The Hierarchy Problem & Extra Dimensions

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Standard Model & hierarchy problem

Structure of Standard Model: chiral gauge theory with Higgs sector,

$$\mathcal{L} = -\frac{1}{2}\mathrm{tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + \overline{\Psi}_L i\gamma^\mu D_\mu \Psi_L + \mathrm{tr}\left[\left(D_\mu H\right)^\dagger D^\mu H\right]$$
$$+\, \mu_{ij}^2\, H_i^\dagger H_j - \lambda_{ijkl} H_i^\dagger H_j^\dagger H_k H_l + \left(\Psi_L^T C y H \Psi_L + \mathrm{h.c.}\right)\ ,$$

$$D_\mu = \partial_\mu + g A_\mu\ , \quad F_{\mu\nu} = -\frac{i}{g}\left[D_\mu, D_\nu\right]\ , \quad SU(3)\times SU(2)\times U(1)\,,$$

$$H^T = (H_1, H_2)\,,$$

$$\Psi_L^T = \Big(\underbrace{q_{L1}, u^c_{R1}, e^c_{R1}, d^c_{R1}, l_{L1}, (n^c_{R1})}_{\text{1st family}}, \underbrace{q_{L2}, \ldots,}_{\text{2nd}} \underbrace{\ldots, (n^c_{R3})}_{\text{3rd}}\Big)\,,$$

spontaneous electroweak symmetry breaking:

$$\langle H_i\rangle \equiv v_i \sim 100 \text{ GeV}$$
$$\ll \Lambda_{GUT} \sim 10^{15} \text{ GeV} \ll M_P \sim 10^{18} \text{ GeV}$$

Hierarchy problems: (1) origin of large hierarchy, $v/M_P \ll 1$, and (2) instability of mass terms in Higgs potential under quantum corrections:

$$\delta\lambda \sim \frac{\alpha}{\pi} \ln \frac{\Lambda^2}{v^2}\,, \quad \delta\left(\frac{m^2}{v^2}\right) \sim \frac{\alpha}{\pi}\frac{\Lambda^2}{v^2}$$

$$\sim 0.5 \qquad\qquad \sim 10^{24}\,, \quad \Lambda \sim \Lambda_{GUT}$$

Many solutions since 40 years (always new degrees of freedom!!):

- dynamical EW symmetry breaking, no fundamental scalar Higgs boson, new strong interactions (technicolor, composite Higgs, ...)

- cancellations: SUSY breaking at EW scale, twin Higgs, ...

- cosmological evolution: relaxion, ...

- extra dimensions provide **new ingredients:** higher-dimensional gravity, branes, infinitely many new (Kaluza-Klein) states, intriguing reformulation of hierarchy problem, with important predictions for the LHC

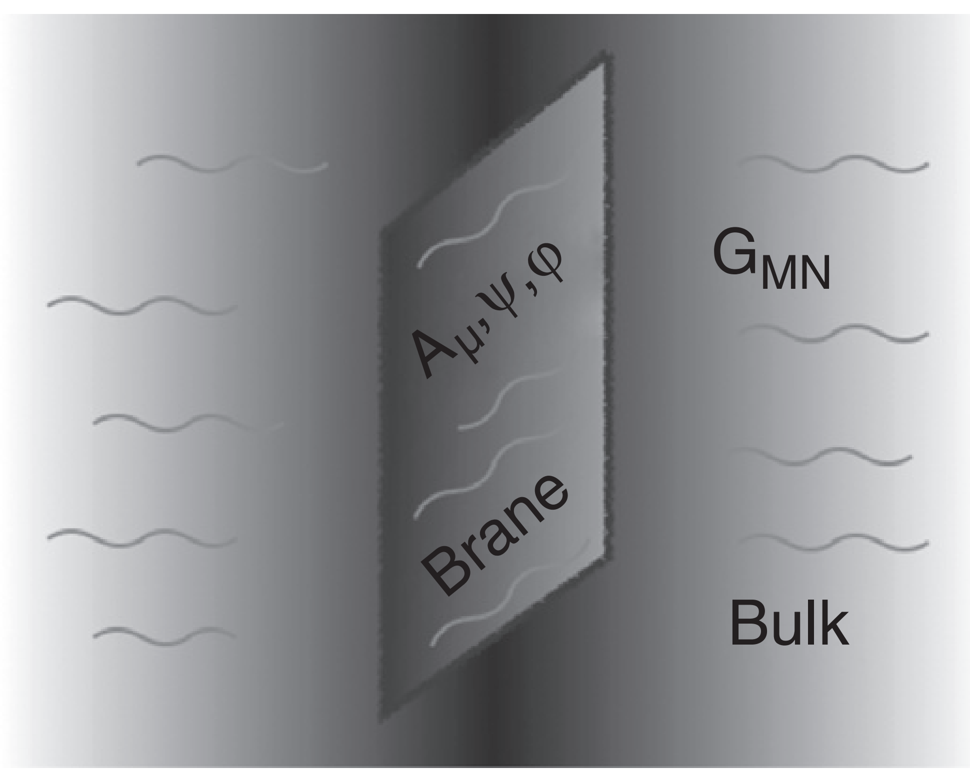
Large extra dimensions

Arkani-Hamed, Dimopoulos, Dvali '98 (ADD), ... RPP review: Gershtein, Pomarol '18

SM on 4-dim brane, gravity in (4+n)-dim bulk, (4+n) Planck mass ~ EW scale:

$$S_{\rm LED} = \frac{M^{2+n}}{2}\int d^4x \int d^n y \sqrt{-G} R_{4+n} + \int d^4 x \sqrt{-g}\mathcal{L}_{\rm SM}$$

$$M^{2+n}(2\pi R)^n = M_P^2, \quad R = \frac{1}{2\pi}\left(\frac{M_P}{M}\right)^{2/n} M^{-1}, \quad M \simeq \text{TeV} \quad \text{(EW scale)}$$



Ibanez, Uranga '12

large ratio of **mass scales** traded for large ratio of **volumes**:

n	2	6
R	0.1 mm	10^{-11} mm
R^{-1}	10^{-3} eV	10 MeV

KK tower of light gravitons:

$$G_{\mu\nu}(x,y) = G^{(0)}_{\mu\nu}(x) + V_n^{-1/2}\sum_{\vec{n}\neq 0} e^{i\vec{n}\cdot\vec{y}/R} G^{(\vec{n})}_{\mu\nu}(x)$$

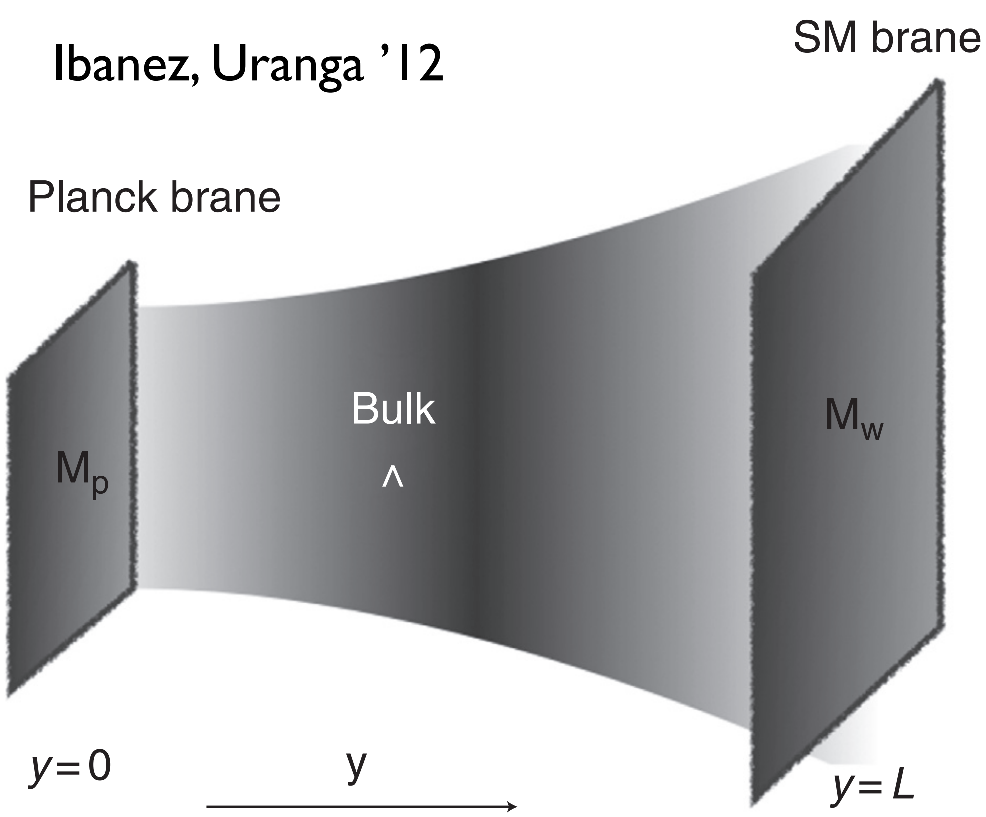
LHC constraints: $M \gtrsim 10$ TeV; **quantum corrections?**

Small extra dimensions

Warped extra dimensions [Randall Sundrum '99 (RS)]: gravity in (4+1)-dim bulk, Higgs on SM brane, warping from bulk-brane gravity:

$$S_5 = -\int d^4x dy \left(\sqrt{-g}\left(\frac{M_5^3}{2}R_5 + \Lambda\right) + \sqrt{-g|_0}\delta(y)\Lambda_0 + \sqrt{-g|_L}\delta(y-L)\Lambda_L\right)$$

Ibanez, Uranga '12



$$ds^2 = a(y)^2\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

$$a(y) = e^{-ky}\,, \quad k = (-\Lambda/6M_5^3)^{1/2}$$

electroweak hierarchy from 5-dim gravity effect:

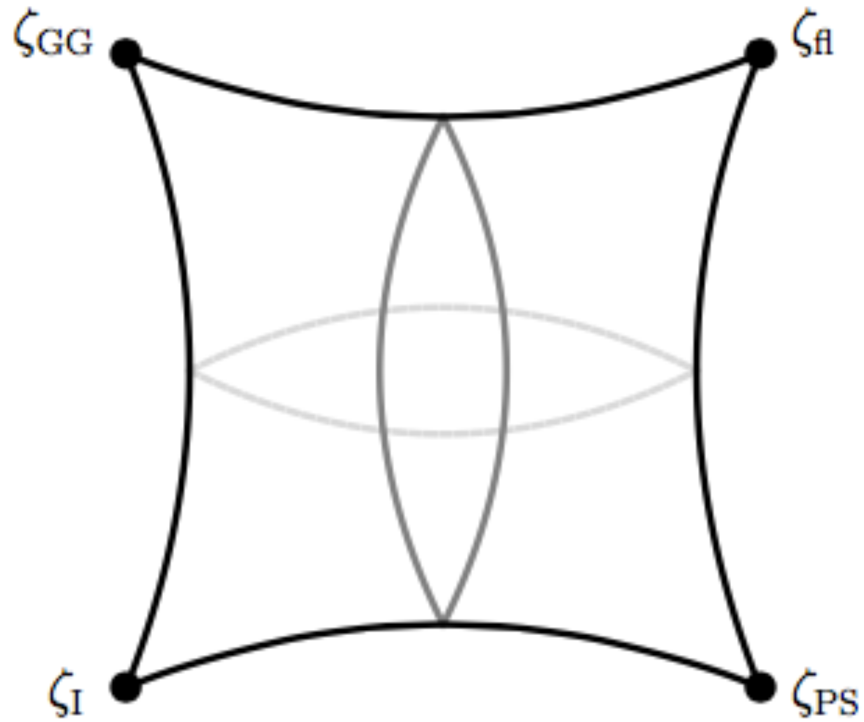
$$m_{\text{Higgs}} \simeq e^{-kL}M_P\,, \quad m_P = \frac{m_5^3}{2k}\left(1 - e^{-2kL}\right)$$

$$m_{\text{grav}} = (n + \tfrac{1}{4})\pi k e^{-kL}\,, \quad k \sim m_5 \sim M_P$$

sufficient hierarchy, i.e. $m_{\text{Higgs}} \simeq \text{TeV}$, for small extra dimensions, $kL \sim 30$, TeV gravitons; **LHC searches**: $m_{\text{grav}} \gtrsim 4\ \text{TeV}$; **quantum corrections?**

GUT-scale extra dimensions

Motivation: SUSY GUTs, string compactifications [Kawamura '00; Hall, Nomura '01; Hebecker, March-Russell '01,...]; consider SO(10) GUT group in 6 dim, compactified on orbifold $T^2/\mathbb{Z}_2$, broken at fixed points to SU(5)xU(1), SU(4)xSU(2)xSU(2), ..., with SM group as intersection; bulk fields 45, 16, 10's [Asaka, WB, Covi '02; Hall, Nomura et al '02; ...]; include U(1) with magnetic flux, distinguishes matter from Higgs [WB, Dierigl, Ruehle, Schweizer '15]:



$$SO(10) \times U(1)_A$$

magnetic flux: N 16's from charged bulk 16-plet and N flux quanta:

$$\mathbf{16}\ [SO(10)] \sim \mathbf{5}^* + \mathbf{10} + \mathbf{1}\ [SU(5)] \sim \mathbf{q}, \mathbf{l}, \mathbf{u^c}, \mathbf{e^c}, \mathbf{d^c}, \nu^{\mathbf{c}}\ [G_{\rm SM}]$$

Higgs fields from two uncharged bulk 10-plets, form split multiplets:

$$H_1 \supset H_u\,, \quad H_2 \supset H_d$$

Flux **breaks supersymmetry** [Bachas '95], soft SUSY breaking only for quark-lepton families:

$$M^2 = m_{\tilde{q}}^2 = m_{\tilde{l}}^2 = \frac{4\pi N}{V_2} \sim (10^{15}\ \mathrm{GeV})^2$$

$$m_{3/2} \sim 10^{14}\ \mathrm{GeV}\,, \quad m_{\tilde{q}}^2 = m_{\tilde{l}}^2 > m_{3/2} \sim m_{1/2} \gg m_{\tilde{h}}$$

Emerging picture of **Split Symmetries** (cf. "split/spread SUSY" [Arkani-Hamed, Dimopoulos; Giudice, Romanino '04; Hall, Nomura '11]):

- *complete* GUT representations (quarks, leptons) come with a multiplicity, *incomplete* GUT representations (Higgs) only once

- masses of scalar quarks and leptons large, *because* they form *complete* GUT multiplets (magnetic flux)

- masses of Higgs/higgsinos small, *because* they form *incomplete* GUT multiplets (THDM); size of **quantum corrections**?

Magnetic flux & quantum corrections

[WB, Dierigl, Dudas, Schweizer, '16; WB, Dierigl, Dudas 1804.07497]

Consider **toy model**: 6-dim **gauge-Higgs unification** (Hosotani `83, Arkani-Hamed et al. `01, Antoniadis et al. `01, ...), original motivation: electroweak symmetry breaking in LED models; Weyl fermion interacting with Abelian gauge field,

$$S_6 = \int d^6x \left(-\frac{1}{4} F^{MN} F_{MN} + i\overline{\Psi}\Gamma^M D_M \Psi \right) ,$$

$$D_M = \partial_M + iqA_M \, , \; F_{MN} = \partial_M A_N - \partial_N A_M \, , \; \Gamma_7 \Psi = -\Psi \, .$$

Compactification to 4 dim Minkowski space on square torus of area L^2, first without magnetic flux, $\langle F_{56} \rangle = 0$; matter and gauge fields have Kaluza-Klein modes with masses $M_{n,m} = 2\pi(m+in)/L$; Wilson-line scalar

$$\phi = \tfrac{1}{2}(A_6 + iA_5)|_{n=m=0}$$

massless at tree level, identified with Higgs field in gauge-Higgs unification (significant differences w.r.t. Standard Model Higgs)

Quantum corrections to tree-level mass,

$$m_\phi^2 = 0$$

one-loop correction, sum over Kaluza-Klein tower of states,

$$\delta m_\phi^2 = -4q^2 \sum_{n,m} \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2 + |M_{n,m}|^2)^2}$$

after some manipulations finite result, due to discrete symmetry, remnant of gauge symmetry in extra dimensions [Antoniadis, Benakli, Quiros '01, Cheng, Matchev, Schmaltz '02, Ghilencea et al. '05; Dierigl '17] ($L = 2\pi R$),

$$m_\phi^2 \simeq 0.19 \, \frac{\alpha}{\pi} \frac{1}{R^2}$$

expected result: loop-factor times cutoff, i.e., small hierarchy; original application: electroweak symmetry breaking in models with large extra dimensions; can one also obtain a scalar much lighter than the cutoff 1/R, i.e. a **large hierarchy** ??

Compactification with magnetic flux

For convenience, rewrite 6d Lagrangian,

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} : \quad \gamma_5 \psi_L = -\psi_L \, , \quad \gamma_5 \psi_R = \psi_R \, ,$$

$$\psi_L = \begin{pmatrix} \psi \\ 0 \end{pmatrix} \, , \quad \psi_R = \begin{pmatrix} 0 \\ \overline{\chi} \end{pmatrix} \, .$$

Complex coordinates and gauge fields,

$$z = \frac{1}{2}(x_5 + i x_6) \, , \quad \partial_z = \partial_5 - i\partial_6 \, , \quad \phi = \frac{1}{\sqrt{2}}(A_6 + i A_5)$$

constant magnetic flux background,

$$\langle A_5 \rangle = -\tfrac{1}{2} f x_6 \, , \; \langle A_6 \rangle = \tfrac{1}{2} f x_5 \, , \quad \langle \phi \rangle = \frac{1}{\sqrt{2}} f \bar{z}$$

shift around quantized magnetic flux,

$$\frac{q}{2\pi} \int_{T^2} F = \frac{q}{2\pi} f = N \in \mathbb{Z} \, , \quad \phi = \frac{f}{\sqrt{2}} \bar{z} + \varphi$$

6-dim action with flux-dependent bilinear term of Weyl fermions,

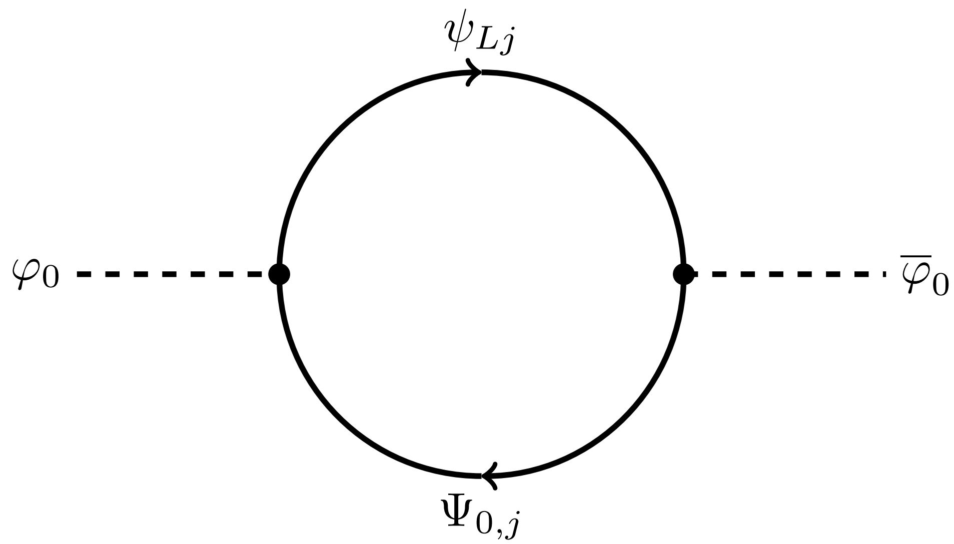
$$
\begin{aligned}
S_6 = \int d^6x \Big(&- \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \partial^\mu \overline{\varphi} \partial_\mu \varphi - \frac{1}{4} \left(\partial_z \overline{\varphi} + \partial_{\bar{z}} \varphi \right)^2 - \frac{1}{2} f^2 \\
&- \frac{1}{2} \partial_{\bar{z}} A^\mu \partial_z A_\mu - \frac{i}{\sqrt{2}} \partial_\mu A^\mu \left(\partial_z \overline{\varphi} - \partial_{\bar{z}} \varphi \right) \\
&- i \psi \sigma^\mu \overline{D}_\mu \overline{\psi} - i \chi \sigma^\mu D_\mu \overline{\chi} \\
&- \chi \left(\partial_z + q f \bar{z} + \sqrt{2} q \varphi \right) \psi - \overline{\chi} \left(\partial_{\bar{z}} + q f z + \sqrt{2} q \overline{\varphi} \right) \overline{\psi} \Big) ,
\end{aligned}
$$

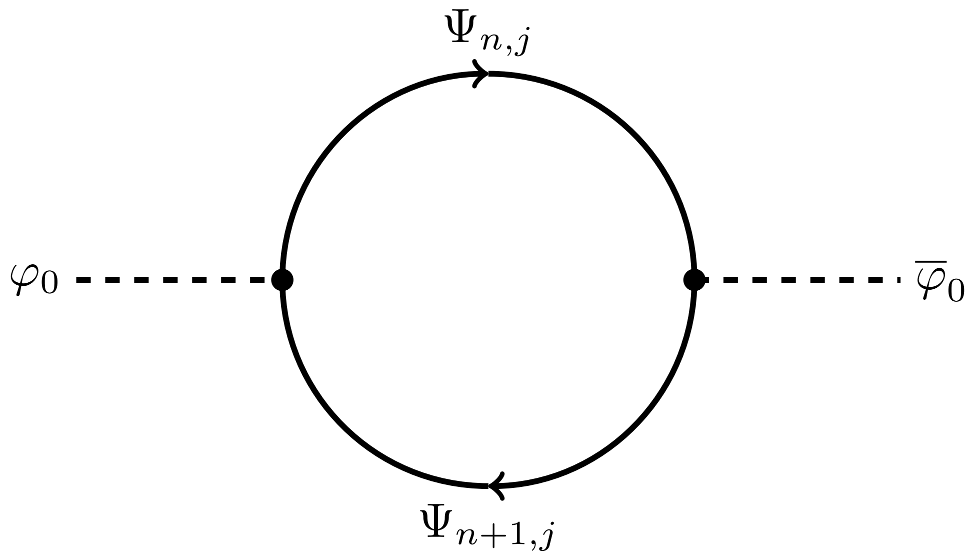
invariance under translations on torus; breaking of translational invariance by background gauge field compensated by **shift** of φ ,

$$
\delta_T \varphi = \left(\epsilon \partial_z + \overline{\epsilon} \partial_{\bar{z}} \right) \varphi + \frac{\bar{\epsilon}}{\sqrt{2}} f
$$

Lagrangian transforms into total divergence; nonlinear realization of translation symmetry on torus, leads to shift symmetry of WL scalar in 4-dim effective theory; Kaluza-Klein tower of fermions: Landau levels, treatable with harmonic oscillator algebra

Quantum corrections & shift symmetry





contribution of zero-mode and 1st Landau level yields usual quadratic divergence:

$$\delta m^2_{\varphi_0} = -2q^2|N| \int \frac{d^4k}{(2\pi)^4} \frac{2k^2}{k^2\,(k^2+2qf)}$$
$$= -\frac{q^2|N|}{4\pi^2}\left(\Lambda^2 - 2qf\ln\left(\frac{\Lambda^2}{2qf}\right) + \ldots\right)$$

Sum over all KK modes leads to cancellation (Schwinger representation of propagators, momentum integrations, perform sum first!):

$$\begin{aligned}
\delta m^2_{\varphi_0} &= -2q^2|N|\sum_n \int \frac{d^4k}{(2\pi)^4}\frac{2k^2}{(k^2+2qfn)\,(k^2+2qf(n+1))} \\
&= \frac{q^2}{4\pi^2}|N|\sum_n \int_0^\infty dt\,\frac{1}{t^2}\left(ne^{-2qfnt}-(n+1)e^{-2qf(n+1)t}\right) \\
&= \frac{q^2}{4\pi^2}|N|\int_0^\infty dt\,\frac{1}{t^2}\left(\frac{e^{2qft}}{(e^{2qft}-1)^2}-\frac{e^{2qft}}{(e^{2qft}-1)^2}\right) \\
&= 0\,.
\end{aligned}$$

WB, Dierigl, Dudas, Schweizer ’16, Ghilencea, Lee ’17

Cancellation can be traced back to shift symmetry of Wilson-line scalar, related to translation invariance of 6-dim action; extends to all orders in perturbation theory; no mass generated for WL scalar! Relevance for hierarchy problem of Higgs? VEV of φ does not generate mass term of chiral fermions, only toy model!

GUT-scale extra dimensions: implications

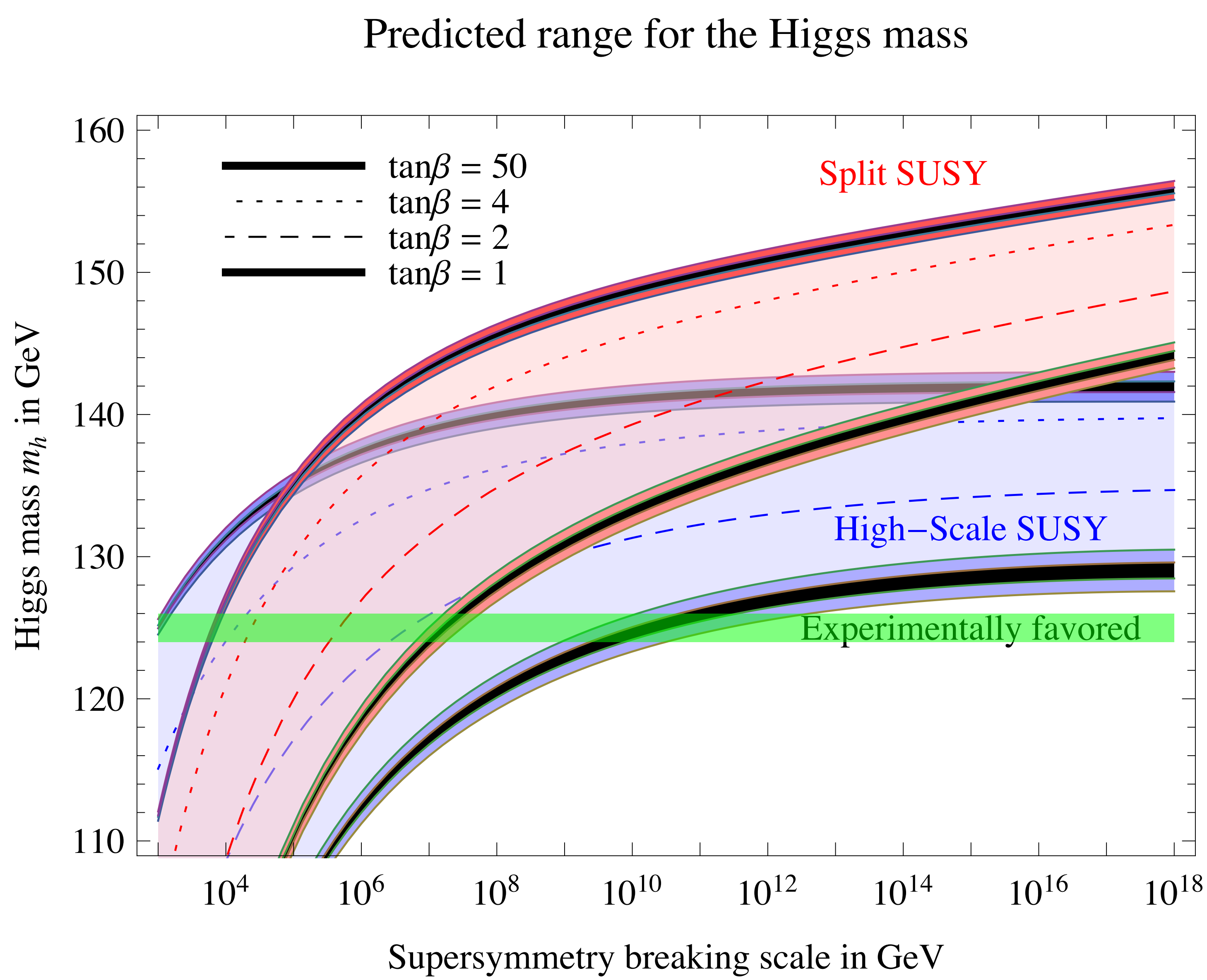
- Suppose, no quadratic divergencies due to cancellations among Kaluza-Klein towers, what predictions remain?

- Compactification: can all moduli be stabilized (D-term breaking, F-term breaking ...) with de Sitter (Minkowski) vacua? yes. Implications for mass spectrum: scalar quarks and leptons, and all gauginos heavy; at low energies THDM (with higgsinos) remains

- Is a matching of THDM to SUSY at GUT scale consistent with RG running and vacuum stability? yes. Implications for Higgs sector

Matching the SM to SUSY at the GUT scale

reminder: one Higgs doublet does not work!



[Degrassi et al '12]

Matching of SM Higgs to MSSM at SUSY breaking scale for \`Split SUSY' (one Higgs doublet, higgsinos and gauginos light) and \`High-scale SUSY'

Strong upper bounds on SUSY breaking scale!

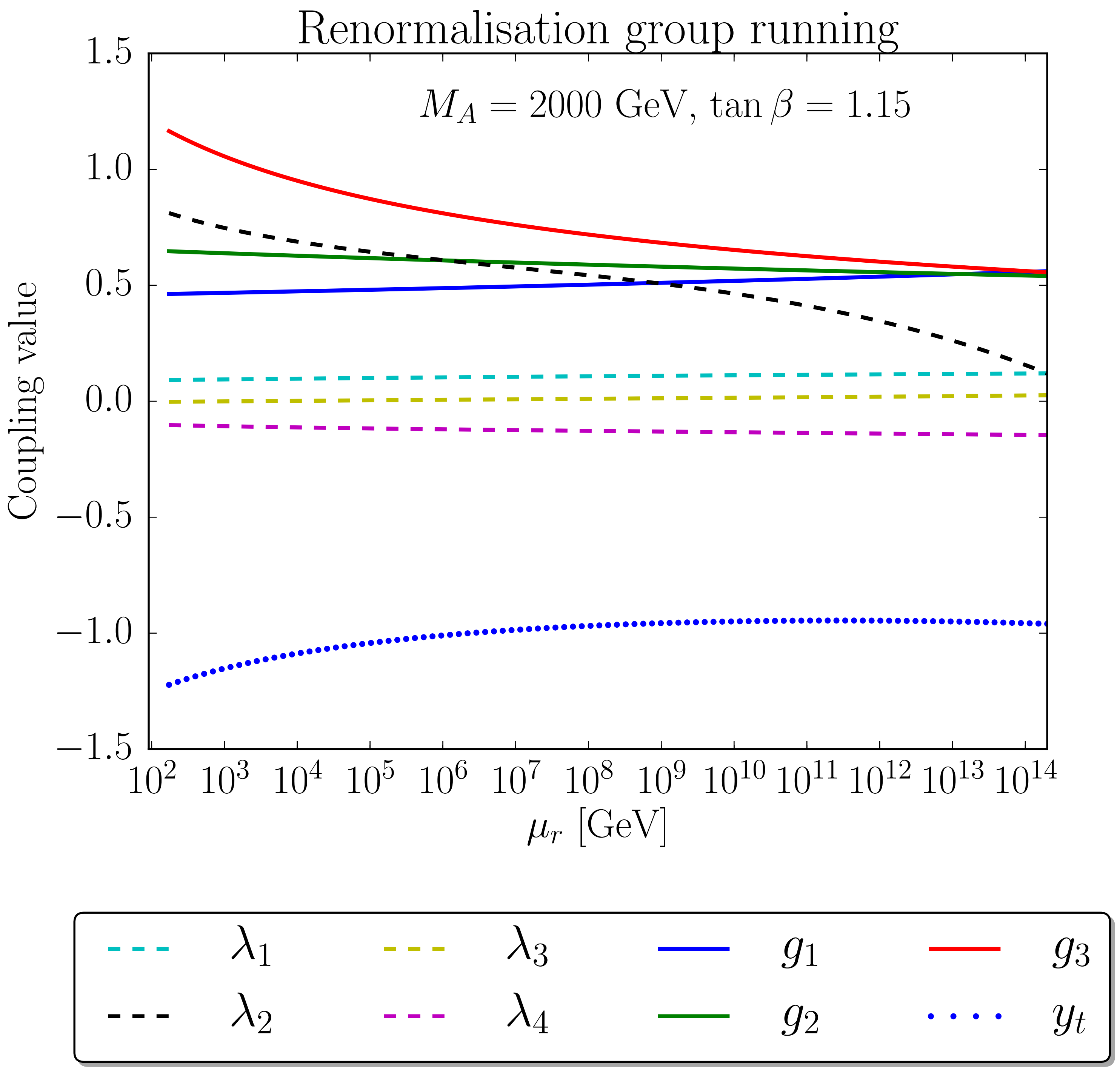
Matching the THDM to SUSY at the GUT scale

Is SUSY breaking at the GUT scale consistent with RG running of couplings and vacuum stability? 6d GUT model yields THDM (tree level), study RG running [Gunion, Haber '03... Lee, Wagner '15; Bagnaschi, Brummer, WB, Voigt, Weiglein '15; Mummidi, Vishnu, Patel '18]):

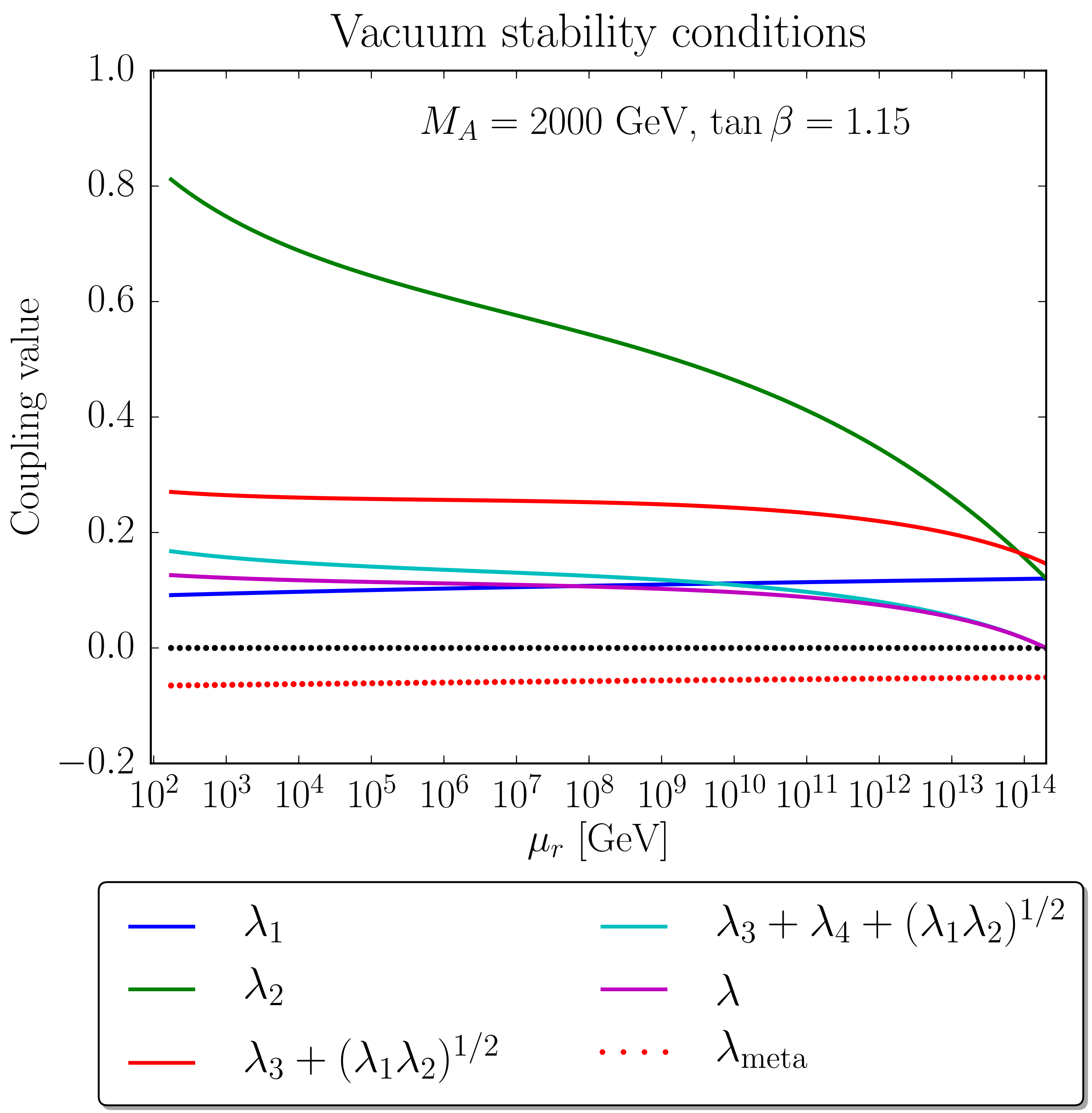
$$
\begin{aligned}
V &= m_1^2\, H_1^\dagger H_1 + m_2^2\, H_2^\dagger H_2 - \left(m_{12}^2\, H_1^\dagger H_2 + \text{h.c.}\right) + V_4\,, \\
V_4 &= \frac{\lambda_1}{2}(H_1^\dagger H_1)^2 + \frac{\lambda_2}{2}(H_2^\dagger H_2)^2 + \lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4 |H_1^\dagger H_2|^2 \\
&\quad + \left(\frac{\lambda_5}{2}(H_1^\dagger H_2)^2 + \lambda_6 (H_1^\dagger H_2)(H_1^\dagger H_1) + \lambda_7 (H_1^\dagger H_2)(H_2^\dagger H_2) + \text{h.c.}\right)
\end{aligned}
$$

Matching conditions at SUSY breaking scale determine quartic couplings:

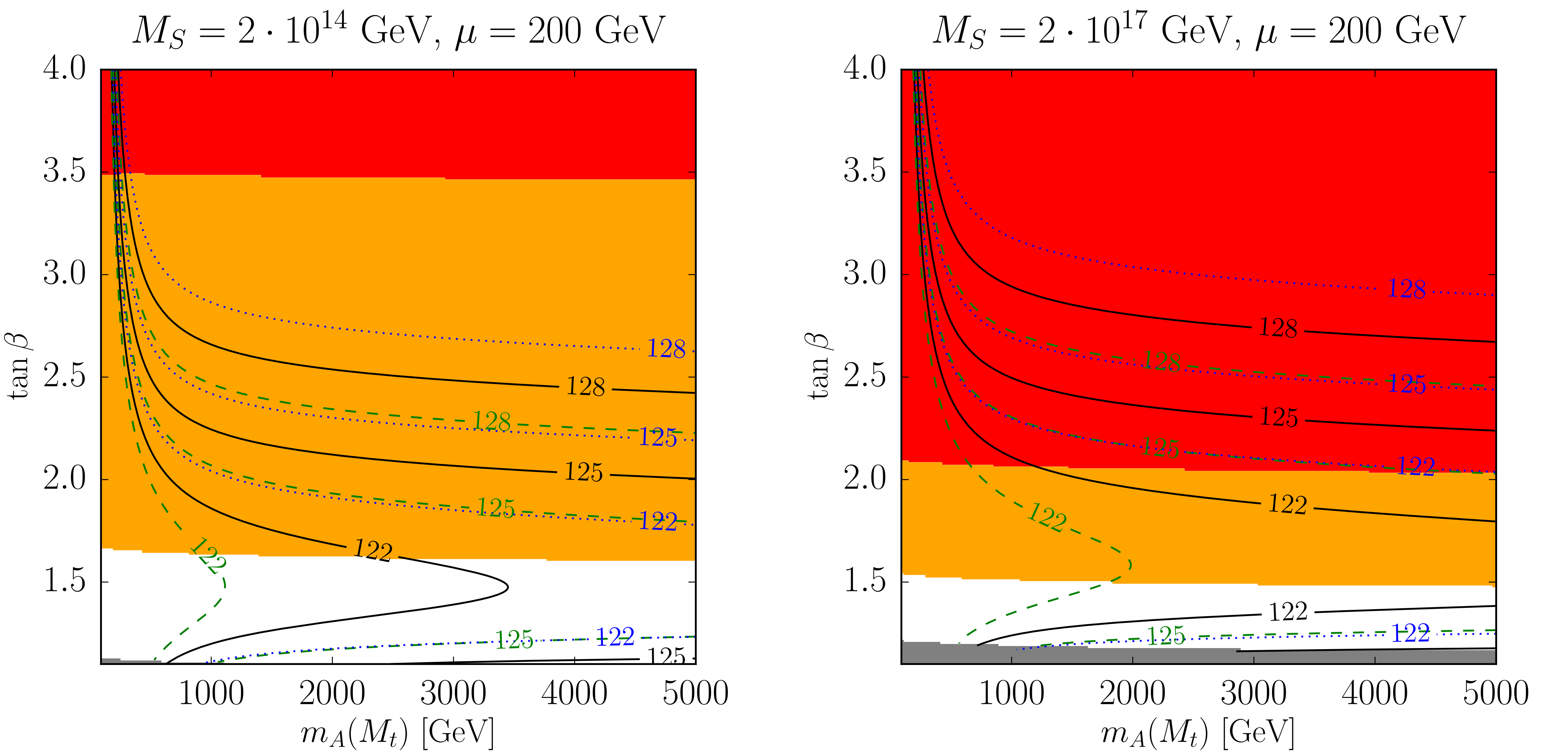
$$
\begin{aligned}
&\lambda_1 = \frac{1}{4}\left(g^2 + {g'}^2\right)\,, \quad \lambda_2 = \frac{1}{4}\left(g^2 + {g'}^2\right) \\
&\lambda_3 = \frac{1}{4}\left(g^2 - {g'}^2\right)\,, \quad \lambda_4 = -\frac{1}{2} g^2\,, \quad \lambda_5 = \lambda_6 = \lambda_7 = 0
\end{aligned}
$$



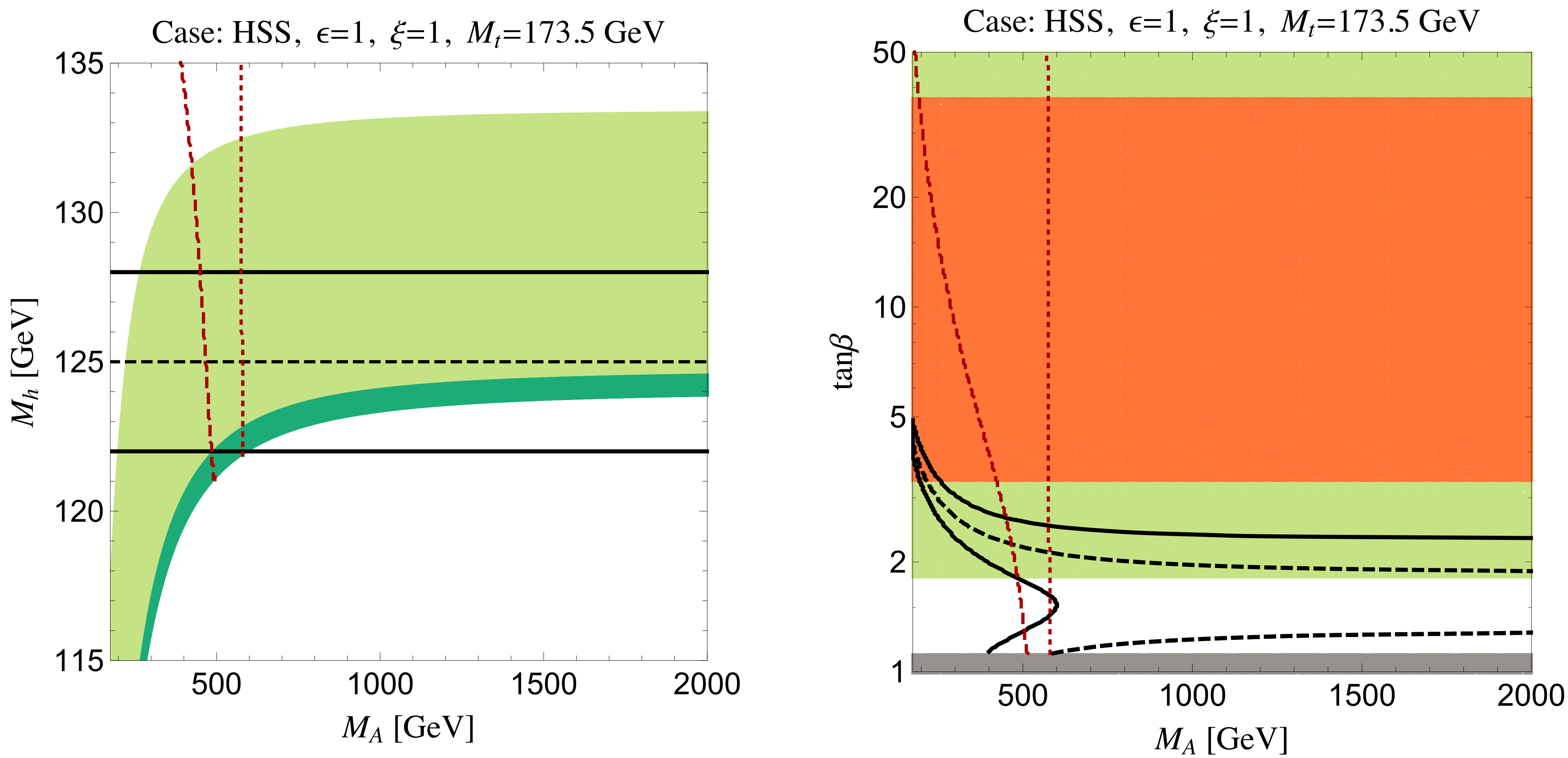
example of RG running of gauge, Yukawa and quartic couplings; reasonable gauge coupling unification



for this example vacuum stability conditions are fulfilled; additional Higgs bosons are heavy!

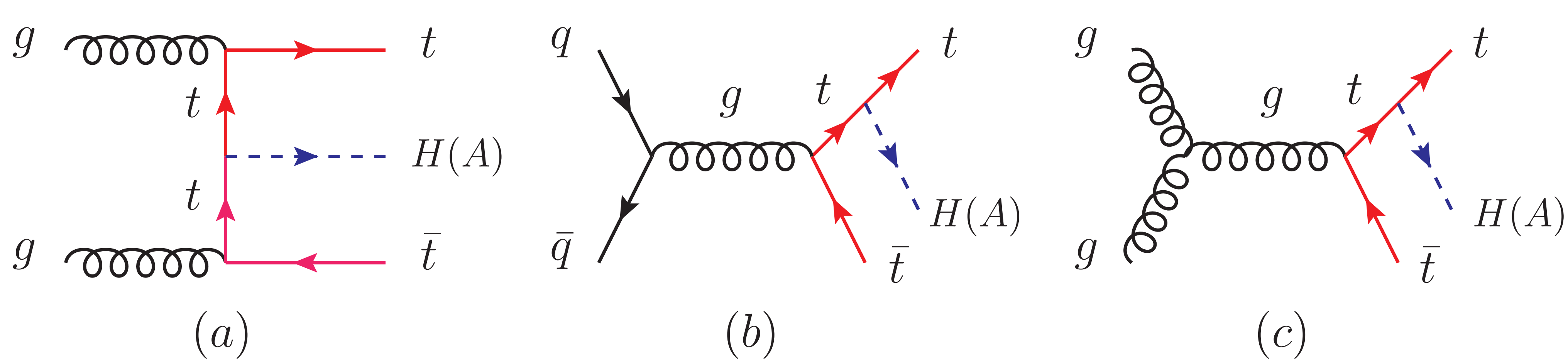


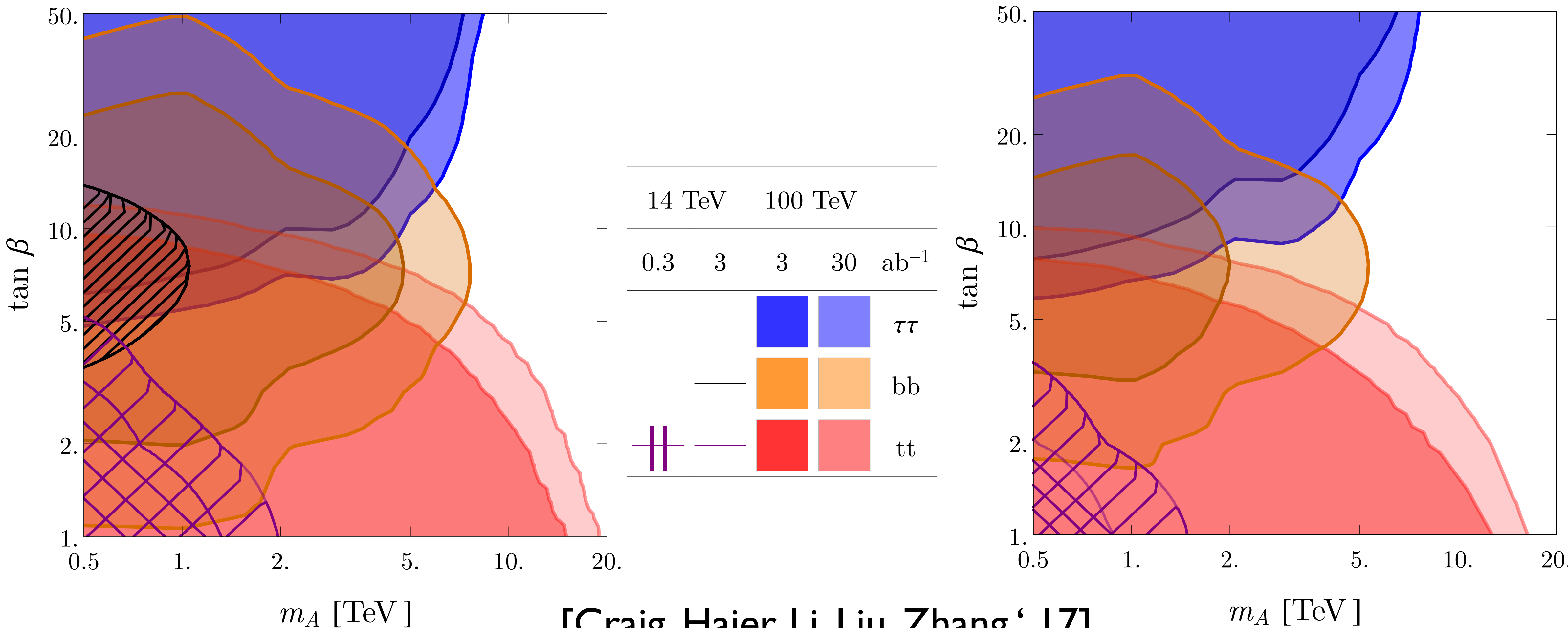
result of parameter scan; red: excluded by vacuum stability; orange: metastable vacuum; large tanβ excluded, small tanβ allowed with $m_A \gtrsim 700$ GeV; **light higgsino** possible, split SUSY inconsistent!



[Mummidi, Vishnu, Patel '18]

Effect of neutrino sector (GUT-scale seesaw) on allowed heavy Higgs mass range (stable, metastable and unstable regions); allowed regions are slightly enlarged





[Craig, Hajer, Li, Liu, Zhang ‘ 17]

exclusion limits (left) and discovery limits (right) for heavy Higgs bosons for 14 TeV LHC and 100 TeV FCC-pp; discovery of heavy Higgses at low tanβ difficult !!

Conclusions

- Hierarchy problem of Higgs sector intriguing puzzle, inspiration for physics beyond the Standard Model

- Solution may have dramatic consequences at TeV energies: ADD & RS scenarios, composite Higgs, ...

- Solution may have modest consequences at TeV energies: piece in puzzle of ultraviolet completion of Standard Model, including GUTs, supersymmetry, **extra dimensions**, ...

- More predictions of flux compactifications: flavour physics, neutrino physics, proton decay (model dependent)

- Detailed study of Higgs sector very important!

Backup Slides

Moduli stabilization & SUSY breaking

Supersymmetric low-energy effective Lagrangian, given in terms of Kahler potential, gauge kinetic function (magnetic flux f induces FI D-term [Quevedo et al '03, Hebecker et al '07,...]):

$$K = -\ln(S + \bar{S} + iX^S V) - \ln(T + \bar{T} + iX^T V) - \ln(U + \bar{U})\,,$$

$$S = \frac{1}{2}(s + ic)\,, \quad T = \frac{1}{2}(t + ib)\,,$$

$$X^T = -i\frac{f}{\ell^2}\,, \quad X^S = -i\frac{N+1}{(2\pi)^2}$$

U is shape modulus; Killing vectors due to quantized flux and Green-Schwarz term, note opposite signs! Gauge kinetic function [cf. Ibanez, Nilles '87]:

$$H = h_S S + h_T T\,, \quad h_S = 2\,, \quad h_T = -\frac{2\ell^2}{(2\pi)^3}$$

Note **opposite sign** of the two contributions! Result: no scale model with gauged shift symmetry, involving S and T!

Gauge invariant KKLT-type superpotential at fixed points (F-term):

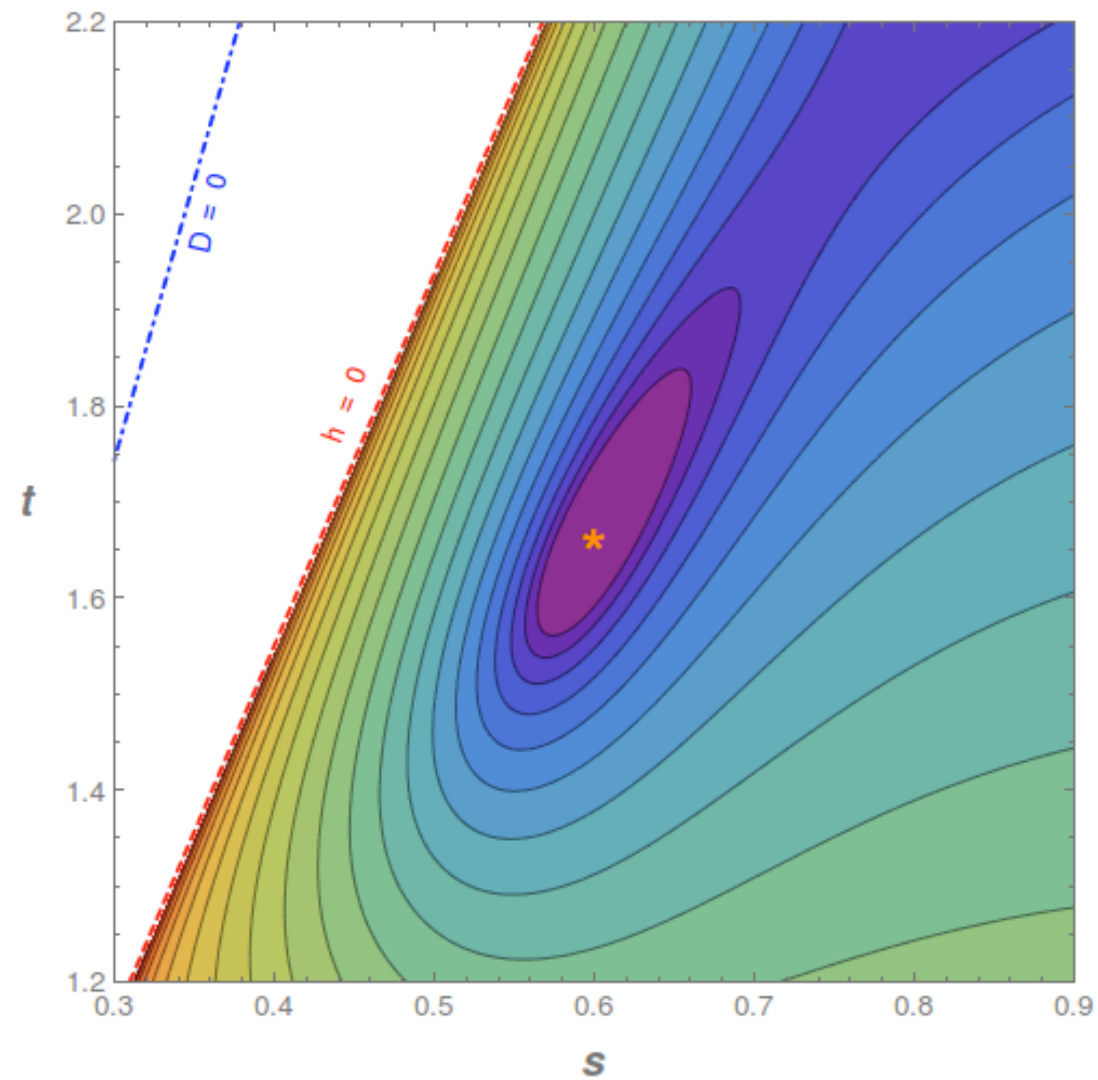
$$W = W(Z,U) = W_0 + W_1\, e^{-aZ} + W_2\, e^{-\tilde{a}U}\,, \quad Z = -iX^T S + iX^S T$$

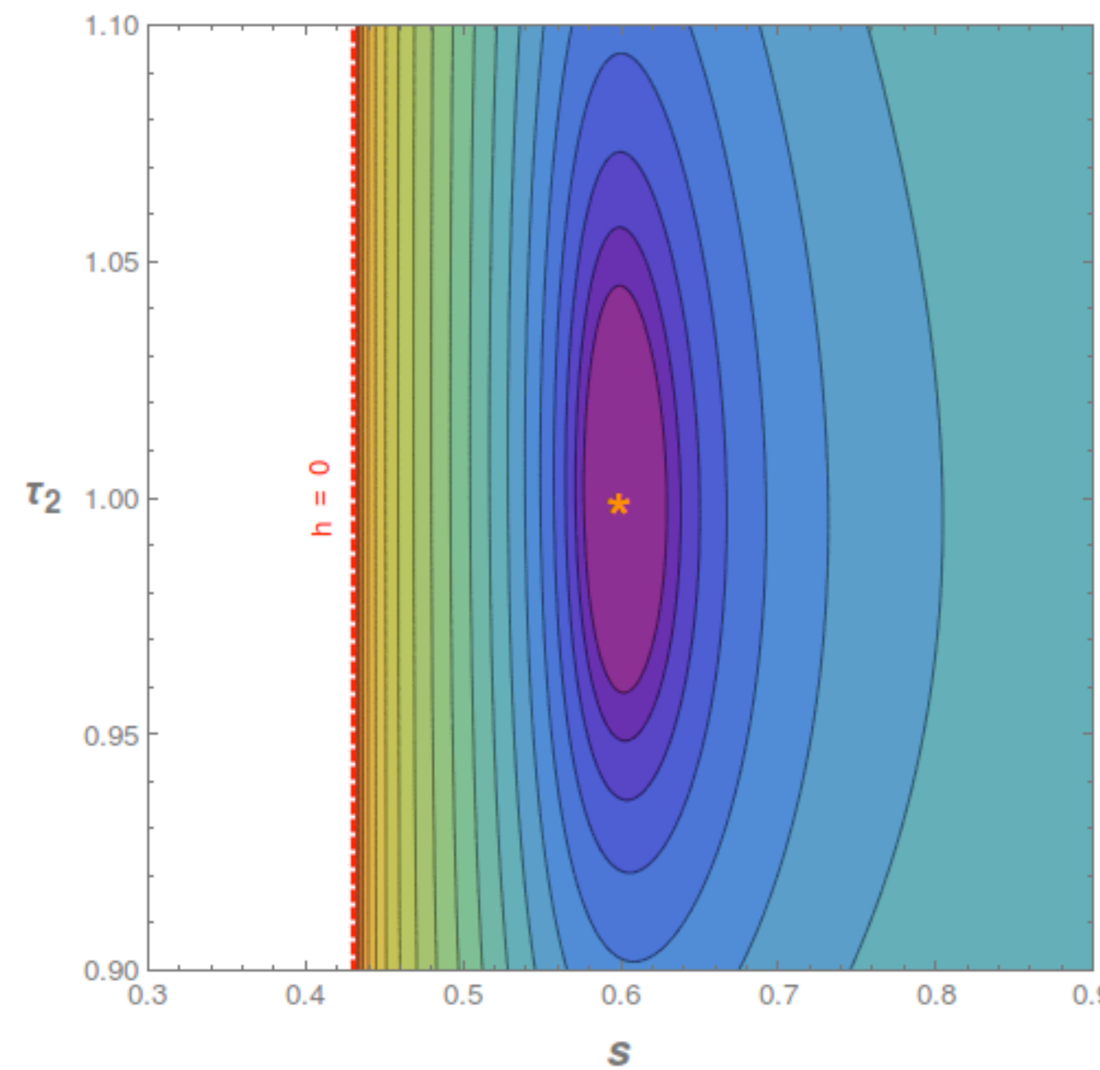
Scalar potential involving F- and D-terms:

$$V = V_F + V_D = e^K (K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2) + \frac{1}{2h} D^2\,,$$

$$D = iK_i X^i = -\frac{i}{s} X^S - \frac{i}{t} X^T\,.$$

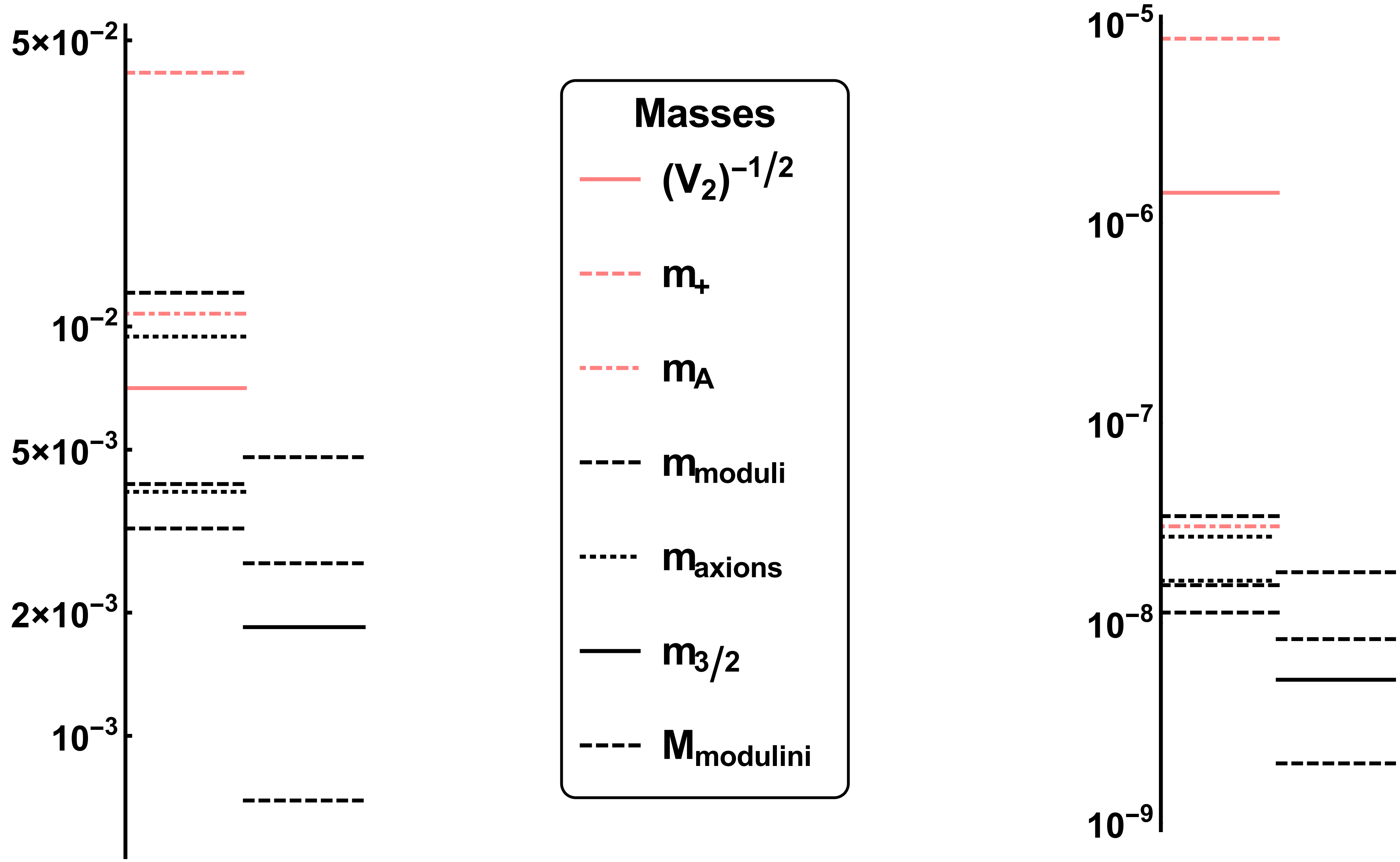
Due to flux AND quantum corrections to gauge kinetic function and Killing vectors, **de Sitter vacua exist** without further F-term uplift (e.g. Polonyi)! Size of extra dimensions determined by parameters of superpotential; example: $W_0 \sim W_1 \sim 10^{-3}\,, a \sim 1 \rightarrow r\ell \sim 10^2$, i.e. GUT scale extra dimensions. Hence most basic ingredients of 6d compactifications sufficient to obtain de Sitter vacua and moduli stabilization!





de Sitter (Minkowski) metastable minimum with GUT scale extra dimensions:

$$g = 0.2\,, \quad L = 200\,, \quad W \sim 10^{-2}$$



boson and fermion masses for GUT scale and “large” extra dimensions:

$$m_A^2 \propto L^{-3}\,, \quad m_{\text{moduli}}^2 \propto L^{-3}\,, \quad m_{\text{axions}}^2 \propto L^{-3}$$
$$m_{3/2} \propto L^{-3/2}\,, \quad M_{\text{modulini}} \propto L^{-3/2}$$