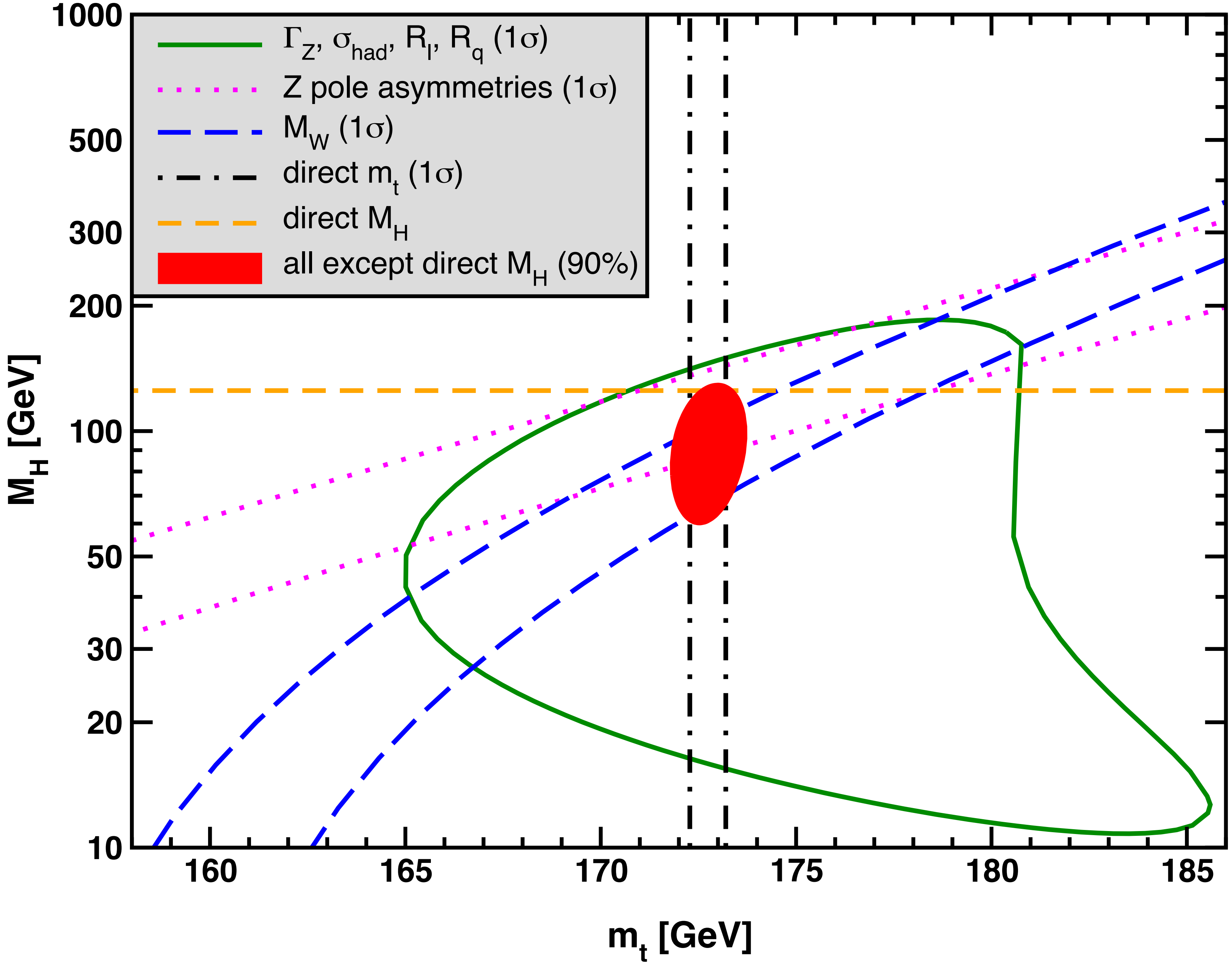
M_H	125.14 ± 0.15 GeV
M_Z	91.1884 ± 0.0020 GeV
$\overline{m}_b(\overline{m}_b)$	4.180 ± 0.021 GeV
$\Delta\alpha_{had}^{(3)}(2\ \mathrm{GeV})$	$(59.0 \pm 0.5)\times 10^{-4}$

$\overline{m}_t(\overline{m}_t)$	163.28 ± 0.44 GeV	1.00	–0.13	–0.28
$\overline{m}_c(\overline{m}_c)$	1.275 ± 0.009 GeV	–0.13	1.00	0.45
$\alpha_S(M_Z)$	0.1187 ± 0.0016	–0.28	0.45	1.00

other correlations small

Freitas & JE, PDG 2018

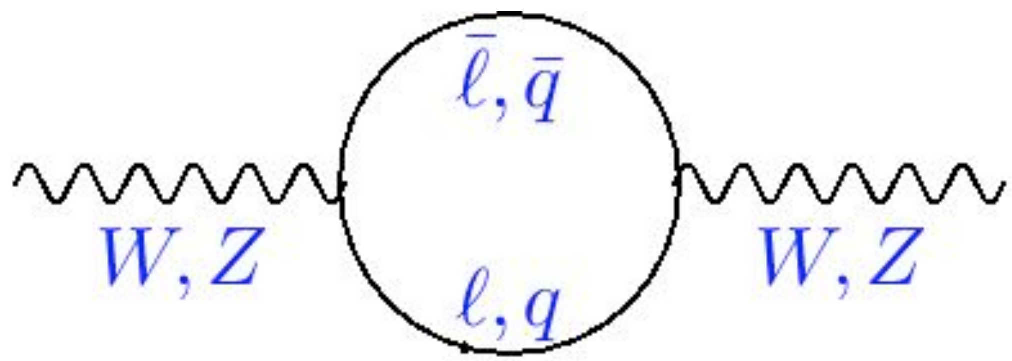
$M_H - m_t$



indirect M_H:
90^{+17}_{-16} GeV
(1.9 σ low)

indirect m_t:
176.4 ± 1.8 GeV
(2.0 σ high)

Oblique physics beyond the SM



- STU describe corrections to gauge-boson self-energies
- T breaks custodial SO(4)
- a multiplet of heavy degenerate chiral fermions contributes $\Delta S = N_C / 3\pi \sum_i [t_{3L}{}^i - t_{3R}{}^i]^2$
- extra degenerate fermion family yields $\Delta S = 2/3\pi \approx 0.21$
- S and T (U) correspond to dimension 6 (8) operators

ρ_0 fit

- $\Delta\rho_0 = G_F \sum_i C_i / (8\sqrt{2}\pi^2)\ \Delta m_i^2$
  - where $\Delta m_i^2 \geq (m_1 - m_2)^2$
  - despite appearance <u>there is</u> decoupling (see-saw type suppression of Δm_i^2)
- $\rho_0 = 1.00039 \pm 0.00019$ (2.0 σ)
  - $(16\ \text{GeV})^2 \leq \sum_i C_i / 3\ \Delta m_i^2 \leq (48\ \text{GeV})^2$ @ 90% CL
  - $Y = 0$ Higgs triplet VEVs v_3 strongly disfavored ($\rho_0 < 1$)
  - consistent with $|Y| = 1$ Higgs triplets if $v_3 \sim 0.01\ v_2$

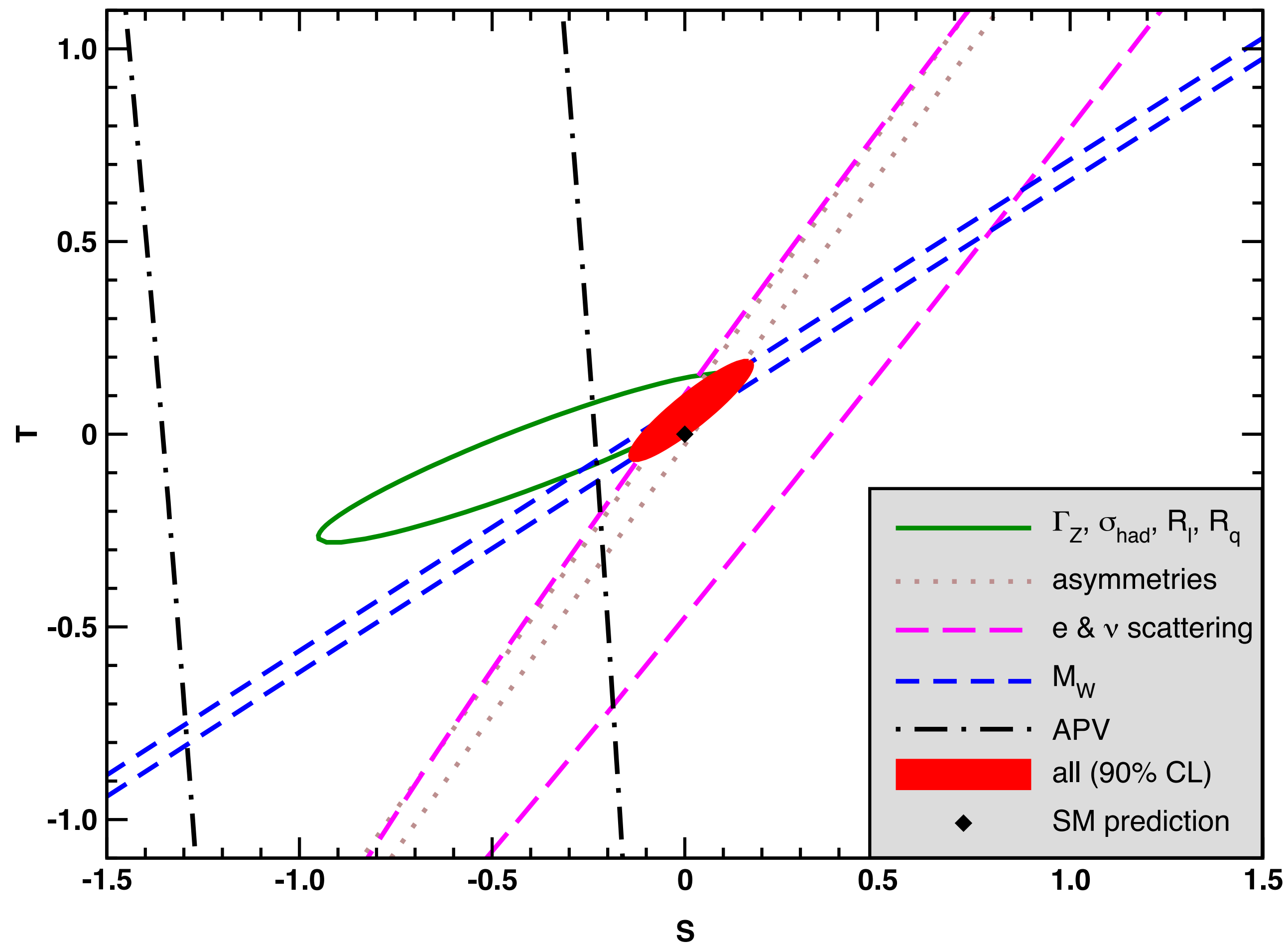
S fit

- S parameter rules out QCD-like technicolor models
- S also constrains extra degenerate fermion families:

➡ $N_F = 2.75 \pm 0.14$ (assuming $T = U = 0$)

- compare with $N_\nu = 2.991 \pm 0.007$ from Γ_Z

S and T



S	0.02 ± 0.07
T	0.06 ± 0.06
$\Delta\chi^2$	– 4.2

- $M_{KK} \gtrsim 3.2$ TeV in warped extra dimension models
- $M_V \gtrsim 4$ TeV in minimal composite Higgs models ***Freitas & JE (PDG 2018)***

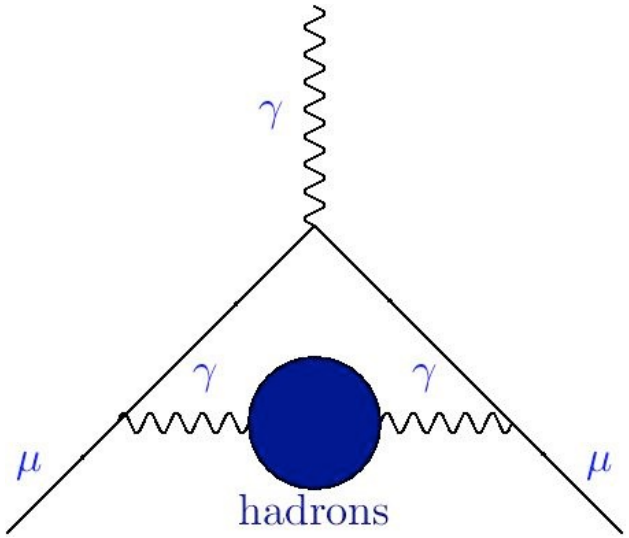
Conclusions

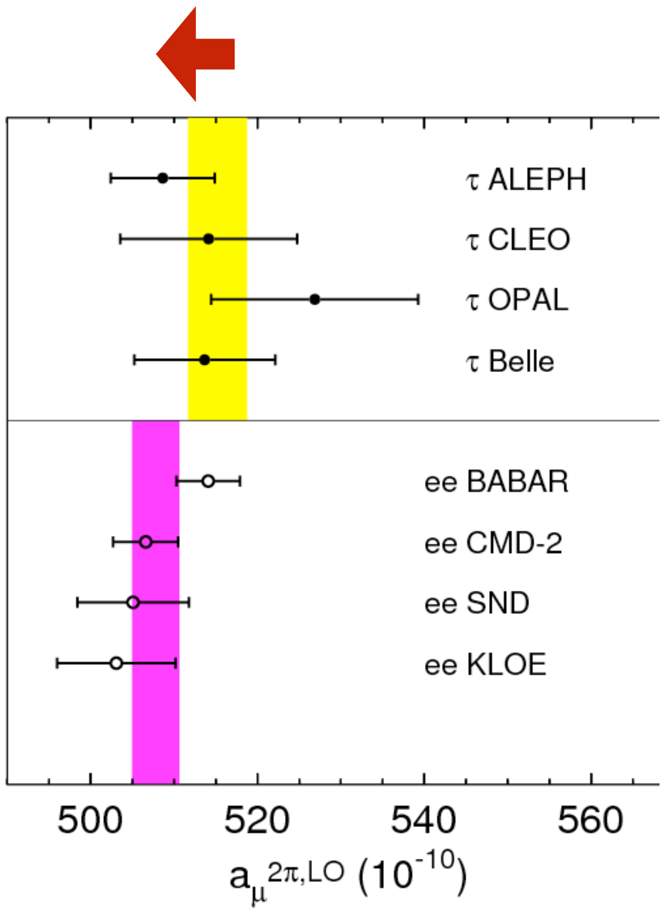
- The SM is 50 years old and in great health — immortal?
- ICHEP 2018: new ATLAS result based on 8 TeV data
  - **$\sin^2\theta_W = 0.23140 \pm 0.00036$**
  - agrees well with SM and world average
- small tension in M_W
  - surely only 2 σ … but in a very special observable
  - simplest possibility: $\rho_0 > 1$
- Precision in $\sin^2\theta_W$ (A_{FB}) & M_W and future polarized e^- scattering experiments at low Q^2 challenge theory → needs major global effort

Backups

$g_\mu - 2$

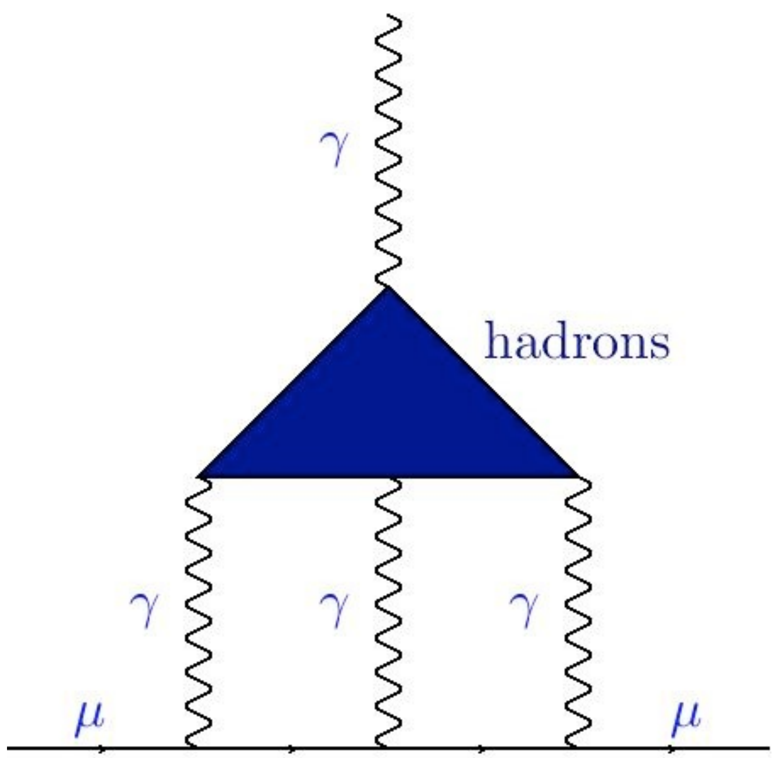
- $a_\mu \equiv (1165920.91 \pm 0.63)\times 10^{-9}$ ***BNL-E821 2004***
- SM: $a_\mu = (1165917.63 \pm 0.46)\times 10^{-9}$ (4.2 σ)
- hadronic vacuum polarization (VP):
  use data + PQCD ***Luo, JE 2002***
  (m_c and m_b needed)
- consistency between experimental $B(\tau^- \to \nu\, \pi^0 \pi^-)$ and prediction from e^+e^- and CVC after accounting for γ-ρ mixing ***Jegerlehner, Szafron 2011***





$g_\mu - 2$ theory prospects

- VP in space-like region from Bhabba *Carloni Calame et al. 2015* and μe-scattering *Abbiendi et al. 2016* using $a_\mu^{had} = \alpha/\pi \int dx\,(1-x)\,\Delta\alpha_{had}[x^2 m_\mu^2/(x-1)]$ *Lautrup et al. 1972*
- hadronic γ×γ error: $\pm 0.32\times10^{-9}$ (30%)
- lattice:
  - 5% statistical error (systematic error under investigation) *Blum et al. 2015*
  - only quark-connected diagrams
  - cross-check: calculation of muonic γ×γ agrees within 2%
  - VP: also few % errors (~1 year to achieve sub-%?)



g_μ–2 hadronic effects

- $a_\mu^{had,LO} = (69.31 \pm 0.34)\times10^{-9}$ *Davier et al., EPJC 77 (2017)*
- $a_\mu^{had,LO} = (68.81 \pm 0.41)\times10^{-9}$ *Jegerlehner, EPJ Web Conf. 166 (2018)*
- $a_\mu^{had,LO} = (68.88 \pm 0.34)\times10^{-9}$ (incl. τ data) *Jegerlehner, EPJ Web Conf. 166 (2018)*
- $a_\mu^{had,LO} = (69.33 \pm 0.25)\times10^{-9}$ *Keshavarzi et al., arXiv:1802.02995*
- $a_\mu^{had,NLO} = (-1.01 \pm 0.01)\times10^{-9}$ (anti-correlated with $a_\mu^{had,LO}$) *Krause, PLB 390 (1997)*
- $a_\mu^{had,NNLO} = (0.124 \pm 0.001)\times10^{-9}$ *Kurz et al., EPJ Web Conf. 118 (2016)*
- $a_\mu^{had,LBLS}(\alpha^3) = (1.05 \pm 0.33)\times10^{-9}$ ($\overline{m}_c$ treatment!) *Toledo-Sánchez & JE, PRL 97 (2006)*
- $a_\mu^{had,LBLS}(\alpha^4) = (0.03 \pm 0.02)\times10^{-9}$ *Colangelo et al., PLB 735 (2014)*
- a_μ (exp.) – a_μ (SM) = $(2.55 \pm 0.77)\times10^{-9}$ ($3.3\ \sigma$) *Freitas & JE, PDG 2018*

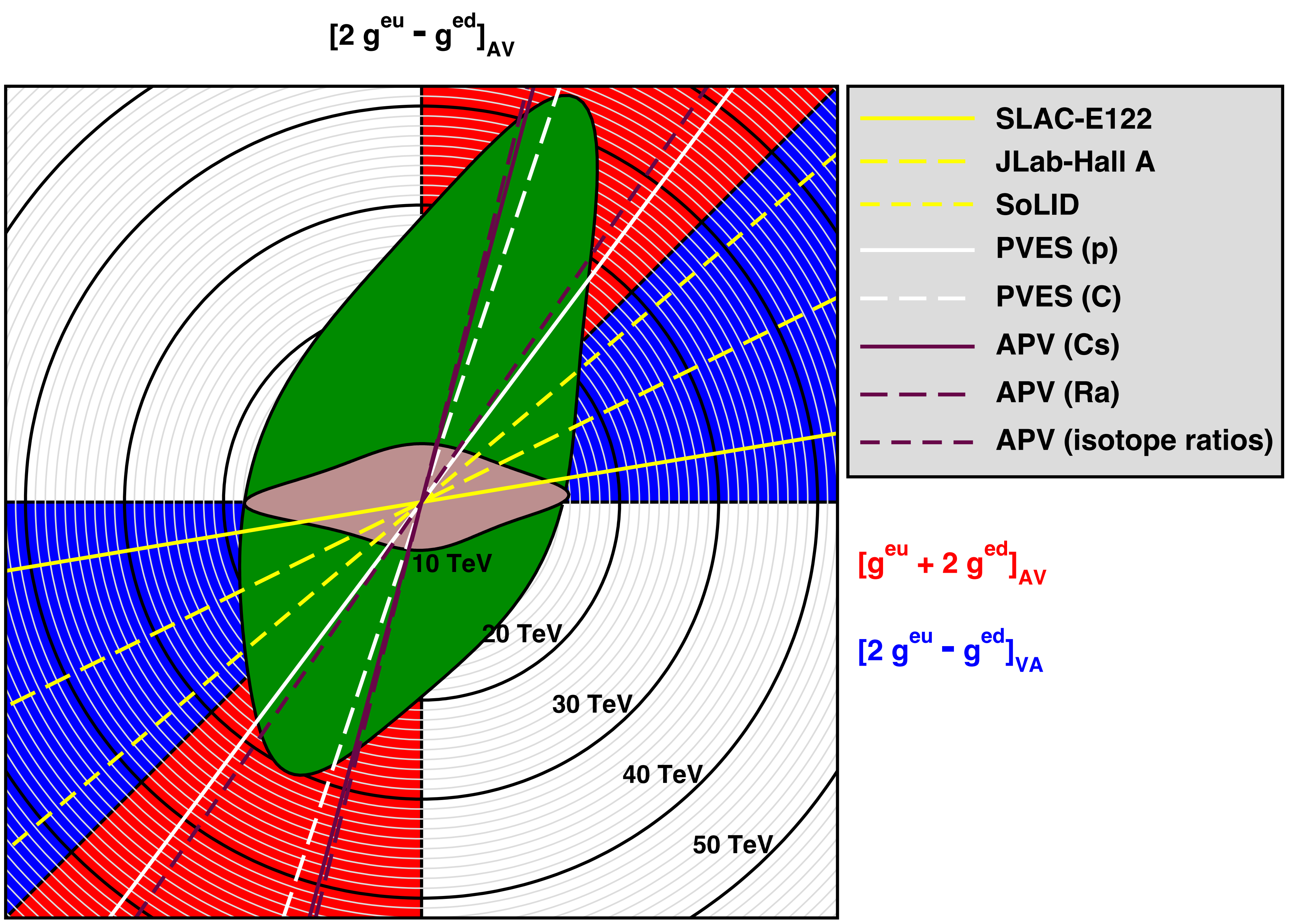
STU fit

$\sin^2\theta_W(M_Z)$	0.23113 ± 0.00014
$\alpha_S(M_Z)$	0.1189 ± 0.0016

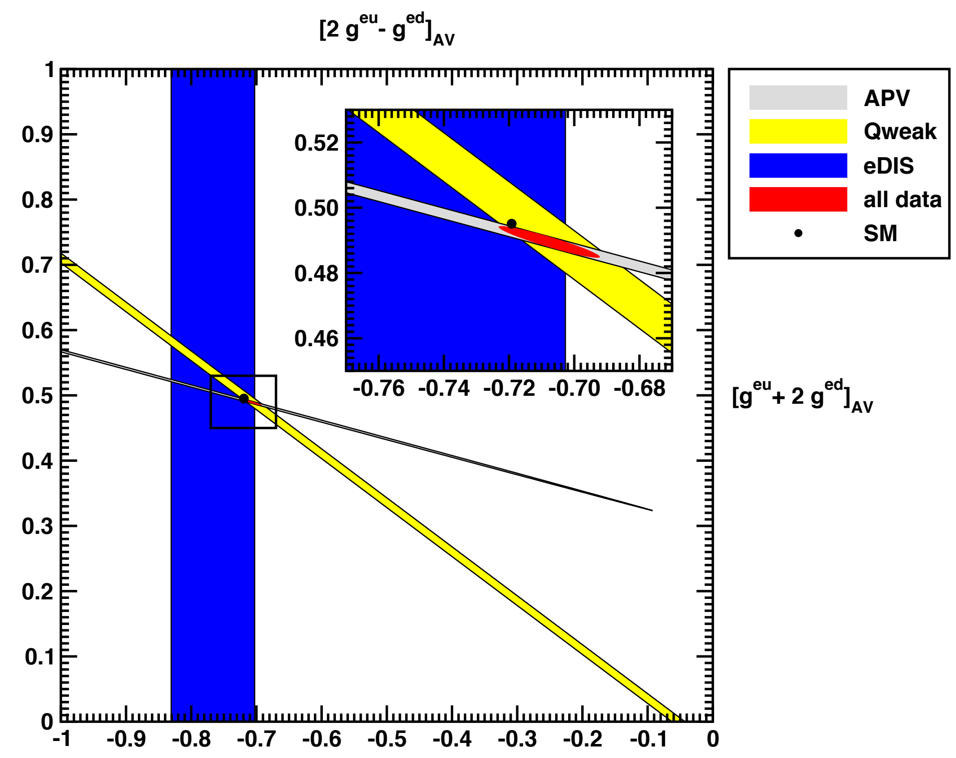
S	0.02 ± 0.10	1.00	0.92	–0.66
T	0.07 ± 0.12	0.92	1.00	–0.86
U	0.00 ± 0.09	–0.66	–0.86	1.00

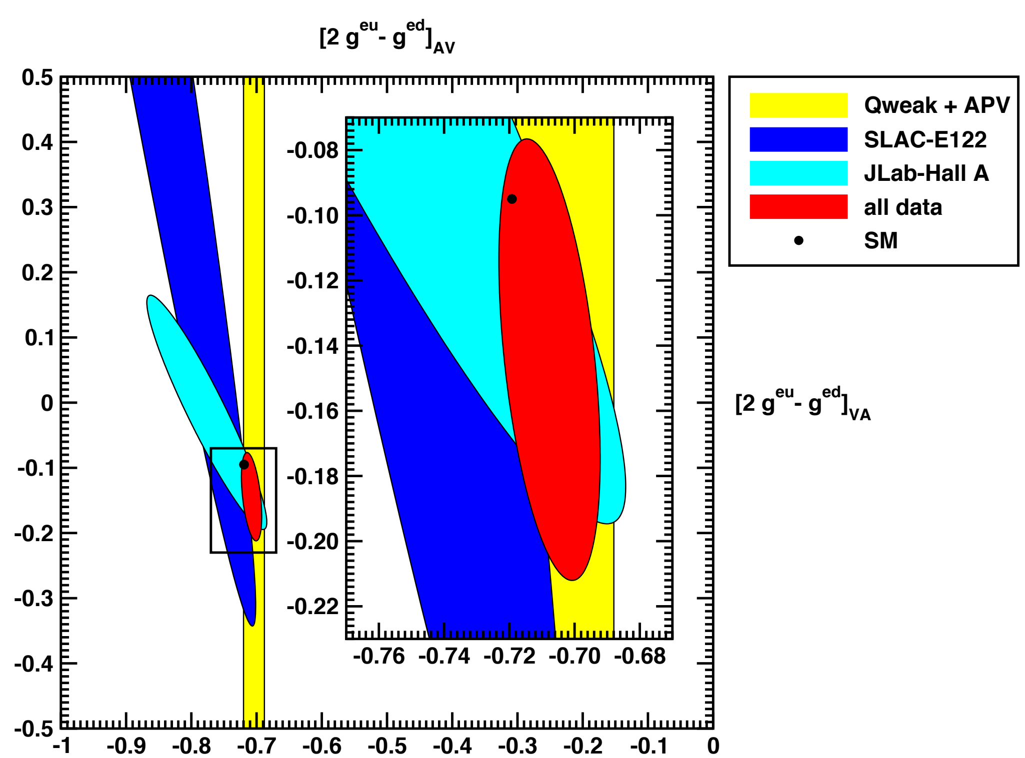
- $M_{KK} \gtrsim 3.2$ TeV in warped extra dimension models
- $M_V \gtrsim 4$ TeV in minimal composite Higgs models ***Freitas & JE (PDG 2018)***

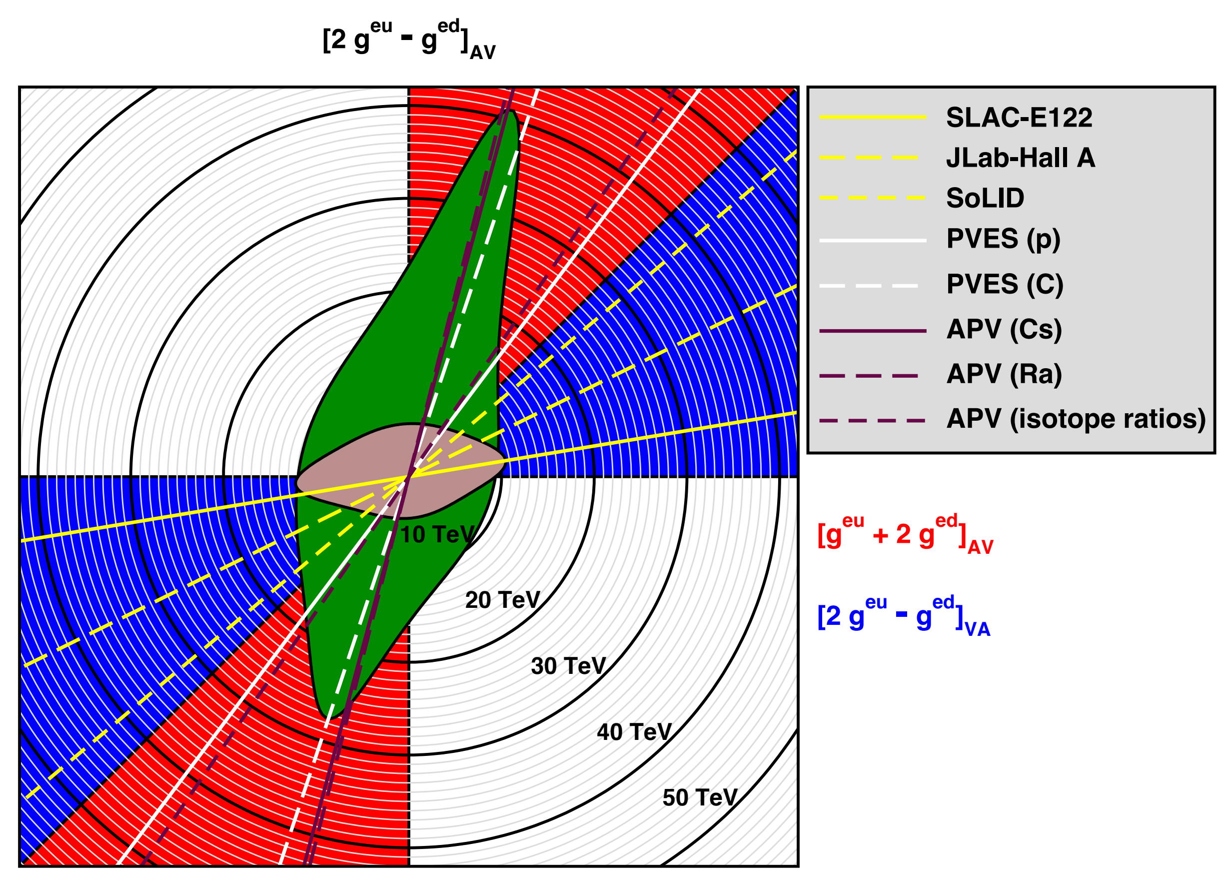
Scale exclusions post Qweak



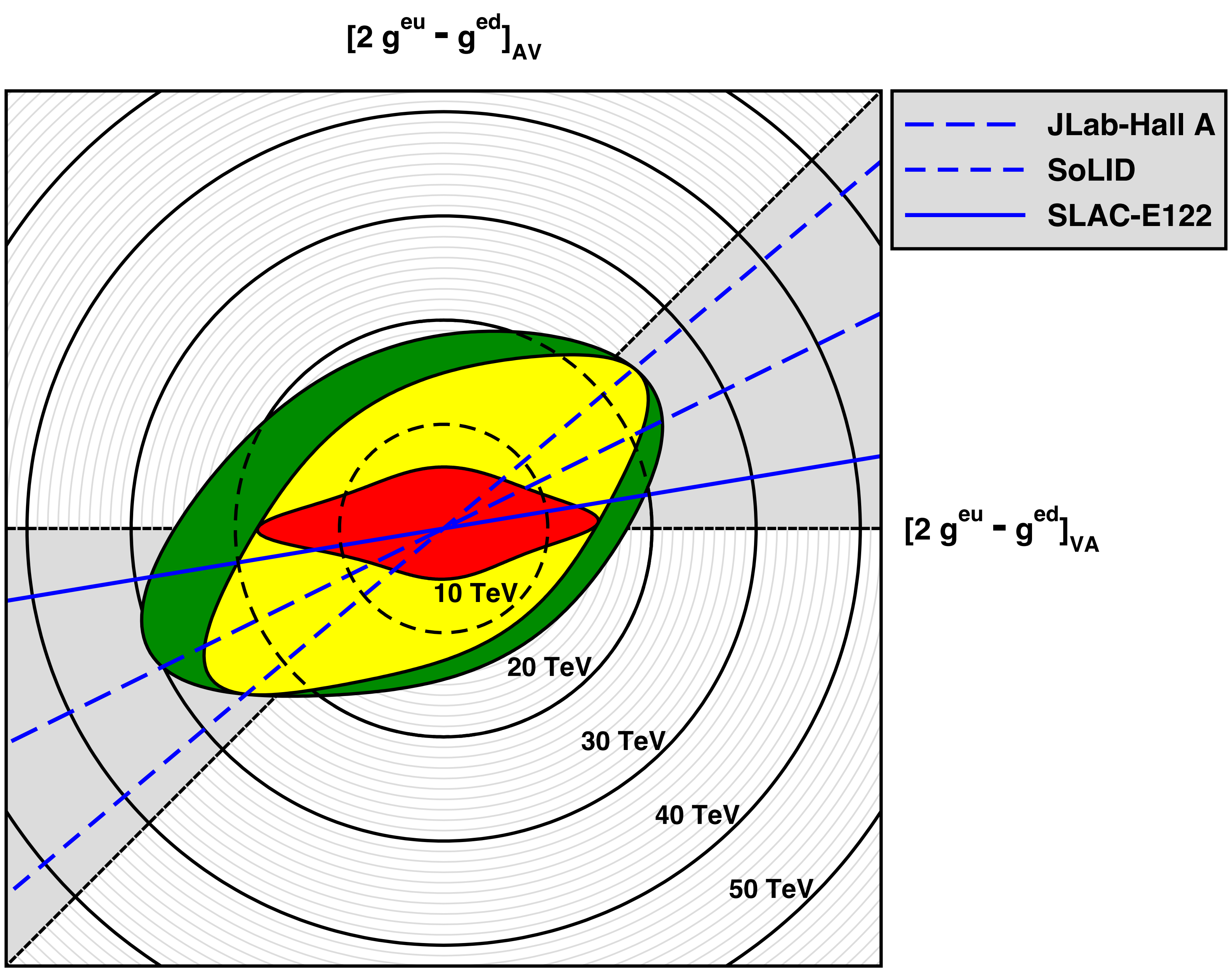
Compositeness scales from low energies



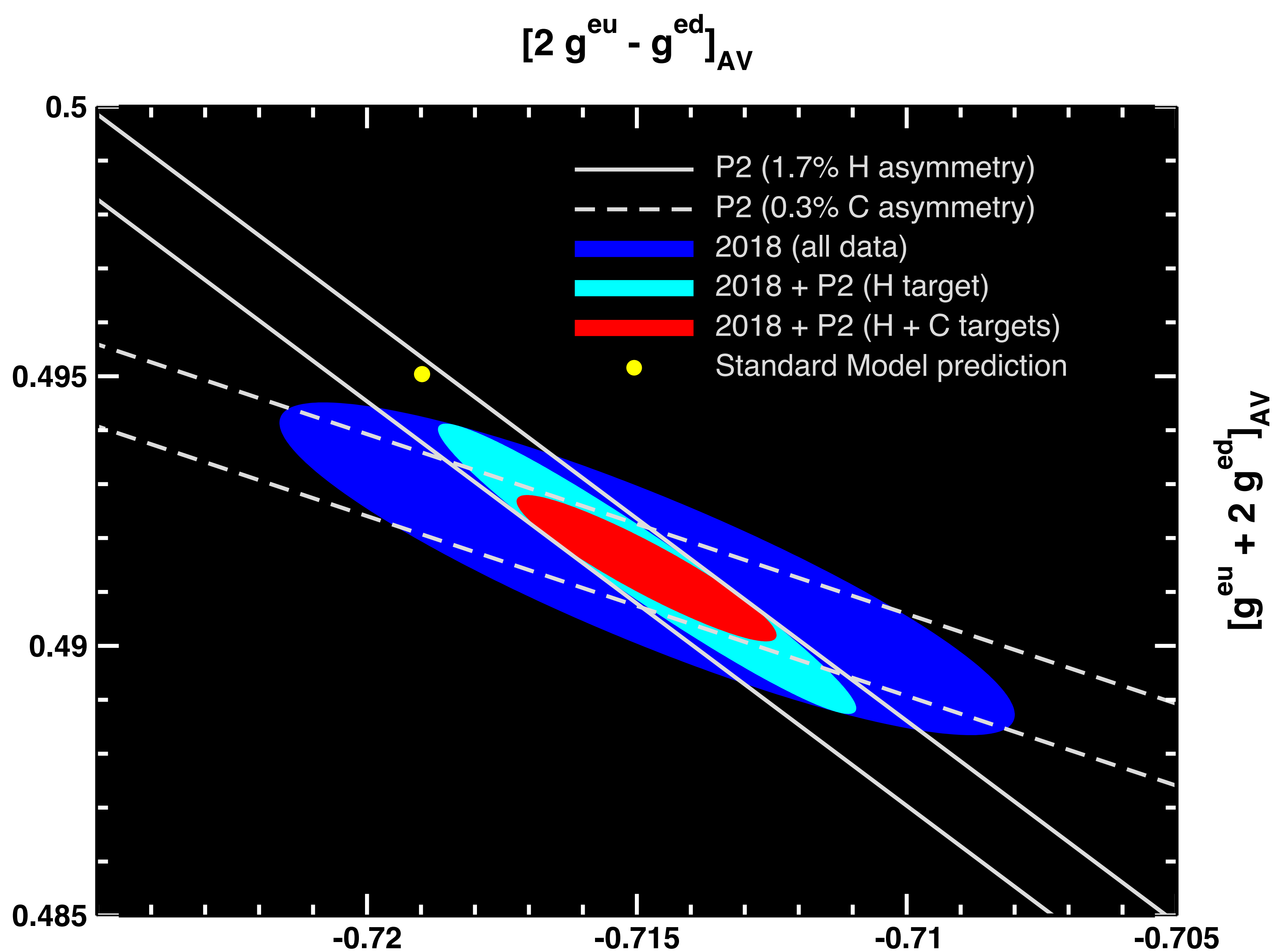




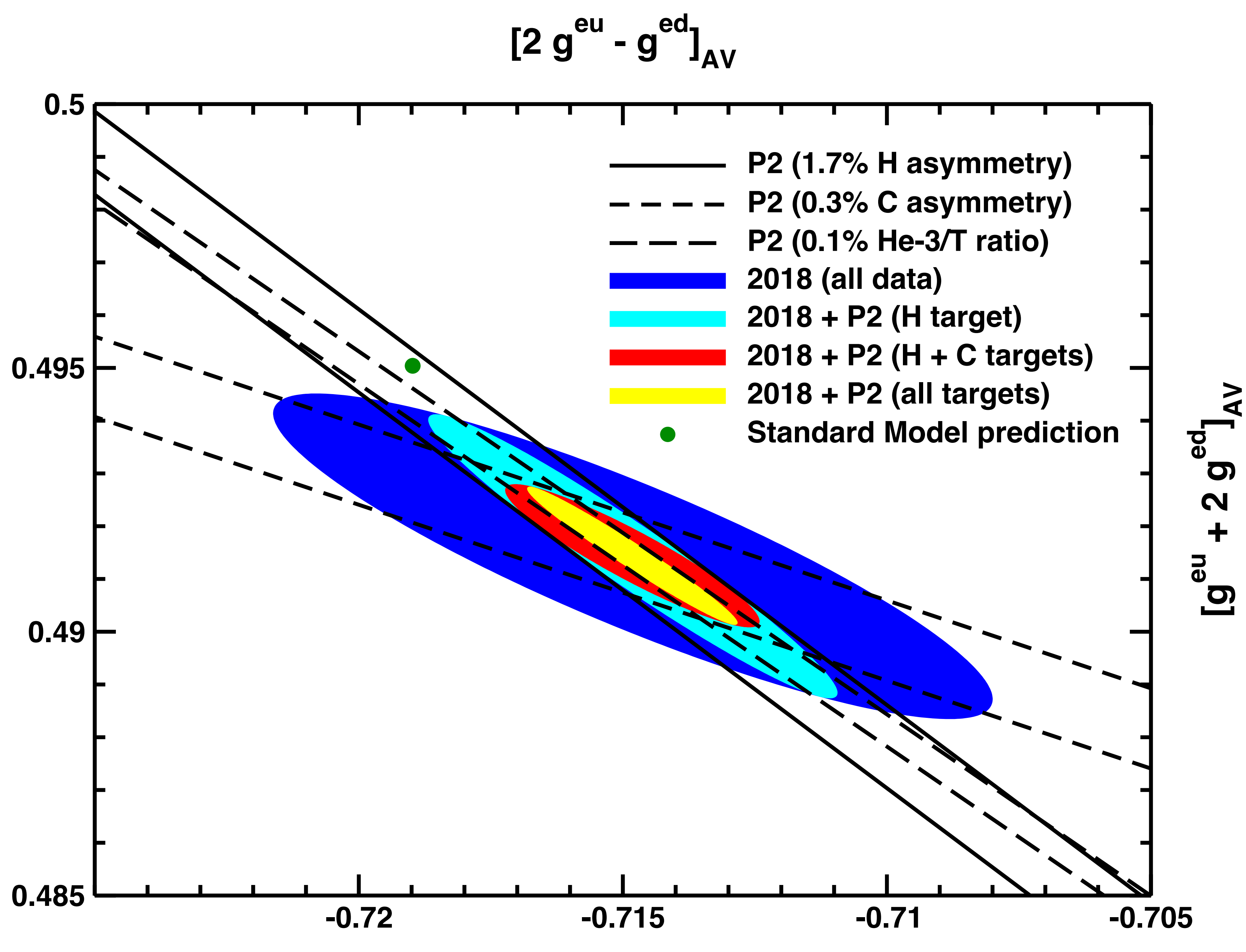
Scale exclusions pre-SoLID / P2



Effective couplings



Effective couplings

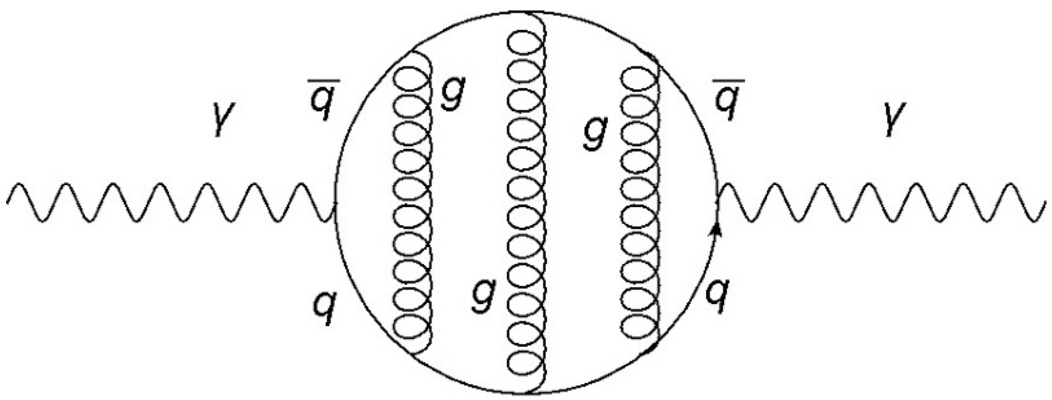


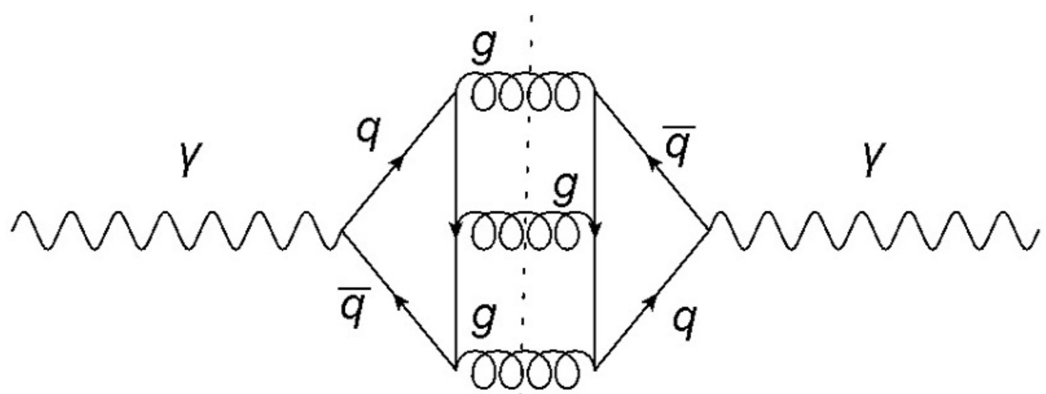
$\sin^2\theta_W(0)$: RGE solution

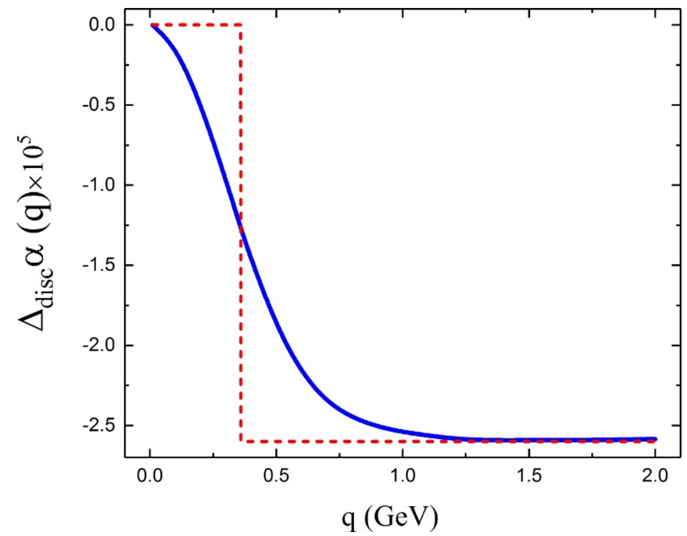
$$\hat{s}^2(\mu) = \hat{s}^2(\mu_0)\frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)} + \lambda_1\left[1 - \frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)}\right] + $$
$$+\frac{\hat{\alpha}(\mu)}{\pi}\left[\frac{\lambda_2}{3}\ln\frac{\mu^2}{\mu_0^2} + \frac{3\lambda_3}{4}\ln\frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)} + \tilde{\sigma}(\mu_0) - \tilde{\sigma}(\mu)\right]$$

- λ_i : rational numbers depending on active particle content of the EFT
- theory uncertainty from RGE running ~ 1.6×10^{-6} (negligible)
- theory error from b and c matching ~ 3×10^{-6} (again using $\overline{m}_c$ & $\overline{m}_b$)
- we recycle the on-shell result for α(2 GeV) ***Davier et al., EPJC 77 (2017)***
  → scheme conversion introducing 4.8×10^{-6} uncertainty
- total uncertainty from PQCD ~ 6×10^{-6} in $\sin^2\theta_W(0) \equiv \overline{s}^2$

$\sin^2\theta_W(0)$: singlet separation







Ferro-Hernández & JE, JHEP 03 (2018)
adapted from lattice g_μ–2 calculation
RBC/UKQCD, PRL 116 (2016)

- use of result for α(2 GeV) needs singlet piece isolation $\Delta_{disc}\,\alpha$(2 GeV)
- then $\Delta_{disc}\,\bar{s}^2 = (\bar{s}^2 \pm 1/20)\;\Delta_{disc}\,\alpha(2\text{ GeV}) = (-\,6 \pm 3)\times 10^{-6}$
- step function ⇒ singlet threshold mass $\bar{m}_s^{disc} \approx 350$ MeV

$\sin^2\theta_W(0)$: flavor separation

strange quark external current	ambiguous external current
Φ	$K\bar{K}$ (non – Φ)
$K\bar{K}\pi$ [almost saturated by $\Phi(1680)$]	$K\bar{K}2\pi$, $K\bar{K}3\pi$
$\eta\Phi$	$K\bar{K}\eta$, $K\bar{K}\omega$

- use of result for α(2 GeV) also needs isolation of strange contribution $\Delta_s\alpha$
- left column assignment assumes OZI rule
- expect right column to originate mostly from strange current ($m_s > m_{u,d}$)
- quantify expectation using averaged $\Delta_s(g_\mu-2)$ from lattices as Bayesian prior
  RBC/UKQCD, JHEP 04 (2016); HPQCD, PRD 89 (2014)
- $\Delta_s\alpha(1.8\text{ GeV}) = (7.09 \pm 0.32)\times10^{-4}$ (threshold mass $\bar{m}_s$ = 342 MeV $\approx \bar{m}_s^{disc}$)